DSA Newscast

Dozenals and Money

The Dozenal Society of America is a voluntary, nonprofit educational corporation, organized for the conduct of research and education of the public in the use of dozenal (also called duodecimal or base twelve) in calculations, mathematics, weights and measures, and other branches of pure and applied science.

SINCE BRITISH currency was decimalized, dozenalists haven't spoken very much about money. However, it's worth some deep and careful consideration. Money is the primary way that most citizens interact with numbers, so the influence of dozenal will likely be greatest here.

For coinage, mathematicians discuss the *optimal coin problem*, and whether coinage is susceptible to the *greedy algorithm*; to which we might add the necessity for *intuitive denominations*:

optimal coin problem   Which denominations will most efficiently make change, with "efficiently" defined as "using the lowest average number of coins"

greedy algorithm   Making change by repeatedly selecting the largest coin that is smaller than the amount of change required

intuitive denominations   Denominations of a size which seems sensible. E.g., "30" rather than "52".

Jeffrey Shallit (*What This Country Needs is an 18¢ Piece*, 11ƵƐ) famously pointed out the algorithm for solving this problem:

$$\frac{1}{L} \sum_{0 <= i < L} \mathrm{opt}(i; e_1, e_2, \ldots e_D)$$

where L is the highest amount of change and $e_1 \ldots e_D$ are the available denominations. Shallit examines American currency, with its limit of 84 cents, and its common denominations of 1, 5, Ƶ, and 21 (technically there is a 42-cent piece, but it is almost never seen or used anymore). The average number of coins per transaction here is about 4;8. Adding an 16¢ piece would make it about 3;Ɛ8. This would improve the efficiency pursuant to the optimal coin problem, but would damage its effectiveness pursuant to the principle of intuitive denominations.

Now, a dozenal currency would naturally have 100 units, rather than 84; this is a rather larger number, and thus we must expect a higher average number of coins per transaction for the same number of denominations. However, due to the much higher divisibility of 100 compared to 84, we will have many more options for intuitive denomination choices, which might make the difference in efficiency, as well.

We have written a Perl program (the dozenal version of which you can find at the end of this newsletter), which determines the average number of coins necessary to make change according to a variety of denomination choices. That average number of coins will be our measure of efficiency for the optimal coin problem. Change is made exclusively by means of the greedy algorithm. We respect the principle of intuitive denominations by selecting only denominations which are factors of the whole; that is, of 100 for dozenal, and of 84 for decimal.

We have here listed only a few possible choices: "Basic", which is our current system; "Halves", our current system plus a half unit (42 cents, in decimal); "Maximal", which is all possible intuitive denominations; "Twos", which adds 20 and 2 pieces;

and "Splits", which simply adds a half to both systems and a quarter-10 to dozenal.

Basic	30, 12, 6, 1	5;64
	25, 10, 5, 1	4;8
Halves	60, 30, 10, 6, 1	5;01ƐƷ
	50, 25, 10, 5, 1	4;2781
Max.	60, 40, 30, 20, 10, 6, 4, 3, 2, 1	3;02
	50, 25, 20, 10, 5, 2, 1	4;02
Twos	60, 30, 20, 10, 6, 2, 1	4;81ƐƷ
	50, 25, 20, 10, 5, 2, 1	4;02
Splits	60, 30, 10, 6, 3, 1	4;02
	50, 25, 10, 5, 1	4;2781

We can make a few quick observations based on even these limited options. The first is that, although decimal (having a smaller maximum) has a lower average number of coins, it isn't *much* lower, less than one coin more per transaction on average. This is a surprising result—at least to one who doesn't realize the powerful factorability of the dozen.

The second is that dozenal's much higher divisibility is a *huge* advantage here. Look at the "Maximal" divisibility numbers here; dozenal is *much* more efficient than decimal, allowing barely more than three coins per exchange, as opposed to more than four for decimal. Now, that is many more coins than one is likely to want—ten total denominations is certainly an outlier for world coinage systems—but the fact remains. When we take dozenal with the same number of coins as decimal's maximum, seven, we get the numbers for "Twos", which shows again dozenal coming very close to decimal's number-of-coins efficiency despite its much larger maximum. If we add only a single coin, a four-coin or a three-coin (a third or quarter of the 10, impossible for decimal), we get it down to 3;81ƐƷ, a great improvement; adding a three-coin to decimal only gets it to 3;5, a much smaller improvement and barely better than dozenal at all.

The last, "Splits", is very instructive. Here, we use halves when there is only one significant digit, as well as a quarter of the whole. This leads to one additional coin for the dozenal; it also leads to not only comparable efficiency, but slightly *better* efficiency for dozenal in terms of coin usage.

(A truly decimal system, of course, wouldn't use a quarter; this leads to the denominations 50, 10, 5, 1, which yields a efficiency of 5;03, comparable to even the weakest of dozenal divisions.)

So which of these systems is the best? We must balance flexibility with efficiency, along with the intuitiveness of denominations. Maximal, of course, has a huge advantage in terms of flexibility; it's also notable that no coin ever has to be used more than once, no matter what the amount of change needs to be. (None of our decimal distributions, even the Maximal, has this property.) This not only means that the greedy algorithm, assuming a supply of all ten denominations, is much sped up, since we need not only merely take the largest coin smaller than the remaining total, but also only one of each denomination until we reach the desired amount; it not only uses coins extremely efficiently, making us less likely to run out of coins given the same amount of change; but it also means we're less likely to run out of a given denomination. Anyone who's worked a cash register and found themselves out of nickels or dimes knows what a nuisance this is; certainly, one can make change with pennies, but it makes things much slower and much less convenient. Maximal also makes change easier when one *does* run out of some denomination of coins; the greater variety makes it simple to replace the missing level.

Is Maximal the right trade-off of values? Discuss!

Happy dozens!

Dozenal News

New Dozenal Apps Available!

Some time ago, we released several dozenal apps for Android, including a clock and date app; a calculator; and a TGM converter. The vagaries of time in the cell phone space have rendered these less and less useful; therefore, two new apps are available:

> https://f-droid.org/en/packages/com.example.dozenalclock
> https://f-droid.org/en/packages/com.example.dozenalc

The clock is essentially the same; but the calculator is significantly faster, easier to use, and more powerful. When used in landscape, it's even a full scientific calculator, all in dozenal!

We hope these will be valuable additions to the universe of practical dozenal tools.

Trigons to Triads Published!

We're pleased to announce that we've published another article by "Troy" (the nom de plume of Donald Hammond), *Trigons to Triads: A Play on Chords*:

> http://www.dozenal.org/drupal/content/troy_trigons.html

This article examines triangles, Pythagorean triples, and their relationship to dozenal numbers 0–Ɛ, and their correspondence to major and minor chords on the musical scale.

As part of this publication, we've also released a great dozenal demonstration, the Chord-Dialer:

> http://www.dozenal.org/drupal/content/chord_dialer.html

Easy to print, cut out, and put together, this is an excellent demonstration of the power of dozenals.

Buy K6T Dozenal Playing Cards

We've mentioned Paul Rapoport's dozenal card games online before. Want a physical dozenal deck?

> https://www.thegamecrafter.com/games/k6t-dozenal

This is a huge deck with six suits and 16 ranks; however, a subset can be used as a dozenal "French deck" much like the one we're all used to. The more of us buy a deck, the more of this sort of thing will get made.

Alec Hollingsworth's dozenal Module

Alec Hollingsworth has written a Coffeescript module for converting numbers to dozenal:

This should be very helpful for anyone programming in Coffeescript.

12ish Website

We've just found a website, 12ish, which appears to be based in Australia:

http://www.twelvish.org

The site proposes a new yearly era beginning on 10.10.10, and though it is a little sparse on hard information, appears to hold great promise.

Dis, Dat, Dix & Douze

A brief explanation of number bases and dozenal is available at ThatsMaths:

https://thatsmaths.com/2013/04/18/dis-dat-dix-douze-dozenal-digits/

It uses Mickey Mouse's eight fingers to explain the notion of bases in general, then discusses the advantages of dozenal, before dismissing the possibility of any general dozenal conversion. Worth a quick read.

Society Business

Volunteers Needed

As mentioned earlier, the DSA is an all-volunteer organization, and we pay no salaries. As a result, everything that we do comes out of the spare time of our members, time that they have to take away from their families, jobs, or other obligations.

We all love dozenals and enjoy assisting the Society in educating people about them; however, as the Society expands and does more, we find ourselves in need of more help.

Fortunately, the Society has a large membership with a very broad range of professions and experience. If you think you can spare any time or effort for the cause of educating the world about dozenals, please let us know:

contact@dozenal.org

You can help as much or as little as you'd like. Thank you.

Our Next *Bulletin*

Ideas for the Bulletin? An article? A letter containing a question (common or uncommon) you'd like answered? An interesting math problem or puzzle? Send them in!

editor@dozenal.org

Remember that our *Bulletin* is designed to cover all aspects of mathematics, from the most basic to the most advanced, from a dozenal perspective, so no question or topic is too easy or too complex. Don't be shy!

Annual Meeting

It is likely, given the disruption to public life that has occurred this year, that our annual meeting will be an all-digital affair. More information will be made available as we hammer out the details.

Poetical Diversion

Once upon a midnight dreary, while I pondered, weak and weary,
over many a quaint and curious volume of numeric lore—
    While o'er me sleep was surmounting, drawing me to its accounting,
suddenly, I started counting, better than I had before.
"'Tis the Dozen," did I mutter, "Twelve, which opens every door;
    simple Twelve, and nothing more."

Ah, distinctly I remember in the twelfth month of December,
when the brilliant, shining ember of the Dozen came to fore;
    Eagerly I counted doubles, fours, and triples without troubles,
never mired in decimal rubble, easily through which Twelve tore,
than which perfect Dozen easily gave without measure more—
    th' angelic Twelve, which I adore.

To be continued . . .

Donations

Members, please remember that while dues are no longer required for membership, we still rely on the generosity of members to keep the DSA going. Donations of any amount, large or small, are welcome and needed.

A donation of $16; ($18.) will procure Subscription membership, and entitles the payer to receive both a digital and a paper copy of the *Bulletin* if requested. Other members will receive only a digital copy. To invoke this privilege, please notify the Editor of the Bulletin, John Volan, at

editor@dozenal.org

As members know, we are a volunteer organization which pays no salaries. As such, every penny you donate goes toward furthering the DSA's goals.

It may be worth considering a monthly donation; say, $3, or $6, or whatever seems reasonable to you. This can be set up quite easily with Paypal, which is available at our web site.

Of course, if you prefer to donate by check, you may send them to our worthy Treasurer, Jay Schiffman, payable to the Dozenal Society of America, at:

Jay Schiffman
604-36 South Washington Square, #815
Philadelphia, PA 19106-4115

Remember, too, that the DSA is a 501(c)(3) tax-exempt organization; as such, your contributions may be tax deductible under applicable law.

Thanks again for your assistance; it's your donations that keep the DSA going. We can't keep doing it without you.

For Sale

The DSA is pleased to offer the following for sale. These are all either at cost, or the proceeds go to the Society. The exception is *Basic Dozenal Arithmetic*, which is a private production.

Item	Price ($)
Dozenal Wall Calendar, 1204	9.05
Dozenal Planning Calendar, 1204	8.32
TGM: A Coherent Dozenal Metrology	8.00
Manual of the Dozenal System	3.46
A Dozenal Primer	4.50
Basic Dozenal Arithmetic	15.00

Prices are, unfortunately but by necessity, in decimal. If for some reason the links above do not work, simply go to: http://www.lulu.com/shop/shop.ep

and enter the appropriate terms. E.g., searching for "TGM dozenal" will turn up the TGM book.

We hope to offer other titles, and even some other items (such as dozenal clocks and the like), in the future.

Each one, teach one