
DSA Newscast

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DOZENAL AND SUBITIZATION

IN THIS ISSUE . . .

Dozenal and Subitization

Dozenal News

| | |
|---|---|
| The Importance of Teaching Alternative Bases | 1 |
| Dozenal in Braille | 2 |
| Short Article on Dozens | 2 |
| Quinquennial, decimal, duodecimal, vigesimal, sexagesimal | 2 |
| Symmetry in Times Tables | 2 |

Society Business

| | |
|------------------------------------|---|
| New DSA Bank Account | 3 |
| Volunteers Needed | 3 |
| Our Next <i>Bulletin</i> | 3 |

Poetical Diversion

| | |
|-----------------------------|---|
| Dozens at the Bat | 3 |
|-----------------------------|---|

Donations

For Sale

The Dozenal Society of America is a voluntary, nonprofit educational corporation, organized for the conduct of research and education of the public in the use of dozenal (also called duodecimal or base twelve) in calculations, mathematics, weights and measures, and other branches of pure and applied science.

DOZENAL HAS A great deal going for it, but one thing that we rarely talk about is the concept of *subitization*. Subitization is a relatively recent concept in mathematics, but it speaks enormously well for the advantages of dozenal in both teaching and using numbers.

Subitizing is the ability to correctly ascertain a quantity without counting it; it's the same skill that allows you to immediately know what number is on the dice you rolled, without having to count the dots. Note that we're not referring to carnival contests correctly guessing the number of beans in a jar; that's not subitization, it's just guessing. Subitization is a very specific educational concept; the term was coined in 1165 (five years after our Society was formed) by Kaufman, Lord, Reese, and Volkmann in *THE AMERICAN JOURNAL OF PSYCHOLOGY, The Discrimination of Visual Number*. That article described the ability of the human mind to distinguish sizes of groups merely on sight; this property of groups is referred to as *numerousness*. Numerousness, the article continued, can be *relative*—greater than, less than, or equal to—or it can be *absolute*—an actual number of objects. To determine whether subitizing is actually occurring, rather than counting or estimation, it's important to ensure that the absolute numerosness judgment be given after a look “so brief that counting is impossible.”

Experimentation, the article shows, has demonstrated that people can accurately report absolute numerosness up to a certain number, and no higher. That number, the dozenalist will be unsurprised to learn, is six. Up to six can be reported, after a glance

of only $\frac{1}{5}$ of a second, accurately and with complete confidence; more than six, and confidence begins to drop while error begins to rise. Seven or eight can be accurately reported with some confidence; beyond eight, all bets are off.

Some studies have been even more interesting, as far as dozenalism goes. One study found that, while repetition of the experiment resulted in accurate judgments up to six objects, on the first round of attempts numbers could only be accurately judged up to *three*. This was with a $\frac{1}{2}$ second exposure, rather than the much briefer exposure of the previous experiment. Regardless, the article found, “something special seems to happen around 6.” The article’s own experiments showed no error in numerosness judgment up through four; at five, errors crept in, and above six became intolerable.

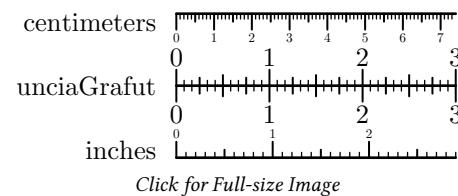
The conclusion of the article is that humans perceive number (that is, make judgments about absolute numerosness) in three different ways: *estimating*, *subitizing*, and *counting*. The first and last we know very well; it's the middle one that's new. The difference in accuracy, speed, and confidence when judging numerosness of groups *above six* and when judging numerosness of groups *below six* is so great that the term *subitizing* was coined. Numbers under six were always subitized; numbers above six were always estimated. Six itself could be done either way, depending on the configuration of the objects in the group. Numbers four and below could be regularly subitized by all subjects without any error.

While this is interesting, how is it relevant for dozenalism? *Subitizing favors the*

dozen. There is considerable debate among scholars where the real discontinuity for subitizing lies; some have placed it as low as three or four. They all place it, though, as some factor of the dozen. This means that we can take ready advantage of subitizing to make our daily arithmetic and counting easier.

A prime example is the way that numbers are grouped. We typically separate them by threes in decimal (1,000,000), or by fours in dozenal (1 0000 0000), though we see a lot of variation in both cases. Both work because subitization of three or four is easy; we can immediately ascertain the correct order of magnitude this way by subitizing each group. You'll never see them grouped by fives, because subitization of five is slower and more error-prone.

Another good example is tallying and grouping. We might, for example, have a ruler divided like so:



Click for Full-size Image

(We have compared decimal customary, decimal SI metric, and dozenal TGM units here.) Note that the two decimal systems, customary and SI metric, can only divide their units into two groups of five. It's hard to subitize individual amounts there. They can certainly be divided by fours and three; but then there is an unequal number of the

fundamental decimal division, making computation more difficult. The TGM rule, however, easily divides into four in equal parts of the fundamental dozenal division, and the quaternary subdivision is immediately subitizable; it's grasped at a glance what fraction of the main unit we're dealing with, without the need to count. This is a huge advantage when working with the superior dozenal base.

There's been a lot of research on subitization; some has even found results indicating that humans subitize *audial* and *tactile* sensory input, as well as visual. Further research into this matter would be valuable and interesting. But the body of research already available makes at least one thing clear: subitization, like all arithmetic, certainly favors the dozen.

DOZENAL NEWS

THE IMPORTANCE OF TEACHING ALTERNATIVE BASES

Erik Engheim at *Medium* has written a piece explaining the importance of teaching non-decimal bases to young math students:

<https://medium.com/@Jernfrost/teaching-kids-about-alternative-number-base-s-36e3cc464250>

The article gives a great summary of number bases in general, explaining them by asking readers to consider how other animals with different numbers of appendages would count, and makes a pretty good (if short) pitch for our favorite base, as well.

Mr. Engheim has also written a piece specific to why twelve is so important and why dozenal is the best:

<https://medium.com/@Jernfrost/why-does-the-number-12-pop-up-so-often-7f33d8b0faa8>

DOZENAL IN BRAILLE

Dozenalists may be interested in this proposal for how to write non-decimal numbers in Braille:

<http://www.brl.org/codes/session03/non-decimal.html>

Not being a Braille-reader, it's hard for me to comment on its quality; but it's certainly interesting. It also seems to be focusing on dozenal, using "T" for ten and "E" for eleven.

SHORT ARTICLE ON DOZENS

The staff of a short-article website has published a brief answer to the question, "Should We Start Using The Base-12 Number System?"

<https://curiosity.com/topics/should-we-start-using-the-base-12-number-system-curiosity/>

It's about a paragraph about dozenal's advantages in divisibility, then a link to Numberphile's famous dozenal video.

QUINQUENNIAL, DECIMAL, DUODECIMAL, VIGESIMAL, SEXAGESIMAL

An intriguing article, translated from its original Spanish, examines some of the Sumerian, Egyptian, and other ancient numbering systems, noting in its title that *nil novum sub sole* ("there's nothing new under the sun"):

<http://en.antiquitatem.com/numeral-system-sexagesimal-sume-dozen>

While the phrasing is sometimes a bit awkward, due to its translation, it's a truly interesting article, and short enough that those without a great deal of knowledge of ancient systems. It's also full of quotations from original sources, which is bound to intrigue any antiquarian.

SYMMETRY IN TIMES TABLES

dozenalsystemandsymmetryinvestigation.com

Chloe Teale, in a truly interesting and richly illustrated blog post, shows some of the lovely symmetries of the dozenal times tables (along with, in fairness, some of the less-lovely symmetries of the decimal tables):

<https://blogs.glowscotland.org.uk/glowblogs/catuodeportfolio1/2015/11/30/du>

While the numbers in the article are entirely in decimal, it notes that the symmetries of the dozenal tables are much more robust, even when viewed in decimal; and points out the relations of those symmetries to the unquagon (SDN; standard “dodecagon”) and the decagon (also SDN; those this term is the same in standard speech, as well). Very interesting.

SOCIETY BUSINESS

NEW DSA BANK ACCOUNT

We mentioned last month that our Paypal donations were stuck in a permanent vortex of strange bank rules related to a recent transfer of our banking to another institution. However, that vortex has been dispelled, and your donations are now flowing properly again. We apologize for the inconvenience this may have caused.

Fortunately, the Society has a large membership with a very broad range of professions and experience. If you think you can spare any time or effort for the cause of educating the world about dozenals, please let us know:

contact@dozenal.org

You can help as much or as little as you'd like. Thank you.

OUR NEXT BULLETIN

VOLUNTEERS NEEDED

As mentioned earlier, the DSA is an all-volunteer organization, and we pay no salaries. As a result, everything that we do comes out of the spare time of our members, time that they have to take away from their families, jobs, or other obligations.

We all love dozenals and enjoy assisting the Society in educating people about them; however, as the Society expands and does more, we find ourselves in need of more help.

Though the delay since our last issue may make this hard to believe, work on the next *Duodecimal Bulletin* continues apace. Have an article? A letter containing a question (common or uncommon) you'd like answered? Send them in!

editor@dozenal.org

Remember that our *Bulletin* is designed to cover all aspects of mathematics, from the most basic to the most advanced, from a dozenal perspective, so no question or topic is too easy or too complex. Don't be shy!

POETICAL DIVERSION

DOZENS AT THE BAT

The outlook wasn't brilliant for the Dozenals that day;
the Decimals had the field; their pitching dominated play;
and though Dozens had the better men, with Six and Four and Three,
and even Two, the Decimals prepp'd the victory to see.

And up to bat were weaknesses, poor Seven and weak Five,
the worst of Dozen's batsmen had to keep its hopes alive;
the Decimals had struck out Eleven and the Dozen's Ten,
and only needed one more out to take the game again.

But Seven drove a single, to the wonderment of all,
and Five, the much despisèd, tore the cover off the ball;
and though his brother's Decimal's only player with some strength,

the Dozen's Five, the lowly, gave the game much-needed length.

For the batsman coming to the plate was Dozen's muscle, Three,
and Decimal Ten's lip quiver'd in his fear the man to see;
this hitter's strengths the Decimals never could arise to meet,
and when Three put bat to shoulder, Decimal knew that he'd been beat.

The first pitch o'er the plate, Three swung and crush'd the Decimal's ball,
which flew right o'er the fences, and seem'd destin'd ne'er to fall;
and finally, though the umpire slanted calls the Decimal's way,
the Dozenals rest easy, for their strengths would win the day.

Of course, in this poem's inspiration, Casey at the Bat, the hometown team loses because Casey strikes out. I hope my fellow dozenalists will forgive the liberty I took of having the Dozens win.

DONATIONS

Members, please remember that while dues are no longer required for membership, we still rely on the generosity of members to keep the DSA going. Donations of any amount, large or small, are welcome and needed.

A donation of \$16; (\$18.) will procure Subscription membership, and entitles the payer to receive both a digital and a paper copy of the *Bulletin* if requested. Other members will receive only a digital copy. To invoke this privilege, please notify the Editor of the Bulletin, John Volan, at editor@dozenal.org

As members know, we are a volunteer organization which pays no salaries. As such, every penny you donate goes toward furthering the DSA's goals.

It may be worth considering a monthly donation; say, \$3, or \$6, or whatever seems reasonable to you. This can be set up quite easily with Paypal, which is available at our web site.

Of course, if you prefer to donate by check, you may send them to our worthy Treasurer, Jay Schiffman, payable to the Dozenal Society of America, at:

Jay Schiffman
604-36 South Washington
Square, #815
Philadelphia, PA 19106-4115

Remember, too, that the DSA is a 501(c)(3) tax-exempt organization; as such, your contributions may be tax deductible under applicable law.

Thanks again for your assistance; it's your donations that keep the DSA going. We can't keep doing it without you.

FOR SALE

The DSA is pleased to offer the following for sale. These are all either at cost, or the proceeds go to the Society. The exception is *Basic Dozenal Arithmetic*, which is a private production.

| Item | Price (\$) |
|-----------------------------------|------------|
| Dozenal Wall Calendar, 1202 | 9.05 |
| Dozenal Planning Calendar, 1202 | 8.32 |
| TGM: A Coherent Dozenal Metrology | 8.00 |
| Manual of the Dozenal System | 3.46 |
| A Dozenal Primer | 4.50 |
| Basic Dozenal Arithmetic | 15.00 |

Prices are, unfortunately but by necessity, in decimal. If for some reason the links above do not work, simply go to: <http://www.lulu.com/shop/shop.ep>

and enter the appropriate terms. E.g., searching for "TGM dozenal" will turn up the TGM book.

We hope to offer other titles, and even some other items (such as dozenal clocks and the like), in the future.

EACH ONE, TEACH ONE

