DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

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THE DUODECIMAL BULLETIN 74;

The Annual Award of the

Dozenal Society of America

is hereby presented posthumously to

Fred Newhall

Member, Officer, Director, Architect,
Author, Speaker & Friend.

For almost one dozen years Fred aided our Society, serving on various committees, rising to the positions of Secretary, Board Chair and then President.

We will greatly miss his leadership, his warmth and his ever-present smile. Without devoted people such as Fred, there would be no DSA.

Given with gratitude by the Board of Directors.

11*2; 1994.

Volume 38;
Number 3;
1995
11*3;

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00 (US) for one calendar year. Student Membership is $3.00 per year, and a Life Membership is $144.00 (US).

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THE DUODECIMAL BULLETIN

Whole Number Seven Dozen Four

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11*3;

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IN THIS ISSUE

NOTICE TO AUTHORS

Our supply of both long and short articles for possible publication in the *Bulletin* is plentiful. A great variety of diversified topics dealing with aspects of number bases is on hand and is much appreciated. In order for us to expedite our work, it would be extremely helpful if all articles, jottings, fillers, and other pieces submitted for inclusion in the *Bulletin* are typed in Word Perfect 5.1 in block form. (It is not necessary to indent for new paragraphs. Simply leave enough spacing to indicate that you are establishing a different trend of thought.) Both a disc copy as well as a paper copy of your manuscript should be included. Furnishing this material enables us to make corrections directly on the disc which aids us immeasurably. Indeed please keep your dozenal material coming to our office and share your good ideas with our readers. Twelve-fold best wishes for a happy 11*3!

DOZENAL SOCIETY OF AMERICA
MINUTES OF THE ANNUAL MEETING - 11*2

Saturday, October 15, 1994
Six Brancatelli Court
West Islip, LI, NY 11795

I. BOARD OF DIRECTORS MEETING

1. Dr. Patricia Zirkel, Board Chair, opened the meeting at 1:40 PM. The following Board members were present:

   Alice Berridge
   Jamison Handy Jr. (by proxy)
   Rafael Marino
   Robert McPherson (by telephone)
   Jay Schiffman
   Gene Zirkel
   Patricia Zirkel

   Member Vera Handy was also in attendance.

2. The minutes of the meeting of October 16, 1993 were approved as published in the *Bulletin*.

3. Members expressed concern about the poor health of Jim Malone, former Board member.

   Members expressed sorrow at the death last year of Fred Newhall who had been so very active in the Society. Fred had a hand in almost every area of DSA activities; his hard work and cheerful manner are sorely missed by us all. We also miss Mary Newhall, his wife, who had been a staunch supporter of the Society, but who has re-located to Virginia.

4. The Nominating Committee (A. Berridge, J. Schiffman) presented the following slate of officers:

Board Chair:	Dr. Patricia Zirkel
President:	Jay Schiffman
Vice President:	Rafael Marino
Secretary:	Alice Berridge
Treasurer:	Alice Berridge

   The slate was elected unanimously.

(Continued)

5. Appointments were made to the following DSA Committees:

   Annual Meeting Committee: Alice Berridge

   Awards Committee: Gene Zirkel, Chair
   Patricia Zirkel
   Rafael Marino

   Volunteers to these committees are welcome at any time.

6. The following appointments were made:

   Editor of The *Duodecimal Bulletin*: Jay Schiffman

   Parliamentarian to the Board Chair: Jamison Handy Jr.

7. Other business of the Board:

   The next Board meeting will be October 14, 1995 at 10:30 AM at either
   Rowan College of New Jersey
   Glassboro, NJ 08028
   or at a Delaware Valley location to be determined soon.

The Board meeting was adjourned at 2:00 PM.

II ANNUAL MEMBERSHIP MEETING

1. DSA President Jay Schiffman gavelled the meeting to order at 2:05 PM.

2. Gene Zirkel moved to accept the minutes of the meeting of October 16, 1993. So approved.

(Continued)

The DSA does **NOT** endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus 1/2 = 0.5 = 0;6.

3. President's Report - Jay Schiffman

Jay noted that the DSA is listed in the 1994 *World Almanac* listing of societies, and that the Ralph Beard article on dozens is printed in the *Encyclopedia Americana*. He showed us a copy of Tim Travis's book, *4000: The Fifth Millennium*. One of the chapters deals with dozens. Jay and Gene have read sections of the book and have found it worthwhile and interesting. Gene has written a review for the *Bulletin*.

There have been an unusual number of inquiries about DSA affairs this year -- more than a gross. This is close to double the usual number of inquiries for a year.

John Porter from Japan has written with ideas for a possible article for the *Bulletin*. Underwood Dudley has mentioned the Society in two of his books. Bill Lauritzen sent notice about his recent lecture: "Nature's Numbers: Round-Earth Number for the New Millennium. Have You Been Brainwashed Mathematically?" Bob McPherson from Gainesville, FL called during the meeting and presented members with an interesting challenge (see below).

President's Appointment:

Parliamentarian to the President: Dr. Patricia Zirkel

4. Treasurer's Report - Alice Berridge

The report indicates a decrease in stock dividends, an increase in fees (extra fees were incurred in replacing stock certificates lost in last year's fire), and a reduction of our Certificate of Deposit. Stock values are lower, reflecting the current status of the financial market. In spite of this bad news, the Society's net worth is $15,788.06. There has been an increase in dues paid. Of the Society's 22 Life Members (who have paid a one-time dues of a gross dollars), eight made extra contributions this year. These extra contributions made a difference. Members were pleased to note that there are a half dozen student members.

5. Editor's Report - Jay Schiffman

Jay said that there are a number of long and short articles for the next several issues. He welcomes all articles, news, puzzles, fillers, etc. Jay will print a "Notice to Authors" concerning style requirements for material submitted. Members praised and thanked Jay for his work.

(Continued)

6. Annual Meeting Committee - Alice Berridge

The next annual meeting will take place on October 14, 1995 at 10:30 AM in New Jersey.

7. Awards Committee - Gene Zirkel

The Ralph Beard Memorial Award will be presented posthumously to Fred Newhall to acknowledge Fred's extraordinary service to the Society. A plaque will be sent to his wife, Mary.

8. Nominating Committee - Alice Berridge

The Committee presented the following slate for the Class of 1997:

John Hansen, Jr.
Rafael Marino
Jay Schiffman
Tim Travis

The slate was elected unanimously.

The names of Alice Berridge, Jay Schiffman & Rafael Marino were presented as our Nominating Committee for the coming year. They were elected unanimously.

9. Other Business

At the instigation of Arthur Whillock of Great Britain, Rafael will begin use of the Internet to establish a DSA connection to same for information purposes. Members were excited at the prospect of increased communications about dozens.

Members agreed that the DSA brochure should be reprinted. The brochure has been useful in promoting the Society and in answering inquiries.

Gene will look into an advertisement from a clock company which might be able to develop a clock with the Society's logo on its face. The price of approximately $7.95 might make this an attractive option for DSAers.

There is a problem with the development of the Index for the *Bulletin*. In the past, Fred Newhall had handled this job. A volunteer is sorely needed to take on this chore.

The meeting was adjourned at 3:00 PM.

(Continued)

III FEATURED SPEAKERS

Presentations began at 11:15 AM in order to accommodate the schedule of Jamison and Vera Handy who had to leave the meeting early.

1. Professor Rafael Marino, a faculty member at Nassau Community College, presented a fascinating talk: "If We Only Had Twelve Fingers." As a young student in Colombia he was impressed with the advantages of base 12. He had contrived an elaborate system similar to Roman numerals, and related to an abacus, to manage computation. He said that when he found the DSA he thought it was "incredible that someone else was thinking the same way." He presented an interesting "Dozenland Clock" and related his ideas to geometry.

2. President Jay Schiffman, faculty member at Rowan College of New Jersey's Camden Campus, spoke on "The Personality of Duodecimal Integers from One to One Gross." Jay categorized number theory characteristics which he discussed and analyzed. Listeners were especially interested in Mersenne Primes, Lucas Numbers, and Fibonacci Numbers. He pointed out that the dozenth term of a Fibonacci series is one gross. He presented us with a classification according to his glossary of numbers from 1 to 100; and a listing of every number from 1 to 100; according to the classifications.

3. Bob McPherson called from Florida with an interesting problem which challenged us ... and amused us:

   Compute (1 - 0.6)(0.2) + (0.6)(0.3) in any base larger than six and compare results.

   We enjoyed evaluating this expression in several different bases.

The group expressed gratitude to Gene and Pat Zirkel for their hospitality and for providing lunch.

Respectfully submitted,

Alice Berridge

□

Do you keep a copy of our DSA brochure or of Andrews' *Excursion* at home and in the car? You never know when you might want to give one to a friend. Be sure to always have one on hand.

THE PERSONALITY OF THE INTEGERS FROM ONE TO ONE GROSS

Jay L. Schiffman
Rowan College of New Jersey, Camden Campus

Introduction: In his fascinating paperback entitled The Lore of Large Numbers [1], author Philip J. Davis devotes a detailed chapter to The Personality of Numbers and provides a list entitled Who's Who Among The Integers from One to One Hundred in the decimal base. The purpose of my paper is to expand his list to the first gross and present additional characteristics of these integers. It is indeed the uniqueness of each integer that lends a special personality to them especially when one resorts to duodecimals. Finally, my goal is to furnish number theoretic characteristics related to each of the integers that are perhaps omitted from the average list. Our initial task is to present the reader with a dictionary which is termed *A Glossary Of Symbols And Terms*. These symbols and terms will be illustrated in this section.

A GLOSSARY OF SYMBOLS AND TERMS

1. $n!$ = *The factorial of the positive integer n.* We define n!, known as *n factorial* recursively as follows: $0! = 1$ and $(n + 1)! = (n + 1)(n!)$. Hence $1! = (0 + 1)(0!) = (1)(1) = 1$ and $5! = (4 + 1)(4!) = (5)(20) = *0$.

2. T_n = *The n-th Triangular Number.* We define the *n-th Triangular Number* T_n as follows: $T_n = 1 + 2 + 3 + ... + n = ((n)(n+ 1)/2$. Thus $T_5 = 1 + 2 + 3 + 4 + 5 = 13 = ((5)(5 + 1))/2$.

3. S_n = *The n-th Square Number.* Define the *n-th Square Number* S_n by the formula $S_n = 1 + 3 + 5 + 7 + ... + (2n-1) = n^2$. Hence $S_4 = 1 + 3 + 5 + 7 = 14 = 4^2$.

4. TH_n = *The n-th Tetrahedral Number.* The *Tetrahedral Numbers* form the sequence {1, 4, *, 18, 2#, 48,...}. The Tetrahedral Numbers are related to the Triangular Numbers as follows: $TH_n = TH_{n-1} + T_n$ for $n \geq 2$ and $TH_1 = 1$. Hence $TH_5 = TH_4 + T_5 = 18 + 13 = 2\#$.

5. F_n = *The n-th Fibonacci Number.* N is a *Fibonacci Number* if N satisfies the Fibonacci Sequence defined recursively as follows: $F_1 = 1$, $F_2 = 1$, and $F_n = F_{n-2} + F_{n-1}$ for $n \geq 3$. Hence $F_3 = F_1 + F_2 = 1 + 1 = 2$.

6. L_n = *The n-th Lucas Number.* N is a *Lucas Number* if N satisfies the Lucas Sequence defined recursively as follows: $L_1 = 1$, $L_2 = 3$, and $L_n = L_{n-2} + L_{n-1}$ for $n \geq 3$. Hence $L_3 = L_1 + L_2 = 1 + 3 = 4$.

7. p_n = *The n-th Prime Number.* Define a *prime number* to be any integer $p > 1$ that has only 1 and p as its factors (divisors). Hence $p_3 = 5$; for the only divisors of 5 are 1 and 5.

(Continued)

8. FE_n = *The n-th Fermat Prime.* N is a *Fermat Prime* if N is prime and N = 2^{2^n} + 1 for any whole number n. To cite an example, $FE_2 = 2^{2^2} + 1 = 15$.

9. CO_n = *The n-th Integer for which a regular n-gon is constructible using solely a straight-edge and compass.* This idea is related to the geometric notion of *Constructible Regular n-gons* in the following sense as can be demonstrated via abstract algebraic techniques: The sole values of N for which a regular n-gon is constructible using only a staight-edge and compass are such that the only odd primes dividing N are the Fermat Primes whose squares do not divide N. Hence a regular duodecahedron (10 sides) is constuctible in the above sense; for 10 = $(2^2)(3)$ and 3 is a Fermat Prime (F_1 = 3 as the reader can readily verify) but 3^2 is not a factor of 10. On the other hand, a regular 16-gon is not constructible since 16 = $(2)(3^2)$ and while 3 is a Fermat Prime, 3^2 is a factor of 16.

*. C_n = *The n-th Composite Number.* N is designated *Composite* if N > 1 and N is not prime. To cite an example, 10 is composite; for 10 possesses a half dozen factors: 1, 2, 3, 4, 6, and 10 while prime numbers possess only a pair of factors (1 and the prime itself).

#. HC_n = *The n-th Hypercomposite Number.* A positive integer N is termed *Hypercomposite* if N possesses a greater number of factors than any of its immediate predecessors. Again 10 serves as a neat example; for 10 has 6 factors while no integer < 10 has more than 4 factors (6, 8, and *).

10. SF_n = *The n-th Square-Free Number.* N is *Square-Free* if N is not divisible by the square of any prime p. To illustrate, 26 = (2)(3)(5) enjoys this property while our favorite integer 10 = $(2^2)(3)$ does not having 2^2 as part of its factorization.

11. $\sigma(n)$ = *The sum of the divisors of a positive integer n including n itself.* To cite some examples, $\sigma(6)$ = 1 + 2 + 3 + 6 = 10, while $\sigma(10)$ = 1 + 2 + 3 + 4 + 6 + 10 = 24 and $\sigma(11)$ = 1 + 11 = 12.

12. P_n = *The n-th Perfect Number.* A positive integer N is styled *Perfect* if it is equal to the sum of all of its aliquot (proper) divisors. For example, 6 is perfect since 6 = 1 + 2 + 3. In short, any integer which coincides with the sum of all its divisors not including itself is classified as perfect. Perfect numbers are rare indeed. The next three perfect numbers in order of magnitude are 24, 354 and 4854. Mathematicians often succinctly employ the equation $\sigma(N) = 2N$ to denote that N is perfect; that is, the sum of all the divisors of N including N itself is precisely twice N. Equivalently, one could write $\sigma_0(N) = N$ to connote that the sum of all the proper divisors of N is precisely N if N is perfect.

13. A_n = *The n-th Abundant Number.* A positive integer N is termed *Abundant* if it is greater than the sum of all its aliquot divisors. For example, 10 is

(Continued)

abundant since 1 + 2 + 3 + 4 + 6 = 14 > 10. The equation $\sigma(N) > 2N$ indicates that N is abundant; that is, the sum of all the divisors of N including N is larger than twice N. Equivalently, one often writes $\sigma_0(N) > N$ to signify that the sum of all the proper divisors of N is greater than N if N is abundant. It can be shown that any integer multiple of a perfect number is necessarily abundant (10 fits this description) while most abundant numbers are even. In fact, the initial odd abundant number is 669 (decimally nine hundred forty five) and we observe that $\sigma(669)$ = 1140 > (2)(669) = 1116. In fact, 1 + 3 + 5 + 7 + 9 + 13 + 19 + 23 + 2# + 39 + 53 + 89 + #3 + 139 + 223 = 693 > 669.

14. D_n = *The n-th Deficient Number.* A positive integer N is classified as *Deficient* if it is less than the sum of its aliquot divisors. To illustrate, 2# is deficient since 1 + 5 + 7 = 11 < 2#. The equation $\sigma(N) < 2N$ signifies that N is deficient; that is, the sum of all the divisors of N including N is smaller than twice N. Equivalently, one generally writes the inequality $\sigma_0(N) < N$ to indicate that the sum of all the proper divisors of N is less than N if N is deficient. It can be shown with relative ease that any prime number is deficient. Moreover, so is any power of a prime and any integer save for 6 = (2)(3) that is expressible as the product of two distinct primes.

15. M_n = *The n-th Mersenne Prime.* N is a *Mersenne Prime* if N is prime and N = $2^p - 1$, where p itself is prime. Mersenne primes lead to perfect numbers. In fact, the form of an even perfect number according to the Greek Mathematician Euclid is $P = (2^{p-1})(2^p - 1)$ where both p and 2^{p-1} are prime. To illustrate if we take p = 3, then 3 is prime and so is $2^3 - 1 = 7$. This leads to the second perfect number $P = (2^{3-1})(2^3 - 1) = (2^2)(7) = 24$.

16. $\tau(n)$ = *The number of divisors of a positive integer n.* To cite some examples, $\tau(10)$ = 6 (1, 2, 3, 4, 6, and 10 are the divisors of 10) while $\tau(6)$ = 4 (1, 2, 3, and 6 are the divisors of 6) and $\tau(5)$ = 2 (1 and 5 are the divisors of 5). It is readily apparent that if p is a prime, then $\tau(p) = 2$, while for any power of a prime p, say p^n, $\tau(p^n) = n + 1$. For example, 8 = 2^3 and $\tau(8) = 4$; for 1, 2, 4, & 8 are the divisors of 8.

17. $\phi(n)$ = *The number of positive integers < n that are relatively prime to n.* To illustrate, $\phi(10)$ = 4 since 1, 5, 7, and # are < 10 and have no factors in common with 10 other than 1, which is equivalent to saying that these integers are relatively prime to 10. In addition, $\phi(7)$ = 6; for 1, 2, 3, 4, 5, and 6 are all < 7 and relatively prime to 7. It is readily apparent that if p is prime, then $\phi(p) = p - 1$. Moreover, if $t = p^n$, (p a prime), then $\phi(p^n) = p^n - p^{n-1}$. Hence $\phi(8) = \phi(2^3) = 2^3 - 2^{3-1} = 2^3 - 2^2 = 8 - 4 = 4$. Observe that this agrees with enumeration; for the integers 1, 3, 5, and 7 are < 8 and are relatively prime to 8. This number-theoretic function known as the *Euler-phi function* or *totient function* possesses extremely important applications in higher algebra and the theory of numbers. In fact recalling

(Continued)

our discussion earlier concerning consructible regular polygons utilizing only a straight-edge and compass, one can demonstrate that the values of N for which a regular N-gon is constructible using only a straight-edge and compass are those values for which [ø (N)/2] (and thus (ø N)) is an integral power of 2. Hence a regular duodecagon is constructible since ø (10) = 4 = 2^2 as is a regular octagon (ø (8) = 4 = 2^2) while a regular 16-gon is not (ø (16) = 6 and 6 is not an integral power of 2).

18. $\pi(n)$ = *The number of positive primes ≤ n.* To cite an example, observe that $\pi(10)$ = 5, since 2, 3, 5, 7, and # are primes not exceeding a dozen. Moreover, $\pi(11)$ = 6; for 2, 3, 5, 7, #, and 11 are primes ≤ 11.

19. $\Pi\, d|n$ = *The product of the divisors of a positive integer n.* To illustrate, Π d|10 = (1)(2)(3)(4)(6)(10)= 1000 = 10^3, while Π d|23 = (1)(3)(9)(23) = 509 and Π d|2# = (1)(5)(7)(2#) = 861. Observe in the latter pair of examples that 23 and 2# coincide with the product of their proper divisors.

1*. PSK_n = *The n-th Perfect Number of the Second Kind.* The idea of a Perfect Number of the Second Kind generalizes the notion of a perfect number. N is styled *Perfect of the Second Kind* if the product of the divisors of N coincides with the square of the number N. Symbolically, one would express this fact via the equation Π d|N = N^2. Citing the examples 23 and 2# from 19. above illustrates this type of behavior; for 509 = 23^2 and 861 = $2\#^2$. Moreover, it is immediate that if N = (p)(q) for distinct primes p and q or if N = p^3 for some prime p, then the product of all the aliquot divisors of N coincides with N and the Product of all the divisors of N is N^2. (Note that (1)(p)(q)(pq) = $(p^2)(q^2)$ = $(pq)^2$ and (1)(p)$(p^2)(p^3)$ = (p^6) = $(p^3)^2$.)

1#. MP_n = *The n-th Multiply Perfect Number.* A second generalization of a perfect number is that of a multiply-perfect number. We define N to be *multiply-perfect* if σ (N) = (k)(N) for some positive integer k ≥ 3. (A perfect number is in essence a multiply-perfect number with k = 2.) Our initial example of a 3- perfect number is *0. σ(*0) = 1 + 2 + 3 + 4 + 5 + 6 + 8 + * + 10 + 13 + 18 + 20 + 26 + 34 + 50 + *0 = 260 = (3)(*0). The reader is invited to show that 480 is a second 3-perfect number. 4-perfect numbers are also known although the smallest one is quite large.

20. A(M,N) = *An amicable or friendly number pair.* A pair of numbers M and N are termed *amicable or friendly* if the sum of the aliquot divisors of one is equal to the other. In essence, we are asserting that A(M,N) is an amicable pair if σ_0(M) = N and σ_0(N) = M. Moreover from this, one deduces that if A(M,N) is an amicable pair, then σ(M) = σ(N) = M + N. If M = 164 and N = 1#8, then σ_0(164) = 1 + 2 + 4 + 5 + * + # + 18 + 1* + 38 + 47 + 92 = 1#8 while σ_0(1#8) = 1 + 2 + 4 + 5# + #* = 164; so that (164,1#8) is an amicable number pair, the smallest of its kind. Note that σ (164) = 360 = σ (1#8) = 164 + 1#8. The reader is invited to check that the number pair (828, 84*) is a second amicable number pair, discovered in 10#6; by a

(Continued)

14; year old Italian school boy. This pair seemingly escaped the great mathematicians of their day.

It should be noted in closing that σ (n), τ (n), and ø (n) are multiplicative number-theoretic functions in the sense that for relatively prime pairs of integers r and s; σ(rs) = σ(r) x σ (s), τ (rs) = τ (r) x τ (s), and ø (rs) = ø (r) x ø (s). This result is extendable to more than two integers provided none of the integers has a common factor apart from 1 among them. To cite an example, we calculate σ (*0), τ (*0), and ø (*0). Initially observe that *0 = $(2^3)(3)(5)$. Now σ (*0) = σ $[(2^3)(3)(5)]$ = (2^3) x σ (3) x σ (5) = $[(2^{3+1} - 1)/(2 - 1)]$ x (3 + 1) x (5 + 1) = $[(2^4 - 1)/ (2 - 1)$ = (14 - 1)/ (2 - 1) x (4) x (6) = (13)/(1) x (4) x (6) = (13) x (4) x (6) = 260, τ (*0) = $[(2^3)(3)(5)]$ = τ (2^3) x τ (3) x τ(5) = (3 + 1) x (1 + 1) x (5 + 1) = 4 x 2 x 2 = 14, and ø (*0) = $[(2^3)(3)(5)]$ = $[2^3 - 2^{3-1}]$ x (3 - 1) x (5 - 1) = $[2^3 - 2^2]$ x 2 x 4 = (8 - 4) x (3 - 1) x (5 - 1) = 4 x 2 x 4 = 28.

Our next goal is to provide an enumeration of a number of essential dozenal sets of numbers in the range from 1 to 100.

DOZENAL SETS OF NUMBERS (RANGE 1-100):

1. *Perfect Numbers:* {6, 24}. (Total: 2)

2. *Deficient Numbers:*

{1, 2, 3, 4, 5, 7, 8, 9, *, #, 11, 12,
13, 14, 15, 17, 19, 1*, 1#, 21, 22, 23, 25, 27, 28, 29, 2*,
2#, 31, 32, 33, 35, 37, 38, 39, 3*, 3#, 41, 42, 43, 44, 45,
47, 49, 4*, 4#, 51, 52, 53, 54, 55, 57, 58, 59, 5#, 61, 62,
63, 64, 65, 67, 69, 6*, 6#, 71, 72, 73, 75, 77, 78, 79, 7*,
7#, 81, 82, 83, 85, 87, 89, 8*, 8#, 91, 92, 93, 95, 97, 98,
99, 9*, 9#, *1, *2, *3, *4, *5, *7, *8, *9, **, *#, #1, #2,
#3, #4, #5, #7, #9, #*, ##}. (Total: 91)

3. *Abundant Numbers:*

{10, 16, 18, 20, 26, 30, 34, 36, 40, 46,
48, 50, 56, 5*, 60, 66, 68, 70, 74, 76, 80, 84, 86, 88, 90,
94, 96, *0, *6, #0, #6, #8, 100} (Total: 29)

4. *Multiply Perfect Numbers:* {*0} (3-Perfect Number). (Total: 1)

5. *Square-Free Numbers:*

{1, 2, 3, 5, 6, 7, *, #, 11, 12, 13,
15, 17, 19, 1*, 1#, 22, 25, 26, 27, 29, 2*, 2#, 31, 32, 33,
35, 36, 37, 3*, 3# 43, 45, 47, 49, 4*, 4#, 51, 52, 55, 56,
57, 59, 5*, 5#, 61, 62, 65, 66, 67, 6*, 6#, 71, 72, 73, 75,
77, 79, 7*, 7#, 81, 85, 86, 87, 89, 8*, 8#, 91, 92, 93, 95,
96, 97, 9*, 9#, *2, *3, *7, *9, **, *#, #1, #2, #5, #6, #7,
#9, #*, ##}. (Total: 75)

(Continued)

6. *Prime Numbers:* {2, 3, 5, 7, #, 11, 15, 17, 1#, 25, 27, 31, 35, 37, 3#, 45, 4#, 51, 57, 5#, 61, 67, 6#, 75, 81, 85, 87, 8#, 91, 95, *7, *#, #5, #7}. (Total: 2*)

7. *Triangular Numbers:* {1, 3, 6, *, 13, 19, 24, 30, 39, 47, 56, 66, 77, 89, *0, #4}.(Total: 14)

8. *Tetrahedral Numbers:* {1, 4, *, 18, 2#, 48, 70, *0}. (Total: 8)

9. *Fibonacci Numbers:*
{1, 1, 2, 3, 5, 8, 11, 19, 2*, 47, 75, 100}. (Total: 10)

*. *Lucas Numbers:* {1, 3, 4, 7, #, 16, 25, 3#, 64, *3}. (Total: *)

#. *Composite Numbers:*
{4, 6, 8, 9, *, 10, 12, 13, 14, 16, 18, 19, 1*, 20, 21, 22, 23, 24, 26, 28, 29, 2*, 2#, 30, 32, 33, 34, 36, 38, 39, 3*, 40, 41, 42, 43, 44, 46, 47, 48, 49, 4*, 50, 52, 53, 54, 55, 56, 58, 59, 5*, 60, 62, 63, 64, 65, 66, 68, 69, 6*, 70, 71, 72, 73, 74, 76, 77, 78, 79, 7*, 7#, 80, 82, 83, 84, 86, 88, 89, 8*, 90, 92, 93, 94, 96, 97, 98, 99, 9*, 9#, *0, *1, *2, *3, *4, *5, *6, *8, *9, **, #0, #1, #2, #3, #4, #6, #8, #9, #*, ##, 100} (Total: 91)

10. *Perfect Numbers of The Second Kind:* {6, 8, *, 12, 13, 19, 1*, 22, 23, 29, 2*, 2#, 32, 33, 3*, 43, 47, 49, 4*, 52, 55, 59, 62, 65, 6*, 71, 72, 73, 77, 79, 7*, 7#, 8*, 93, 97, 9*, 9#, *2, *3, *5, *9, #1, #2, #9, #*, ##}. (Total: 3*)

11. *Set of Values such that a Regular n-gon is constructible using only a straight-edge and compass:*
{3, 4, 5, 6, 8, *, 10, 13, 14, 15, 18, 20, 26, 28, 2*, 34, 40, 43, 50, 54, 58, 68, 71, 80, 86, *0, *8, #4}. (Total: 24)

12. *Fermat Primes:* {3, 5, 15}. (Total: 3)

13. *Hypercomposite Numbers:* {2, 4, 8, 10, 20, 30, 40, 50, *0}. (Total: 9)

14. *Mersenne Primes:* {$M_2 = 3$, $M_3 = 7$, $M_5 = 27$, $M_7 = *7$}. (Total: 4)

15. *Square Numbers:* {1, 4, 9, 14, 21, 30, 41, 54, 69, 84, *1, 100}. (Total: 10)

(Continued)

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THE CATALOGUE OF INTEGERS FROM ONE TO ONE GROSS CHARACTERISTICS

In what follows, we shall make use of the symbols described in our glossary above. Hence F_n will denote the n-th Fibonacci Number and C_n will connote the n-th Composite Number. Thus $F_5 = 5$, since the fifth Fibonacci Number in the sequence is five while C. = 16, as the tenth composite integer in the sequence is one dozen six. The other codes are obtained in similar fashion, and we cap the article with the catalogue.

1. Unit; Multiplicative Identity; Neither Prime nor Composite;
F_1; F_2; 1^n for all counting integers n; 0!; 1!; T_1; TH_1; L_1; S_1; SF_1; D_1; $\tau(1) = 1$; $\varnothing(1) = 1$; $\sigma(1) = 1$; $\pi(1) = 0$; $\Pi\, d|1 = 1$.

2. p_1; Binary Base; $1^n + 1^n$ for all counting integers n;
HC_1; F_3; 2!; $F_1 + F_2$; D_2; SF_2; $\tau(2) = 2$; $\varnothing(2) = 1$; $\sigma(2) = 3$; $\pi(2) = 1$; $\Pi\, d|2 = 2$.

3. p_2; Ternary Base; M_2; FE_1; CO_1; F_4; T_2; L_2; 1! + 2!; $F_1 + F_3$; SF_3; D_3; $\tau(3) = 2$; $\varnothing(3) = 2$; $\sigma(3) = 4$; $\pi(3) = 2$; $\Pi\, d|3 = 3$.

4. 2^2; C_1; HC_2; TH_2; L_3; S_2; CO_2; $F_1 + F_2 + F_3$; $F_2 + F_4$; D_4; $L_1 + L_2$; $\tau(4) = 3$; $\varnothing(4) = 2$; $\sigma(4) = 7$; $\pi(4) = 2$; $\Pi\, d|4 = 8$.

5. p_3; $2^2 + 1^2$; FE_2; F_5; CO_3; SF_4; D_5; $L_1 + L_3$; $\tau(5) = 2$; $\varnothing(5) = 4$; $\sigma(5) = 6$; $\pi(5) = 3$; $\Pi\, d|5 = 5$.

6. 2×3; P_1; H_3; C_2; PSK_1; SF_5; CO_4; T_3; 3!; $\tau(6) = 4$; $\varnothing(6) = 2$; $\sigma(6) = 10$; $\pi(6) = 3$; $\Pi\, d|6 = 30$.

7. p_4; L_4; $F_1 + F_2 + F_3 + F_4$; M_3; SF_6; D_6; $\tau(7) = 2$; $\varnothing(7) = 6$; $\sigma(7) = 8$; $\pi(7) = 4$; $\Pi\, d|7 = 7$.

8. 2^3; C_3; PSK_2; Octal Base; F_6; 2! + 3!; $2^2 + 2^2$; CO_5; $F_1 + F_3 + F_5$; D_7; $L_1 + L_2 + L_3$; HC_3; $\tau(8) = 4$; $\varnothing(8) = 4$; $\sigma(8) = 13$; $\pi(8) = 4$; $\Pi\, d|8 = 54$.

9. 3^2; C_4; S_3; $1^3 + 2^3$; 1! + 2! + 3!; D_8; $\tau(9) = 3$; $\varnothing(9) = 6$; $\sigma(9) = 11$; $\pi(9) = 4$; $\Pi\, d|9 = 23$.

*. 2×5; C_5; CO_6; PSK_3; SF_7; Decimal Base; $1^2 + 3^2$; T_4; TH_3; D_9; $L_2 + L_4$; $\tau(*) = 4$; $\varnothing(*) = 4$; $\sigma(*) = 16$; $\pi(*) = 4$; $\Pi\, d|* = 84$.

#. p_5; SF_8; L_5; D.; $\tau(\#) = 2$, $\varnothing(\#) = *$; $\sigma(\#) = 10$; $\pi(\#) = 5$; $\Pi\, d|\# = \#$.

10. $2^2 \times 3$; Duodecimal Base; HC_4; C_6; CO_7; A_1; 2! × 3!; $F_1 + F_2 + F_3 + F_4 + F_5$; $F_2 + F_4 + F_6$; $\tau(10) = 6$; $\varnothing(10) = 4$; $\sigma(10) = 28$; $\pi(10) = 5$; $\Pi\, d|10$; = 1,000.

11. p_6; F_7; $2^2 + 3^2$; $D_{\#}$; SF_9; $\tau(11) = 2$; $\varnothing(11) = 10$; $\sigma(11) = 12$; $\pi(11) = 6$, $\Pi\, d|11 = 11$.

(Continued)

12. 2×7; C_7; SF_*; PSK_4; $1^2 + 2^2 + 3^2$; D_{10}; τ (12) = 4; ø (12) = 6; σ (12) = 20; π (12) = 6, Π d|12 = 144.

13. 3×5; C_8; $SF_\#$; PSK_5; CO_8; T_5; D_{11}; $L_1 + L_2 + L_3 + L_4$; τ (13) = 4; ø (13) = 8; σ (13) = 20; π (13) = 6; Π d|13 = 179.

14. 2^4; Hexadecimal Base; 4^2; S_4; C_9; $2^3 + 2^3$; CO_9; D_{12}; $L_1 + L_3 + L_5$; τ (14) = 5; ø (14) = 8; σ (14) = 27; π (14) = 6; Π d|14 = 714.

15. p_7; $1^2 + 4^2$; $1^4 + 2^4$; FE_3; CO_*; D_{13}; SF_{10}; τ (15) = 2; ø (15) = 14; σ (15) = 16; π (15) = 7; Π d|15 = 15.

16. 2×3^2; C_*; A_2; $3^2 + 3^2$; L_6; τ(16) = 6; ø (16) = 6; σ (16) = 33; p (16) = 7; Π d|16 = 3,460.

17. p_8; D_{14}; SF_{11}; τ (17) = 2; ø (17) = 16; σ (17) = 18; π (17) = 8; Π d|17 = 17.

18. $2^2 \times 5$; A_3; $C_\#$; $CO_\#$; score; TH_4; $2^2 + 4^2$; $F_1 + F_2 + F_3 + F_4 + F_5 + F_6$; τ (18) = 6; ø (18) = 8; σ (18) = 36; π (18) = 8; Π d|18 = 4,768.

19. 3×7; C_{10}; PSK_6; F_8; SF_{12}; T_6; $F_1 + F_3 + F_5 + F_7$; D_{15}; τ (19) = 4; ø (19) = 10; σ (19) = 28; π (19) = 8; Π d|19 = 309.

1*. $2 \times \#$; C_{11}; PSK_7; SF_{13}; D_{16}; τ(1*) = 4; ø (1*) = *; σ (1*) = 30; π (1*) = 8; Π d|1* = 344.

1#. p_9; SF_{14}; D_{17}; τ (1#) = 2; ø (1#) = 1*; σ (1#) = 20; π (1#) = 9; Π d|1# = 1#.

20. $2^3 \times 3$; HC_5; C_{12}; A_4; CO_{10}; 4!; τ (20) = 8; ø (20) = 8; σ (20) = 50; π (20) = 9; Π d|20 = 140,000.

21. 5^2; $3^2 + 4^2$; C_{13}; D_{18}; S_5; τ (21) = 3; ø (21) = 18; σ (21) = 27; π (21) = 9; Π d|21 = *5.

22. 2×11; SF_{15}; C_{14}; $1^2 + 5^2$; PSK_8; D_{19}; $L_1 + L_2 + L_3 + L_4 + L_5$; τ (22) = 4; ø (22) = 10; σ (22) = 36; π (22) = 9; Π d|22 = 484.

23. 3^3; PSK_9; C_{15}; D_{1*}; τ (23) = 4; ø (23) = 16; σ (23) = 34; π (23) = 9; Π d|23 = 509.

24. $2^2 \times 7$; P_2; C_{16}; $1^3 + 3^3$; T_7; $L_2 + L_4 + L_6$; π (24) = 6; ø (24) = 10; σ (24) = 48; π (24) = 9; Π d|24 = 10,854.

25. p_*; $2^2 + 5^2$; L_7; SF_{16}; $D_{1\#}$; $2^2 + 3^2 + 4^2$; τ (25) = 2; ø (25) = 24; σ (25) = 26; π (25) = *; Π d|25 = 25.

26. $2 \times 3 \times 5$; C_{17}; A_5; CO_{11}; $1^2 + 2^2 + 3^2 + 4^2$; SF_{17}; 3! + 4!; τ (26) = 8; ø (26) = 8; σ (26) = 60; π (26) = *; Π d|26 = 330,900.

(Continued)

27. $p_\#$; M_5; SF_{18}; D_{20}; τ (27) = 2; ø (27) = 26; σ (27) = 28; π (27) = #; Π d|27 = 27.

28. 2^5; C_{18}; $2^4 + 2^4$; $4^2 + 4^2$; CO_{12}; D_{21}; 2! + 3! + 4!; τ (28) = 6; ø (28) = 14; σ (28) = 53; π (28) = #; Π d|28 = 16,#68.

29. $3 \times \#$; SF_{19}; C_{19}; PSK_*; D_{22}; $1^5 + 2^5$; 1! + 2! + 3! + 4!; $F_2 + F_4 + F_6 + F_8$; $F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7$; τ (29) = 4; ø (29) = 18; σ (29) = 40; π (29) = #; Π d|29 = 769.

2*. 2×15; SF_{1*}; D_{23}; C_{1*}; $PSK_\#$; $3^2 + 5^2$; F_9; CO_{13}; τ (2*) = 4; ø (2*) = 14; σ (2*) = 46; π (2*) = #; Π d|2* = 804.

2#. 5×7; $SF_{1\#}$; D_{24}; $C_{1\#}$; PSK_{10}; $1^2 + 3^2 + 5^2$; $2^3 + 3^3$; TH_5; τ (2#) = 4; ø (2#) = 20; σ (2#) = 40; π (2#) = #; Π d|2# = 861.

30. $2^2 \times 3^2$; C_{20}; HC_6; $1^3 + 2^3 + 3^3$; 6^2; S_6; T_8; A_6; τ (30) = 9; ø (30) = 10; σ (30) = 77; π (30) = #; Π d|30 = 3,460,000.

31. p_{10}; SF_{20}; D_{25}; $1^2 + 6^2$; τ (31) = 2; ø (31) = 30; σ (31) = 32; π (31) = 10; Π d|31 = 31.

32. 2×17; SF_{21}; C_{21}; D_{26}; PSK_{11}; τ (32) = 4; ø (32) = 16; σ (32) = 50; π (32) = 10; Π d|32 = *04.

33. 3×11; SF_{22}; C_{22}; D_{27}; PSK_{12}; τ (33) = 4; ø (33) = 20; σ (33) = 48; π (33) = 10; Π d|33 = *69.

34. $2^3 \times 5$; C_{23}; CO_{14}; $2^2 + 6^2$; A_7; τ (34) = 8; ø (34) = 14; σ (34) = 76; π (34) = 10; Π d|34 = *35,594.

35. p_{11}; SF_{23}; D_{28}; $4^2 + 5^2$; τ (35) = 2; ø (35) = 34; σ (35) = 36; π (35) = 11; Π d|35 = 35.

36. $2 \times 3 \times 7$; A_8; SF_{24}; C_{24}; τ (36) = 8; ø (36) = 10; σ (36) = 80; π (36) = 11; Π d|36 = 1,060,900.

37. p_{12}; SF_{25}; D_{29}; τ (37) = 2; ø (37) = 36; σ (37) = 38; π (37) = 12; Π d|37 = 37.

38. $2^2 \times \#$; C_{25}; D_{2*}; $L_1 + L_2 + L_3 + L_4 + L_5 + L_6$; τ (38) = 6; ø (38) = 18; σ (38) = 70; π (38) = 12; Π d|38 = 41,368.

39. $3^2 \times 5$; C_{26}; $D_{2\#}$; $3^2 + 6^2$; T_9; $L_1 + L_3 + L_5 + L_7$; τ (39) = 6; ø (39) = 20; σ (39) = 66; π (39) = 12; Π d|39 = 44,899.

3*. $2 \times 1\#$; C_{27}; D_{30}; SF_{26}; PSK_{13}; τ (3*) = 4; ø (3*) = 1*; σ (3*) = 60; π (3*) = 12; Π d|3* = 1,284.

3#. p_{13}; D_{31}; SF_{27}; L_8; τ (3#) = 2; ø (3#) = 3*; σ (3#) = 40; π (3#) = 13; Π d|3# = 3#.

(Continued)

40. $2^4 \times 3$; C_{28}; A_9; HC_7; CO_{15}; τ (40) = *; ø (40) = 14; σ (40) = *4; π (40) = 13; Π d|40 = 71,400,000.

41. 7^2; C_{29}; D_{32}; S_7; τ (41) = 3; ø (41) = 36; σ (41) = 49; π (41) = 13; Π d|41 = 247.

42. 2×5^2; $5^2 + 5^2$; $1^2 + 7^2$; C_{2*}; D_{33}; $3^2 + 4^2 + 5^2$; τ (42) = 6; ø (42) = 18; σ (42) = 79; π (42) = 13; Π d|42 = 60,408.

43. 3×15; $C_{2\#}$; SF_{28}; D_{34}; PSK_{14}; CO_{16}; τ (43) = 4; ø (43) = 28; σ (43) = 60; π (43) = 13; Π d|43 = 1,609.

44. $2^2 \times 11$; C_{30}; D_{35}; $4^2 + 6^2$; τ (44) = 6; ø (44) = 20; σ (44) = 82; π (44) = 13; Π d|44 = 69,454.

45. p_{14}; SF_{29}; D_{36}; $2^2 + 7^2$; τ (45) = 2; ø (45) = 44; σ (45) = 46; π (45) = 14; Π d|45 = 45.

46. 2×3^3; $3^3 + 3^3$; C_{31}; A_*; $2^2 + 3^2 + 4^2 + 5^2$; $F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8$; τ (46) = 8; ø (46) = 16; σ (46) = *0; π (46) = 14; Π d|46 = 2,*20,900.

47. $5 \times \#$; C_{32}; SF_{2*}; D_{37}; PSK_{15}; F_*; $1^2 + 2^2 + 3^2 + 4^2 + 5^2$; T_*; $F_1 + F_3 + F_5 + F_7 + F_9$; τ (47) = 4; ø (47) = 34; σ (47) = 60; π (47) = 14; Π d|47 = 1,901.

48. $2^3 \times 7$; C_{33}; $A_{\#}$; $2^2 + 4^2 + 6^2$; TH_6; τ (48) = 8; ø (48) = 20; σ (48) = *0; π (48) = 14; Π d|48 = 3,363,314.

49. 3×17; C_{34}; $SF_{2\#}$; PSK_{16}; D_{38}; τ (49) = 4; ø (49) = 30; σ (49) = 68; π (49) = 14; Π d|49 = 1,*69.

4*. 2×25; C_{35}; SF_{30}; D_{39}; PSK_{17}; $3^2 + 7^2$; τ (4*) = 4; ø (4*) = 24; σ (4*) = 76; π (4*) = 14; Π d|4* = 1,#44.

4#. p_{15}; SF_{31}; D_{3*}; τ (4#) = 2; ø (4#) = 4*; σ (4#) = 50; π (4#) = 15; Π d|4# = 4#.

50. $2^2 \times 3 \times 5$; HC_8; C_{36}; CO_{17}; A_{10}; Sexagesimal Base; τ (50) = 10; ø (50) = 14; σ (50) = 120; π (50) = 15; Π d|50 = 9,061,000,000.

51. p_{16}; SF_{32}; $D_{3\#}$; $5^2 + 6^2$; τ (51) = 2; ø (51) = 50; σ (51) = 52; π (51) = 16; Π d|51 = 51.

52. 2×27; C_{37}; SF_{33}; D_{40}; PSK_{18}; τ (52) = 4; ø (52) = 26; σ (52) = 80; π (52) = 16; Π d|52 = 2,284.

53. $3^2 \times 7$; C_{38}; D_{41}; τ (53) = 6; ø (53) = 30; σ (53) = 88; π (53) = 16; Π d|53 = 100,853.

54. 2^6; 4^3; 8^2; $2^5 + 2^5$; C_{39}; CO_{18}; D_{42}; S_8; τ (54) = 7; ø (54) = 28; σ (54) = *7; π (54) = 16; Π d|54 = 851,768.

(Continued)

55. 5×11; C_{3*}; SF_{34}; D_{43}; PSK_{19}; $1^2 + 8^2$; $4^2 + 7^2$; $1^3 + 4^3$; $1^6 + 2^6$; (55) = 4; ø (55) = 40; σ (55) = 70; π (55) = 16; Π d|55 = 2,541.

56. $2 \times 3 \times \#$; $C_{3\#}$; A_{11}; SF_{35}; $T_{\#}$; τ (56) = 8; ø (56) = 18; σ (56) = 100; π (56) = 16; Π d|56 = 6,430,900.

57. p_{17}; SF_{36}; D_{44}; τ (57) = 2; ø (57) = 56; σ (57) = 58; π (57) = 17; Π d|57 = 57.

58. $2^2 \times 15$; C_{40}; $2^2 + 8^2$; D_{45}; CO_{19}; τ (58) = 6; ø (58) = 28; σ (58) = *6; π (58) = 17; Π d|58 = 131,*68.

59. $3 \times 1\#$; C_{41}; SF_{37}; D_{46}; PSK_{1*}; τ (59) = 4; ø (59) = 38; σ (59) = 80; π (59) = 17; Π d|59 = 2,909.

5*. $2 \times 5 \times 7$; C_{42}; SF_{38}; A_{12}; τ (5*) = 8; ø (5*) = 20; σ (5*) = 100; π (5*) = 17; Π d|5* = 8,05*,814.

5#. p_{18}; SF_{39}; D_{47}; τ (5#) = 2; ø (5#) = 5*; σ (5#) = 60; π (5#) = 18; Π d|5# = 5#.

60. $2^3 \times 3^2$; C_{43}; A_{13}; $6^2 + 6^2$; $2^3 + 4^3$; τ (60) = 10; ø (60) = 20; σ (60) = 143; π (60) = 18; Π d|60 = 23,000,000,000.

61. p_{19}; SF_{3*}; D_{48}; $3^2 + 8^2$; $L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7$; τ (61) = 2; ø (61) = 60; σ (61) = 62; π (61) = 19; Π d|61 = 61.

62. 2×31; C_{44}; $SF_{3\#}$; D_{49}; $PSK_{1\#}$; $5^2 + 7^2$; τ (62) = 4; ø (62) = 30; σ (62) = 96; π (62) = 19; Π d|62 = 3,204.

63. 3×5^2; C_{45}; D_{4*}; $L_2 + L_4 + L_6 + L_8$; τ (63) = 6; ø (63) = 34; σ (63) = *4; π (63) = 19; Π d|63 = 184,183.

64. $2^2 \times 17$; C_{46}; $D_{4\#}$; L_9; τ (64) = 6; ø (64) = 30; σ (64) = #8; π (64) = 19; Π d|64 = 192,054.

65. $7 \times \#$; C_{47}; D_{50}; SF_{40}; PSK_{20}; $4^2 + 5^2 + 6^2$; τ (65) = 4; ø (65) = 50; σ (65) = 80; π (65) = 19; Π d|65 = 3,521.

66. $2 \times 3 \times 11$; C_{48}; A_{14}; SF_{41}; T_{10}; τ (66) = 8; ø (66) = 20; σ (66) = 120; π (66) = 19; Π d|66 = 10,490,900.

67. p_{1*}; SF_{42}; D_{51}; τ (67) = 2; ø (67) = 66; σ (67) = 68; π (67) = 1*; Π d|67 = 67.

68. $2^4 \times 5$; C_{49}; A_{15}; CO_{1*}; $4^2 + 8^2$; τ (68) = *; ø (68) = 28; σ (68) = 136; π (68) = 1*; Π d|68 = 775,488,368.

69. 3^4; 9^2; S_9; C_{4*}; D_{52}; τ (69) = 5; ø (69) = 46; σ (69) = *1; π (69) = 1*; Π d|69 = 2*209.

(Continued)

6*. 2 × 35; $C_{4\#}$; SF_{43}; D_{53}; PSK_{21}; $1^4 + 3^4$; $1^2 + 9^2$; τ (6*) = 4; ø (6*) = 34; σ (6*) = *6; π (6*) = 1*; Π d|6* = 3*84.

6#. $p_{1\#}$; SF_{44}; D_{54}; τ (6#) = 2; ø (6#) = 6*; σ (6#) = 70; π (6#) = 1#; Π d|6# = 6#.

70. $2^2 \times 3 \times 7$; C_{50}; A_{16}; TH_7; $1^2 + 3^2 + 5^2 + 7^2$; τ (70) = 10; ø (70) = 20; σ (70) = 168; π (70) = 1#; Π d|70 = 58,101,000,000.

71. 5 × 15; C_{51}; SF_{45}; D_{55}; PSK_{22}; $6^2 + 7^2$; $2^2 + 9^2$; $CO_{1\#}$; τ (71) = 4; ø (71) = 54; σ (71) = 90; π (71) = 1#; Π d|71 = 4,221.

72. 2 × 37; C_{52}; SF_{46}; D_{56}; PSK_{23}; $3^2 + 4^2 + 5^2 + 6^2$; τ (72) = 4; ø (72) = 36; σ (72) = #0; π (72) = 1#; Π d|72 = 4,344.

73. 3 × 25; C_{53}; SF_{47}; D_{57}; PSK_{24}; τ (73) = 4; ø (73) = 48; σ (73) = *0; π (73) = 1#; Π d|73 = 4,469.

74. $2^3 \times \#$; C_{54}; A_{17}; $F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8 + F_9$; $F_2 + F_4 + F_6 + F_8 + F_*$; τ (74) = 8; ø (74) = 34; σ (74) = 130; π (74) = 1#; Π d|74 = 18,100,714.

75. p_{20}; SF_{48}; D_{58}; $F_\#$; $5^2 + 8^2$; τ (75) = 2; ø (75) = 74; σ (75) = 76; π (75) = 20; Π d|75 = 75.

76. $2 \times 3^2 \times 5$; C_{55}; A_{18}; $3^2 + 9^2$; $2^2 + 3^2 + 4^2 + 5^2 + 6^2$; τ (76) = 10; ø (76) = 20; σ (76) = 176; π (76) = 20; Π d|76 = 86,##6,623,000.

77. 7 × 11; C_{56}; SF_{49}; D_{59}; PSK_{25}; $3^3 + 4^3$; $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$; T_{11}; τ (77) = 4; ø (77) = 60; σ (77) = 94; π (77) = 20; Π d|77 = 4,961.

78. $2^2 \times 1\#$; C_{57}; D_{5*}; τ (78) = 6; ø (78) = 38; σ (78) = 120; π(78) = 20; Π d|78 = 316,768.

79. 3 × 27; C_{58}; SF_{4*}; $D_{5\#}$; PSK_{26}; τ (79) = 4; ø (79) = 50; σ (79) = *8; π(79) = 20; Π d|79 = 5,009.

7*. 2 × 3#; C_{59}; $SF_{4\#}$; D_{60}; PSK_{27}; τ (7*) = 4; ø (7*) = 3*; σ (7*) = 100; π (7*) = 20; Π d|7* = 5,144.

7#. 5 × 17; C_{5*}; SF_{50}; D_{61}; PSK_{28}; τ (7#) = 4; ø (7#) = 60; σ (7#) = *0; π (7#) = 20; Π d|7# = 5,281.

80. $2^5 \times 3$; $C_{5\#}$; A_{19}; CO_{20}; τ (80) = 10; ø (80) = 28; σ (80) = 190; π (80) = 20; Π d|80 = 107,854,000,000.

81. p_{21}; $4^2 + 9^2$; $2^4 + 3^4$; SF_{51}; D_{62}; τ (81) = 2; ø (81) = 80; σ (81) = 82; π (81) = 21; Π d|81 = 81.

(Continued)

82. 2×7^2; $7^2 + 7^2$; C_{60}; D_{63}; $1^4 + 2^4 + 3^4$; τ (82) = 6; ø (82) = 36; σ (82) = 123; π (82) = 21; Π d|82 = 394,808.

83. $3^2 \times \#$; C_{61}; D_{64}; $2^3 + 3^3 + 4^3$; τ (83) = 6; ø (83) = 50; σ (83) = 110; π (83) = 21; Π d|83 = 3*9,623.

84. $2^2 \times 5^2$; C_{62}; A_{1*}; $6^2 + 8^2$; $*^2$; S_*; $1^3 + 2^3 + 3^3 + 4^3$; τ (84) = 9; ø (84) = 34; σ (84) = 161; π (84) = 21; Π d|84 = 23*,*93,854.

85. p_{22}; SF_{52}; D_{65}; $1^2 + *^2$; τ (85) = 2; ø (85) = 84; σ (85) = 86; π (85) = 22; Π d|85 = 85.

86. 2 × 3 × 15; C_{63}; SF_{53}; $A_{1\#}$; CO_{21}; τ (86) = 8; ø (86) = 28; σ (86) = 160; π (86) = 22; Π d|86 = 30,300,900.

87. p_{23}; SF_{54}; D_{66}; τ (87) = 2; ø (87) = 86; σ (87) = 88; π (87) = 23; Π d|87 = 87.

88. $2^3 \times 11$; C_{64}; A_{20}; $2^2 + *^2$; τ (88) = 8; ø (88) = 40; σ (88) = 156; π (88) = 23; Π d|88 = 33,218,194.

89. 3 × 5 × 7; C_{65}; SF_{55}; D_{67}; T_{12}; τ (89) = 8; ø (89) = 40; σ (89) = 140; π (89) = 23; Π d|89 = 34,859,969.

8*. 2 × 45; C_{66}; SF_{56}; D_{68}; PSK_{29}; $5^2 + 9^2$; τ (8*) = 4; ø (8*) = 44; σ (8*) = 116; π (8*) = 23; Π d|8* = 6,604.

8#. p_{24}; SF_{57}; D_{69}; τ (8#) = 2; ø (8#) = 8*; σ (8#) = 90; π (8#) = 24; Π d|8# = 8#.

90. $2^2 \times 3^3$; C_{67}; A_{21}; τ (90) = 10; ø (90) = 30; σ (90) = 1#4; π (90) = 24; Π d|90 = 217,669,000,000.

91. p_{25}; SF_{58}; D_{6*}; $3^2 + *^2$; τ (91) = 2; ø (91) = 90; σ (91) = 92; π (91) = 25; Π d|91 = 91.

92. 2 × 5 × #; C_{68}; SF_{59}; $D_{6\#}$; $5^2 + 6^2 + 7^2$; τ (92) = 8; ø (92) = 34; σ (92) = 160; π (92) = 25; Π d|92 = 41,048,014.

93. 3 × 31; C_{69}; SF_{5*}; D_{70}; PSK_{2*}; τ (93) = 4; ø (93) = 60; σ (93) = 108; π (93) = 25; Π d|93 = 7,169.

94. $2^4 \times 7$; C_{6*}; A_{22}; τ (94) = *; ø (94) = 40; σ (94) = 188; π (94) = 25; Π d|94 = 3,4#*,068,714.

95. p_{26}; $SF_{5\#}$; D_{71}; $7^2 + 8^2$; τ (95) = 2; ø (95) = 94; σ (95) = 96; π (95) = 26; Π d|95 = 95.

96. 2 × 3 × 17; $C_{6\#}$; SF_{60}; A_{23}; τ (96) = 8; ø (96) = 30; σ (96) = 180; π (96) = 26; Π d|96 = 48,690,900.

(Continued)

97. 5 × 1#; C_{70}; SF_{61}; D_{72}; $PSK_{2\#}$; τ (97) = 4; ø (97) = 74; σ (97) = 100; π (97) = 26; Π d|97 = 7,7*1.

98. 2^2 × 25; C_{71}; D_{73}; $4^2 + *^2$; τ (98) = 6; ø (98) = 48; σ (98) = 156; π (98) = 26; Π d|98 = 633,368.

99. 3^2 × 11; C_{72}; D_{74}; $6^2 + 9^2$; τ (99) = 6; ø (99) = 60; σ (99) =132; π (99) = 26; Π d|99 = 652,*39.

9*. 2 × 4#; C_{73}; SF_{62}; D_{75}; PSK_{30}; τ (9*) = 4; ø (9*) = 4*; σ (9#) = 100; π (9#) = 26; Π d|9# = 8,241.

*0. $2^3 \times 3 \times 5$; 5!; C_{75}; HC_9; A_{24}; MP_1(3-Perfect); T_{13}; CO_{22}; TH_8; $L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8$; τ (*0) = 14; ø (*0) = 28; σ (*0) = 260; π (*0) = 26; Π d|*0 = 2,95*,645,400,000,000.

*1. $\#^2$; C_{76}; D_{77}; $S_{\#}$; $L_1 + L_3 + L_5 + L_7 + L_9$; τ (*1) = 3; ø (*1) = 92; σ (*1) = #1; π (*1) = 26; Π d|*1 = 92#.

*2. 2 × 51; C_{77}; SF_{64}; D_{78}; PSK_{32}; $1^2 + \#^2$; τ (*2) = 4; ø (*2) = 50; σ (*2) = 136; π (*2) = 26; Π d|*2 = 8,744.

*3. 3 × 35; C_{78}; SF_{65}; D_{79}; PSK_{33}; L_*; τ (*3) = 4; ø (*3) = 68; σ (*3) = 120; π (*3) = 26; Π d|*3 = 8,909.

*4. 2^2 × 27; C_{79}; D_{7*}; τ (*4) = 6; ø (*4) = 50; σ (*4) = 168; π (*4) = 26; Π d|*4 = 77#,454.

*5. 5^3; C_{7*}; $D_{7\#}$; PSK_{34}; $5^2 + *^2$; $2^2 + \#^2$; τ (*5) = 3; ø (*5) = 84; σ (*5) = 110; π (*5) = 26; Π d|*5 = 9,061.

*6. $2 \times 3^2 \times 7$; $C_{7\#}$; A_{25}; $1^3 + 5^3$; $4^2 + 5^2 + 6^2 + 7^2$; τ (*6) = 10; ø (*6) = 30; σ (*6) = 220; π (*6) = 26; Π d|*6 = 547,627,783,000.

*7. p_{27}; M_7; SF_{66}; D_{80}; τ (*7) = 2; ø (*7) = *6; σ (*7) = *8; π (*7) = 27; Π d|*7 = *7.

*8. 2^7; $8^2 + 8^2$; $4^3 + 4^3$; $2^6 + 2^6$; C_{80}; D_{81}; CO_{23}; τ (*8) = 8; ø (*8) = 54; σ (*8) = 193; π (*8) = 27; Π d|*8 = 75,*94,714.

*9. 3 × 37; C_{81}; SF_{67}; D_{82}; PSK_{35}; τ (*9) = 4; ø (*9) = 70; σ (*9) = 128; π (*9) = 27; Π d|*9 = 9,769.

. 2 × 5 × 11; C_{82}; SF_{68}; D_{83}; $7^2 + 9^2$; $3^2 + \#^2$; τ (**) = 8; ø (**) = 40; σ (**) = 190; π (**) = 27; Π d| = 7#,797,694.

#. p_{28}; SF_{69}; D_{84}; τ (#) = 2; ø (*#) = **; σ (*#) = #0; π (*#) = 28; Π d|*# = *#.

(Continued)

#0. 2^2 × 3 × #; C_{83}; A_{26}; τ (#0) = 10; ø (#0) = 34; σ (#0) = 240; π (#0) = 28; Π d|#0 = 715,261,000,000.

#1. 7 × 17; C_{84}; SF_{6*}; D_{85}; PSK_{36}; $2^3 + 5^3$; τ (#1) = 4; ø (#1) = 90; σ (#1) = 114; π (#1) = 28; Π d|#1 = *,2*1.

#2. 2 × 57; C_{85}; $SF_{6\#}$; D_{86}; PSK_{37}; τ (#2) = 4; ø (#2) = 56; σ (#2) = 150; π (#2) = 28; Π d|#2 = *,484.

#3. 3^3 × 5; C_{86}; D_{87}; $3^2 + 4^2 + 5^2 + 6^2 + 7^2$; τ (#3) = 8; ø (#3) = 60; σ (#3) = 180; π (#3) = 28; Π d|#3 = 93,2*0,969.

#4. 2^3 × 15; C_{87}; D_{88}; CO_{24}; $6^2 + *^2$; T_{14}; τ (#4) = 8; ø (#4) = 54; σ (#4) = 180; π (#4) = 28; Π d|#4 = 96,69#,854.

#5. p_{29}; SF_{70}; D_{89}; $4^2 + \#^2$; τ (#5) = 2; ø (#5) = #4; σ (#5) = #6; π (#5) = 29; Π d|#5 = #5.

#6. 2 × 3 × 1#; C_{88}; SF_{71}; A_{27}; τ (#6) = 8; ø (#6) = 38; σ (#6) = 200; π (#6) = 29; Π d|#6 = *1,560,900.

#7. p_{2*}; SF_{72}; D_{8*}; $2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2$; τ (#7) = 2; ø (#7) = #6; σ (#7) = #8; π (#7) = 2*; Π d|#7 = #7.

#8. 2^2 × 5 × 7; C_{89}; A_{28}; $1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2$; τ (#8) = 10; ø (#8) = 40; σ (#8) = 240; π (#8) = 2*; Π d|#8 = *17,336,456,454.

#9. 3 × 3#; C_{8*}; SF_{73}; $D_{8\#}$; PSK_{38}; τ (#9) = 4; ø (#9) = 78; σ (#9) = 140; π (#9) = 2*; Π d|#9 = #,609.

#*. 2 × 5#; $C_{8\#}$; SF_{74}; D_{90}; PSK_{39}; τ (#*) = 4; ø (#*) = 5*; σ (#*) = 160; π (#*) = 2*; Π d|#* = #,804.

##. # × 11; C_{90}; SF_{75}; D_{91}; PSK_{3*}; $F_1 + F_2 + F_3 + F_4 + F_5 + F_6 + F_7 + F_8 + F_9 + F_*$; τ (##) = 4; ø (##) = *0; σ (##) = 120; π (##) = 2*; Π d|## = #,*01.

100. $2^4 \times 3^2$; One Gross; C_{91}; A_{29}; F_{10}; 10^2; 4! + 5!; 3! × 4!; $F_1 + F_3 + F_5 + F_7 + F_9 + F_{\#}$; S_{10}; τ(100) = 13; ø(100) = 40; σ(100) = 297; π(100) = 2*; Π d|100 = 1,000,000,000,000,000.

Reference

[1.] Philip J. Davis, The Lore of Large Numbers, The New Mathematical Library, New York, 1961.

□

BOOK REVIEW

Gene Zirkel

4000
The Fifth Milenium
Six Revolooshunairy Iedeeas[1]

by Timothy F. Travis. Published 1994 by Aster Esprit Press, Box 1615, El Toro CA 92630. 187; pages. $19.95

When Board Chair, Dr. Pat Zirkel, spotted an ad in the *NY Times* for a book treating dozenal counting we contacted Aster Esprit and obtained a gem of a treatise on base twelve counting, and a new idea-filled member for our Society (number *3 gro 4 doz 2* in his symbolism).

In this interesting volume Timothy Travis deals with a half dozen topics: the dozen system, phonetic spelling, calendar reform, academic certification, the positive reinforcement of citizenship, and the unification of the technological and the spiritual aspects of humanity.

In keeping with the nature of this *Bulletin* my review will deal with the his first theme (although I found much to enjoy in the entire book).

The presentation of the case for dozenals is done in an attractive manner. The use of color and good diagrams enhances what is a clear presentation. It begins with some very apt questions and answers pertaining to duodecimals followed by many simple examples. A treatment of weights and measures, the dozenal circle and the dozenal clock are included.

Travis uses the following seven segment display numerals for his presentation:

referring to the digits for ten and eleven to as 'dek' and 'brad' and pronouncing 10; as doz.

I found his nomenclature for higher numbers confusing as he mixed old and new words together. The table below uses our usual notation for dek (*) and el(#) to present his pronunciation.

[1]One of the author's ideas is phonetic spelling.

(Continued)

base ten numeral	base twelve numeral	pronunciation
13	11	eleven
14	12	twelve
15	13	thirteen
16	14	fourteen
17	15	fifteen
18	16	sixteen
19	17	doza seven
20	18	doza eight
21	19	doza nine
22	1*	doza dek
23	1#	doza brad
24	20	twenty
...	...	...
117	99	ninety nine
118	9*	ninety dek
119	9#	ninety brad
120	*0	deka
121	*1	deka one
...	...	...
132	#0	brada
133	#1	brada one
...	...	...
144	100	gro
145	101	gro one
...	...	...

This could lead to confusion, for example:

'Eleven is thirteen, but thirteen is not eleven, it's fifteen, while fifteen, of course is thirteen'!

However this is a minor point of nomenclature. The ideas presented here are sound.

Apparently, Travis was unfamiliar with our Society and our suggestions for a standardized notation. Yet in at least three instances he chose terms that we use, namely '*dek*' for ten, '*gro*' for one gross, and the prefix '*e-*' for fractions, such as *egro* for 0;01 or 1/100;.

This certainly attests to the wisdom of our predecessors who chose these names dozens of years ago.

His treatment of the circle coincides with previous suggestions, and his ideas for weights and measures and the clock are similar in nature to former proposals such as the Doremic System of Weights and Measures[2].

(Continued)

DOZENAL JOTTINGS

From Members and Friends

The Dozenal Society of America is pleased to welcome the following new members:

340	Christian McCormick,	Corona, NY
341	Tim Maurer,	Belle Mead, NJ
342	Timothy Travis,	El Toro, CA
343	Sharon Ditter,	Austin, TX
344	Miles F. McKee,	Grosse Point Park, MI

□

Book Review *(Concluded)*

I suggest that this wonderful volume belongs in every dozenalist's library.

The second and third revolutionary ideas presented by Travis are
phonetic spelling and calendar reform. It is interesting to note that many of our members have been interested in calendar reform and/or language improvements. In particular, stalwarts such as our founder Ralph Beard and our recently deceased past president Fred Newhall immediately come to mind. It would seem that many of those who wish to make the world a more rational place arithmetically, are also interested in simplifying our calendar and our language.

The fourth and fifth ideas: academic certification and the positive reinforcement of citizenship urge us to keep on learning, a wonderful idea.

The only fault I find is the sixth idea - the unification of the technological and the spiritual aspects of humanity. Here the author takes gratuitous and unwarranted swipes at religion in general and catholicism in particular. I found this to be offensive and inappropriate in an otherwise logical book.

[2]See "The Do-Re-Mi System of Dozenal Nomenclature" by Henry Clarence Churchman, this *Bulletin*, Volume #, Number 1, pages 1-5, May 1955. Also "Duodenal Arithmetic and Metrology" by John W. Nystrom, C.E., Volume 6, Number 1, pp 3-10, February 1950.

□

DETACH HERE-- OR-- PHOTOCOPY

We extend an invitation to membership in our society.
Dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______________________________
LAST FIRST MIDDLE

Mailing Address (For DSA items) ______________________________

____
(See below for alternate address)

Telephone: Home ____________ Business ____________

Date & Place of Birth ______________________________

College Degrees ______________________________

Business or Profession ______________________________

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______________________________

Address ______________________________

Year & Math Class ______________________________

Instructor ____________ Dept. ____________

Other Society Memberships ______________________________

____

Alternate Address (Indicate whether home, office, school, other)

____

____

____

Signed ____________ Date ____________

My interest in duodecimals arose from ______________________________

____

____

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
%Math Department
Nassau Community College
Garden City, LI, NY 11530