COUNTING IN DOZENS

1 2 3 4 5 6 7 8 9 \times ε 10 one two three four five six seven eight nine dek el do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written 10, and is called do, for dozen. The quantity one gross is written 100, and is called gro. 1000 is called mo, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, 265 represents 5 units, 6 dozen, and 2 dozen-dozen, or gross. This number would be called 2 gro 6 do 5, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal srithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5;91
31	694	Three ft. two in.	3:21
96	3£2	Two ft. eight in.	2:81
19£	<u> 1000</u>	Eleven ft. seven in.	5.71

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, which is 5 dozen and 3; so set down 53. Using this "which is" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only $2\mathfrak{L}$, which is two dozen and eleven. For larger numbers, the depth is uccessive remainders are the desired dozenal numbers.

12) 365
12) 30 + 5
12) 2 + 6
12) 2 + 6
13) 4 - 2

14 Answer: 265

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by \mathcal{X} , and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or χ .

Numerical		Progression	on Multiplication Tabl						le	le				
1	One			1	2	3	4	5	6	7	8	9	χ	٤
10	Do	;1	Edo							12 19				
100	Gro	;01	Egro	4	8	10	14	18	20	24	28	30	34	38
1,000	Mo	;001	Emo	5	10	13	20	21 26	26 30	<u>2£</u> 36	34	<u>39</u>	50	<u>47</u> 56
10,000	Do-mo	;000,1	Edo-mo	7	12	19	24	2Σ	36	41	48	53	5%	65
100,000	Gro-mo	;000,01	Egro-mo							48 53				
1,000,000	Bî-mo	;000,001	Ebi-mo	χ	18	26	34	42	50	5χ	68	76	84	92
1,000,000,000	Tri-mo	and so on.		£	1χ	29	38	47	56	65	74	83	92	X 1

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THE DUODECIMAL SOCIETY OF AMERICA

Office, 20 Carlton Place, Staten Island, N. Y. 10304 Secretary, 11561 Candy Lane, Garden Grove, Cal 92640

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve base arithmetic. The lessons and examinations are free to those whose entrance application is accepted. Remittance of \$6, dues for one year, must accompany application. Forms free on request.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island, New York 10304. Kingsland Camp, Chairman of the Board of Directors; Charles S. Bagley, President; Jamison Blandy, Jr., Editor; Henry C. Churchman, Associate Editor. Permission for reproduction may be granted upon application. Separate subscription rate \$3 per year, \$1 per copy.

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The Duodecimal Bulletin

All figures in italics are duodecimal.

MEETING OF THE BOARD

AND

ANNUAL MEETING OF 1968

The Duodecimal Society of America was called to order by President Charles S. Bagley in the beautiful Yorktown Room at the O'Hare Concord Motor Inn, 6565 North Mannheim Road, Des Plaines, Illinois 60018, in the year of 1180 April 7;700 (1968 April 7 at 1400 hours), in keeping with written notice mailed to the whole membership at least ten days prior to that meeting.

A roll call disclosed more than one half of the total number of directors present, as follows:

F. Emerson Andrews, of Tenafly, N.J.
Charles S. Bagley, of Alamogordo, N.M.
Ralph H. Beard, of Staten Island, N.Y.
Kingsland Camp, of New York, N.Y.
Henry C. Churchman, of Council Bluffs, Iowa.
Jamison Handy, Jr., of Pacific Palisades, Cal.
Tom B. Linton, of Garden Grove, California.

Other members and our guests included: Dr. B. A. M. Moon, of Christchurch, N. Z.; Dr. Kenneth Mears, of Oakville, Ontario, Prof. John Selfridge, Math Department, University of Illinois, Urbana, Illinois; Stan Bumpus, of Bonnie, Illinois; Miss Peggy Dwyer, of Anderson, Indiana; James Dixon, of Anderson, Indiana; Erich Kothe, of Schiller Park, Illinois and his guest Clarence Seamans, of Elmhurst, Illinois.

This being the initial day of our meetings, it was devoted to an enjoyable series of enlightening speeches and pleas centering around duodecimals.

The gathering was privileged to hear a penetrating address by Dr. Kenneth Mears on his work, in cooperation with Dr. B. A. M. Moon, of Canterbury University, Christchurch, New Zealand, in attempting to stay the fixed determination of the New Zealand Parliament to move that nation from shillings and pence to a decimal currency base. His subject was "Bureaucrats," which, in itself, is quite descriptive of his talk. It was made quite inescapable that if any reform movement can persuade the-powersthat-be its cause is desirable, those powers will override all objections --- and that logic is not necessarily the overpowering influence in politics as generally assumed. Their experiences in that part of the English speaking world might well furnish us a useful modus operandi. Perhaps we should concentrate more on enlightening our Councils, our Parliaments, and our Congress, by furnishing them not with our logic but our facts, facts, facts. Committees of Congress are always looking for new and more efficient ways of doing time honored chores. And it is not always what a majority of the people think they should have. It might be observed that logic is propaganda, but facts are facts and our tax-free status might not be endangered if we merely became congressional committees' "little helpers."

Tom B. Linton next was presented by President Bagley, and gave an interesting presentation of the relation of number bases in which corresponding powers of two different bases are related by the ratio of their logarithms. It is hoped this scholarly talk will be put in the form of a Bulletin article and appear in the next issue.

James Dixon, a student member and guest speaker, was then introduced by President Bagley. He is a student at Purdue and has worked with computers. His experiments were presented in a very interesting way. We are looking forward to the receipt of some worthwhile papers from his hand.

Stan Bumpus, of Bonnie, Illinois, a student member at the University of Illinois, was presented by President Bagley. His experiences with computers proved interesting. He compared some notes and results of experiments worked out by Mr. Dixon, and found mutually favorable results. We are looking forward to his putting some of his experiences in an equally interesting paper.

At nine gross Moments (1800 hours) the meeting recessed until $\chi 00$ (2000 h) and enjoyed a dinner in the banquet room of the famous Allgauer's Restaurant, in the O'Hare Concord Motor Inn.

Evening Events

At X gross Moments (twenty hundred Hours), members and guests returned to the Yorktown Room. President Bagley again presented Dr. Kenneth Mears, who now spoke on the subject of "Hardware." He presented an impassioned plea, urging the society to purchase and make available for use of members a dozenal electronic computer.

President Bagley next introduced guest speaker Dr. B.A.M. Moon of Christchurch, New Zealand, who gave an exceptionally erudite presentation on the subject of a system of "Negative Numbers." It is hoped we may be able to produce a part at least of his efforts in either the December or next March bulletin.

Meeting adjourned at eleven-one-ought (£10) or twenty-two hundred ten (2210) Hours, until a Quarter of five, 8 April 1180;490 (8 April 1968, 0930 H).

Proceedings of 1180 April 8.

Chairman Kingsland Camp now called to order the Annual Meeting of the Board of Directors of DSA, at four-nine-ought (490 Moments or 0930 Hours), in the Yorktown Room of the O'Hare Concord Motor Inn, in the Chicago, Illinois, metropolitan area.

The following members of the Board of Directors were present:

F. Emerson Andrews Charles S. Bagley Ralph H. Beard Kingsland Camp

Henry C. Churchman Jamison Handy, Jr. Tom B. Linton.

Chairman Camp noted that a working quorum was present and legally able to conduct all proper business to come before it.

The minutes of the last meeting were read, discussed, and approved.

Reports of Officers of Society

Chairman Kingsland Camp reported that he had received a constructive letter during the past year from Ralph H. Beard, suggesting among other that the President select a Director of Personnal. Also suggesting that a permanent office for the DSA be purchased in a low tax area, possibly in the Rocky Mountain region, in which to store DSA business records, library, etc. Letter received for discussion under New Business. Report approved.

Vice President Henry Churchman reported with no other articles available he had prepared and distributed Dozenal Essays of 1966 early in 1967. Also Dozenal Essays of 1967 were distributed to members in January 1968, in the further absence of The Bulletin due to unavoidable circumstances surrounding the Editor. It was moved by Andrews, seconded by Beard, that the report be accepted and the Vice President commended for filling the gap. Notion carried.

The Secretary, Tom B. Linton, reported we now have the following membership:

- 8 Honorary Members
- 9 Life Members
- 18 Fellows
- 41 Senior Members
- 33 Members
- 57 Student Members.

Of these, the honorary and life members pay no dues but some send gifts from time to time. This leaves about 127 dues paying members, under normal conditions. Dues of student members is half, but a majority are in arrears. But because of a lack of Bulletins since 1966, no pruning for nonpayment of dues has been done. Pruning will begin in January of 1969. Membership lists are brought to date and issued twice a year.

In 1172 (1967) we lost three members by death. Their names are Cora Fellows, Lesbia Beard, and B. K. Humphrey.

The Board of Directors now numbers ten, after the death of ${\tt H.}$ K. Humphrey. They are grouped as follows:

Class of 1180 (1968) Class of 1181 (1969) Class of 1182 (1970)

Mr. Bagley Mr. F. E. Andrews Mr. Camp
Mr. Beard Mr. Churchman Mr. Linton
Mr. Terry Mr. Handy Mr. Lyman
Mr. Humphrey,dec'd Mr. Scifres Vacancy

Four directors are to be elected this year for the Class of 1180-1183 (1968-71). H. K. Humphrey's death removes one of the four from consideration.

The officers of DSA during 1172 (1967) were:

Kingsland Camp, Chairman of the Board, Charles S. Bagley, President, Henry C. Churchman, Vice President, Eugene Scifres, Treasurer, Tom B. Linton, Secretary, Jamison Handy, Jr., Editor. The Society had 10X (154) paid up members at end of 117X. And 114 (160) paying members in December of $117\mathfrak{L}$ (1967).

Moved by Handy, seconded by Andrews, report be approved. Carried.

Editor's report made by Jamison Handy, Jr. All material for two Bulletins ready for the printer. Due to press of private demands on time, final dispatch in the mail unavoidably delayed. Dozenal Doings discontinued for the present. Moved by Linton, seconded by Churchman, Editor's report be approved. Carried.

The report of F. Emerson Andrews as chairman of the DSA Finance Committee is as follows:

Members of committee: F. Emerson Andrews, Chairman,
Henry C. Churchman,
Jamison Handy, Jr.,
H. K. Humphrey (now deceased),
Tom B. Linton,
Eugene Scifres,
George S. Terry.

No meeting of committee in 1967 was necessary. The George Terry bonds, due in 1972, equal \$2,000.00. The Lewis Carl Seelbach bonds, due 15 August 1968, equal principal sum \$3,000.00. Report approved.

Emerson Andrews moved, seconded by Churchman, the following Resolution:

Resolved that the Board of Directors of The Duodecimal Society of America hereby expresses its profound regret over the recent deaths of its faithful Treasurer, Kay Humphrey, and members Lesbia Beard and Cora Fellows, recalling with great appreciation their contributions to the progress of the Society; and hereby extends to the members of their families its deep sympathy.

That this Resolution be spread on our permanent records, and that the Secretary be instructed to forward a copy of same to their survivors.

Carried.

Chairman Camp produced, and Ralph Beard read, letter of October 29, 1967 addressed to the board chairman. Its contents were discussed item by item. Moved by Linton, seconded by Handy, the contents be taken up one at a time as new business. Carried.

Election of Board Members Class of 1183 (1971)

It was moved by Andrews, seconded by Handy, that Charles S. Bagley, Ralph H. Beard, and George S. Terry be nominated as members of the Board of Directors DSA, Class of 1183 (1971); that nominations be closed, and that the Secretary be directed to cast the unanimous vote of the directors for the motion. The question was put by Kingsland Camp, Chairman of the Board. Cartied. The chairman declared the following elected:

Class of 1183 (1971)

Charles S. Bagley, Ralph H. Beard, George S. Terry. It was then moved by Henry Churchman, seconded by Ralph Beard, that the fourth position on the board, in the Class of 1183 be filled by Peter Andrews, of Pittsburgh, Pennsylvania; that nominations be closed, and that the Secretary be directed to cast the unanimous vote of the directors for the motion. Carried. Emerson Andrews expressed a wish to be recorded in the minutes as abstaining, and it was so ordered by the chairman. Ralph H. Beard was requested by the chairman to inform Peter Andrews.

A vacancy being noted on the Board, Class of 1182 (1970), it was moved by Ralph Beard, seconded by Churchman, that the fourth position on the board, Class of 1182 (1970) be filled by Theodore Baumeister, of Younges Island, S. C., that nominations be closed, and that the Secretary be directed to cast the unanimous vote of the directors for the motion. Carried. Ralph H. Beard was requested to inform Theodore Baumeister of this action.

It was then moved by Ralph Beard, seconded by Tom Linton, that if and when a vacancy on the Board of Directors occurs more than ninety days before a regular annual meeting of the board, and has not otherwise been filled by the board, the following members, in the order in which their names here appear and equal to the number of vacancies, shall stand elected to fill such vacancy immediately upon acceptance, after notice given by either the Chairman of the Board or the President, to-wit: 1. John L. Selfridge, of Urbana, Illinois; 2. Thomas H. Goodman, of Baltimore, Maryland; 3. Elliott M. Hightower of La Marque, Texas; 4. David H. Scull of Annandale, Virginia; 5. Erich Kothe of Schiller Park in Illinois; 6. Douglas Colton of Smohomish, Washington; that nominations be closed, and the Secretary be directed to cast the unanimous vote of the directors for the motion. Carried.

It was moved by F. Emerson Andrews and seconded by Ralph H. Beard that the following members of the Board of Directors be nominated and stand elected to the following offices:

Kingsland Camp, Chairman of Board; Tom B. Linton, Secretary; Charles S. Bagley, President; Eugene Scifres, Treasurer; Henry C. Churchman, V. President; Jamison Handy, Jr., Editor;

that nominations be closed, and that the Secretary be instructed to cast the unanimous vote of the directors for the motion. Carried. $\qquad \qquad \text{New Business}$

At this point Tom Linton extended an invitation and urgent desire to see the 1181 Annual Meeting of the DSA take place in the State of California. Time and place were discussed, and the possibility of holding some part or all of the three day gathering on some university campus in southern California was seriously urged as a means of getting more support for DSA among college math departments as well as shaping our direction to coincide with the needs of math programs in our universities. It was moved by Churchman, seconded by Andrews, that the DSA hold its next annual meeting in California, sometime between January and May 1969, the time and place of meeting to be selected by Tom Linton. Carried.

It was moved by Beard, seconded by Churchman, that the 1967 Award of DSA be made to Brian Bishop of Leigh-on-Sea, Essex, England, for his untiring work to create The Duodecimal Society of Great Britain and to successfully advance its dozenal programs and publish its Newscast until his resignation as Secretary for reasons of health. Ralph Beard has form and will have award set up in proper shape. Kingsland Camp, if possible, will personally present it. Carried.

It was moved by Kingsland Camp, seconded by Beard, that the 1968 Award of DSA be made to Tom Linton, of Garden Grove, California, for his early and successful efforts to create the Duodecimal Circular Slide Rule, for his many and tireless efforts to advance the work and membership of the DSA, and for his many contributions of worthwhile articles for the Bulletin. Ralph H. Beard has the form and will have the award prepared. Kingsland Camp will present it personally.

Lessons and Tests

Moved by Churchman, seconded by Handy, that the Secretary is hereby authorized to designate a person or persons to assist him in giving lessons and tests for prospective members to be rated as eligible for Senior Membership. Carried.

Hardware

It was moved by Churchman, seconded by Emerson Andrews, that the Secretary is hereby authorized to complete the purchase of an electronic computer to be chosen by him at an approximate cost of \$2,000.00, when total sum of purchase money becomes available from special gifts to close the deal. Carried.

Tom B. Linton announced that the 6-inch Circular Slide Rule is now an accomplished fact and can be purchased at \$7.50 each.

Moved by Churchman, seconded by Handy, that the Secretary is hereby authorized to purchase a supply of 6-inch Duodecimal Plastic Scales related to millimetres and/or inches, and 6-inch Slide Rule Stick-ons, with the voluntary aid of Kingsland Camp, at approximate cost of \$500.00, when total sum of money becomes available from special gifts to close the deal. Carried.

Additional Groups Abroad

Some discussion but no present action.

Associate Editor

It was moved by Emerson Andrews, seconded by Beard, that Henry Churchman is hereby named as Associate Editor of The Bulletin, effective immediately, to cooperate with Editor Jamison Handy, Jr., in getting Bulletins printed and distributed. Carried.

Comments and Letters Department

Informally discussed and urged, without immediate action, that a department of Comments and Letters to the Editor he instituted in the Bulletin. Adjournment

Moved by Emerson Andrews, seconded by Beard, that the Board of Directors Meeting for April 1180 (1968) stand adjourned sine die and the motion Carried, at 8 gross (1600 Hours).

Dozenal Dinner at Allgauers

At 1180 April 8;960 (1900 Hours) DSA again met at Allgauers to dine and enjoy new friendships. In addition to board members present, there also were Senior Member John Selfridge, Math Department, University of Illinois, Urbana, Illinois; Miss Peggy Dwyer and James Dixon of Purdue University; and Stan Bumpus of the University of Illinois, current student members DSA; also present were our guests Dr. B. A. M. Moon, University of Canterbury, Christchurch, New Zealand; Dr. Kenneth Mears, of Oakville. Ontario; and Clarence Seamans, of Elmhurst, Illinois, guest of DSA member Erich Kothe, of Schiller Park, Illinois.

Letter from George S. Terry

At dinner, a letter of greetings and expression of regret at not being able to attend, from one of the Founding Fathers of DSA, George S. Terry, of Sonoita, Arizona, was read by Henry C. Churchman.

Evening Meeting

The membership meeting of DSA was again called to order by the President, Charles S. Bagley, in the Yorktown Room, at 1180 Apr 8; X60 (2100 Hours).

With blackboard, and a printed description of his method, Member Erich Kothe, of Schiller Park, Illinois, presented a paper on Chromatic Musical Scales and Dozenal Notations, that was very much relished, discussed, and appreciated. (More on this in the next issue).

At £30 (2230 H) Monday evening, the meeting adjourned till 460 (0900 Hours) 9 April. Tuesday Workshop

At 1180 April 9;490 (0930 Hours) membership meeting of DSA was called to order by President Charles S. Bagley. Among those present were

> Charles S. Bagley Ralph H. Beard Kingsland Camp Henry Churchman

Jamison Handy, Jr Tom B. Linton Bruce Moon Stan Bumpus

Tom B. Linton talked on the digital aspects of slide rules and demonstrated that a dozenal slide rule can be constructed by using the standard decimal sliderule gradations and decimal logarithms. He labeled it another breakthru for engineers moving to base-twelve. A welcome development.

Member Stan Bumpus gave a detailed blackboard demonstration of Pi in dozenals. Showed an excellent grasp of work in base-12. Talk well received.

Dr. B. A. M. Moon, of Christchurch, N.Z., eulogized the late Prof. A. C. Aitken, of Edinburgh, Scotland.

Discussed possible 1969 meeting of DSA near Griffin Observatory in Los Angeles area. No action. Matter remains in hands of Tom B. Linton, with full authority to select place and time of 1181 annual meeting.

(Concluded on page twodo-five)

By Tom Linton

The binary base of two is so much better than the base ten for electronic digital computors that practically all such computors use the base two for internal computations. The base ten is so seriously inadequate in high speed counting that all programming of the computor is either done in base two, or input and output means are used to convert from the awkward base ten and back aeain.

Is there a parallel to this awkwardness in our everyday work? The difficulty of switching a drafting department from the common fractional methods to uniform decimal usage led me to examine other possibilities. The fractional scale is easy; can we improve on the decimal?

One difficulty in using decimals in engineering drafting is the exasperating division by three where one divided by three yields the continuous decimal .333 . . I think it was Randolph Churchill who commented on "those damned dots," and with the frequency of needing thirds they do indeed seem to be damned but a necessary evil in the base ten.

Another difficulty lies in the frequent basic need of binary division; that is, successive divisions by two. The logic of the binary division is clearly seen on the draftsman's scale dividing inches into halves, quarters, eighths, sixteenths, and thirty-seconds. The difficulties arise in conversion from fractional to decimal values. Conversion for 1/32 = .03125 and 1/64= .015625 are particularly obnoxious since "rounding off" to three or four decimal places is usually required, thus necessitating inaccurate conversions. These difficulties present a real challenge.

TABLE I: BINARY FRACTIONS (expressed in decimals and dozenals)

	Decimal	Common	Dozenal	
five tenths	.5	1/2	;6	six twelfths
twenty-five hundredths	.25 1/4		; 3	three twelfths
hundred twenty-five thousandths	.125	1/8	;16	dozen six per gross
six hundred twenty-five ten thousandths	.0625	1/16 1/3	14 ;09	nine per gross
three thousand one hundred twenty-five hundred thousandths	.03125	1/32 1/2	28 ;046	four dozen six per great gross
fifteen thousand six hundred twenty-five millionths	.015625	1/64 1/8	54 ;023	two dozen three per great gross

Obviously if the number base could be chosen such as to be divisible by more factors than is ten, some of the awkwardness of the base ten could be avoided. A brief scan shows twelve is such a number.

Twelve is evenly divisible by 2, 3, 4, and 6. Since ten is divisible only by 2 and 5, we double the number of divisors by the selection of the dozen as a possible number base. This apparently minor advantage is seen to have a large cumulative effect in our daily engineering and other number requirements.

As an example of the relative simplicity of the common binary divisions (successive halving), in the base twelve, note their comparison with the base ten shown in Table I. The relative simplicity and exactness of the dozenal twelve-base equivalents is striking. How many hours are wasted in fractional-decimal drafting in dropping or adding the decimal tag-endings to make dimensions add up to a given value?

The natural next thought is to avoid the use of fractions entirely by using decimals only, and I have tried that "natural" step with a group of designers. It proved to be about as natural as walking sideways!

Among the drawbacks of decimal dimensioning were these: The decimal scale was slower to read especially when spotting a succession of points; associated with the slowness were more frequent errors; checking a drawing was more tedious and subject to error and differences of opinion; more time was needed to correlate with tooling, vendors, and the material and production control departments. The fine decimal divisions (.020 on the decimal scale), and the division by 5's instead of the more natural division by 2's, are physical handicaps which may be reduced but not eliminated by practice and familiarity.

Look at two drafting scales, one divided fractionally down to 1/32 and the other divided decimally to .020 as in the SAE preferred scale. The greater legibility of the fractional scale is unquestionable; the resulting speed and accuracy of reading follows inevitably. (See Figure 1)

Then note the "dozenal" scale of Figure 1. Divisions are as legible as the fractional scale but illustrate the dozenal squivalent of the decimal scale. Reference to the binary equivalents of Table I shows the easy conversion from the fractional to the dozenal scale values; it follows that THE GREAT ARGUMENT between the fractional and decimal adherents has disappeared in the versatile base twelve usage. In its place we have the compatibility of the fractional-dozenal scales. On the dozenal scale the small circles indicate the 1/3 and 2/3 place, or dozenally .4 and .8 (since 4 and 8 are respectively 1/3 and 2/3 of twelve). At the right end of the dozenal scale (see Figure 1) the binary divisions of twelfths are shown as an alternate smallest division; such flexibility is denied us on the decimal scale.

I once firmly believed the ultimate answer lay in the metric system. Most if not all science students come first to this er-(Continued on page do-mine)

WELCOME, HEXADECIMALISTS!

In order to display the dozenal numeration in relation to some others the following table was compiled by Henry Churchman, a fellow of The Duodecimal Society of America, by counting in separate columns in sequence to the largest single hexadecimal symbol, plus one.

The hexadecimal names for B, C, D, E, F, and 10, and the hexadecimal point, are the writer's own, no claim is made that they are conventional, and any one is welcome to use or change them.

Binary	y Decimal		Duc	odecimal	Не	Hexadecimal		
0000	0		0		0			
0001	1		1		1			
0010	2		2		2			
0011	3		3		3			
0100	4		4		4			
0101	5		5		5			
0110	6		6		6			
0111	7		7		7		Age	
1000	8		8		8			
1001	9 (r	nine)	9	(nine)	9	(nine)	9	
1010	10. (1	ten)	χ	(dek)	Α	(ann)	10.	
1011	11. (6	eleven)	E	(el)	В	(bel)	11.	
1100	12. (1	twelve)	10;	(doe)	C	(cal)	12.	
1101	13. (1	thirteen)	11;	(do-one)	D	(nob)	13.	
1110	14. (1	fourteen)	12;	(do-two)	E	(erl)	14.	
1111	15. (1	fifteen)	13;	(do-three)	F	(fan)	15.	
10000	16. (sixteen)	14;	(do-four)	10!	(hex)	16.	

In duodecimal counting "teen" and "ty", which are derivatives of ten, are avoided. Thus, 11, the equal of thirteen in quantity, is called "doe-one" (written "do-one" or "a dozen and one). And 20 is described as "two-doe", written "two-do". "Twenty" belongs to base-ten and should mean two tens to every educated person who has reached the age of reason.

Hexadecimally, if 10 be called ten, it is equally logical that 11 be called eleven and 12 described as twelve---altho the quantities actually exceed those named if one is working in sixteen-base. Eventually hexadecimalists may emulate the dozeners and (as duodecimalists call 10 "doe") speak of the hexadecimal 10 as "hex". Then, 11 becomes "hex-one" and 12 "hex-two", for such they are in their numerical values, and so they might be imprinted on the hexadecimalist's mind. "Teen" then becomes obsolete in base-sixteen, as it is now avoided, as one avoids smallpox, in base-twelve.

Without the slightest knowledge of psychology, a hexadecimalist (even so a duodecimalist) might describe 10 as ten simply because he was not given a name which described its properties. And his thinking processes force him to utter a term for 10 with which he already is familiar, even tho he knows the quantity exceeds ten. So, a drowning man grasps for a straw, not because of confidence in its power to save him, but for the reason that no one has thrown him a suitable float. He is forced to a false

Figure 1

A possible hexadecimal numeration for experimental use is as follows. Other, more euphonic, 3-letter, one-syllable names may be found, but we should, perhaps, avoid names ending with "t" or "s" if we would eliminate "thex" and "shex" articulations.

1	one	AO annhex
2	two	BO belhex
9	nine	CO calhex
Α	ann (Anna's age is ten.)	DO donhex
	bel (Belle's " eleven.)	
C	cal (Calvin's " twelve.)	FO fanhex
D	don (Donald's " thirteen.)	FF fanhex-fan, or simply fanfan
E	erl (Erlander is fourteen.)	100! rex
F	fan (Fanny is fifteen.)	10A rexann
10!	hex (Hexadecimal, sixteen.)	10F rexfan
11!	hex-one is sixteen and one.	F00 fanrex
121	hex-two	FFF fan, fanfan
19!	hex-nine	1000! mex
1A	hexann	1FFF hexfan fanfan
1B	hexbel	FFFF fanfan fanfan
1C	hexcal	10 000! hexmex
1D	hexdon	1F FFF hexmex fanfan fanfan
1E	hexerl	100 000! rexmex
1F	hexfan	1 000 000! bimex
20!	two-hex	1 000 000 000! trimex
90!	nine-hex	and so forth, and so forth!

move because it is the best he can do in his circumstances. Dozeners should be forever grateful for dek, el, and dozen, altho, in their case ten, eleven, and twelve are not misleading.

We must continue to live with decimalists for some time to come. And to call 40! "forty" when we know it surpasses four tens (is the equal of six tens and four) tends to cause a decimalist to believe such speaker is an arithmetical anarchist or a nihilist---against all law and order in arithmetic, which, certainly, does not describe any hexadecimalist. Why not call 40! four-hex? A definite identification of base-sixteen in itself. See Figure 1 for other examples of hexadecimal terminology.

One might also suggest a hexadecimal identification point, not to be confused with the period, comma, or semicolon now used by the decimalists and duodecimalists, or the colon used in reporting sexagesimal minutes and seconds of time.

In base-twelve, for example, the <u>dozenal identification point</u>, a semicolon, may be pronounced by the initials only, if we shorten the expression to "dip". For instance, 12; might be called "one two dip", or the shorter term "do-two" which is more often used. And ";6" might be called "dip six" or one-half, or sixtwelfths which it is in fact.

(Please turn to page twodo-five)

Until you can quickly and naturally convert from 12-base to base-16; and from base-sixteen to base-twelve with equal ease, base-sixteen might seem mysterious and simply unachievable, beyour ability to grasp. This simply is not so. Rework the following samples once every day for one week, and not only may the mystery disappear, but you could become quite proficient at division and addition in base-sixteen. First, to understand and grasp the values represented in 16-base, let us temporarily substitute the following symbols for values:

Let 10! hex = sixteen decimally. $10!^2 = 100! = rex = 256$ decimally. (Hex square) $10!^3 = 1000! = mex = 4096$ in base-ten. (Hex to 3rd power) $10!^6 = 1\ 000\ 000! = bimex = 16,777,216$ base-ten. $10!^9 = 1\ 000\ 000\ 000! = trimex = 68,719,476,736 base-ten.$

It is necessary to remember that the dozenal point (;) is a semicolon. And here let the hexadecimal point (!) be an exclamation point. Also remember that, temporarily,

A mass F	e called	ann (Anna's age is ten)	IO in	base-ten.
B	11	bel (Belle's " eleven)	11	• • • • • • • • • • • • • • • • • • • •
C	14	cal (Calvin's " twelve)	12	(1
D	11	don (Donald's " thirteen)	13	11
E	- 11	erl (Erlander is fourteen)	14	11
F	11	fan (Fanny is fifteen)	15	**
10!	*1	hex (Hexadecimal, sixteen)	16	11
20!	11	two-hex	32	11

To convert from 12-base to 16-base numbers, divide the number successively by hex or 10! (sixteen). The successive remainders, stated in reverse order, are the base-16 number.

For example, change 2X27X4;0 to base-16. (D)

To convert from 16-base to 12-base numbers, divide the number successively by cal or C (twelve). The successive remainders, stated in reverse order, are the base-12 number.

For example, change BlOAC! to base-12. (E) (12) C)B 1 ${}^{9}0$ ${}^{0}A$ ${}^{A}C!$ = 2X£7X4;0 = 725164.0 C)E ${}^{2}C$ ${}^{8}0$ ${}^{8}E!$ + 4 Proof: We consider that ${}^{2}C$ ${}^{2}C$ ${}^{3}A$ ${}^{2}B!$ + X verted that Proof: We have merely reconverted the same value from base-16 back to base-12. See c) 2 2! + ε (D) above. ---H. C. Churchman

TIME FOR ALL SEASONS

ALL OVER the earth, people experience the annual recurrence

of the Mean Southern Solstice day, December twenty-first of the present Gregorian Common Calendar.

It is the middle of night at the North Pole, the middle of day at the South Pole, the initial day of the Winter Season in the North Temperate Zone (of the Summer Season in the South Temperate Zone); and at the Equator, the sun will rise and set as far south as it will ever get on any day, year after year.

If the Mean Southern Solstice (abbreviated to "Mesusol") day be designated as Zero or "Mesusol" day, and if we number every day thereafter as "Post Mesusol" or PM days, every regular year will contain four equal quarters, of 91 days each---no more and no less.

These 91 days in a quarter year equal a dozen and one Heptads or periods of seven days each. Every Heptad contains a Sunday, Monday, Tuesday, Wednesday, Thursday, Friday, and Saturday; but Sunday is not the first day of a heptad in every year. Heptads are commercial periods of time, useful in accounting and computer work. A fiscal year might equal four quarters, any four.

If you work seven days in every heptad, your working record will show 7 dozen and 7 days in each Quarter. If you work five days in every heptad, your record should show 5 dozen and 5; or if you work four days in every heptad, your work record shows a work total of 4 dozen and 4 days in each Quarter year.

Whether you use base-ten or base-twelve notation, Mesusol day in every year is Zero day. No longer is the question to be put whether the commercial and the fiscal years shall be based on a twelve-month or thirteen-month calendar. Months may become curiosities in the next century, and the Quarter year (as quite often now in England) would become the basis for figuring rents and fixed wages, partial tax returns and settling for public utility services, and other things where the period of a year is considered too long or too unrealistic.

Weeks are not changed in any way but continue as now. In a way, this system of time measurement supplements and strengthens the week, alters its continuity in no way whatever, and is not intended to replace the week or even duplicate it.

Davs

Each day is divided successively, in the metric system of time control, into twelve equal parts, down to the momente (50 present seconds) of time. (See pp 13-16, Dozenal Essays 1967). It is divided by successive minus powers of twelve, to the instante (25/864 part of one second) for more careful measurement of time; and by the same method of subdividing, to any dozenal fraction of an instante demanded by science.

An introduction to and immediate employment of metric time is long overdue among scientists in all fields; and in countdowns. (Continued on page 14)

TIME SEASONS

A Correlation of the Earth's : between Mean Southern Solstice : lar or Leap Year Mean Southern : in either the present Base-ten : : normal SEASONAL QUARTERS and : Days of a Regular or Leap Year : Solstice Day; and counting the : or ever growing popular system : : the Gregorian Common Calendar : and the next succeeding Regu- : successive days of each Year : of Base-twelve symbols. Gregorian Calendar Year 2001 AND ALL MEXICAR TEARS. Days of the MU FEELOD Years 11x8 and 11x9, OF ALL Days of the MU PERICO Years 2000 and 2001, OF ALL Oregorian Calendar Year 2000 AND ALL LEAP YEARS, PRIOR AND OF ALL SUBSEQUEET TRASS. from preceding Hean Southern Solstice Day. PHOOR AND OF ALL SUBSHOUNT YEARS. from preceding Mean Southern Solstice Day. Menta4 Ш IV III IV ш Days Hean Southern Solstice Day, of every year, Dec Hoan Southern Solstice Day, of every year, Moun Southern Solstice Day, of every year, Dec Mean Southern Solstice Day, of every year, 3 10 30 30 Jan 15 20 27 32 39 44 55 61 68 28 35 42 49 56 63 70 37 44 51 58 65 7 57 64 N 20 61 75 6£ 76 30 111 118 125 132 139 112 119 126 133 140 147 599712174CE 113 120 127 134 141 148 115 122 129 136 143 150 22 29 6 2音 27 May 113 Jun Third Quarter Jæ Jum 147 152 159 164 202 209 216 223 230 237 244 251 259 146 151 158 163 161 175 180 6 208 215 Jul 231 Lug 245 18X 193 191 Sep Fourth 100 107 202 209 214 226 231 238 243 241 28 286 293 300 307 314 321 328 335 342 349 356 363 Sep 294 301 308 206 216 221 Oct 208 213 21X 225 230 237 242 249 Oct 303 310 317 302 309 316 323 330 337 30 6 30% 311 318 325 332 339

(Repeat lat Quarter as 2nd, 3rd and 4th, if you wish Base-ten

occurs only in

353 360

(Gregorian Leap Years

(Repeat 1st Quarter as 2nd, 3rd and 4tb, if you wish to count by Quarters.) Base-twelve

Supplemental Day, occurs only in Leap Years,

256 25X

23X

250

Regular Year.) Regular Years

(Oregorian

17

Dec

TIME FOR ALL SEASONS (Cont'd from page do-one)

Metric time control might prove a boon to astronomers. One advantage in using a metric timepiece in astronomy is found when we determine the length of the cycle of any recurring event. As an example, let us say an event occurred in the Julian "Momente" 998 134;576. This is equal to J. D. 244 0120, or 21 September 1968.

The momente of 0;576 is equal to 1115 Hours (either U.T. or G.M.T., depending on what scheme you are using). If this event repeated itself, let us assume for brevity's sake, at 244 0211 (Dec. 21, 1968), but earlier in the day——in Metric Time at 998 $1X\mathcal{E};210$, then, to determine its cycle we can subtract direct the time of its earlier occurrence from the time of its recurrence, thus: $998\ 1X\mathcal{E};210$ (recurrence)

- <u>998 134;576</u> (next preceding occurrence) = 76;856 = cycle.

Note, to subtract 576 from 210, we borrow one dozen, as in base-ten we borrow ten, from one place to our left.

Now, at what momente will this event recur in a dozen cycles? Simply move the dozenal point one place to our right, thus: 768;560. Then add 768;560 to the recurred Julian Momente:

998 1X£; 210 (recurrence)

- + 768;560 (dozen cycles)
- = 998 957; 770 (J.D. 244 1299 = Dec. 14, 1971, in the 770 momente of a Julian Day---1510 h).

Date Stamps

In common use, we may keep track of Seasonal Time by at least three methods. A date stamp at a bankteller's window might indicate September 21, 1968 as: 1968 PM 275. Check the baseten Seasonal Quarters on the back of this page with the Gregorian Leapyears date of 21 September. 1968 being a leapyear.

A turnpike tollhouse might use: 1180 PM 1X2;576, or just the last six digits, to show, in base-twelve (the year is one dozen and one gross, eight dozen years), the PM day number (1X2 is the equal of one gross, ten dozen, and eleven days, or 275 days in base-ten), and the Local Time (576 equals 1115 H). Momentes are 50 seconds in length, while minutes of time are 60 seconds long. Minutes of time are 1/5 less precise.

Or one might, in astronomy, use 09981×256 , a Julian Momente date stamp, to indicate, in base-twelve, the Julian Day $998~1 \times 250$ and the total number of momentes (five gross, seven dozen, and six, equal to eleven hours and fifteen minutes of time) elapsed, of the Julian Period day.

If you were one of those fortunate enough to watch on television the countdown and blastoff of the latest U. S. spacecraft, you may have noticed the sudden burst of speed between 6:00 and 5:59 seconds, especially the fleeting 5:90, 5:80, 5:70 and 5:60 which fled past in less than one second. Normetric time.

---From papers of H. C. Churchman

SINGLE SYMBOLS FOR TEN AND ELEVEN

When you count by dozens it is necessary to develop two single symbols for ten and eleven. This is so because we add up to a dozen before we carry one. Difficult designs delay dozenals.

In England, today, a half-rolled two is tentatively employed to represent ten. Another possibility is the Roman numeral ten, quite generally used in America. It has stood for ten units for more than 2500 years. Most people today still call it ten as it appears on Big Ben's face in London, and European tower clocks.

For eleven, the Duodecimal Society of America has continuously since 1945 (and Mr. F. E. Andrews since 1934) used a "swash $\mathfrak E$." That symbol is called el. It is quite universally employed, today, in Britain and America, to represent eleven.

One base-twelve proponent in America, GROVER CLEVELAND PERRY, nearly a half century ago, suggested a capital L to represent eleven. It is, in effect, a half-rolled 7. A half-rolled 7 for one of the two new symbols was again suggested a dozen or more years ago, in France. See Douze Notre Dix Futur, by JEAN ESSIG, 1955, Dunod, Paris. And L for eleven has been suggested by more than one alert duodecimalist in Britain. Not our idea, but---

The printers' upper case L has two advantages over the £ symbol. L is already on the standard keyboard of every student who possesses a typewriter at any American or European university; and no one is likely to call it anything except el. Any suggestion that X and L belong only with the alphabet, that they have little place in mathematics save algebra, is not accepted by the hexadecimalists, who have seized A, B, C, D, E, and F for their symbols ten thru fifteen. No one to our knowledge has rebelled against the printers' lower case letter 1 to represent one, employed in Europe almost from adoption of the Arabic system.

If L be pressed into service for lack of an apt "swash 2" to represent eleven, it could be called either el or eleven. For instance, L2 could be called eldo-two, or eleven-two, as well as el-two. And 4L could be called four-do-el, or four dozen and el, or four-el, or four eleven.

By custom, extending back to 1152, THE DUODECIMAL BULLETIN employs the italic X to represent dek or ten; and an italic "swash 2" to represent el or eleven. But all correspondents are encouraged to write in duodecimals, and about duodecimals, employing any of the symbols for ten and eleven which may be found on their typewriter. These can be transmuted into THE BULLETIN's symbols whenever the article appears in this publication, in keeping with the custom just mentioned, and to promote a continuity of symbols well understood by all members around the earth.

Send to The Duodecimal Society of America, 20 Carlton Place, Staten Island, N. Y. 10304, your papers about dozenals, how best to promote the use of base-twelve; what makes base-twelve, for the man in the street, superior to the base with only two factors and more practical than base-16 (which is primarily a bin in which to store binary sums, as we store grain in a granary).

H.C.C.

METRONIC MEASURES AS EMPLOYED TODAY

In the United States of America, every day in the year, a multiple of the distance of one metron is unofficially employed. And no linear dozenal unit can be more exactly determined in the laboratory (and remembered), since one metron is, by definition, the exact length of 75 000;0 krypton 86 atomic light waves. One international metre, today, is defined as equal to 1,650,763.73 such wavelengths.

One dozen metrons equal the unit called one Dometron. And one gross dometrons are a length equal to 528 Canadian statute feet, plus two and a fraction inches, called an Edon in the metronic system of units of length. One duodécamètre duodécimal might, without unbearable stress, be said to be equal to one dozen dométrons; and twelve duodécamètres equal to one edon---due to the unsurpassed vision of M. Jean Essig to be found in his far ahead book, DOUZE NOTRE DIX FUTUR, 1955, Dunod, Paris.

Thruout the State of Iowa, at scores and scores of unsuspected places on interstate public highways, we might spot, when too late to do anything about it, two transverse white lines painted across our direction of travel, a distance of 528 statutory feet between them. The white stripes, themselves, are perhaps six to twelve inches wide.

Of course, a highly trained and carefully equipped engineering crew would be required to measure an exact 528 feet between such cross lines, but, between the far sides of the two white stripes let no one doubt there are to be found 528 feet plus a surplus of one or more inches on each end——a dimension equal to one exact Edon measured by krypton 86 atomic light wave standards.

Some motor vehicle traffic within the State of Iowa is supervised and the rate of travel controlled in daylight by use of airborne patrolmen flying above a spot, leaving only to return at unexpected moments. They carefully check the elapsed time between the instants a vehicle crosses each of the two stripes.

If the posted rate of travel is 60 miles per hour, this is equal to one mile a minute; and one edon (one-tenth statute mile) should be negotiated in six seconds of time.

A tolerance of one or two miles per hour is allowed in fact to cover slight errors, but if that Edon be traversed by time check in five seconds, then the vehicle operator is clearly moving at a one-fifth faster rate than permitted, or, at 72 miles per hour in a 60 m.p.h. zone.

A bothway radio will convey the readings and the motor vehicle description to a highway patrolman in his car a few miles ahead, and the driver will be flashed a stop signal---this is the "Edon trap." Motorists are coming to believe Someone above is watching over their shoulder, whether or not they believe in a deity. A salutary effect is noted wherever this method is used.

Now this writer does not claim that the words "edon" and "Aeromile" ("Navinaut" or "Kilomètre Duodécimal") are found any place in the Iowa state statutes, but, the state Public Safety Department regulations do describe in feet what turns out to be an actual distance equal to one edon if it were measured with that degree of precision suitable, from a practical standpoint, to achieve a proper control. And a dozen edons might be called one aeromile or navinaut, le kilomètre duodécimal, or a Nante.

Another employment of the edon is found in interstate highway placement of reflector holders, to indicate the right edge of the shoulder or emergency parking space. Usually, these are to be found three to the edon and rather equally spaced. Officially, they were placed 176 feet apart.

It might be worthwhile to reiterate that one Edon of distance, when multiplied by the fifth power of twelve, is the equal of a Great Circle of the earth. And every plane that travels 172.8 statute miles, or 1728 edons, might be said to have moved the equal of one per gross part of the length of a Great Circle. In other words, 144 such arcs would have taken it around the earth if it had flown on a great circle, deviating neither right nor left of a straight course.

Many officially posted signs along public highways in Iowa and other states advise the motorist: "Detour begins 500 Feet", or "1000 Feet", or "1500 Feet", but when one measures this distance on the mileage meter, it turns out to be 528 feet, or 1056 feet, or 1584 feet, more or less. One "Edon" could have been substituted for each "500 Feet" with less loss of exactitude.

Every lorry or truck operator, who reports the number of miles traveled on each trip, or in each day, unknowingly reports the distance in edons, for the simple reason that the mileage meter keeps count in one-tenths of a statute mile, the most practical.

If one pulls out on a reading of 74210.3 in the morning, and puts down 74843.9 in the logbook that night, or at the end of a trip, one obviously has covered 633.6 statute miles, or 6336 edons, on that particular day, or trip. You or I might quibble about a difference of some two or three inches per edon but 6336 inches equal 528 feet, a difference of two and a fraction edons fewer in a day or trip involving 6,336. A difference in air pressure in the wheels could create that much variance.

The only thing we need change is the name, but that need not happen until it becomes economically or legislatively advantageous to do so. No hurry, but when we move to a metric system of measurements, let us do it dozenally and at less initial cost and inconvenience. You know what one-tenth of a mile equals—and you know the length of 3-2/3 inches (one metron in a 12-base metric system, so far as the naked eye can tell). You might say decimetres are not very tasty in describing distances today; but when they come, dometrons can not be far behind. Metrons and edons are already here—underground!

The Duodecimal Bulletin

A PAGE FROM DOUZE NOTRE DIX FUTUR

Below is found an offset of page one hundred six of Douze Notre Dix Futur, published by JEAN ESSIG, 1955, Dunod, Paris. It illustrates the broad vision of M. Essig succinctly. No one is wholly oriented in duodecimal dimensions until the book has been fully read and digested, including the footnote regarding a definition of the dozenal mètre (precursor or successor of the dométron?) by its comparison, pursuing the latest thinking, with a selected wavelength.

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DOUZE NOTRE DIX FUTUR

L'hectomètre duodécimal, le mètre duodécimal, le duodécimètre duodécimal, le centimètre duodécimal se déduisent aisément de cette première définition, et on donne ci-dessous les dimensions que représenteraient ces nouvelles unités par rapport à celles que nous utilisons aujourd'hui.

I hmd = hectomètre duodécimal =
$$\frac{1929.01}{12}$$
 = $160.75 \,\mathrm{m} = 1.607 \,\mathrm{hm}$
I md = mètre duodécimal = $\frac{1929.01}{1728}$ = 1,116 m
I dmd = duodécimètre duodécimal = $\frac{1,116}{12}$ = 0,093 m = 0,93 dm
I cmd = centimètre duodécimal = $\frac{0.93}{12}$ = 0,0775 dm = 0,775 cm

L'unité de longueur fondamentale serait le mètre duodécimal, douze millionième partie duodécimale de la circonférence terrestre, mètre qu'il conviendrait (1) de refaire en platine iridié de la manière la plus exacte que nous permettent nos mesures actuelles, et qui serait à nouveau conservé au Pavillon de Breteuil.

Bien entendu, on montre immédiatement qu'en numération duodécimale, toutes ces nouvelles unités dériveraient les unes des autres d'une formule extrêmement simple :

On n'a pas poussé davantage l'analyse des unités de longueur. Il est bien évident que les nouveaux millimètre, mu, angstrom, dériveraient des unités précédentes, selon les mêmes lois.

(1) Ou de définir, selon les idées les plus modernes, par son rapport avec une longueur d'onde déterminée.

Numbers enclosed in a rectangle are twelve-base numerals.

A DIMENSIONING CHALLENGE (Continued from page 9)

roneous conclusion after a period of working simple problems in the metric units. There are obvious advantages based almost totally on one characteristic: the units are related by exponential expressions of the base ten. A kilometer is a thousand meters; a kilogram is a thousand grams; and this pleasant relationship holds for many, but by no means all, metric units. Only two other advantages are unique to the metric system, and while minor, they are helpful and significant.

The nomenclature is tidy. From a given unit such as the meter most others are derived by adding Greek prefixes to designate larger units, as kilometer, while Roman prefixes are used to designate smaller units, as centimeter. The other advantage is in the numerical equality of specific gravity and density at standard conditions of temperature and (sea level) gravity.

Why not the metric system? Mainly because it is based on what we have indicated is the avkward ten-base. Because of this awkwardness, the metric system is not yet a homogeneous system. Metric countries use the 24 hour day, and the minutes and seconds of time. People buy eggs by the dozen. The year has a dozen months. The astronomical parsec and light year measures of distance may be expressed in either kilometers or miles, but are not exponentials of the ten base. The derived physical units of the cgs (centimeter gram second) and of the mks (meter kilogram second) systems fail to live up to the simple relationship arguments of metric proponents.

While physical relationships involving the transcendentals π and ϵ are not significantly affected by a change of the numerical base in which they are expressed, the ease of handling calculations including them may be greatly affected. If we want to avoid the obnoxious pitfalls of the base ten metric system, and maybe we should want to, we may further examine base twelve.

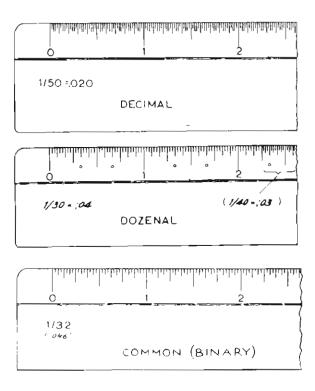
To figure by twelves we need two new symbols for ten and eleven. Modified and used here are the Roman symbol X for ten and the capital script $\mathcal L$ for eleven.

While good unique names have been suggested and used for the two extra digits, such as dek for ten and el for eleven, no big objection exists to retaining the present names ten and eleven for the names of the quantities they represent. For twelve I personally find it easy to use the name "dozen" to designate the 10 spot in the dozenal number series, since it seems to be a little more flexible than twelve, and perhaps less ambiguous. "Two dozen," for instance, can mean only one number. In Table II, dozenal and decimal counting are compared.

To help understand this number form, it is important to see that numbers (as a quantity concept) do not necessarily belong to the numerals, digits or other symbols used to represent them. For example, eleven is eleven, whether written as XI, 11, or 2. It took nearly a millenium for European people to generally accept Arabic 11 for the Roman XI. And even now (since the latter was not outlawed, as the Arabic once was) many still find employment for Roman numerals where identification rather than

computation is the purpose (such as for outlining or organizing material, and such inertial use as for chapter headings, or cornerstone dates for buildings).

Fig. 1.



What we have learned from the Arabs is not limited to base ten alone. The electronic computer engineers found this out to their great advantage when they cultivated the ability to switch from decimal to binary thinking. And then, finding that while straight binary was good for optimum use of computer insides but clumsy for pencil and paper, they went to a more convenient derivative called octal or base eight (count to seven, then in the space for 8 write 0 and carry one eight) for detail work.

Inasmuch as mechanical computers (e.g. ordinary adding machines) could be made to any reasonably wanted base---just a matter of things like how many teeth on a gear or rack---it made good sense to make them fit their ten fingered human users. But little and inexpensive electronic basic components for digital counting are intrinsically binary (such as "on" or "off", or, + or -). Human inertia was so great in this matter that even in this day of rapid progress, a lot of possible computer capacity

TABLE II: COUNTING COMPARISON

DOZENAL			DEC	IMAL	
OI	ne 1	1	one		
	70 Z	2	two		
niı	ne 9	9	nine		
te	en X	10	ten		
eleve	en E	11	eleven		
doze	en 10	12	twelve		
dozen or		13	thirteen		
dozen tv	JO 12	14	fourteen		
dozen eigh		20	twenty		
dozen nin		21	twenty on		
dozen te		22	twenty tw		
dozen el		23	twenty th		
two doze		24	twenty fo		
two dozen or	ne 21	25	twenty fi	ve	
eight dozen fou	ir 84	100	one hundr	ed	
one gros	ss 100	144	one hundr	ed forty	four
six gross el dozen fou	ir 624	1000	one thous	and	
seven gros	ss 700	1008	one thous	and eigh	t**
one great gros	s 1000	1728	seventeen	hundred	
				twenty e	ight
* Short for "a dozen	and ele	even".			
** A convenient close	approxi	mation i	for thousand	or kilo	units.
Fractions of ten (de	cimals)	and fra	actions of t	welve (d	07672161
are seen to compare th		and III	iccions of E	werve (a	ozenais)
are seen to compare c.		DECIMAL			
1/10 2/10 3/10	4/10	5/10	6/10 7/10	8/10	9/10
.1 .2 .3	. 4	. 5	.6 .7	.8	.9
.1 .2 .3	. 4		.0 .7	.0	• 7
thre tenth two tenth	⊕ ₩	e r	te te	e e	6 3
three tenths two tenths one tenth	four	five tenths	seventenths	eight tenths	nine tenths
a ha has	hs	e hs	hs hs	eight enths	hs le
		DOZENA			
		DOZENAL			

5/10

; 5

four twelfths 6/10

7/10

seven twelfths 8/10

:8

eight twelfth: 9/10

nine

X/10

\$/10

; ξ

eleven twėlfths

The Duodecimal Bulletin
TABLE III: RECIPROCALS

was wasted for some twenty years before, and during, and after World War II before it dawned on many beyond a few pioneers that the savings of straight binary on big jobs more than paid for the in-and-out conversion; and an intermediate hybrid, binary-coded-decimal (which can count to sixteen per unit, but deliberately "wasted" eleven through sixteen to avoid conversions), was worthwhile for other applications.

So, to go on with this dimensioning challenge, if we realize that $\underline{\text{two}}$ in the base $\underline{\text{two}}$ is written $\underline{10}$, $\underline{\text{three}}$ in the base $\underline{\text{three}}$ is $\underline{10}$, $\underline{\text{eight}}$ in the base $\underline{\text{eight}}$ is $\underline{10}$, besides ten in the base ten being $\underline{10}$, we are ready to find the advantages of writing dozen in the $\underline{\text{twelve}}$ base as $\underline{10}$, and read and think of it as one dozen.

Similarly in fractions: .1 indicates one half in the base two, one third in the base three, one eighth in the base eight, etc. Again we separate the quantity represented from a particular form or way of representing it. For example, if someone says "one tenth", you may write either "1/10" or ".1" and could read it back either the same or distinguish by reading digitally "one per one-oh" or "point one" respectively. (The digital or "telephone style" of spelling figures is obviously safer in this day and age if the radix is not obvious or understood!) So, in the dozenal fraction form, the respective denominators of twelfths, per gross, per great gross, etc., may be implied by the use of the dozenal point and successive places.

When we compare Table I with the scale drawings, Figure 1, we see the compatibility of the dozenal-fractional scales compared with the awkwardness of the decimal-fractional combination. On a typical opposite bevelled 2-face scale, the dozenal divisions on one side and fractional divisions on the other side make a smoothly functioning pair.

In Table III two advantages of the base twelve are apparent: fewer continuing "dozenal" fractions and simpler equivalents for the commonly used fractions 1/3, 1/4, and 1/6.

Among our present system of units, in the base twelve:

```
1 month = ;1 year (instead of .08333'' decimally).
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In accurate machine protractors the vernier usually reads to 5 minutes of arc. This is 1/12 degree, or in the base twelve, ; l degree.

The 30-60 degree triangle is frequently used; in the base-12, 30° is ;1 (1/10) circle, or ;2 π radians.

In a larger number base, the same number of digits can express higher integers. For example:

1000 (great gross) in the base twelve is 1728 in the base ten, so there is a saving of one digit for all numbers between it and 999.

DOZENAL		DECIMAL
<i>; 6</i>	1/2	.5
<i>4</i>	1/3	.33333
; 3	1/4	.25
; 2497 * * *	1/5	. 2
; 2	1/6	.16666
;186X35'''	1/7	.142857
; 16	1/8	.125
; 14	1/9	.11111
;12497***	1/X = 1/10	.1
;11111'''	$1/\mathcal{E} = 1/11$.090909**
; 1	1/10 = 1/12	.08333

TABLE IV: MULTIPLICATION TABLES

				P	Base	TWEL	VE				
1	2	3	4	5	6	7	8	9	χ	ε	10
2	4	6	8	χ	10	12	14	16	18	1 X	20
3	6	9	10	13	16	19	20	23	26	29	30
4	8	10	14	18	20	24	28	30	34	38	40
5	χ	13	18	21	26	2ε	34	39	42	47	50
6	10	16	20	26	30	36	40	46	50	56	60
7	12	19	24	28	36	41	48	53	5 X	65	70
8	14	20	28	34	40	48	54	60	68	74	80
9	16	23	30	39	46	53	60	69	76	83	90
χ	18	26	34	42	50	5 X	68	76	84	92	XO
٤	1 X	29	38	47	56	65	74	83	92	X1	20
10	20	30	40	50	60	70	80	90	XO	80	100
					Base	TEN					
1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

¹ hour = :06 day or :1 the clock (half) day.

¹ inch = ;1 foot.

¹ troy ounce weight = ;1 pound (decimally .08333 ...).

¹ avoirdupois ounce = :09 pound (decimally .0625).

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Similarly, for fractionals (if we can coin this word as a noun to generalize the term "decimals" in a larger number base) the same number of places to the right can express a smaller dimension. For example:

;0001 in the twelve base is 1/20,736, or about .000,0482 in base ten.

Stated another way, the successive digits representing twelfths, per gross, per great gross, represent finer as well as more useful divisions than tenths, hundredths, thousandths, etc. In a very convenient special case (from Table I), six hundred twenty five ten-thousandths exactly equal nine per gross (.0625 = ;09).

A comparison of the multiplication tables, Table IV, shows interesting advantages in the base twelve. Note more zero endings in the base twelve table, and more 6-endings than the comparative 5-endings of the tens table.

Also note in Table IV the simplicity of the multiples of 3, 4, 6, 8, and 9 in the base twelve, compared with only one simple series of multiples for 5 in the comparable area of base-ten.

In the light of the singularly usable numerical characteristics of the base twelve, perhaps I shouldn't have been surprised to find the idea is not new.

The number 60, as the lowest number divisible by both ten and twelve, and its use in arc and time division, was used by the Babylonians and probably by the Sumerians before that. As trade and attendant commercial computations developed beyond the fingercounting (certainly the only reason we have the base ten at all) the invention of the abacus preserved the unwieldy base ten through at least the later stages of the 2000 years of Roman numeral usage.

When the Arabs introduced their superior Arabic notation with the zero into Spain, it took some five hundred years for that usage to spread over Europe, and the spreading was in the face of hysterical arguments against the strange new symbols. Everywhere, it seemed, decent citizens banded together to stamp out these unholy numbers with their alien cipher, clearly instruments of the devil.*

Gradually, however, the Arabic numbers were accepted, then helped to beget a resurgence of mathematical activity reminiscent of the earlier Greece, but continuing on to and including our day. It seems quite certain that the numerical facility afforded by the Arabic notations was a necessary basis for that leap forward in mathematics, though the change was not necessary for many of the calculations of that day, nor are numbers required in large segments of modern mathematics. But the change served first as a strong catalyst, then as an essential ingredient to the total mathematical and scientific efforts to follow.

Simon Stevin published the first decimal notation (late 16th century) and is reported to have recognized the superiority of the base twelve, but as today, was faced with overwhelming odds, so went along with finger counting base ten. A few years later (1614) Napier published his tables of logarithms (but the natur-

al logarithms sometimes called Naperian were published a little later, by others). Almost certainly the logarithms would never have been invented without first having the decimal (or dozenal) notation with its zero.

When Jean Picard, and later James Watt of steam engine fame, suggested the basis of the metric system, no rush to acceptance occurred, but in the wake of the French Revolution the method was adopted and improved upon by a committee appointed by the French Assembly. It is reported that Lagrange, president of the committee, argued eloquently and successfully against the adoption of the base twelve, so it may be presumed that some of the other committee members, such as Lavoisier and Laplace, must have been in favor of the twelve base.**

For the century and a half (or shall we say "gross years"?) after the adoption by France of the Metric system we have had many arguments favoring the base twelve, culminating in the publications: NEW NUMBERS, a very readable account by F. Emerson Andrews; DOUDECIMAL ARITHMETIC, a monumental volume of tables by George S. Terry; DOUZE, NOTRE DIX FUTUR (TWELVE, OUR FUTURE TEN) by Jean Essig. The fact that this last came in comparatively recent times (1955) from France, initiator of the metric system, is particularly noteworthy.***

The base ten will not be displaced soon, maybe never, but perhaps the increasing complexity of computations and the increasing volume of them will eventually cry out for that simplification which counting by the dozen most certainly offers, especially to those of us who are NOT always able to live in constant access to a computer.

Some 900 years have elapsed since the Arabic numerals with the "cipher" catalyzed mathematics and science. Do we need another catalyst? My decimally divided drafting scale seems to say YES.

*For an interesting account, see DANTZIG; Number, the Language of Science.

**BELL; Men of Mathematics, p. 169.

***Response to ANDREWS's New Numbers led to the formation of the Duodecimal Society of America, 20 Carlton Place, Staten Island, New York 10304.

REFERENCES:

- 1. Andrews, F. Emerson; NEW NUMBERS.
- 2. Terry, George S.; DUODECIMAL ARITHMETIC.
- 3. Essig, Jean; DOUZE, NOTRE DIX FUTUR.
- 4. BULLETIN, a periodic publication of THE DUODECIMAL SOCIETY OF AMERICA, 20 Carlton Place, Staten Island, N. Y. 10304.
- 5. Whiteside, John E.; Complaint Against the Decimal System, PRODUCT ENGINEERING, Vol. 23, No. 6, June 1952, p. 264.
- 6. Schumacher, William C.; More 12 Place Math, PRODUCT ENGI-NEERING, Vol. 23, No. 9, Sept. 1952, p. 274.

The Duodecimal Bulletin

The Duodecimal Bulletin 20 Carlton Place Staten Island, New York 10304 17 August 1180

Gentlemen:

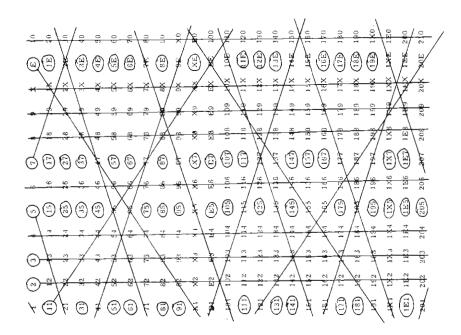
My colleague Mr. Merritt Elmore scratched out the enclosed version of the Sieve of Eratosthenes while giving a test one day. I found it interesting. It shows some of the prime number patterns mentioned in the Manual of the Dozen System. We might ask if this is the smallest number of lines that will scratch out the unwanted numbers.

I received Mr. Elmore's permission to send this on to you.

Very tryly yours,

Eliot Wirt Department of Mathematics San Jose City College

SIEVE OF ERATOSTHENES---DUODECIMALWISE
By Merritt Elmore,
San Jose City College.



WELCOME, HEXADECIMALISTS! (Cont'd from p. 2)

Hexadecimal Point

The hexadecimal identification point (!) might be called "hip" by employing the initial letter of each descriptive word. That sounds like a snappy signal in a fast moving football game. Any hexadecimal point not only identifies a whole number, such as in hex-four (14!), but also a fraction such as eight per hex (!8) for one-half, as belonging to base-sixteen alone.

Hexadecimal Numeration in Powers

As a possible hexadecimal numeration scheme, one might try the following terms.

hex = 10! rex = $100! = 10!^2$ mex = $1000! = 10!^3$ bimex = $1.000.000! = 10!^6$ trimex = $1.000.000.000! = 10!^9$

As one studies in more detail the duodecimal and the hexadecimal numeration systems, the shortcomings of base-ten become more evident. Dozeners and Hexadecimalists should depend on the base of ten in no way whatever, as we attempt to build up these more perfect systems in base-twelve and base-sixteen. H. C. C.

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ANNUAL MEETING. (Continued from p. 7)

New dozenal groups in Canada, Italy, Mexico, discussed. No action.

Discussed Ralph Beard's suggestion to establish a permanent headquarters for DSA some where in the midwest or Rocky Mountain area. Action deferred.

President Charles S. Bagley announced the appointment of Ralph H. Beard as Director of Personnel, and of Stan Bumpus as Associate Director of Personnel for Student Members.

At 1180 April 9;5%0 (1968 April 9, 1140 Hours) it was moved by Beard and seconded by Handy that the 1180 Annual Meeting of the membership of DSA at the O'Hare Concord Motor Inn, near Chicago, Illinois, stand adjourned sine die. Carried.

By seven gro (1400 Hours) DSA checking out at the motor inn, and dispersing to New York, Canada, New Mexico, California, and Dr. Moon to New Zealand via London.

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(In the next issue will appear Letters to the Editor and Comment from alert dozenalists. Speak your piece. Mail to The Duodecimal Bulletin associate editor, 10 State Street, Council Bluffs, Iowa 51501.)