COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	X	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3Ɛ2*	Two ft. eight in.	*2;8'*
19Ɛ	*1000*	Eleven ft. seven in.	*Ɛ;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
 12 ) 30  + 5
  12 ) 2  + 6
       0  + 2     Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by X, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or X.

Numerical Progression

1	One		
10	Do	*;1*	Edo
100	Gro	*;01*	Egro
1,000	Mo	*;001*	Emo
10,000	Do-mo	*;000,1*	Edo-mo
100,000	Gro-mo	*;000,01*	Egro-mo
1,000,000	Bi-mo	*;000,001*	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	Ɛ
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
Ɛ	1X	29	38	47	56	65	74	83	92	X1

The Duodecimal Bulletin

Whole Number *30*

Volume *16*, No. *1*
October 1962 (*1176*)

THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island 4, New York. F. Emerson Andrews, Chairman of the Board of Directors; Charles S. Bagley, President; Ralph H. Beard, Editor. Separate subscriptions $2.00 a year, 50¢ a copy.

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All figures in italics are duodecimal.

ALAMOGORDO MEETING OF THE BOARD OF DIRECTORS 12 APRIL 1962

At the suggestion of Chairman Emeritus George S. Terry, this meeting of the Board was held at the Desert Aire Motel in Alamogordo, New Mexico. This city is the home of President Charles S. Bagley, - and he and Mr. Terry staged an impromptu duodecimal demonstration there in October 1961, with such good effect that Mr. Terry favored further cultivation of the possibilities.

Mr. Bagley reserved accommodations for the Directors at the luxurious two-story Desert Aire Motel which has a beautifully decorated spacious lounge, a lovely dining room, and excellent cuisine. The delights of its heated outdoor pool were much enjoyed.

The directors began to foregather in the early part of the week to participate in the program of tours of the scenic and scientific wonders of the locality, that President Bagley had arranged.

Among them were visits to the High Altitude Astronomical Observatory on Sacramento Peak, with its background of snow-covered ten-thousand-foot mountains of the Sacramento Range, and to the White Sands National Monument in the Tularosa Valley. In addition to the blazing white gypsum dunes, in this valley are also located the Holloman Air Force Base and the White Sands Missile Range. The drifting dunes uncover historic relics from time to time, and these recoveries are housed in the reservation's interesting museum, reminding one of the historic revelations from melting glaciers.

There was a public meeting in the splendid new Sierra High School on Wednesday Evening through the courtesy of Principal Walter Wier. Charles Bagley presided, and the features of the meeting were talks by Henry Churchman on "A New Approach to Unity in Weights, Measures, Time and Angles, and by F. Emerson Andrews on the history of the Society.

The formal meeting of the Board of Directors occurred Thursday Morning, April 12th. Chairman Andrews called the

Meeting to order at 9:00, suggesting that the reading of the minutes might be dispensed with, as these were adequately reported in the Duodecimal Bulletin.

The attendance at the meeting was exceptionally full, those present in addition to Chairman Andrews being:

Directors	Bagley	Humphrey
	Beard	Linton
	Churchman	Terry
	Handy	and Member Eugene Scifres.

The year's activities were covered in the report of Secretary Beard as follows. It is to be remembered that our report for 1960 gave us a membership of 82. The related figure this year is 116. This total is composed of 70 members, 12 student members, total 82 members; 13 aspirants, 21 student aspirants, total 34 aspirants. This is our largest total enrollment so far, - and it represents only those in good financial standing. The gain in the membership for the year, of 34, is attributable largely to the excellent publicity duodecimals have enjoyed.

First, the syndicated column of Arnold Hagen, INFORMATION FREE, has run an item in the prominent newspapers of eight localities such as Long Beach, Calif.; Oak Park, Ill.; and St. Petersburg, Fla. In each of these appearances, the response has run from 15 to 30 a day for about a week, making a sizable total.

Then, Mr. Brice Shaw, a math instructor of the Central High School of Flint, Mich., had nearly all of his pupils mail requests for duodecimal information. There has been about 300 such requests, and from them we have secured 10 student aspirants. We have corresponded with Mr. Shaw and Mr. Hagen, and they know of our appreciation.

Then, one of the world's outstanding mechanical engineers wrote a paper for the magazine POWER, which discoursed very clearly on the merits of the metric system, but said that, before we went overboard about the decimetric system, we had better investigate the merits of an octal metric system, or, more especially, a duodecimal metric system. This paper was briefed for an editorial reprinting in Product Engineering, and has resulted in many inquiries.

The consulting engineer who wrote the article is Theodore Baumeister, executive head of the Engineering Dept. of Columbia University, and editor, or consulting editors, of nearly every mechanical engineering handbook. I wrote to Mr. Baumeister to express our appreciation, and he has joined our Society. We have in our files a copy of his article, and of the issue of Product Engineering which has the abstract.

Perhaps the outstanding publicity event of the year was the publication in the NEW YORKER magazine of a duodecimal article, "Do, Gro, Mo". For many years we have mailed Mr. William Shawn, of their editorial staff, all of our publications, and then, in April 1961, Jamison Handy wrote an excellent letter to the Editor, emphasizing the merits of the duodecimal system.

At the Annual Meeting in Carnegie Center in New York last April, Paul Brodeur, staff writer for the New Yorker, assiduously interviewed every officer of the Society, and every speaker. Later, by telephone and mail, New Yorker staff people checked every detail of his report. And, finally, in the issue of the New Yorker for 5 August 1961, the article appeared.

This was a very fair representation, with none of the irony or sarcasm which we had dreaded, but nevertheless welcomed. The circulation of the New Yorker runs into millions, and this good article has been a tremendous asset. There was no flood of response to its publication. But the character and standing of the respondents has been excellent, and from all parts of the country. Much of this correspondence was addressed to President Bagley, who answered every inquiry at length with care and skill. As a result, we have enlisted a number of fine people in our undertaking.

Another important element in our good publicity was the publication of Numbers, Please, by F. Emerson Andrews. This fine book addressed to young people, devotes considerable text to number bases, and the whole final chapter is on duodecimals. We are indebted to Mr. Andrews for far more than this attractive book. His position of great influence puts him into intimate touch with people of the forward echelons, and he constantly introduces duodecimals into their discussions.

Furthermore, in January we had another fine bit of unanticipated publicity. The Listener, a magazine published weekly by the British Broadcasting Co., had in its issue of 25 January 1962, a two-page article, "Twelves and Tens," by A. C. Aitken, Prof. of Mathematics at Edinburgh University, and member of the Duodecimal Society of Great Britain. Dr. Aitken has been a duodecimal enthusiast for many years. His strongly duodecimal paper has sparked further discussions in the columns of the Listener since then. All of this is excellent nourishment for us and for the Duodecimal Society of Great Britain.

There had been only one issue of the Duodecimal Bulletin so far for 1961. Since then another issue, Vol. *15*; No. *2*; has been released which contains an index of Volumes *10* to *15*; inclusive, facilitating the binding of these twelve issues into a third volume. The costs of the Bulletin have grown alarmingly. The bill for the past issue was $754. For this reason, we are restricting its distribution to those clearly interested, in order to make our publications dollars go farther.

The demand for our literature has been heavier than ever. We have been compelled to print another 5M of An Excursion in Numbers, Mr. Andrews' fine article. This is our 9th printing, and we will probably need another this year.

One interesting development of the year is the publication of the magazine, Recreational Mathematics, by Joseph S. Madachy, Idaho Falls, Idaho. It is as fresh and refreshing as pussywillows in Spring, and completely unstodgy. It pays a lot of attention to material on duodecimals, and Mr. Andrews has written several articles for it. Several of its issues carried a formal advertisement of the Manual of the Dozen System, but the response from them has been unnoticeable.

One of the continuing pleasures of this office is the correspondence with our members. For many, this forms their major contact with the Society. This year, ten aspirants have gone through their tests and advanced to full member status.

Sales of the Manual of the Dozen System are slowly rising. We have not been accorded adequate reviews and notices of this important work in the magazines, scientific or other. But its total sales to date are about 400, and a reprint may be necessary in six years, which would furnish us with the opportunity to correct some of the regrettable errors in this edition.

Treasurer Humphrey reported receipts for the year 1961 of $1340 against expenditures of $1710. The deficit of $370 reduced our reserves to $6600. Since the coming years project ever-increasing budgets, every effort must be made to increase the Society's income.

President Bagley advised the Board that William Crehl, an associate on the missile development project, has found available time on the Univac 1103-A Digital Computer to develop expanded tables of the natural functions. These have one-sixth of the interval of the Terrytables. An initial run showed that the Single Precision Routine produced no improvement on Mr. Terry's tables, except as to the finer steps. Now Mr. Crehl has done the tables of Sines and of Cosines on the Double Precision Routine. These are now available. Only minor corrections have developed, but we now possess these tables in expanded form with quite fine intervals.

Secretary Beard reported that no change in our official personnel was contemplated. But Chairman Andrews drew attention to the fact that one of the offices specified in our constitution was unfilled, - that of Vice-President. Mr. Henry C. Churchman was elected to that office, with the special assignment of heading a 3-man committee, of his own selection, for elaborating the By-Laws of the Society. These measures of our corporate structure have been until now embraced in the provisions of our constitution, - and should be formally and separately covered as our functioning routine.

Mr. Andrews also advised that he has agreed to continue as Chairman of the Board for the ensuing year, but that, effective next year, he wished to be relieved of that office, and that other provision should be made at that time.

Mr. Churchman then spoke on his motion for changes in the conditions and grades of membership, and the dues involved. His ideas had been embodied in a written proposal, copies of which had been furnished to each of the Directors.

The discussion became general. From the discussion the shape of the changes desired began to emerge:

Elimination of the grade of Aspirant. All applicants are to become Members upon acceptance.

The grade of Senior Member is to be introduced to include those who have demonstrated their knowledge of duodecimals.

The grade of Fellow is to remain as applying to those of our Members who have been recognized as having made major contributions to the literature, the application, or the general development of the duodecimal system.

Voting privileges are to be limited to Senior Members and Fellows.

Fees and dues and student status are to be covered in the By-Laws.

The By-Laws are to provide for the following changes, as of January 1st, 1963,

1. No initiation fee.
2. Dues $6.00 a year.
3. Dues for Students $3.00 a year. Students are to be entered as Student Members upon completion of the tests.
4. Memberships of any grade become life-memberships when donations, in excess of dues, amount to $150. Thereafter no dues apply.
5. Subscriptions to the Duodecimal Bulletin are to be $1.00 per issue, though supplied to members free.
6. A copy of the Manual of the Dozen System is supplied to new members without charge.

Copies of the amended constitution and of the by-laws are to be supplied to all of the membership as soon as practical.

As there was no further business to be acted upon at this time, the Board expressed its appreciation of the splendid sponsoring of this meeting by George Terry, and of the completeness of President Bagley's organization of the hospitality of Alamogordo, - and Chairman Andrews declared the meeting adjourned.

THE ANNUAL MEMBERSHIP MEETING

The Annual Meeting of the membership was held 17 May 1962 in the Carnegie International Center in N.Y.C. In the absence of the President and the Vice-President, Secretary Beard presided.

Prior to the meeting, the Directors Dinner at the Hotel Beekman Towers was greatly enjoyed by all present: Mr. and Mrs. Andrews, Peter B. Andrews, Kingsland Camp, and Mr. and Mrs. Beard.

Secretary Beard opened the meeting at 20:40 with a cordial welcome to the modest attendance, and a review of the year's activities. This review has been detailed in the report of the Alamogordo Meeting, and need not be repeated here. This applies, as well, to the report of Treasurer Humphrey which followed.

Chairman Andrews narrated the transactions of the Board at Alamogordo, and the amendments made in our constitution, which have considerably changed the grades of membership. The new grades are Members, Senior Members, and Fellows, - the grade of Aspirant having been eliminated. Except for Students, the requirement of passing initial tests has been omitted, and new entrants are admitted as Members. Senior Members are those Members who have demonstrated their competence in duodecimal computations. Fellows are those Members who have been recognized as having made major contributions to the literature, the application,or the general development of the duodecimal system.

Chairman Andrews reported that Henry C. Churchman had been elected Vice-President, heading a committee responsible for drawing up the By-Laws of the Society.

The meeting proceeded to reelect the Directors of the Class of 1962 as the Class of 1965, and Van Allen Lyman, who was appointed last year a Director of the Class of 1964 was duly elected to that Class.

As an important news-item for dozeners, Mr. Beard called attention to the action of Pope John in approving the programming of the consideration of a fixed date for Easter by the meeting of the Ecumenical Council scheduled for October 11th, as a step toward the establishment of a perpetual world calendar. This is the most important action favoring the World Calendar, sponsored by Elisabeth Achelis, and the World Calendar Association, in recent years.

The feature of the meeting was a talk by Richard Stern, about the change to a decimal currency by the Union of South Africa. He played the official recording prepared by the Union for the education of its people in connection with the change. After "Decimal Joe," Mr. Stern commented on Earl Russell's endorsement of duodecimals. The Earl has recently been much in the news, (in his 90th year), about his active resistance to the use of the atom bomb by England.

With the conclusion of Mr. Stern's talk, the meeting was formally adjourned for the usual refreshments, and the lingering friendly discussions shared by all.

NEW DOZENAL PROPOSAL OF CHARLES H. BRITTAIN

Another Dozener who has worked on dozenals independently through long years has recently joined our ranks. Charles H. Brittain, P. O. Box 81, Lanham, Md., became interested in duodecimals in 1937. He believes that it is necessary to use an entirely new set of number symbols and names for duodecimals in order to avoid confusion in the long period when both decimals and duodecimals will be in simultaneous use.

Mr. Brittain is a building contractor, and this may have had some bearing on his choice of the 1/32-inch as the unit of his linear scale, since standard screw threads and sizes can be fitted to this scale without change. His co-ordination of mass and capacity occurs with a unit of volume of approximately 1½ quarts, this volume of water weighing about 3 pounds.

The measures of time in his system embrace a day of twelve double-hours, and a year of twelve months. The coins of his duodecimal currency increase in a series of steps of three and four, to a dollar of a gross of cents.

Mr. Brittain has sought a fitting duodecimal symbol, and finds most promise in hexagonal designs. He has submitted a number of such designs in striking colors.

PALINDROMIC NUMBERS

by

F. Emerson Andrews

A palindrome, says Mr. Webster, is "a word, verse, or sentence, that is the same when read backward or forward." He gives HANNAH as an example, and with a bit of forcing:

LEWD DID I LIVE & EVIL I DID DWEL

I have seen pleasanter ones, including:

STEP ON NO PETS

If you don't worry about where the words break, one of the most amazing is the "general Goethals" palindrome:

A MAN, A PLAN, A CANAL, PANAMA

Numbers, too, can be palindromic, having the same values backward or forward. We get them in dates, usually every century plus decade: 1661, 1771, 1881. For many persons who read this, there will be two quite close together, 1991 and 2002.

Incidentally, we just passed a "palindromic" year of a different sort: 1961 happens to read the same upside down. But 1881 was both truly palindromic and read upside down without change.

With words, the creation of a palindrome is a mental exercise requiring no little ingenuity. But with numbers, certain of the mathematical processes appear to produce palindromes automatically. In any number base using zero and place value, any number to which its reversal is added trends toward a palindromic form.

As an exercise in duodecimals, take for examples the year in which America was discovered, *X44* (1492) and the year these words were written, *1176* (1962):

	X44	
	44X	*1176*
	1292	*6711*
Add again:	*2921*	*7887*
	3ƐƐ3	

"Very cute," says the sophisticated mathematician, "but a bit obvious. What else could the numbers do when reversed? Look at it algebraically, with each column treated separately:

a	b	c	...x
...x	c	b	a
a + ...x	b+c	b+c	a ÷ ...x "

Quite true. A number of any length to which its reversal is added will assume full palindromic form at once, provided there is no carry in any column. But if there is a carry, it will "trend" toward palindromic form, but the full palindrome may be near or very far. In the Columbus date it was near. But take such a two-place number as *ƐΧ; 12* summations are required before the first full palindrome, *689Χ6Χ986*, is produced.

Presumably randomly-chosen long numbers would have a greater chance than small numbers of including one or more carries, and would therefore, on average, require many more operations to produce full palindromes. But a little experimenting--witness the *ƐΧ* example--will demonstrate that performance is quite erratic.

It is not even true that only sums involving no carries produce perfect palindromes in the first step. See:

2Ɛ	*607*
Ɛ2	*706*
121	*1111*

One would anticipate that the larger the number base, the easier (on averages) the process of finding a full palindrome for a given quantity. The original number might be one or more places shorter, and the probability of carries ought to decrease. For example, would a substantial sample of random quantities expressed as duodecimals produce palindromes with fewer summations than the same quantities decimally expressed?

The erratic behavior of some of these numbers complicates any experimental proof. For example, I believe the first hundred (*84*) quantities require *118* summations decimally, only *8Ɛ* duodecimally. (Numbers like, 3, 33, 303 are already palindromic and are scored at zero.) But if one goes on to the first gross (*100*) quantities, the decimal system slips into many low summations in the quantities

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MORE ABACUS

by Tom B. Linton

11561 Candy Lane, Garden Grove, Calif.

Supplementing Robert W. Edelen's excellent paper on a Duodecimal Abacus, perhaps some notes of mine might be useful.

Both Japanese and Chinese abacus (soroban) users I have observed in this area use a rather large "desk" abacus which they operate with almost unbelievable speed — not unlike a fast typist or "hot" pianist as far as fast finger movements are concerned. Obviously these movements automatically parallel the thought processes. The abacuses I have seen being *used* had one bead up and four down at each upright. One instruction booklet "Maid (sic) in Japan" *Soroban for Everyone* illustrates such an arrangement, as does the more extensive book Takashi Kojima's *The Japanese Abacus, Its Use and Theory*, $1.00, Charles E. Tuttle Company, Rutland, Vermont and Tokyo, Japan. This latter book includes some of the history and present status of the abacus and the type of tests given to applicants for the various grades of abacus operators' licenses in Japan.

Mr. Kojima identifies the modern Japanese abacus as the one-up and four-down arrangement. He identifies the one-up and five-down arrangement as "Older-Type Japanese Abacus", and the round-bead two-up and five-down arrangement as "Modern Chinese", although I have observed Chinese shopkeepers (presumably modern) using the one-up four-down soroban. Soroban seems to be the Japanese term for the double-cone shape of bead abacus which they developed from the rounded-bead abacus of the Chinese. This change is said to increase speed of operation.

Both types of sorobans (Japanese abacus) are available in this area (Southern California); I get most of mine at the Nippon Book Company, 364 East First Street, Los Angeles, Calif. From there I obtained the #1301, 11-place (Ɛ;-place) sorobans distributed at the board meeting in Alamogordo. The white beads make for easy "reading" in the small pocket size, and the price was a modest $1.45 without instructions. Both smaller and larger abacuses are available. Twenty one-place units simplify multiplying and dividing by providing for entering the multipliers or divisors at the left end, while doing the arithmetic at the right end.

Note that dozenal computations are performed without modifications on the "one-up, five-down" abacus with the same efficiency as the shopkeepers demonstrated in decimal work on the "one-up, four-down" abacus. While the "one-up, five down" arrangement does not satisfy Mr. Edelen's criteria for the least number of beads for a duodecimal abacus, I suggest it to be quite acceptable, and in my personal opinion, easier to learn and faster to operate than the "least beads" abacus of Edelen's article. However, I speak as a rank amateur in abacus operation; at the proficient level I am only an observer (after I watch an expert on the abacus, my wife says "now pull your eyeballs back in, Pop").

While the abacus has been termed a digital computor, I rather think of it as comparable to the memory portion of the computor. On the abacus we can record the progress of computation, although at the end we have only the result, and no memory of the steps. The "memory" merely holds the last operation until the next move is made, then holds that. Certainly the abacus is nearer in concept and function to the digital computor than to the analog or "Slide Rule" computor. Only a mistake or conscious "rounding off" can cause other than a precise answer.

Whether Edelen's "efficient-bead" or my "efficient-operation" abacus is used, "efficient duodecimal calculations" are a cinch with a moderate amount of "efficient-practice".

Duodecimally....GO!

PALINDROMIC NUMBERS

(Continued from page X)

just over 100, and the advantage shifts--*152* summations decimally but *16X* duodecimally.

I shall leave the logical and the experimental proof of this theorem to the reader with more talent and time. It is doubtful whether palindromic numbers have any mathematical significance; but they may offer useful exercise in duodecimal arithmetic, and some of the exasperating charms of the better forms of solitaire.

DUODECIMALS AND BRITISH COINAGE

"The Case against Decimalisation," by A. C. Aitken, Professor of Mathematics in the University of Edinburgh, Oliver and Boyd, Edinburgh and London, 1962. 22 p.

A proposal is before the British Parliament, and in some of the Dominions as well, to decimalize the British currency, which presently is partly duodecimalized, with 12 pence to the shilling, and 20 shillings to the pound. When this reviewer was in London in December, 1961, he discussed with Mr. Brian R. Bishop, President of the Duodecimal Society of Great Britain, measures which might be taken to counter this proposal and use it as an opportunity for explaining the advantages of dozenal counting.

What part of the British Society may have had in important new developments is not known, but opposition to the Parliamentary proposal has sprung up, not only from the quarters always opposed to change, but in terms of adoption, perhaps gradually, of the whole duodecimal system.

One of the most important strokes in this direction was a talk over BBC (printed in "The Listener" for 25 January 1962) by A.C. Aitken, a mathematics professor in Edinburgh University, in which Professor Aitken not only opposed the decimalization of the currency, but proposed its further duodecimalization (12 shillings to a new pound, perhaps called a royal), with a view to present ease in handling and eventual adoption of a complete duodecimal system. He has now issued the pamphlet noted above, in which these ideas are solidly presented.

"The Case against Decimalisation" begins with a substantial history of numeration, the accidental origin of the ten base, sexagesimals among the Sumerians and Babylonians, problems of the circle, the slow introduction of the "Arabic" numerals and place value, early objectors to the decimal base, and some counter proposals. He comes then to the duodecimal system:

> That the system of Leonardo is not the final word but that the duodecimal system with appropriate notation is appreciably superior again, is held at the present time by a relatively small number of persons in the whole world. (It is true, of course, that the

vast majority of the rest are entirely ignorant of the whole issue.) One may mention the Duodecimal Society of America, counting in its membership distinguished actuaries and other prominent men--and it is symptomatic that such a society should take its origin in a country devoted since 1786 to decimal currency, though not, and this is again symptomatic, to decimal metric; there is a Duodecimal Society of Great Britain, recently founded, small in membership and resources . . .

He observes that the coming of electronic computers, though based on a binary system, will transform not merely arithmetic but education in arithmetic. "A younger generation, familiar with binary and octonary systems as well as with decimal, will be sure to ask: What, reckoned in terms of time and efficiency, is the worth of the decimal system, and is there a better?"

He believes that Russia and America, almost simultaneously, will conclude that there is a better, while Britain, "who of all nations in the world *are* in the special and most favorable position to make the change, may be left behind." This especially favorable position is based largely on the immense familiarity all persons in England necessarily have with elementary duodecimal computation in making change with 12 pence to the shilling.

At this point Professor Aitken comes to the heart of his immediate proposal. Instead of decimalizing the present currency, make one simple change. Keep twelve pence to the shilling, and add to it twelve shillings to a new pound, to be called a royal. He does not claim the proposal is original, but believes it will have substantial present and long-range advantages. The pound as well as the shilling would acquire splendid divisibility. In the usual British notation, the new pound would then be R1:0:0. Its quarter, R0:3:0. Its eighth, R0:1:6. No new symbols for old ten and eleven would be immediately introduced, but by this "phased gradualness, an interregnum of years of quiet habituation and consolidation, we may bring in the more efficient system."

The pamphlet presents briefly the general arithmetic of duodecimals, stressing, as do most treatises, the superiority of duodecimal expression of fractionals. Although difficulties of a change-over are recognized, Professor Aitken estimates the inefficiency of the 10-based system at a

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ABOUT THAT LENGTH

by Henry Clarence Churchman
404 Wickham Bldg., Council Bluffs, Iowa

This short exposition of a systemic dozenal scale of dimensions attempts to avoid all duodecimal signs and symbols other than the dozenal (;) point to indicate a twelve-base number, for it is anticipated that in time it will be distributed in every clime (and especially ridiculed and disparaged) among decimalists. Let them at least understand what they dislike.

Even among duodecimalists there are a few who fear that because the dozenal point is a semicolon it might be taken for a punctuation sign occurring in the middle of a sentence containing dozenal figures. Yet the rank and file of Europeans and Americans for centuries have accepted the punctuation sign of a period or comma within or without a sentence as their decimal point. And a colon (:) for the sexagesimal point.

As this paper was prepared for the rank and file it will employ the semicolon as a dozenal point when it is found between digits wheresoever. And will continue to use the period (.) in America as a decimal point when it appears between digits anywhere.

In that manner in this paper it is not difficult for any reader to distinguish a twelve-base from a ten-base numeral, and a dozenal fraction from a decimal fraction. The reverse is equally true.

For instance, the denominate number *$10;00* is not just ten dollars but is here equal to one dozen dollars because of the dozenal point. And *$16;00* is equal to one dozen and six dollars---actually eighteen dollars, decimally speaking.

So when you see a dozenal (;) point between digits (thus, *7;5*, pronounced seven dozenal point five or 7-5/12), think in dozens only. It will open your mind to new horizons; yet you will find no difficulty in returning, I assure you, to the classic ten-base arithmetic and decimal points whence we have set out.

We are merely confronting here another arithmetical language.

LENGTH

Almost every idea of man is concerned with length. Often a distance is deliberately indefinite or helplessly compared with the average, as when one uses the adjective big, or little, or tall or short.

When a young boy describes a fish he caught he finds an advantage by adding the modifier "about" to the dimension indicated between his two hands, which he describes as "that length." Inches and centimeters are farthest from his mind.

And the farther he gets away from witnesses, the more he tends to lean upon "about" and to avoid anything like exactness. Many adults are no different.

But every century seems to find man getting more precise about his dimensions. Accuracy in distances is one of the marks of a scientist; a very stern requirement of architects and builders.

Manufacturers now are rejecting bolts and screws which fail to meet a specification within a ten-thousandth of an inch, and the tendency is to tighten up.

"About that length" can be quite inaccurate. When we start describing a dimension in Krypton 86 wavelengths for the dozenal system we must not deal through intermediaries. Let us freely discard them all and tie with Kr 86 only.

In the latter part of the eighteenth century a totally new system of measurements was conceived. It tossed overboard all empirical dimensions then known to mankind. It brought into being a basic length described as a quarter meridian circle of the earth. What could appear more appealing and scientific?

A ten-millionth part of this arc extending from equator to north pole was given the name of "meter." From the definition of a meter it is plain that it was subservient---a minute fractional part of another unit. The basic unit of length was unquestionably the quarter meridian circle of the earth. From this grand arc the meter was to be derived.

ADVANCEMENT OF THE METER

But when a king is deposed for weakness, as sometimes happens, the able servant becomes master. A great circle of the earth, even a quarter great circle, appears too vast for immediate comprehension. The common man can not "look at it."

Another weakness, we now are told, was the shape of the earth's crust which is still changing.

When the more practical meter was made the supreme ruler in place of the quarter meridian circle of the earth, its dimension was scratched on changeless metal and it was housed in royal style in an international building with temperature controls long before common citizens began to enjoy air conditioning.

It was in, and the yard was out, but not quite all the way. The yard had something the meter lacked.

The English people time and again have heroically demonstrated that they do not know when they are licked and thereby win many international conflicts. They retained their Imperial Yard for over a gross-year after the meter was initiated.

In fact they did not transfer allegiance from the Imperial Yard as their basic unit of length and relate their yard to the meter until the middle of 1959.

By that time the meter itself was no longer recognized as an exact ten-millionth part of a quarter meridian circle of the earth nor as that certain bar housed at Sevres, near Paris, in an international enclave.

The international meter, as was true of the deposed Imperial Yard, was never more than an empiric unit itself, important simply because it is an agreed length. The bar now has been displaced by an agreed number of certain light waves---and that number could and might be adjusted for scientific gains in microscopic dimensional accuracy.

This essay was written to suggest a definite move in that direction. It might or might not come in our lifetime. Here time is not of the essence.

ECLIPSE OF THE METER

When in 1959 the six English speaking industrial nations in concert announced an agreement that the length of one international yard shall after July 1, 1959, equal 91.44 centimeters, the length of the international foot became exactly 30.48 centimeters. And the international inch was thereby made equal to 2.54 centimeters.

This was expected to produce harmony in dimensions among all English speaking persons. It did more. It solemnly recognized by international agreement that there now is no accepted basic dimension save the meter. It now is agreed that all other dimensions are dependent on and draw their exactness and their accuracy from the agreed length of the meter. The royal crown, Caesar's scepter, now rests on the meter.

If the meter be now slightly modified (see "A Comparison") to aid the precision demands of scientists in microscopic work, the English system of measurements would follow, in view of its international tie with the meter. Nevertheless, do not expect any adjustment in the length of the meter in this century. Scientists in the meantime might come to rely wholly upon the "metronic system" of dimensions hereinafter described for their minute measurements in terms of Krypton 86.

The Roman government and its people may be said to have reached their highest peak of greatness at a time when nothing was settled, all was turmoil, and there was constant danger of invasion and national destruction.

When peace came to that empire, when "all the world" was at peace with Rome, its fall was unknowingly set in motion. Is it possible now, with peace established between the meter and the inch, that the 18th Century meter dimension is about to undergo its own metamorphosis, to be displaced by a simple "dometron" described below, the "New Yard" and the "New Meter?"

But, first, let us study the meter.

WHAT IS A METRIC SYSTEM?

The present (1962) basic system of measures owes its growth not to its dimension lengths at all. Its usefulness is due entirely to its parallel relationship with the number system now in use---ten-base. For pointing the way to a metric system---for factually demonstrating that the people will accept a whole new change---we shall be forever indebted to France. Observe now its pattern.

Starting with the meter as its final basic length unit, this dimension under French statutory law was increased tenfold by attaching a prefix called "deka" to the word "meter" so that one "dekameter" was a unit of length equal to ten meters.

In similar fashion higher units of measurement were obtained by adding other Greek words for hundred and thousand as a prefix to the word "meter." This produced "hektometer" and "kilometer" equal, respectively, to 100 meters and 1000 meters.

Too, the meter was subdivided into ten equal parts again and again. This was accomplished by adding Latin words for ten and hundred and thousand as a prefix to the word "meter" so as to obtain the units of "decimeter" and "centimeter" and "millimeter."

Because we have accepted the metric system definitions for these fractions without qualification we think of "deci" as one-tenth part rather than ten units envisioned in the designation of "decimal numeration." At the same time we think of "duodecimal notation" as twelve-base.

Further, we think of "centimeter" as a hundredth part of a meter, even though a "centipede" suggests a hundred legged creature. Too, we think of a "millimeter" as a thousandth part of a meter although the Latin "milli" is the word for a thousand, and a "millipede" is supposed to have something like a thousand legs---not one-thousandth of one leg, heaven forbid!

Droll as metric terms might have been originally declared to be, everyone who has accepted metric system definitions knows their intended values today. This thought is mentioned here in passing, to prepare one against possible prejudices when the dozenal systemic measurements are set forth. Of course dozenal systemic names will at first seem obnoxious. And certainly they will be unusual.

What first we abhor, then tolerate, eventually we embrace. So say philosophers in every age. Habitual use of doremique terms lessens the horror of them, weakens the will to resist them, and obscures the awareness of a new expression. Dare to use them and live in a brave new world.

METRONIC SYSTEM OF DIMENSIONS

Now that a dozenal system of notation is on the horizon, and many persons are already employing dozens in counting, in packaging, and in shipping, it might be thought suitable here to take a glance at what this writer earlier called the "dometron" or "eminante" unit of length (1.1176 meter), and now adds two synonyms, the "New Yard" and the "New Meter."

Each smaller or larger systemic unit of length (each unit composed of *7;5* times a different power of a dozen Krypton 86 wavelengths) will now bear three or four names---select the unit or synonym that pleases you alone, that helps you understand a particular length. The length's the thing.

All of them are considered exactly equal to their mates. And everyone of them is divided not into ten but into twelve equal parts. This brings to each of them the benefits of a dozenal system of counting. This the present (1962) meter lacks.

For example, one-twelfth dometron is equal to one "metron." One-twelfth of the New Meter is equal to one "New Decimeter." One-twelfth of the New Yard is equal to one "New Palm." And one "metron," one "New Decimeter," and one "New Palm" are equal to each other. They are mates. They are buddies, they are inseparable equals.

Let us now examine the exact lengths of these proposed new dozenal units of dimension in relation to something every working scientist will recognize.

First let us note that, even as the original metric system unit was related to a quarter meridian circle of the earth, so the basic dozenal unit of length initially may be said to have been an Assumed Great Circle of the Earth. This basic distance extending all the way around the earth is designated by this writer the equal of one "dominante" unit of length.

It might be considered a dominant dimension. (See Nante Units). It is slightly less than the equatorial great circle of the earth today. In Krypton 86 dimensions it is exactly equal to *7 500 000 000 000;0*, or *7;5* (7-5/12) times the twelfth power of twelve, such wavelengths.

But even as the Quarter Meridian Circle yielded place to its child, the meter, so this Assumed Great Circle of the Earth is seen by this writer as too cumbersome for accurate duodecimal measurements, and it is here suggested that it be withdrawn in favor of the more scientific and exact basic dimension of *75 000;0* (seven dozen and five dozen-dozen-dozen) Krypton 86 wavelengths, the equal of one "metron."

The "metron" is systematically related to the "dominante" unit of length above described. For example, if we multiply the "dominante" unit by the negative fourth power of twelve (12^{-4}) a length is obtained equal to *750 000 000;0* (decimally, 7-5/12 times the 8th power of a dozen) Krypton 86 wavelengths. This intermediate unit of length is designated by this writer as one "domimetron" or "Nante" or "New Mile" or "New Kilometer." It is about 1.931 present (1962) kilometer.

SOME ASPECTS OF THE METRON

If we multiply one "dominante" unit of length by the negative eighth power of twelve (12^{-8}) we can achieve what is here called one "edominante" unit, a dimension equal to 3-2/3 inches or 44 lines, equated with *75 000;0*, or *7;5* (7-5/12) times the fourth power of twelve, Krypton 86 wavelengths. (See Metronic Units and Nante Units).

In order to achieve systemic exactness both up and down the range of values, this "edominante" unit of dimension is here said to equal *75 000;0* Krypton 86 wavelengths EXACTLY. This dimension can be stated with great exactness in six significant figures, the last four figures being ciphers.

It is this edominante unit of dimension, the basic "metron," which when cubed (one cubic metron) equals the metronic unit of capacity, here called one "jon" or "New Pint" or "New Liter".

The "jon" is equal to about 4/5 of the present (1962) liter, slightly over 800 cubic centimeters. It is some 3 cubic inches greater than a "fifth" of bottled spirits as now sold in the U.S.A. bearing the imprint of "4/5 Quart" on container.

Initially, a "jon" of volume might be bottled and sold, as is the Fifth, in any of the 50 States of America today by simply imprinting the legend "4/5 Liter" instead of "4/5 Quart" on its container. It would contain one "jon" by volume. The "4/5 Liter" imprint is entirely legal throughout the U.S.A. under our federal statutes making use of the metric system units lawful.

And it is this exact volume of water, one "jon," the weight of which equals the new unit of mass. It is the weight of one cubic metron of water under certain controlled conditions. This weight is equal to one metronic "Kalbab" or cubic stone, here also called the "New Pound" of the "New Kilogram" or one "Khal" or "calc."

NANTE SYSTEM OF DIMENSIONS

Let us now consider the NANTE dimension heretofore described as one New Mile or one New Kilometer. If we multiply the Nante dimension by twelve raised to the fourth power (12^4) we will find the equal of one "dominante" dimension or an Assumed Great Circle of the Earth heretofore described.

And if on the other hand we multiply the Nante dimension by the negative fourth power of twelve (12^{-4}) we would find the equal of one "edominante" unit, or three and two-thirds inches equal to ninety-three thousand one hundred thirty-three and one-third microns (each micron here again equated with 1.65131 Krypton 86 wavelength), each "edominante" equal to *75 000;0* such wavelengths.

This "edominante" unit is the exact equal of one "metron" mentioned above. If you will refer to the Nante Units table it will be observed that each unit is twelve times larger than its next adjoining smaller unit.

Naturally many persons will find it difficult to memorize both the new dimensions and the systemic new names. Others will want to avoid a systemic appearance at all costs, and to use ancient unit names. For all of them we have here associated each new systemic dimension unit with its nearest unit of length in English or French current use.

It may prove to be a bridge upon which traffic passes both ways in coming and going from and to the systemic units. If this helps you, use it. If it confuses, drop back to the metron or the Nante to visualize systemic dimensions in dozen-based notation.

It is very probable that in some lines of endeavor concerned more with dimensions below the "New Chain" or "New Dekameter" and down to the "New Decimeter" or the "New Palm" (indeed, to the New Angstrom), the systemic units of metron, dometron, etc., will be utilized.

On the other hand, dealing more with distances ranging around the earth, such as transportation and travel, trucking and air and rail lines, and navigation on the high seas, persons are more likely to rely more upon the Nante systemic units. Both are systemically related, and are interchangeable within limited range between themselves.

Only when the Old Eighteenth Century meter is eventually adjusted scientifically and considered equal to 1 651 310.00 round decimal unit wavelengths in place of the present (1962) accepted 1 650 763.73 Krypton 86 waves, will the "New Meter" and the "New Yard" or "dometron" or "eminante" equal 44 international inches or 1.1176 meter EXACTLY and all greater units would be exactly interchangeable with Old English and Old Metric adjusted units of length.

At all events, decimally speaking, if one "emimetron" is equal to 89 Krypton 86 wavelengths EXACTLY (which would appear to be as near as any human being can repeatedly determine its length in Krypton 86 glowing light), then one "dometron" or "New Meter" or "New Yard" must be found equal to 1 845 504 Krypton 86 light waves, again decimally speaking, or *750 000;0* wavelengths dozenally.

The "New Micron" (shown m *0;00001*, called one "eremitron"), which is equal to *0;75* (seven dozen five per gross of one single) Krypton 86 light wave, is smaller than one single such wavelength (decimally 89/144 part or 5/8 wavelength).

The "New Angstrom" is one-twelfth the size of the "New Micron" or *0;075* part of one single Krypton 86 wavelength (decimally 89/1728 or 0.0515 fraction in place of the present 0.1651 wavelength.

A COMPARISON OF MODIFIED OLD AND NEW METRIC UNITS ADJUSTED TO KRYPTON 86 WAVELENGTHS DECIMALLY

1 Old Meter will be	1 651 310 waves.	New Meter is equal to	1 845 504 waves.
1 Old Decimeter	165 131 "	New Decimeter "	153 792 "
1 Old Centimeter	16 513.1 "	New Centimeter "	12 816 "
1 Old Millimeter	1 651.31 "	New Millimeter "	1 068 "
1 Old Micron	1.65131 wave.	New Micron "	89/144 wave.
1 Old Angstrom*	0.165131 "	New Angstrom "	89/1728 "

*The above modified Old Angstrom dimension (0.165131 Krypton 86 wavelength), if properly rounded to four significant figures (0.1651) is no different than the present Angstrom measurement (0.165076373) properly rounded to the same four significant figures (0.1651).

METRONIC PREFIXES AND POWERS AND DENOMINATIONS

Prefixes				Powers	Denominations		
bimi,	equals the	sixth	power of one dozen,	$10;^{6}$	1 000 000;0,	called	1 bimo
remi	"	fifth	" "	$10;^{5}$	100 000;0	"	1 gromo
domi	"	fourth	" "	$10;^{4}$	10 000;0	"	1 domo
mi	"	third	" "	$10;^{3}$	1 000;0	"	1 mo
re	"	second	" "	$10;^{2}$	100;0	"	1 gro
do	"	first	" "	$10;^{1}$	10;0	"	1 do
unit	"	zero	" "	$10;^{0}$	1;0	"	1 unit
edo	" negative	first	" "	$10;^{-1}$	0;1	"	1 edo
ere	" "	second	" "	$10;^{-2}$	0;01	"	1 egro
emi	" "	third	" "	$10;^{-3}$	0;001	"	1 emo
edomi	" "	fourth	" "	$10;^{-4}$	0;0001	"	1 edomo
eremi	" "	fifth	" "	$10;^{-5}$	0;00001	"	1 egromo
ebimi	" "	sixth	" "	$10;^{-6}$	0;000001	"	1 ebimo

Note that "e" is the initial letter of every prefix indicating a negative exponent. Also, that one bimo multiplied by one ebimo reduces the exponent to zero power of one dozen---1 unit.

METRONIC UNITS RELATED DOZENALLY TO KRYPTON 86 WAVES AND NEW YARD OR NEW METER

Systemic Dozenal Units		Equated With	English Units	French Units
1 domimetron,	m 10 000;0 or	750 000 000;0 Kr.86	New Mile	New Kilometer
1 mimetron	m 1 000;0	75 000 000;0	New Furlong	New Hektometer
1 remetron	m 100;0	7 500 000;0	New Chain	New Dekameter
1 dometron	m 10;0	750 000;0	New Yard	New Meter
1 metron	m 1;0	75 000;0	New Palm	New Decimeter
1 edometron	m 0;1	7 500;0	New Quan	New Centimeter
1 eremetron	m 0;01	750;0	New Karl	New Millimeter
1 emimetron	m 0;001	75;0		
1 edomimetron	m 0;0001	7;5		
1 eremimetron	m 0;00001	0;75		New Micron
1 ebimimetron	m 0;000001	0;075		New Angstrom

METRONIC UNITS RELATED TO NANTE, NEW MILE, AND NEW KILOMETER DOZENALLY

Systemic Units	Metronic	Doremic	English Unit	French Unit
1 domimetron,	m 10 000;0 or	1 Nante	New Mile	New Kilometer
1 mimetron	m 1 000;0	1 edonante	New Furlong	New Hektometer
1 remetron	m 100;0	1 erenante	New Chain	New Dekameter
1 dometron	m 10;0	1 eminante	New Yard	New Meter
1 metron	m 1;0	1 edominante	New Palm	New Decimeter
1 edometron	m 0;1	1 ereminante	New Quan	New Centimeter
1 eremetron	m 0;01	1 ebiminante	New Karl	New Millimeter
1 emimetron	m 0;001	75;0 Krypton 86 waves		
1 edomimetron	m 0;0001	7;5 " "		
1 eremimetron	m 0;00001	0;75 " wave		New Micron
1 ebimimetron	m 0;000001	0;075 " "		New Angstrom

NANTE UNITS RELATED TO FOOT (INCH, LINE) AND MICRON

Nante Unit and Value		International Feet	Microns
1 dominante,	N 10 000;0	or 131 383 296	or 40 045 628 620 800
1 minante	N 1 000;0	10 948 608	3 337 135 718 400
1 renante	N 100;0	912 384	278 094 643 200
1 donante	N 10;0	76 032	23 174 553 600
1 Nante	N 1;0	6 336	1 931 212 800
1 edonante	N 0;1	528	160 934 400
1 erenante	N 0;01	44	13 411 200
1 eminante	N 0;001	44 inches	1 117 600*
1 edominante	N 0;0001	44 lines.	93 133-1/3

*This number of microns (1 117600), in these seven significant figures in the metronic system of dimensions, is here equated with 1 845 504 Krypton 86 light waves. This action, entirely within the duodecimal system of dimensions, will increase the length of the Eighteenth Century meter by less than 550 krypton waves, the modified meter being then equal to 1 651 310 wavelengths in place of the presently agreed 1 650 763.73. It is necessary to mention the matter here but it is not imperative that any change be made in the present length of the meter at this time, since it eventually will be replaced by the New Meter, the New Kilometer, and the New Domikilometer. Thus, the 1962 dometron is here defined as a dimension equal to 1 845 504 wavelengths of the orange line of glowing krypton 86 under certain controlled conditions, speaking decimally; and equal to 750 000;0 (seven dozen five dozen-dozen-dozen-dozen) such wavelengths, speaking dozenally. This exacting dimension is here defined equal to one New Yard and also one New Meter, in the metronic systemic arrangement of dozenal units of length.

CASTING OUT Ɛ's

by Wm. Bruce Knapp
President Radicands Math Club
San Fernando High School
San Fernando, California

One of the worst aspects of working with any math system is the possibility of error. Working the problem a second time is tedious and serves little purpose since the error (if there is one) will often be repeated. The system proposed in this article provides a fast check of the duodecimal basic operations (addition, subtraction, multiplication, and division) by a method similar to "casting out nines" in the decimal system. The principle applied to any number system of base n will work for "casting (n-1)'s" in that system. Therefore, for purposes of convention, we shall refer to the check as "casting out Ɛ's." Once mastered, the system provides an almost foolproof check of ten or twenty-digit multiplication or division problems in a matter of seconds.

ADDITION AND SUBTRACTION

Steps	Addition	Subtraction
1. Add the digits of each addend (subtrahend and minuend for subtraction) to obtain the sums a_1, a_2, etc.	3Ɛ4 = 16 Ӿ97 = 22 +116 = 8 13Ӿ5	7Ӿ80 = 21 −Ɛ12 = 12 6Ɛ6Ӿ
2. Add the digits of each sum (a_1, a_2, etc.) and continue until each sum is one digit.	3Ɛ4 = 16 = 7 Ӿ97 = 22 = 4 +116 = 8 = 8 13Ӿ5	7Ӿ80 = 21 = 3 −Ɛ12 = 12 = 3 6Ɛ6Ӿ
3. Add the one-digit sums (subtract them if it is a subtraction problem) and add the digits in the result until a one-digit number is produced.***	3Ɛ4 = 7 Ӿ97 = 4 +116 = +8 13Ӿ5 17 = (8)	7Ӿ80 = 3 −Ɛ12 = −3 6Ɛ6Ӿ 0 = (0)
4. This one-digit result will equal the one-digit number found by adding the digits of	3Ɛ4 Ӿ97 +116 13Ӿ5 = 17 = (8)	7Ӿ80 −Ɛ12 6Ɛ6Ӿ = 29 = Ɛ = (0)

the sum (or difference, in subtraction problems) if the problem is correct. If Ɛ is obtained as a one-digit sum, convert it directly to zero (see example in the subtraction problem).

Both examples are correct because the numbers shown agree. (Don't forget that if Ɛ is obtained for either of the circled numbers it should be changed to 0.)

***Note: If a negative difference is obtained in step 3 for a subtraction check, add Ɛ to the number; i.e., if -4 had been the one-digit number produced by step 3 in the subtraction example above, we would add Ɛ to -4 and obtain 7.

MULTIPLICATION AND DIVISION

Steps

1. Find the one-digit sums of the digits in the multiplicand and multiplier (divisor and quotient for division) as in steps 1 and 2 for addition.

```
Multiplication                          Division

  27,18Ɛ,540=32=5                            4X3;X=23=5
       x 848=18=9                     110)53,21X;0
  1891Ɛ768
   X46Ɛ994                                =2=2
    1891Ɛ768
  18933941X80
```

2. Multiply the one-digit sums of the multiplicand and multiplier (multiply the one-digit sums of the divisor and quotient for division); add the digits of this product until a one-digit number is obtained.

```
  27,18Ɛ,540=5                               4X3;X =5
         848=9                        110)53,21 ;0  2
  1891Ɛ768     39=10=(1)                           X=(X)
   X46Ɛ994
    1891Ɛ768
  18933941X80
```

3. This one digit number will equal the one-digit number found by adding the digits of the product.

```
  27,18Ɛ,540                                 4X3;X
         848                          110)53,21 ;0=19=(X)
  1891Ɛ768
   X46Ɛ994
    1891Ɛ768
  18933941X80=48=10=(1)
```


This method is not foolproof (for instance, if the last zero in the product of the multiplication example above had been deleted, it would not have changed the one-digit sum) but it will work Ɛ0 e/g of the time. The proof is based upon the modulo concept in basic number theory.

By mathematical induction we may easily show that the special case of the Fermat-Euler theorem

$$10^k \equiv 1 \pmod{Ɛ}$$

is true when k is any integer. By multiplying both sides by a constant, n, we obtain

$$n10^k \equiv n \pmod{Ɛ}.$$

Since any duodecimal number may be expressed as $n10^k + r$, [for example, 1X65 would be $1(10^4) + X(10^3) + 6(10^2) + 5$] by the addition axiom we have

$$n10^k + \quad \equiv \quad + r \pmod{Ɛ}.$$

Since n and r represent the digits, the above equation tells us that a number ($n10^k$ r) is congruent to the sum of its digits (n + r) for modulo Ɛ. If we add, subtract, or divide two numbers, a and b, to obtain a sum, difference, product or quotient respectively, c, by the basic axioms of number theory the sum[1] of the modulos of a and b must equal the modulo of the sum[2] c. Since the modulo of the digits' sum of a, b, etc. is the same as the modulo of a, b, or c, it follows that these one-digit sum modulos must correspond as do the modulo of the entire numbers.

----

1. sum, difference, product or quotient as the case may be.
2. sum, difference, product or quotient respectively.

DUODECIMALS AND BRITISH COINAGE

(Continued from page 12)

wastage of "350 hours in every 1000" as compared with use of a 12-based arithmetic. In the second millenium, now closely approaching, there is bound to be incredible technological progress, and such inefficiency may well be regarded as intolerable.

> The transition from a defective system of numeration and metric, to a new one, attainable by easy and gradual phase, will be viewed in remote retrospect as one of the most ordinary pieces of belated tidying-up that ever was delayed for so long past its due time.
>
> -- F. Emerson Andrews

THE MATHEMATICS JOURNAL

Under a date line of 18 October 1962, there was issued Volume 1, Number 1, of the Mathematics Journal. This lively and refreshing publication is produced by a group of the mathematics students of the Dwight D. Eisenhower High School, of Blue Island, Illinois.

To add to our pleasure, one of its first editorials is a clear and strong statement of the advantages of duodecimals. The editorial regrets the delay in putting the powerful advantages of a duodecimal metric system into the service of man.

We delight in the initiative of these young mathematicians. The flame of their fervor will brighten the road forward. Viva!

1962 AND ITS DIGITS SALUTE DO

by Charles W. Trigg
Los Angeles City College

$$-1 + 9 + 6 - 2 = 10 \qquad 1(9 - 6)!(2) = 10$$

$$(1)(9) + 6/2 = 10 \qquad 1(!\sqrt{9})(6/2)! = 10$$

$$1 + \sqrt{9} + 6 + 2 = 10 \qquad 1(9)(6)(.\dot{2}) = 10$$

$$1^{9}(6)(2) = 10 \qquad (.\dot{1})(9)(6)(2) = 10$$

$$-\begin{vmatrix} 19 \\ 26 \end{vmatrix} = \begin{vmatrix} 21 \\ 69 \end{vmatrix} = -\begin{vmatrix} 62 \\ 91 \end{vmatrix} = \begin{vmatrix} 96 \\ 12 \end{vmatrix} = 10$$

In duodecimal notation this is the year 1176, and

$$1(-1 + 7 + 6) = 10 \qquad 11 - 7 + 6 = 10$$

$$-1(1) + 7 + 6 = 10 \qquad -1 - 17 - 6 = 10$$

In each of the equations the digits are in the same order as they are in the year. In the determinants the digits are in clockwise order.

NEW STANDARD ATOMIC NUMBER 12 FOR CARBON

Reported by Peter B. Andrews
115 Linden Lane, Princeton, N. J.

A new basis for the expression of atomic weights has been adopted by the International Union of Pure and Applied Chemistry, parallelling the action of the International Union of Pure and Applied Physics.

The action adopts the exact number 12 as the assigned atomic (nuclidic) mass of the principal isotope of carbon, carbon 12. This eliminates the difference that has existed between atomic weights used by chemists and those used by physicists.

An official table of the atomic weights, based on the new standard, was published in October, 1961, by the IUPAC, with the recommendation that it be placed in universal use as of January, 1962.