Advance copy

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THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island 4, New York. George S. Terry, Chairman of the Board of Directors. F. Emerson Andrews, President. Ralph H. Beard, Editor.

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THE SOCIETY'S ANNUAL AWARD

by F. Emerson Andrews

The officers of the Society have pre-empted this page, over the protests of Editor Beard, to make a special announcement. The Awards Committee, consisting of George S. Terry, chairman, Eugene M. Scifres, and Lewis Carl Seelbach, has unanimously conferred the Society's 1947 Award upon Ralph H. Beard "for his significant contributions to duodecimal literature, especially in the field of weights and measures, and for his invaluable services to the Society as its Secretary Treasurer and as founder and Editor of its Bulletin."

Probably every member of the Society has had reason to be personally aware of Mr. Beard's high talent and unflagging labors, and will share your officers' undisguised pleasure at this year's choice. Fewer of us, perhaps, knew how recently Mr. Beard became deeply interested in duodecimals, and how rapidly he has become a master in several of its special fields.

In September, 1941, Mr. Beard addressed a letter to the author of NEW NUMBERS in care of its publisher, with carbons to four persons mentioned in that book. He saw special need, in view of the international situation, to educate the public in the advantages of duodecimals. "Is there an organization for this purpose?" he wrote. "If no organization exists, I would like to help, unprofessionally, in starting one. . ."

Three of the persons he thus addressed were Mr. Terry, the late Mr. Seely, and myself. We had already formed a correspondence group which called itself the Duodecimal Society of America, in informal existence since 1935. We needed the fresh enthusiasm and the organizing genius of Mr. Beard, and we invited him to join us. The place he has made for himself in the five years which have followed is known to us all.

In giving Ralph H. Beard its award for 1947, the Society honors itself and belatedly recognizes a great contribution of time and talent to its organization and its continuing work.

OFFICIAL TRANSACTIONS

The Annual Meeting of the Society, as well as a meeting of the Board of Directors, was held at the Russell Sage Foundation Building, 130 East 22nd St., New York City, on the 23rd of January, 1947.

The reports of officers and committees indicate that the Society is making steady gains in membership, in finances, in the exploration of duodecimal history, and in public effect. Especially notable is the more general inclusion of material on duodecimals in the new mathematical texts being published.

Plans for the year include enlargement of the membership, the formation of a mathematics research group, the publication of a bibliography of duodecimal tables, and the preparation of further duodecimal articles for publication.

The officers and committee chairmen for 1947 are:

Chairman of the Board	George S. Terry, 507 Main St., Hingham, Mass.
President	F. Emerson Andrews, 34 Oak St., Tenafly, N.J.
Vice-President	Paul E. Friedemann, 4124 Greensburg Pike, Forest Hill, Pittsburgh 25, Pa.
Secretary-Treasurer	Ralph H. Beard, 20 Carlton Place, Staten Island 4, N.Y.
Committee on Awards	George S. Terry, as above.
Mathematical Recreations	Mary Lloyd, 2304 Forest Park Ave., Baltimore 7, Md.
Mathematical Research	Harry C. Robert, Jr. 1683 Johnson Rd.N.E., Atlanta 6, Ga.
Member Qualification	Mary Lloyd, as above.
Nomenclature	Kingsland Camp, Room 1009, 393 7th Ave., N.Y. 1, N.Y.
Public Information	Jamison Handy, Jr. 3422 West 59th Place, Los Angeles 43, Calif.
Weights and Measures	Ralph H. Beard, as above.

THE FIBONACCI SERIES

by Harry C. Robert, Jr.

Leonardus Pisanus (Leonardo of Pisa,) frequently called Fibonacci, very probably was one of those individuals whose fate it was to live centuries ahead of their times. Whether such a fate is a matter of ill fortune for the individual or one of extreme good fortune for the rest of mankind is an interesting matter for speculation which we will leave to the philosophers.

Living during the first blush of dawn of the Renaissance, in the first two or three decades of the 13th century, Fibonacci's history is still obscured by the dark cloud of ignorance that marked the Dark Ages. Certainly he was the greatest problem solver of his day. As an Algebraist, his methods and his notation were not improved on for several centuries. Without doubt he played a very important part in the introduction of the Arabic numerals and the most important Hindu "zero" into Europe.

Although many of our energetic delvers into the past have traced some of his problems and methods to the Egyptians, Arabs, and others, and therefore suspect him of having a secret source for others which they have been unable to trace, they have not yet been able to rob him of all claim to fame as an original mathematician. Possibly he was only a messenger boy, but if so he did his job well, delivered the right message at the right place and at the appropriate time, and although we do not know how he departed this life, he left the world his debtor.

Today we have little to remind us of Fibonacci and his important part in the development of mathematics, - a paragraph or two in our larger encyclopedias, sometimes a footnote in popular mathematical works, little else. But for one thing the world might forget this worthy pioneer. This is the happy circumstance that has resulted in his nickname, "Fibonacci," being applied to that most interesting summation series -

Base X	1	1	2	3	5	8	13	21	34	55	89	144	233	. . .
Base XII	*1*	*1*	*2*	*3*	*5*	*8*	*11*	*19*	*2X*	*47*	*75*	*100*	*175*	. . .

where each term is the sum of the two preceding ones.

The important place of this series in phyllotaxis, its other close relationships to nature, such as the ratio of planetary periods, sunflower and seashell spirals, and the use of its terms to form approximations of the Golden Mean, have been rather fully covered in very capable works, including Lick's "Recreations in Mathematics:" Northrop's "Riddles in Mathematics;" A. H. Church's "On the Interpretation of Phenomena of Phyllotaxis;" and Jay Hambidge's "Practical Applications of Dynamic Symmetry."

Thus far, we have divided our worthy algebraist into several phases and left him to the mercy of the philosopher, historian, botanist, and artist. But he was a mathematician. True, most works that mention the Fibonacci series say that it has little use in mathematics, and so far I have found only one instance,- Uspensky and Heaslet use it in the proof of Lamés' Theorem, in their work on "Elementary Number Theory." Is it possible that we have overlooked something? Let us look at the series.

Term No.	Value	Factors	New Primes
1	1	-	-
2	1	-	-
3	2	-	2
4	3	-	3
5	5	-	5
6	8	2^3	-
7	11	-	11
8	19	3·7	7
9	2X	2·15	15
X	47	5·Ɛ	Ɛ
Ɛ	75	-	75
10	100	2^4·3^2	-
11	175	-	175
12	275	11·25	25
13	42X	2·5·51	51
14	6X3	3·7·3Ɛ	3Ɛ
15	Ɛ11	-	Ɛ11
16	1 5Ɛ4	2^3·15·17	17
17	2 505	31·95	31·95
18	3 XƐ9	3·5·Ɛ·35	35
19	6 402	2·11·2Ɛ1	2Ɛ1
1X	X 22Ɛ	75·147	147
1Ɛ	14 701	-	14 701
20	22 X00	2^5·3^2·7·1Ɛ	1Ɛ
21	37 501	5^2·18X1	1 8X1
22	5X 301	175·375	375
23	95 802	2·15·45·91	45·91
24	133 Ɛ03	3·11·25·1Ɛ5	1Ɛ5
25	209 705	-	209 705*
26	341 608	2^3·5·Ɛ·51·27	27
27	54Ɛ 111	-	54Ɛ 111*
28	890 719	3·7·3Ɛ·133Ɛ	1 33Ɛ
29	1 21Ɛ 82X	2·75·Ɛ561	Ɛ 561
2X	1 XƐ0 347	Ɛ11·2097	2 097
2Ɛ	3 10Ɛ Ɛ75	5·11·6X1X1	6X 1X1*
30	5 000 300	2^4 3^3·15·17·8Ɛ	8Ɛ

* These numbers only partially checked as Primes.

Now it is not the purpose of this paper to demonstrate any startling new proofs or truths. The fact is, I don't have any. Perhaps there are none. However this series either involves or produces a great many interesting relationships, several of them being of an unexpected character. There is no evidence in the ordinary literature to indicate that these relationships have been investigated to any extent. The probability is that most of them lead into blind alleys, yet certain of the characteristics of this series could conceivably produce the key to unlock some of our unsolved problems concerning primes.

So, without proof, and as briefly as possible, I will summarize a few of the most obvious relationships. The proof of some is self evident, others are a little more subtle, and probably some are beyond our ability to prove at this time. If any one or more of these simple characteristics strikes your fancy, investigate it carefully yourself. You will at least find it as interesting, instructive, and as profitable as your favorite revolving numbers.

Recurrences and Other Relationships

1. Terminations: The do and unit place figures repeat every two-do terms. This charecteristic is true only for the dozenal base. On base ten, units (only) cycle in sixty terms.

2. Designating the number of the term as "n" and the corresponding value by F_n, we have:

$$F_n = F_{n-1} + F_{n-2}, \text{ the basis of the series.}$$

And also

$$F_n = (2)F_{n-2} + F_{n-3}$$

$$F_n = (3)F_{n-3} + (2)F_{n-4}$$

$$F_n = (5)F_{n-4} + (3)F_{n-5}$$

Or in general

$$F_n = (F_{p+1})F_{n-p} + (F_p)F_{n-p-1}$$

3. In addition to the foregoing there appear to be a great number of other relationships not directly connected to those given in (2). As just one example:

$$F_n = (4)F_{n-3} + F_{n-6}$$

4. Returning to the general form given in (2) and making n even (or 2n) and p = n, we obtain:

$$F_{2n} = (F_{n+1})F_n + (F_n)F_{n-1}$$

$$= F_n(F_{n+1} + F_{n-1})$$

or, since $F_n = F_{n+1} - F_{n-1}$,

$$F_{2n} = F_{n+1}^2 - F_{n-1}^2$$

and the corresponding expression for odd terms is:

$$F_{2n+1} = F_{n+1}^2 - F_n^2$$

5. In like manner:

$$F_{3n} = F_{n+1}^3 - F_{n-1}^3 + F_n^3$$

where, since $F_{n+1}^3 - F_{n-1}^3$ is divisible by $F_{n+1} - F_{n-1}$, which is equal to F_n, it is evident that F_{3n} is divisible by F_n.

6. From *(4)* and *(5)* we see that F_{2n} is divisible by F_n, and F_{3n} is likewise divisible by F_n. From observation of the series, it is evident also that F_{kn} will be divisible by F_n, and also by F_k. It appears that F_{kn} will also be divisible by $F_k(F_n)$, if k and n are relatively prime. In general, every prime appearing as a factor of one of the terms will recur in regular cycles. That is, if a factor, p, first occurs in the mth term, it will recur in the 2mth, 3mth, kmth terms and will not be a factor of any other terms in the series.

7. Because of *(6)*, for $n > 4$, no term in the series will be a prime unless n is a prime. If n is prime, and the corresponding term of the series is not, it must be composed of prime factors which have not previously occurred in the series. For example, look at term *17*.

8. With the exception of the *6*th and *10*th terms, it appears that every term of the series, starting with the *3*rd, introduces one or more new prime factors into the series.

9. With the exception of the *5*th term, each new prime factor introduced will be of the form $kn \pm 1$, where n is the number of the term and apparently k can be any integer.

χ. It appears that sooner or later every prime will appear as a factor of one of the terms of the series. From *(9)*, any prime p must appear not later than the (p+1) term.

£. For $n < 10$, $F_n < n^2$

For $n = 10$, $F_n = n^2 = 10^2 = 100$

For $n > 10$, $F_n > n^2$.

From the general appearance of the series, it is very unlikely that any other terms of the series are squares. In fact, with the exception of *2*, *3*, and *5*, it is unlikely that powers of the prime factors will be greater than one. At points in the series where it might be expected that squares of *7* or *11* would occur, only the first power shows up. This statement has, however, by no means been proven.

10. Uspensky and Heaslet use, as an exercise in their "Elementary Number Theory," the proposition that any two consecutive terms of the Fibonacci series are relatively prime. Not only is it apparent from the foregoing that this is true, but also that any three consecutive terms of the series are also relatively prime.

11. Several writers have shown that any two consecutive terms produce a convergent of the continued fraction:

$$\frac{\sqrt{5}+1}{2} = 1 + \frac{1}{1+}\ \frac{1}{1+}\ \frac{1}{1+}\ \cdots = \frac{F_n}{F_{n-1}}$$

or $\frac{F_n}{F_{n+1}}$ produces convergents of the continued fraction:

$$\frac{\sqrt{5}-1}{2} = \frac{1}{1+}\ \frac{1}{1+}\ \frac{1}{1+}\ \frac{1}{1+}\ \cdots$$

12. Many other relationships involving the same ratios can be developed by such expressions as:

$$\frac{F_n}{F_{n\pm 2}},\ \frac{F_n}{F_{n\pm 3}},\ \text{or in general } \frac{F_n}{F_{n\pm p}}.$$

13. It also appears that the series may be defined as an approximation in whole numbers (integers) of a geometrical progression whose first term is:

$\frac{\sqrt{5}+1}{2\sqrt{5}}$, and whose ratio is $\frac{\sqrt{5}+1}{2}$. That is, it may be written: $\left|\frac{\sqrt{5}+1}{2\sqrt{5}}\right|$, $\left|\frac{1}{\sqrt{5}}\left(\frac{\sqrt{5}+1}{2}\right)^2\right|$, $\left|\frac{1}{\sqrt{5}}\left(\frac{\sqrt{5}+1}{2}\right)^3\right|$, etc.,

where $|A|$ indicates the nearest integer to A.

14. By means of *(13)* any term may be found by the expression:

$$F_n \text{ is the nearest integer to } \frac{1}{\sqrt{5}}\left(\frac{\sqrt{5}+1}{2}\right)^n.$$

Another expression which does not involve approximations, but is somewhat more complicated for large values of n, arises from the expansion of:

$\left(\frac{\sqrt{5}+1}{2}\right)^n$, which may be put in the form: $A + B\left(\frac{\sqrt{5}+1}{2}\right)$, in which it may be shown that $A = F_{n-1}$ and $B = F_n$. That is:

$$\left(\frac{\sqrt{5}+1}{2}\right)^n = F_{n-1} + F_n\left(\frac{\sqrt{5}+1}{2}\right)$$

15. Relationships similar to those in (*13*) and (*14*) can be set up for the expression $\frac{\sqrt{5}-1}{2}$.

16. It should be noted that:

$$\frac{\sqrt{5}+1}{2} - \frac{2}{\sqrt{5}+1} = \frac{(\sqrt{5}+1)^2 - 4}{2(\sqrt{5}+1)} = 1 .$$

Also that: $\frac{2}{\sqrt{5}+1} = \frac{\sqrt{5}-1}{2} = \frac{\sqrt{5}+1}{2} - 1$. And,

using our series:

$$\frac{F_{n+1}}{F_n} - \frac{F_n}{F_{n+1}} = 1 + \frac{(-1)^n}{F_nF_{n+1}} .$$

17. A few of the other relationships between terms are:

$$F_{n-1}(F_{n+1}) - F_n^2 = (-1)^n$$

$$F_{n-2}(F_{n+2}) - F_n^2 = (-1)^{n+1}$$

$$F_{n-3}(F_{n+3}) - F_n^2 = (-1)^{n+2}(4) ,$$

or in general:

$$F_{n-p}(F_{n+p}) - F_n^2 = (-1)^{n+p-1}(F_p)^2 .$$

And a still different series:

$$F_{n-1}(F_n) - F_{n-2}(F_{n+1}) = (-1)^n$$

18. Finally, just one of the many expressions for the sum of the first n terms of the series:

$$\sum_{z=1}^{z=n} F_z = F_{n+2} - 1 .$$

The foregoing should suffice to introduce to those not already familiar with the series of Fibonacci some idea of its possibilities, its parallel to our system of all numbers, and its wealth of recurrences and other relationships. Perhaps I have omitted or have not yet found some relationship of greater importance than any of those mentioned. Perhaps you will find it.

In all fairness, it should be mentioned that such an eminent authority as Dr. Eric T. Bell considers any investigation of the Fibonacci series aside from Leonardo's original use in a problem concerning the progeny of rabbits, as much ado about nothing, or at least very little. Dr. Bell, in his "Development of Mathematics," also says that there is a very extensive literature on the subject, much of which borders on the eccentric, and then adds that the most interesting modern work is one by E. A. Lucas (in French) published 1878. Since none of the other readily available reference works even hints that there is a literature on the subject, I am left in the dark, both as to what extent various phases of this interesting series have been thoroughly investigated, and also as to what Dr. Bell considers eccentric. He could mean me.

. . . . .

Mr. Terry adds an artistic footnote to Paragraph (*1*) of Mr. Robert's article:

"Not only do the last two figures cycle in *20* terms, but they are arranged in an interesting symmetry about the *10*th, *30*th, *50*th, etc., terms. Travelling in either direction from these terms, the series of the endings of the odd terms is *01*, *02*, *05*, *11*, *2X*, *75*, and repeat. The arrangement of the even terms is in complementary pairs, as *47* and *75*.

"Note also that the *1*st differences (and the *2*nd, and the nth) repeat the original series."

It is an interesting co-incidence, that Lewis Carl Seelbach submitted a comment, paralleling Mr. Robert's Paragraph (*4*), at about the same time that Mr. Robert's paper was received.

MICROFILM OF TERRY'S "DUODECIMAL ARITHMETIC"

Eugene M. Scifres has had George S. Terry's "Duodecimal Arithmetic" recorded on microfilm with the consent of the author and of the publishers, Longmans, Green & Co., and has donated a copy of the film to the Society. The film reproduces the book, page by page, each exposure occupying a space 1" x 1" on the same standard 35 mm. film as fits the Leica, Contax, etc. This film will fit the standard slide-film projector, and does not require a special projector, or "reader," as it is sometimes called. For lectures and group discussions, this will make an excellent visual aid, and we are deeply grateful to Mr. Scifres for his foresight and generosity in making this donation to the Society.

DUODECIMAL LOG SINES

Arguments are Temins (.01°) and Minettes (.001°), paralleled with Degrees and Minutes of Arc.

Degrees	Minutes	Temins	Minutes 0 / Minettes 0	12.5 / 1	25 / 2	37.5 / 3	50 / 4	62.5 / 5	75 / 6	87.5 / 7	100 / 8	112.5 / 9	125 / X	137.5 / £	150 / Ʊ			
00		00		9. 8X61	X. 0281	X. 2220	X. 36X1	X. 4793	X. 5640	X. 6332	X. 6£00	X. 759£	X. 7££1	X. 8554	X. 8X5X	2£	87	30
02	30	01	X. 8X5X	X. 9316	X. 974£	X. 9£4£	X. X319	X. X67X	X. X926	X. £111	X. £408	X. £6X6	X. £96X	£. 0018	£. 0272	2X	85	
05		02	£. 0272	£. 04£6	£. 0728	£. 094X	£. 0£61	£. 1164	£. 135X	£. 1547	£. 1727	£. 18£X	£. 1X86	£. 2047	£. 2201	29	82	30
07	30	03	£. 2201	£. 2370	£. 2515	£. 2674	£. 280X	£. 295£	£. 2XX6	£. 3029	£. 3167	£. 32X1	£. 3413	£. 3541	£. 3667	28	80	
10		04	£. 3667	£. 3789	£. 38X8	£. 3X03	£. 3£17	£. 4028	£. 4136	£. 4241	£. 4346	£. 4447	£. 4546	£. 4643	£. 4739	27	77	30
12	30	05	£. 4739	£. 4830	£. 4922	£. 4X11	£. 4X£X	£. 4£X5	£. 508X	£. 5171	£. 5252	£. 5332	£. 540£	£. 54X7	£. 5581	26	75	
15		06	£. 5581	£. 5655	£. 5728	£. 57£9	£. 5888	£. 5957	£. 5X24	£. 5X£0	£. 5£76	£. 603X	£. 6102	£. 6184	£. 6245	25	72	30
17	30	07	£. 6245	£. 6304	£. 6383	£. 6440	£. 64£8	£. 6573	£. 6629	£. 66X2	£. 6756	£. 6809	£. 687X	£. 692£	£. 699£	24	70	
20		08	£. 699£	£. 6X4X	£. 6X£8	£. 6£65	£. 7011	£. 7078	£. 7123	£. 7188	£. 7231	£. 7295	£. 7338	£. 739£	£. 7440	23	67	30
22	30	09	£. 7440	£. 74X1	£. 7541	£. 75X1	£. 7640	£. 769X	£. 7737	£. 7793	£. 782£	£. 7887	£. 7921	£. 7977	£. 7X10	22	65	
25		0X	£. 7X10	£. 7X66	£. 7X£X	£. 7£51	£. 7£X4	£. 8037	£. 8088	£. 811X	£. 816X	£. 81£X	£. 824X	£. 8299	£. 8328	21	62	30
27	30	0£	£. 8328	£. 8376	£. 8403	£. 8450	£. 8499	£. 8525	£. 8570	£. 85£7	£. 8642	£. 8688	£. 8712	£. 8757	£. 87X0	20	60	
30		10	£. 87X0	£. 8824	£. 8868	£. 88£0	£. 8933	£. 8975	£. 89£7	£. 8X39	£. 8X7£	£. 8£00	£. 8£40	£. 8£80	£. 9000	1£	57	30
32	30	11	£. 9000	£. 9040	£. 907£	£. 90£9	£. 9138	£. 9176	£. 91£3	£. 9231	£. 9268	£. 92X6	£. 9322	£. 935X	£. 9395	1X	55	
35		12	£. 9395	£. 9411	£. 9447	£. 9482	£. 94£8	£. 9532	£. 9568	£. 95X1	£. 9616	£. 964X	£. 9683	£. 96£7	£. 972X	19	52	30
37	30	13	£. 972X	£. 9762	£. 9795	£. 9808	£. 983X	£. 9870	£. 98X2	£. 9914	£. 9945	£. 9978	£. 99X7	£. 9X18	£. 9X48	18	50	
40		14	£. 9X48	£. 9X78	£. 9XX8	£. 9£17	£. 9£47	£. 9£76	£. 9£X4	£. X013	£. X041	£. X06£	£. X099	£. X106	£. X133	17	47	30
42	30	15	£. X133	£. X160	£. X189	£. X1£6	£. X222	£. X24X	£. X276	£. X2X1	£. X309	£. X334	£. X35£	£. X385	£. X3£0	16	45	
45		16	£. X3£0	£. X416	£. X440	£. X466	£. X490	£. X4£5	£. X51X	£. X543	£. X568	£. X590	£. X5£5	£. X619	£. X641	15	42	30
47	30	17	£. X641	£. X664	£. X688	£. X6X£	£. X712	£. X735	£. X758	£. X77X	£. X7X1	£. X803	£. X825	£. X846	£. X868	14	40	
50		18	£. X868	£. X889	£. X8X£	£. X910	£. X930	£. X951	£. X971	£. X992	£. X9£2	£. XX12	£. XX31	£. XX51	£. XX70	13	37	30
52	30	19	£. XX70	£. XX8£	£. XXX£	£. X£09	£. X£28	£. X£47	£. X£65	£. X£83	£. X£X1	£. X£££	£. £018	£. £036	£. £053	12	35	
55		1X	£. £053	£. £071	£. £08X	£. £0X6	£. £103	£. £120	£. £138	£. £154	£. £170	£. £188	£. £1X4	£. £1££	£. £217	11	32	30
57	30	1£	£. £217	£. £232	£. £249	£. £264	£. £27£	£. £295	£. £2£0	£. £306	£. £320	£. £336	£. £350	£. £366	£. £380	10	30	
60		20	£. £380	£. £395	£. £3XX	£. £404	£. £419	£. £431	£. £446	£. £46£	£. £473	£. £487	£. £4X0	£. £4£4	£. £507	0£	27	30
62	30	21	£. £507	£. £51£	£. £533	£. £546	£. £55X	£. £571	£. £584	£. £597	£. £5X9	£. £600	£. £613	£. £625	£. £637	0X	25	
65		22	£. £637	£. £649	£. £65£	£. £671	£. £683	£. £694	£. £6X6	£. £6£7	£. £708	£. £719	£. £72X	£. £73£	£. £74£	09	22	30
67	30	23	£. £74£	£. £760	£. £770	£. £780	£. £791	£. £7X1	£. £7£0	£. £800	£. £810	£. £81£	£. £82£	£. £83X	£. £849	08	20	
70		24	£. £849	£. £858	£. £867	£. £876	£. £884	£. £893	£. £8X1	£. £8X£	£. £8£9	£. £907	£. £915	£. £923	£. £930	07	17	30
72	30	25	£. £930	£. £93X	£. £947	£. £955	£. £962	£. £96£	£. £978	£. £985	£. £991	£. £99X	£. £9X6	£. £9£3	£. £9££	06	15	
75		26	£. £9££	£. £X07	£. £X13	£. £X1£	£. £X26	£. £X32	£. £X39	£. £X45	£. £X50	£. £X57	£. £X62	£. £X69	£. £X74	05	12	30
77	30	27	£. £X74	£. £X7X	£. £X85	£. £X8£	£. £X96	£. £XX0	£. £XX6	£. £X£0	£. £X£6	£. ££00	£. ££05	£. ££0£	£. ££14	04	10	
80		28	£. ££14	£. ££1X	£. ££23	£. ££28	£. ££31	£. ££36	£. ££3X	£. ££43	£. ££47	£. ££50	£. ££54	£. ££58	£. ££60	03	07	30
82	30	29	£. ££60	£. ££64	£. ££68	£. ££70	£. ££73	£. ££77	£. ££7X	£. ££82	£. ££85	£. ££88	£. ££8£	£. ££91	£. ££94	02	05	
85		2X	£. ££94	£. ££97	£. ££99	£. ££X0	£. ££X2	£. ££X4	£. ££X6	£. ££X8	£. ££XX	£. £££0	£. £££1	£. £££3	£. £££4	01	02	30
87	30	2£	£. £££4	£. £££5	£. £££6	£. £££7	£. £££8	£. £££9	£. £££X	£. ££££	£. ££££	0. 0000	0. 0000	0. 0000	0. 0000	00	00	
Minettes			Ʊ	£	X	9	8	7	6	5	4	3	2	1	0	Temins	Degrees	Minutes
Minutes			150	137.5	125	112.5	100	87.5	75	62.5	50	37.5	25	12.5	0			
			DUODECIMAL LOG COSINES															

DUODECIMAL LOG TANGENTS

Arguments are Temins (.01°) and Minettes (.001°), paralleled with Degrees and Minutes of Arc.

Degrees	Minutes	Temins	Minutes 0 / Minettes 0	12.5 / 1	25 / 2	37.5 / 3	50 / 4	62.5 / 5	75 / 6	87.5 / 7	100 / 8	112.5 / 9	125 / X	137.5 / £	150 / Ʊ			
00		00		8. 8X61	X. 0281	X. 2221	X. 36X2	X. 4794	X. 5642	X. 6335	X. 6£04	X. 75X3	X. 7££7	X. 855£	X. 8X66	2£	87	30
02	30	01	X. 8X66	X. 9323	X. 975X	X. 9£5£	X. X32£	X. X692	X. XX10	X. £129	X. £426	X. £707	X. £991	£. 0041	£. 029X	2X	85	
05		02	£. 029X	£. 0525	£. 075X	£. 0983	£. 0£98	£. 11X3	£. 13X0	£. 1590	£. 1773	£. 194X	£. 1£1X	£. 20X3	£. 2260	29	82	30
07	30	03	£. 2260	£. 2414	£. 2581	£. 2724	£. 2883	£. 2X18	£. 2£68	£. 30£3	£. 3237	£. 3375	£. 34£0	£. 3623	£. 3752	28	80	
10		04	£. 3752	£. 387X	£. 39X2	£. 3£03	£. 4021	£. 4138	£. 4250	£. 4361	£. 4470	£. 4578	£. 4681	£. 4784	£. 4885	27	77	30
12	30	05	£. 4885	£. 4983	£. 4X80	£. 4£76	£. 506X	£. 5160	£. 5251	£. 533£	£. 5428	£. 5513	£. 55£8	£. 56X0	£. 5782	26	75	
15		06	£. 5782	£. 5863	£. 5942	£. 5X20	£. 5X£8	£. 5£93	£. 6069	£. 6141	£. 6214	£. 62X6	£. 6376	£. 6446	£. 6514	25	72	30
17	30	07	£. 6514	£. 65X1	£. 666X	£. 6735	£. 67££	£. 6884	£. 6948	£. 6X0£	£. 6X92	£. 6£53	£. 7014	£. 7093	£. 7152	24	70	
20		08	£. 7152	£. 7210	£. 7289	£. 7346	£. 7401	£. 7478	£. 7532	£. 75X8	£. 7661	£. 7715	£. 7788	£. 783£	£. 78£1	23	67	30
22	30	09	£. 78£1	£. 7963	£. 7X14	£. 7X84	£. 7£34	£. 7£X3	£. 8051	£. 80££	£. 8169	£. 8216	£. 8282	£. 832X	£. 8396	22	65	
25		0X	£. 8396	£. 8441	£. 84X7	£. 8551	£. 85£7	£. 8660	£. 8705	£. 8769	£. 8811	£. 8874	£. 8917	£. 897X	£. 8X20	21	62	30
27	30	0£	£. 8X20	£. 8X82	£. 8£24	£. 8£86	£. 9026	£. 9086	£. 9126	£. 9186	£. 9225	£. 9285	£. 9323	£. 9382	£. 9420	20	60	
30		10	£. 9420	£. 947X	£. 9518	£. 9575	£. 9612	£. 966£	£. 9708	£. 9764	£. 9800	£. 9858	£. 98£3	£. 994X	£. 99X6	1£	57	30
32	30	11	£. 99X6	£. 9X40	£. 9X97	£. 9£31	£. 9£88	£. X022	£. X077	£. X111	£. X168	£. X200	£. X255	£. X2X9	£. X342	1X	55	
35		12	£. X342	£. X397	£. X42£	£. X483	£. X517	£. X562	£. X603	£. X656	£. X6XX	£. X741	£. X794	£. X827	£. X87X	19	52	30
37	30	13	£. X87X	£. X911	£. X963	£. X9£6	£. XX48	£. XX9£	£. X£31	£. X£83	£. £015	£. £067	£. £0£9	£. £14X	£. £1X0	18	50	
40		14	£. £1X0	£. £232	£. £283	£. £315	£. £368	£. £3£7	£. £449	£. £49X	£. £52£	£. £580	£. £611	£. £662	£. £6£3	17	47	30
42	30	15	£. £6£3	£. £744	£. £795	£. £825	£. £876	£. £907	£. £958	£. £9X9	£. £X39	£. £X8X	£. ££1£	£. ££6£	0. 0000	16	45	
45		16	0. 0000	0. 0051	0. 00X1	0. 0132	0. 0183	0. 0213	0. 0264	0. 02£5	0. 0346	0. 0397	0. 0428	0. 0478	0. 0509	15	42	30
47	30	17	0. 0509	0. 055X	0. 05X£	0. 0640	0. 0691	0. 0722	0. 0774	0. 0805	0. 0856	0. 08X7	0. 0939	0. 098X	0. 0X20	14	40	
50		18	0. 0X20	0. 0X71	0. 0£03	0. 0£55	0. 0£X7	0. 1039	0. 108£	0. 1121	0. 1174	0. 1206	0. 1259	0. 12X£	0. 1342	13	37	30
52	30	19	0. 1342	0. 1395	0. 1428	0. 147£	0. 1512	0. 1566	0. 15£9	0. 1651	0. 16X5	0. 1739	0. 1791	0. 1825	0. 187X	12	35	
55		1X	0. 187X	0. 1913	0. 1967	0. 1X01	0. 1X56	0. 1XX£	0. 1£45	0. 1£9X	0. 2035	0. 208£	0. 2125	0. 2180	0. 2216	11	32	30
57	30	1£	0. 2216	0. 2272	0. 2309	0. 2364	0. 2400	0. 2458	0. 24£5	0. 2551	0. 25XX	0. 2647	0. 26X4	0. 2742	0. 27X0	10	30	
60		20	0. 27X0	0. 283X	0. 2899	0. 2937	0. 2996	0. 2X36	0. 2X96	0. 2£36	0. 2£96	0. 3037	0. 3098	0. 313X	0. 31X0	0£	27	30
62	30	21	0. 31X0	0. 3243	0. 32X5	0. 3348	0. 33X£	0. 3463	0. 34£7	0. 3580	0. 3605	0. 366£	0. 3716	0. 377£	0. 3828	0X	25	
65		22	0. 3828	0. 3892	0. 393X	0. 39X6	0. 3X53	0. 3£01	0. 3£8£	0. 4019	0. 4088	0. 4138	0. 41X8	0. 4259	0. 430£	09	22	30
67	30	23	0. 430£	0. 4382	0. 4434	0. 44X7	0. 455£	0. 4614	0. 468X	0. 4744	0. 47££	0. 4876	0. 4934	0. 49£0	0. 4X6X	08	20	
70		24	0. 4X6X	0. 4£29	0. 4£X8	0. 5069	0. 512X	0. 51£1	0. 5274	0. 533X	0. 5401	0. 5487	0. 5564	0. 561£	0. 56X8	07	17	30
72	30	25	0. 56X8	0. 5778	0. 5845	0. 5916	0. 59X8	0. 5X7£	0. 5£54	0. 6029	0. 6104	0. 61X0	0. 627X	0. 6359	0. 643X	06	15	
75		26	0. 643X	0. 6520	0. 6604	0. 66X9	0. 6794	0. 6881	0. 698£	0. 6X60	0. 6£52	0. 7046	0. 7140	0. 7239	0. 7337	05	12	30
77	30	27	0. 7337	0. 7438	0. 753£	0. 7644	0. 7750	0. 785£	0. 7970	0. 7X84	0. 7£9£	0. 80£9	0. 821X	0. 8342	0. 846X	04	10	
80		28	0. 846X	0. 8598	0. 8710	0. 8847	0. 8985	0. 8£09	0. 9064	0. 91X5	0. 9339	0. 9498	0. 963£	0. 97X8	0. 9962	03	07	30
82	30	29	0. 9962	0. 9£1X	0. X0X2	0. X271	0. X449	0. X630	0. X820	0. XX19	0. £024	0. £239	0. £462	0. £698	0. £922	02	05	
85		2X	0. £922	0. ££7£	1. 0230	1. 04£5	1. 0786	1. 0X93	1. 11£0	1. 152X	1. 1891	1. 2061	1. 2462	1. 2899	1. 3156	01	02	30
87	30	2£	1. 3156	1. 3661	1. 4005	1. 481X	1. 50£9	1. 5887	1. 657X	1. 742X	1. 8520	1. 89X3	1. £950	2. 3198		00	00	
Minettes			Ʊ	£	X	9	8	7	6	5	4	3	2	1	0	Temins	Degrees	Minutes
Minutes			150	137.5	125	112.5	100	87.5	75	62.5	50	37.5	25	12.5	0			
			DUODECIMAL LOG COTANGENTS															

FERMAT'S FACTORING FORMULA

One of the many advantages of the duodecimal number base is the greater ease in separating prime and composite numbers, and in identifying possible factors and roots through terminal figures.

Extensive lists of primes have been published, the work of D. H. Lehmer being widely recognized in this field. But, frequently, in checking the results of some formula or hypothesis, the number in question will exceed the scope of the available tables, and the labor of determining whether it is prime or composite is considerable.

Fermat developed an operation to perform this task which curtails the work involved. In the following extract from his works, we have taken the liberty of transposing his figures to the duodecimal base.

. . . . .

"An odd number not a square can be expressed as the difference of two squares in as many ways as it is the product of two factors, and if the squares are relatively prime, the factors are. But if the squares have a common divisor, the given number is divisible by the common divisor, and the factors by its square-root.

"Given a number, (for example, *487 081 381,*) to find if it be a prime or composite, and the factors in the latter case.

"Extract the square root. I get *22 085*, with the remainder *1Ɛ 4X0*. Subtracting the latter from twice the root plus one, I get *24 86Ɛ*, which is not a square as seen from its ending. Hence I add *44 151* (= *2 + 2r + 1*) to it. Since the sum, *68 X00*, is not a square (as determined from the last three digits,) I add the same number increased by two, *44 153*. And I continue until the sum becomes a square.

```
         2  2  0  8  5
 2   )4 87 08 13 81
      4
 42  )  87
        84
 4408)  3 08 13
        2 X8 54
 44145)   1Ɛ 7Ɛ 81
          19 86 X1
           1 Ɛ4 X0
```

"This does not happen until we reach *422 100*, which is the square of *710*.

```
          44 14Ɛ
        - 1Ɛ 4X0
          24 86Ɛ
        + 44 151
          68 X00
        + 44 153
            etc.
          44 169
710² =   422 100
```

"To find the factors of the given number, I subtract the first number added, (*44 151*, or, *2 + 2r + 1*,) from the last addend, (or *44 169*,) and to half the difference I add *2*. The result is *10*. The sum of *10* and the root (r) is *22 095*. Adding and subtracting the root of the final sum (*710*,) I get *22 7X5* and *21 585*, which are the two numbers nearest to r whose product is the given number. They are the only factors, since they are primes."

```
  22 095     22 095
 +   710    -   710
  22 7X5     21 585
```


. . . . .

The preceding operation, while lengthy in some cases, is not laborious. Because of the clearer identification of the terminal figures of squares in duodecimals (as compared with decimals,) it is necessary to test very few of the intermediate sums as possible squares.

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YOUR MAILING LIST

We are eager that every one who is interested in duodecimals learns of the existence of the Society, and of what it is doing. We maintain a list of such names, and mail to them some one or other of the Society's publications from time to time.

Probably most of us know of several individuals in the circle of our acquaintances who should be added to this list. If you will send us their names and addresses, we will be happy to send them an occasional mailing.

Each of us, too, should keep a sufficient supply of duodecimal material on hand so that we can give adequate encouragement to an interested inquiry.

DUODECIMAL NOMENCLATURE - Further Notes

By Kingsland Camp

This article presents a rearrangement and possible improvement of the ideas set forth in my previous essay in Vol. 1 No. 2, page 4 of our Bulletin. We may create at the outset the following "coincidences:"

(1) The sound No shall signify none or zero, as it already does in several languages and in some contexts also in English.

(2) The sound Doe shall signify twelve, as is already the usage in our Society.

(3) The scheme of consonants, vowels and diphthongs shall lend themselves to a mnemonic not too difficult to memorize.

The new suggested arrangement follows. It shows for certain consonants alternate forms that may be preferred by those speaking some other languages. English vowels and diphthongs are spelled out in the least ambiguous two- or three-letter form. Hard ḡ is overscored to prevent ambiguity; with the same end in view, the vowels and diphthongs have been chosen to be recognizable whether pronounced long or short. The inevitable future importance of the Russian tongue makes it worth while to show close equivalents in it also.

Numerals	Consonants	As in	Russian form	Vowels, Diphthongs	Pronunciation	Russian form	Mnemonic
0	N		н	oh		о	00 no но
1	D,T		д,т	eh		э	11 day дэ
2	M		м	ee		и	22 me ми
3	F		ф	ah		а	33 fah фа
4	S,Z		с,з	oo		у	44 soo су
5	R,L		р,л	ia'	ee-ah', yah	я	55 ria' ря
6	B,P		б,п	iay'	ee-ay', yea	е	66 biay' бе
7	K		к	io'	ee-oh', yo	ё	77 kio' кё
8	Sh		ш	ai	I, eye	ай	88 shy шай
9	J,Ch		ч	oy		ой	99 joy чой
X	V		в	au	ah'-oo	ау	XX vow вау
E	Ḡ	ḡeese	г	iu'	ee-oo', you	ю	EE ḡiu' гю

Some may find it easy to pronounce the first eight words of the mnemonic with a rhythm not far from that of the familiar musical scale (do, re, mi, fa, sol, la, say, do).

We then have the following names for digit-pairs; the mnemonic of course is along the diagonal from upper left to lower right:

0 or 00 no но	10 doe до	20 Moe мо	30 foe фо	40 so со	50 roe ро	60 beau бо	70 ko ко	80 show шо	90 Joe чо	X0 voe во	E0 go го
1 or 01 nay нэ	11 day дэ	21 may мэ	31 fay фэ	41 say сэ	51 ray рэ	61 bay бэ	71 kay кэ	81 shay шэ	91 jay чэ	X1 vay вэ	E1 gay гэ
2 or 02 knee ни	12 Dee ди	22 me ми	32 fee фи	42 see си	52 ree ри	62 bee би	72 key ки	82 she ши	92 jee чи	X2 vee ви	E2 ḡee ги
3 or 03 nah на	13 dah да	23 Ma ма	33 fah фа	43 sah са	53 rah ра	63 bah ба	73 kah ка	83 shah ша	93 jah ча	X3 vah ва	E3 gah га
4 or 04 noo ну	14 doo ду	24 moo му	34 foo фу	44 soo су	54 roo ру	64 boo бу	74 coo ку	84 shoo шу	94 Jew чу	X4 voo ву	E4 goo гу
5 or 05 nia' ня	15 dia' дя	25 mia' мя	35 fia' фя	45 sia' ся	55 ria' ря	65 bia' бя	75 kia' кя	85 shia' шя	95 jia' чя	X5 via' вя	E5 ḡia' гя
6 or 06 niay не	16 diay де	26 miay ме	36 fiay фе	46 siay се	56 riay ре	66 biay бе	76 kiay ке	86 shiay ше	96 jiay че	X6 viay ве	E6 ḡiay ге
7 or 07 nio' нё	17 dio' дё	27 mio' мё	37 fio' фё	47 sio' сё	57 Rio' рё	67 bio' бё	77 kio' кё	87 shio' шё	97 jio' чё	X7 vio' вё	E7 ḡio' гё
8 or 08 nigh най	18 dye дай	28 my май	38 fie фай	48 sigh сай	58 rye рай	68 buy бай	78 kai кай	88 shy шай	98 jai чай	X8 vie вай	E8 guy гай
9 or 09 noy ной	19 doy дой	29 moy мой	39 foy фой	49 soy сой	59 Roy рой	69 boy бой	79 coy кой	89 shoy шой	99 joy чой	X9 voy вой	E9 goy гой
X or 0X now нау	1X dau дау	2X mau мау	3X fau фау	4X sau сау	5X rau рау	6X bau бау	7X cow кау	8X shau шау	9X jau чау	XX vow вау	EX ḡau гау
E or 0E niu' ню	1E diu' дю	2E miu' мю	3E fiu' фю	4E siu' сю	5E riu' рю	6E biu' бю	7E kiu' кю	8E shiu' шю	9E jiu' чю	XE view вю	EE ḡiu' гю

And the following briefly recited multiplication table. Pauses, suggested by the punctuation marks, may facilitate memorizing.

01 *nay* нэ	02 *knee* ни	03 *nah* на	04 *noo* ну	05 *nia'* ня	06 *niay* не	07 *nio'* нё	08 *nigh* най	09 *noy* ной	0X *now* нау	0E *niu'* ню
02 *knee* ни	04 *noo;* ну	06 *niay* не	08 *nigh* най	0X *now,* нау	10 *doe* до	12 *dee* ди	14 *doo,* ду	16 *diay* де	18 *dye* дай	1X *dau* дау
03 *nah* на	06 *niay* не	09 *noy;* ной	10 *doe* до	13 *dah* да	16 *diay* де	19 *doy,* дой	20 *Moe* мо	23 *Ma* ма	26 *miay* ме	29 *moy* мой
04 *noo* ну	08 *nigh;* най	10 *doe* до	14 *doo* ду	18 *dye,* дай	20 *Moe* мо	24 *moo* му	28 *my,* май	30 *foe* фо	34 *foo* фу	38 *fie* фай
05 *nia'* ня	0X *now* нау	13 *dah* да	18 *dye* дай	21 *may* мэ	26 *miay* ме	2E *miu'* мю	34 *foo* фу	39 *foy* фой	42 *see* си	47 *sio'* сё
06 *niay;* не	10 *doe* до	16 *diay,* де	20 *Moe* мо	26 *miay,* ме	30 *foe* фо	36 *fiay,* фе	40 *so* со	46 *siay,* се	50 *roe* ро	56 *riay* ре
07 *nio'* пё	12 *dee* ди	19 *doy* дой	24 *moo* му	2E *miu'* мю	36 *fiay* фе	41 *say* сэ	48 *sigh* сай	53 *rah* ра	5X *rau* рау	65 *bia'* бя
08 *nigh* най	14 *doo;* ду	20 *Moe* мо	28 *my* май	34 *foo,* фу	40 *so* со	48 *sigh* сай	54 *roo,* ру	60 *beau* бо	68 *buy* бай	74 *coo* ку
09 *noy* ной	16 *diay* де	23 *Ma;* ма	30 *foe* фо	39 *foy* фой	46 *siay* се	53 *rah,* ра	60 *beau* бо	69 *boy* бой	76 *kiay* ке	83 *shah* ша
0X *now* нау	18 *dye;* дай	26 *miay* ме	34 *foo* фу	42 *see,* си	50 *roe* ро	5X *rau* рау	68 *buy,* бай	76 *kiay* ке	84 *shoo* шу	92 *jee* чи
0E *niu'* ню	1X *dau* дау	29 *moy* мой	38 *fie* фай	47 *sio'* сё	56 *riay* ре	65 *bia'* бя	74 *coo* ку	83 *shah* ша	92 *jee* чи	X1 *vay* вэ

Duodecimal Place Names

My previous article on nomenclature suggested that such a scheme of number-names (consonant plus a vowel or diphthong for every possible digit pair) would, together with suitable duodecimal subdivision of the globe, give cities and mountains and other points of importance thereon, alternate four-syllable place names that assign position within a nautical mile of distance each way (and incidentally tell the time-zone of the place within the very first syllable).

Thus we would substitute for longitude E or W of Greenwich in degrees (or hours), minutes and seconds, simply the duodecimal fraction (from *0;0000* to but not including *1;0000*) of the circumference west of a new prime meridian, preferably one crossing the least or the least-inhabited land areas. In the examples below, this is taken as the (present) 170th W meridian; perhaps the 169th would have been slightly better. For latitude we should substitute the polar distance, or duodecimal fraction of a circumference from the North Pole; the maximum such distance is of course *0;6000*, that of the South Pole. Converting the Nautical Almanac positions of the following observatories, we have their alternate place names.

Place	Longitude W (of Greenwich) Latitude N	In Duodecimals	Pronunciation English Russian
Washington (Naval Obs)	05h 08m 15.78s 38° 55' 14".0	*;8X9E ;1852*	*Shaujiu' dairee* шаучю дайри
New York (Columbia U.)	04h 55m 50 s 40° 48' 34".6	*;8970 ;1781*	*Shoyko dio'shay* шойко дёшэ
Mt.Wilson,Cal. (Astroph.Obs.)	07h 52m 14.33s 34° 12' 59".5	*;X328 ;1X39*	*Vahmai daufoy* вамай дауфой
Ottawa, Canada	05h 02m 51.94s 45° 23' 38".1	*;8X35 ;15X1*	*Shaufia' dia'vay* Шауфя дявэ
Greenwich, England	00h 00m 00.00s 51° 28' 38".2	*;6400 ;134E*	*Boono dahsiu'* буно дасю
Leningrad, Russia	-02h 01m 11.4s 59° 56' 32".0	*;53X7 ;1003*	*Rahvio' doenah* равё дона
Manila, P.I.	-08h 03m 54.71s 14° 34' 42".1	*;2374 ;2620*	*Mahcoo miay'moe* маку мемо
Rio de Janeiro, Brazil	02h 52m 53.5s -22° 53' 42".2	*;7936 ;391E*	*Coyfiay' foydiu'* койфе фойдю

The strangeness of these alternate names should not repel us. Every new idea, as previously remarked, has an appearance of foolishness about it when first introduced.

It should be mentioned that even with the decimal system, skilled computers often add and subtract in digit-pairs; probably many of us half-consciously do this with fractions of a dollar. One-syllable names would make it easier for the mind to handle columns of figures in pairs more generally.

A Russian lady, Miss Alexandra Kalmykow (last syllable pronounced -koff) submits the following well-taken comments and criticisms: the kind that lead to constructive changes.

(1) "Why two symbols for some consonants but not all? Combinations D-T and B-P are just as valid as K-G and F-V yet the former are joined, the latter disjoined."

Answer: The plan of my nomenclature is to pick that set of twelve consonant sounds, with that set of twelve vowel or diphthong sounds, that are pronounceable distinctly by the greatest number of people. We all have met people from other lands whose pronunciation softens B and D into P and T, or vice-versa; these pairs, therefore, were given one significance each. It seems to me that K-G and F-V are less often so confused in pronunciation, therefore I gave these letters separate significances on the theory that more people will easily learn to pronounce them distinctly. I may of course be mistaken in this, and welcome evidence pro or con, with improving suggestions.

(2) "Sh would be a proper combination with Zh (that is, Z as in azure). S-Z (Z as in zero) is all right, if no confusion with Z as in azure."

I agree. If I had grasped this criticism of yours earlier, I would have included the Zh sound as alternative to Sh, and I expect to do so in any future revisions of my system.

(3) "J, Ch is objectionable as a combination since J often has the value of Y in year (jaeger) or Dz (Jabot) or H (juramentado)."

Answer: I am sure you are mistaken here. Consult the dictionary and you will see that it is only very rarely, and in words appropriated from foreign languages - your own illustrations are from German, French and Spanish respectively - that the letter J represents any other sound than that in the word joy, in our English tongue. I well remember meeting, many years ago, a fellow from "Choimenny." It took me some minutes before I realized that that was as near as he could come to saying "Germany" - of course he pronounced our J-sound like Ch.

(4) "Why cater to English spelling in some cases?"

Answer: The object was not uniformity but to make the idea as perfectly clear to English-speaking readers as possible. The phonetic alphabet you suggested would have been better if more than a handful of people could read it; but it also is criticizable for illogical construction, and for seldom suggesting interrelationships between the sounds its letters represent.

THE "EXPANSIBLE INTEGERS"

of Charles Q. De France

Among the oldest friends of the Duodecimal Society is elderly Charles Q. De France of Lincoln, Nebraska. Being ripe in years, he finds the use of duodecimals burdensome, and prefers to use decimals in his calculations, though he recognizes the possibilities in duodecimals.

He is deeply interested in the investigation of prime numbers, and uses an interesting algorithm in checking them, and their reciprocals. This can be illustrated duodecimally as follows:-

N = *1X7*   N - *1* = $2 \cdot 3^3 \cdot 5$

Place	Rem.	Place	Rem.	P + P	R x R	÷ N = Q	Rem.
1	*10*	*2*	*100*	*3*	*1000*	*6*	*86*
3	*86*	*3*	*86*	*6*	*6030*	*32*	*8X*
6	*8X*	*6*	*8X*	*10*	*6604*	*35*	*X5*
6	*8X*	*3*	*86*	*9*	*6310*	*33*	*183*
9	*183*	*9*	*183*	*16*	*2X209*	*161*	*182*
9	*183*	*16*	*182*	*23*	*2X046*	*160*	*1X6*
23	*1X6*	*23*	*1X6*	*46*	*36230*	*1X5*	*1*

indicating that *1X7* is a prime with a *46*-place reciprocal, and that it conforms to $(N-1)^2 = N(N-2) + 1$.

Mr. De France's research in primes focussed his attention on numbers that represent powers of the base plus or minus one, $(B^n \pm 1)$, their reciprocals, and the numbers that result when they are divided by $(B \pm 1)$. We will let extracts from his many letters tell about them.

"For some time I have been reviewing my earlier work on what I call 'Expansible Integers.' I discovered these numbers (after a fashion) a great many years ago, but at that time I had only a portion of the truth about them. Later, thanks to Mr. Terry, I got hold of two books by Allan Cunningham, of England, which have been very helpful. Cunningham uses congruences in his search for primes, but that is too deep for me.

"There are two great divisions of expansible integers. One is of the type: *10101*, or *1001001*, consisting of a number of ones separated by one or more zeros. It will be observed that the first number is divisible by 7, 17, and 111, and the other by 31 and 3X891. The first yields a 6-place revolving reciprocal, .0̇00 ƐƐƐ̇ , and the other, nine places, .0̇00 000 ƐƐƐ̇ . These numbers are of the form: $10^{0n} + 10^{2n} + 10^{4n}$ . . . etc.

"The second division of expansible integers is of the type: Ɛ0Ɛ1, ƐƐ01, Ɛ00ƐƐ1, and Ɛ00Ɛ0ƐƐ1 . It will be found that every odd power of 10, plus 1, can be divided by 11, producing numbers of the type of Ɛ0Ɛ1 . This prime number is the quotient of the division of $(10^5 + 1)$ by 11, and yields a reciprocal of X (or 2n,) places: .0̇00 10Ɛ ƐƐX Ɛ̇ . The seventh power, or $(10^7 + 1)$ divided by 11, gives the quotient: Ɛ0Ɛ0Ɛ1, a composite with a 12-place (or 2n,) reciprocal: .0̇00 001 0ƐƐ ƐƐƐ XƐ̇ , and is divisible by 157 and 7687, both of these having 12-place reciprocals. Further odd powers have a larger number of preceding Ɛ0's. The third power, $(10^3 + 1) \div 11$, produces the quotient Ɛ1, which yields a 6-place (or 2n,) reciprocal, and the factors 7, and 17, both of which have 6-place reciprocals.

"The powers of the even powers of 10, plus 1, $(10^{2^n} + 1)$, are of the same pattern, both as to factors and reciprocals:

Power	Number	Factors	Places of Recip.
$10^2 + 1$ =	101	5·25	4
$10^4 + 1$ =	10 001	75·175	8
$10^8 + 1$ =	100 000 001	15·81·106X95	14

" 'Expansion' of these numbers is accomplished by doubling, trebling, etc., (not multiplying by 2 or 3,) always retaining the final 1. While my own investigation of these numbers has been limited largely to the decimal base, the construction seems equally valid for duodecimals. There should be considerable study given to the extent of the correspondences and the differences between similar constructions on these two bases.

"For example, I have not taken the time to check $(10^{14} + 1)$ for duodecimal validity. In decimals, $(10^{16} + 1)$ has the factors: 353·449·641·1409·69857 . The number has a 32-place reciprocal, and so have the factors. Some one should check this out. Other elements in the foregoing need examination and exploration, like the differences between $(10^6 + 1)$, $(10^X + 1)$, $(10^{10} + 1)$ and the $(10^{3n} + 1)$ family. There are many others.

"Since I am approaching my 6Xth birthday, it is really too late for me to undertake to duplicate in duodecimals the work that I have done in decimals, but the Society shall always have my kindest regards and best wishes."

Charles Q. De France.

CALCULATING π

by Eugene M. Scifres

A while ago, I spent some time calculating the dozenal π, and found it an engrossing subject. As a point of departure, the angle whose tangent is one, first suggested itself. but the series does not converge well for a number that large. So, I broke that angle in half, using the 0.16^c, and the associated equation:

$$\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{3} = \tan^{-1} 1 = 0.16^c$$

This gave me two series, each of which converged rapidly enough that I could get the value to ten dozzenal places on one piece of paper this size. But that was still a lot of figuring, and there were many chances for error. So I tried another series, this time based on the angle 0.1^c, and my results were somewhat as follows:

$$\tan\ 0.1^c = \frac{1}{\sqrt{3}}$$

$$\tan^{-1}\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} - \frac{1}{3}\left(\frac{1}{\sqrt{3}}\right)^3 + \frac{1}{5}\left(\frac{1}{\sqrt{3}}\right)^5 - \frac{1}{7}\left(\frac{1}{\sqrt{3}}\right)^7 + \ . \ . \ .$$

However, the angle I want to determine is 0.6^c, or π radians, and that is six times as large. I believed it simpler to introduce this factor of 6 into each term, and then, when the series is evaluated, there is no further computation. Nor does the cumulative error from slight differences in the last duodecimal place amount to so much. Including the factor 6, the series is:

$$\pi = 6\tan^{-1}\frac{1}{\sqrt{3}} = \frac{6}{\sqrt{3}} - \frac{6}{3}\left(\frac{1}{\sqrt{3}}\right)^3 + \frac{6}{5}\left(\frac{1}{\sqrt{3}}\right)^5 - \frac{6}{7}\left(\frac{1}{\sqrt{3}}\right)^7 + \ . \ .$$

The quantity $\frac{6}{\sqrt{3}}$ may be factored out of the series, leaving:

$$\pi = 6\tan^{-1}\frac{1}{\sqrt{3}} = \frac{6}{\sqrt{3}}\left[1 - \frac{1}{3}\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{5}\left(\frac{1}{\sqrt{3}}\right)^4 - \frac{1}{7}\left(\frac{1}{\sqrt{3}}\right)^6 + \ \right]$$

$$= \sqrt{10}\left[1 - \frac{1}{3}\left(-\frac{1}{3}\right)^1 + \frac{1}{5}\left(-\frac{1}{3}\right)^2 - \frac{1}{7}\left(-\frac{1}{3}\right)^3 + \ \right]$$

You will notice that each term of the last equation contains two factors one of which is the successive powers of $(-\frac{1}{3})$, and the other is the reciprocal of successive odd numbers. The evaluation of this series may be tabulated in a simple manner. First, however, it is necessary to extract the square-root of *10*, or do , which is *3.569 £74* .

In the first column below, each number results from dividing its predecessor by *-3* , thus providing for one part of the series.

The odd numbers in the second column are divided into the relative terms in the first column, and these quotients are listed in the third and fourth columns, the third for positive, and the fourth for negative results. The third and fourth columns are then added together, algebraicly, yielding the value of π .

3.569 £74	1	3.569 £74	
-1.1X3 3X5	3		.475 136
.475 136	5	.0£1 031	
- .165 852	7		.027 809
.061 X99	9	.008 265	
- .020 773	£		.002 2X6
.008 265	11	.000 76£	
- .002 8X2	13		.000 223
.000 X£5	15	.000 079	
- .000 37X	17		.000 024
.000 127	19	.000 008	
- .000 04X	1£		.000 003
.000 017	21	.000 001	
- .000 006	23		.000 000
		3.667 X83	-0.4X3 277
		-0.4X3 277	
π	=	3.184 808	

By actual long division, the circle value of one radian is obtained. The radian is used in determining angles by the $\tan^{-1}$ formula, and equals $\frac{1^c}{2\pi}$, and , from the above,

$$1 \text{ radian} = 0.1X£\ 808^c$$

. . . . .

THE ABBREVIATION OF π

The error involved in using the customary duodecimal abbreviation for π of *3.1848* is less than half of the error in the corresponding decimal abbreviation, 3.1416 .

The correct values, to twelve places, are:

Duodecimally, π = *3.184 809 493 £92*

Decimally, π = 3.141 592 653 596

MATHEMATICAL RECREATIONS

Mary Lloyd, Editor

The article on the Fibonacci Series in this issue makes it apropos that we should submit a recreational problem that is credited to Fibonacci.

> What square, when augmented and diminished by *5*, generates two other squares? That is:
>
> $$x^2 + 5 = y^2$$
>
> $$x^2 - 5 = z^2$$
>
> To find x , y , and z .

By an amusing mischance, one of the cryptarithms in the last issue, the one on multiplication, was printed without its sum. But the omission improved the puzzle instead of maiming it.

The first crypt in that issue was a problem in addition. It simply reeked with leads.

```
  I Z S U O E N E S I
+     Z A O E A E S V
  V U R L L L L D U R
```

Take the first two figures. Only a second look is necessary to see that Z is £. and U is *0*. From this, the sequence I V is confirmed because V must be one more than I ; and S is now known to be *6*, since the sum of *6* and *6* ends *0* . The rest of the puzzle falls apart to show that the code used was: Universal Doz. The codes for the other cryptarithms were: Dozen Triumph, Numerical Doz., and Dozenal Quirk.

The puzzle about The Monkey and The Cocoanuts elicited the best response that we've had yet. An orchid to Mr. Robert with our congratulations. There must be lots more gold in them thar hills, Podner. So, how's about it?

Eugene M. Scifres writes that he has done more travelling since leaving the service than before. He works for the Abrams Aerial Survey Corp., of Lansing, Mich., and his recent mapping assignments have included:- the upper Michigan peninsula, both shores of Lake Michigan, and the Sault Sainte Marie; the area around Fort McClellan, Ala; 1500 miles of roadway for the Georgia Highway Dept.; the New Orleans area; and timber areas in Mississippi.

Commenting on Harry Robert's Cocoanut Problem, he says that he first met this problem while at the University of Louisville, but that in his version "there was one cocoanut for the monkey at the final division as well as one at each of the previous divisions. I recall that I worked a couple of days on the solution, trying different approaches, and that the final method whereby I arrived at a solution was very similar to Mr. Robert's.

"With the added condition that the monkey should have one at the final division, the minimum answer is *9059* cocoanuts. A variation of this same problem is given on Page 32 of the 'Mathematical Recreations,' of Maurice Kraitchik. In this book, however, there are three men instead of five. The general solution in which any number of men may be present, and providing that the monkey gets one unit at each division and one unit at the final division, is, for the minimum:

$$\text{Number of Objects} = N^{(N+1)} - N + 1$$

from which the following table can be compiled:

Number of Men	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Number of Cocoanuts	*1*	*7*	*67*	*1 451*	*9 059*	*115 ƐƐ7*	*1 Ɛ20 137*

and if there were an even dozen men, the answer is the astronomical figure *Ɛ ƐƐƐ ƐƐƐ ƐƐƐ ƐƐ1*, which is a lot of cocoanuts in any man's language."

This time, because you've been such a nice responsive class, Teacher is going to let you off with only two cryptarithms. Happy days!

Mary.

```
            H I E
I C T ) H U U N U O
        W Y T U
          T H U U
          A I D U
            E W N O
            Y U C N
              U D O
```

```
                A O M
W E M ) T O N N S E
        T W C C
            U R S
            C R S
            T S S E
              E U M
                O I
```

THE MAIL BAG

We always read with interest the releases of the American Institute of Weights and Measures, for their comprehensive coverage of the developments in this fiels, and for their lack of bias. Recently, we were delighted to discover that they had cited part of our comment on Dr. Ingalls' latest brochure in their Bulletin of October 1st, 1946. We wish to express our thanks for the gracious compliment.

. . The items of greatest importance in the present weights

and measures situation, are the international factors emphasized by the formation of the United Nations Standards Co-ordinating Committee, and domestically, the consideration of a proposal for dimension control for the building construction and equipment industries. This is spoken of as Modular Control, and conceives modular units of 4", 16", and 40".

The general progress that has been accomplished through the standardization program of the American Standards Association is remarkable, and of great benefit to the American public, as a whole. The construction industries can be depended upon to accomplish a practical rationalization of this part of that program. Personally, that 40" item would cause me concern as to the vulnerability of our wood-piles to invasion. Either 36" or 48" would seem to afford greater accomodation, as a modular standard.

Paul Van Buskirk has been active in these matters, setting forth the advantages of the duodecimal division of construction standards in talks before discussion groups, as well as in technical papers. Several of his articles have appeared in the section, "Our Readers Say," of Civil Engineering.

. . We have had a letter from Alfred Norland taking us to task for proposing a system of weights and measures which leaves the astronomical measurements out of consideration. There is some truth in his accusation. The fact that the light-year measures some *809* trimo miles seems to us to have little weight in determining units of convenient size.

This sort of thing was tried in the French decimal metric system, and not only failed of accurate definition, but also resulted in units of awkward size. On the other hand, we would remind Mr. Norland of the tremendous improvement in astronomical metrology afforded by the unified time and angle measure, suggested by Nystrom and Terry, and included in the Do-Metric proposal.

There is some possibility that Mr. Norland may have an idea to suggest. He is, as most Dodekaphiles know, the author of an original duodecimal proposal that is very interesting. We would like to publish an article on his "Twe System," and only the lack of sufficient time has deferred our preparing a resumé.

. . Jorge Carreras Codrington, who has undertaken the translation of some of the duodecimal literature into Spanish, is at present in the United States in connection with the marketing of certain American products in Cuba. Business matters have forced him to postpone further work on the translation until a more convenient time.

Mr. Codrington was able to spend some time with us in New York, en route to Worcester, Mass. He discussed with us the weights and measures used in Cuba, and the teaching in their more advanced schools of the use of other number bases than ten. We look forward to seeing Mr. Codrington again, on his way back to Habana.

. . We also had the pleasure of meeting Mr. and Mrs. William Shaw Crosby on their recent visit to New York. Nothing affords us more enjoyment than these personal contacts with our members. Once we've met, letters acquire a greater intimacy for all concerned, and the distances between us become less of a limitation.

. . George S. Murphy has raised a novel point. He reports: "Several times my work has emphasized the complete absence of a symbol for a non-significant zero. The obvious existing symbols that might be appropriate are - theta, phi, and Q. Or perhaps some combination of all three might be simplest. Thus we might write 'one bimo' with four significant figures as *1 000* ~~*000*~~ . On the other hand, this might be misunderstood as a correction. I must confess that I have found difficulty in thinking up even one extra symbol that would be unique, simple to write, and easy to read."

. . The year's end has come and gone, and Volume II of the Bulletin is closed with only two issues. We are deeply chagrined. Instead of improving on the performance of previous years, we have done worse. And this in spite of our fervent resolutions to the contrary. Woe is us!

But here we go again. We hereby highly resolve . . . ., or something. Perhaps we have done too much work, and too little planning. That sounds as though there might be something in it.

Let's just imagine that there will be four issues this year. There would be, first of all, the need for nearly twice as much copy as we have been getting. We have more members than ever before, and if each member would write one more paper than he did last year, we'd probably be well on the way to attaining that objective. There is the additional benefit that the Bulletin would be more interesting than it is.

Perhaps you have been deterred by the thought that you could not write anything of sufficient interest. Don't let that stop you. If necessary, we'll edit your material, and secure your approval of the final result before publication. Let us judge the value of your ideas. You send them in, and we'll let you know how we appraise them. "Open the door, Richard."

. . Lewis Carl Seelbach advises us that we erred in crediting him with the proposal of a new world language. He states that the Lingveroj material, of which we spoke, related to a proposed compound polyglot dictionary, using Dr. Arvid Reuterdahl's world alphabet as a means of cataloging elements from many languages. We regret our error and trust that it has not been a source of embarrassment to Mr. Seelbach.

Ye Ed.

Our common number system is decimal - based on ten. The dozen system uses twelve as the base. This requires two additional symbols: X, called *dek*, is used for ten, and £, called *el*, is used for eleven. Twelve is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

Modern numeration employs one of the greatest of man's inventions, the zero - symbol for nothing. It permits the use of place values. In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5.9'*
31	*694*	Three ft. two in.	*3.2'*
96	*3£2*	Two ft. eight in.	*2.8'*
19£	*1000*	Eleven ft.seven in.	*£.7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2£*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal number.

```
12 ) 365
  12 ) 30  + 5
    12 ) 2 + 6
         0 + 2      Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by X, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or X.

Numerical Progression

1	One		
10	Do	*.1*	Edo
100	Gro	*.01*	Egro
1,000	Mo	*.001*	Emo
10,000	Do-mo	*.000,1*	Edo-mo
100,000	Gro-mo	*.000,01*	Egro-mo
1,000,000	Bi-mo	*.000,001*	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	£
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2£	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2£	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
£	1X	29	38	47	56	65	74	83	92	X1