The Dozenal Society of America

Rationality

by "Troy" (Donald Hammond)

Terms must be defined. In doing so here, I realize that much of what I write will be well-known to many members — though not all — but it does no harm to confirm these matters.

The word 'rational' is directly related to 'rate' and 'ratio', to 'ratify' and 'ratiocination'. It implies the use of reasoning, of proportion, of due measure; it suggests pragmatism and common sense. A mathematician will call common fractions 'rational numbers' because they are *ratios* and so can be used for practical calculations involving only the simple arithmetic which is (or should be) available to most ordinary people in their daily affairs.

Rationals are 'sensible' numbers in the same way, perhaps, that *heat* which causes a change of temperature is called, by engineers and physicians, 'sensible heat'. The other kind of heat — latent or 'hidden' heat — causes a change of state (if you continue to heat boiling water it doesn't get any hotter but instead turns to steam) but no change of temperature; the only way we can *measure* latent heat is to convert it to the sensible variety. In the same way, *irrational*, or 'hidden' numbers, like π or $\sqrt{2}$, need to be measured by conversion into rational approximations if they are to be used for practical purposes.

There was a somewhat far-fetched story in which a factory work-robot was instructed to electroplate a copper disc with platinum. Before issuing an ingot of platinum to form the anode, the stores department insisted on an exact specification for the amount of precious metal to be used. After some days had passed with no sign of the work an enquiring manager discovered the hapless robot trying to find an *exact* value for π; it had not been told to use a rational approximation… Again, on photocopying machines the enlargement ratio from A4 to A3 is given as '141%', never as the $\sqrt{2}$ it is supposed to be.

So rational numbers are the *only* kind we can use for measurement. It follows, surely, that measurement *systems* should accommodate, as fully as they can, those rational numbers — ratios — which arise naturally from basic, everyday considerations of geometry, proportion and practical economy. In this context, American author Donald Kingsbury once observed that traditions are 'solutions for which we have forgotten the problems', with the corollary that discarding the traditions without due thought brings the problems back again… In this article I shall look at examples of traditional measuring systems and how the problems they solved are now returning to bedevil us as decimal-

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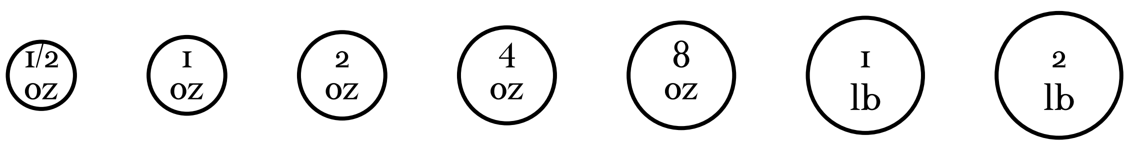
ization wreaks its damage; and suggest how the Rational approach can both preserve and enhance these hard-won and — yes! — advanced principles which our political masters would like us to forget.

All approximations are, speaking mathematically, rational numbers; but they are not always very *sensible* numbers to use if simpler ones can be chosen. While no-one can avoid approximations for irrational numbers, it *is* often possible to eliminate nuisances like 1.166 . . . and shorten such as 0.4375 by using numbering and/or measuring methods more suited to the work and ratios demanded. I use the term 'Rational' — with capital R — to denote scales or units which are not only ratio-based but are also sensible (i.e. as simple as possible). It is this principle, this mode of thought, which I call 'rationality'.

How to Lose Weights

Mathematics tells us that, in many cases, *binary* is best. The powers of two constitute the 'minimum' power-series for weights on a two-pan balance (with weights on [one] side and goods the other) since it specifies the least number of different weights needed to cover all numbers of units. The rule about weights is that the subsidiary pieces should be simple *unit* fractions whose denominators are factors of the basic standard. Hence, for example, including ¼ lb. and ½ lb. pieces makes a ¾ lb. piece unnecessary.

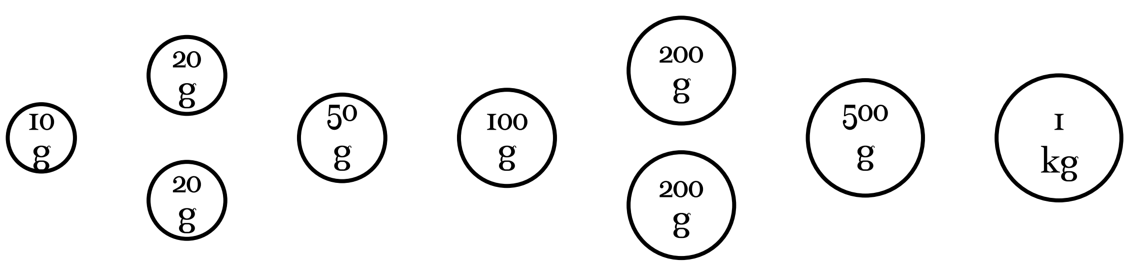
Here is a 7-piece set of English kitchen-weights (still much used in English kitchens) of the system dating from c. 1290 (8ɛ6).



Avoirdupois (Binary)

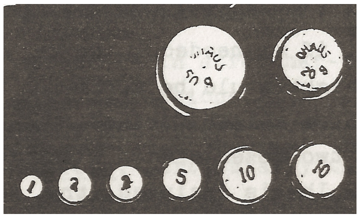
All intermediate weights, in ½-oz steps, can be made from combinations of these pieces, so the set requies only one of each. Use of this system is now effectively forbidden in British schools.

By comparison, the decimal-metric set of weights needs to *duplicate* some:



Metric (Denary)

Thus, the binary set, needing only three pieces for every four in metric, is more efficient in use and cheaper to make than is the denary set.

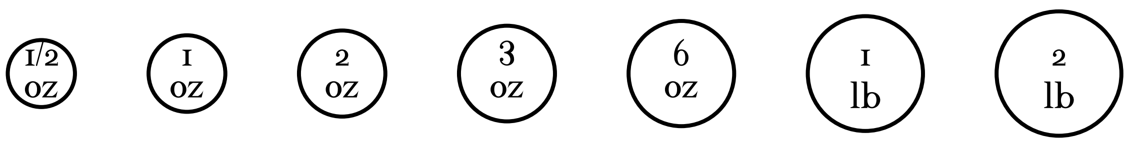


Photo□ at left is of a set of 'Student' laboratory weights. In this case, the 2g and 10g are duplicated. This is cheaper than having two 20g, but the need for duplicates remains.

We can see the basic flaw in the decimal arrangement if we remember that subsidiary weights have to be *unit* fractions of the standard, so the next larger piece after 100g (1/5 kg) has to be 500g (1/2 kg). *There are no unit denary fractions in between.* Hence, the only ways to make 400g are to use two 200g or two 100g plus one 200g.

'Aha!' One hears the decimalists cry, 'Your beloved twelve is not a power of two, either; so you dozenists are stuck with the same problem.'

Well, not quite. Twelve accepts 2 and 4 as factors; it accepts also 3 and 6. Noting that 4 is *twice* 2 and that 6 is *twice* 3, we see that it is possible to use a binary *multiplier* (and so retain binary efficiency) and simultaneously introduce the second prime, 3, to design a set of weights for a dozenal system. Let us now incorporate this scheme into something like the Troy pound of twelve ounces (even older than Avoirdupois):



Troy (Binary-Ternary)

Just as with pure binary, all intermediate weights can be achieved by combining others, so we need only one of each size.

There is more. It will not have gone unobserved that 3oz, 6oz and 1 lb. can be made from combinations of lower values; in fact, if we needed to go only as far as a dozen ounces, the 1 lb. weight would be superfluous. Including the 1 lb., therefore, allows further weighing up to and including 2 lb. or two dozen ounces *without the need for a 2 lb. piece.* If the 2 lb. is included, the range extends to 4 lb. *inclusive.*

The binary (Avoir.) and dozenal (Troy) sets are easy to use and need only seven weights each. The dozenal set has an added advantage in that it can give the full 4 lb. while the binary misses by ½ oz. The decimal set is not quite so easy to use and — more seriously — involves nine weights rather than seven and is thus more bulky to store and uses more metal in manufacture.

Note also that with a twelve-ounce pound so divided, a more flexible range of fractions is available, including thirds and sixths as well as halves, quarters and eighths.

Both the pure-binary Avoirdupois and the binary-ternary Troy systems would be acceptable to most dozenists (who would simply write 14 instead of 16, or 10 instead of 12) though we should naturally prefer the latter. What is not acceptable is the absurd wastefulness arising from doctrinaire decimalization. That is the Rational approach to kitchen-weighing.

Brickbats

The housebrick has been mentioned before in the Journal (see No. 3), but deserves a closer look; embodying as it does a solid, three-dimensional actuality, this humbly yet essential artifact illustrates to perfection the need for Rational measure.

The Imperial Standard brick is based on the yard. Its effective size, which includes the mortar joints when laid, gives dimensions of length, width and height as one-quarter, one-eighth and one-twelfth of a yard respectively.

Figure B.1 shows a modest brick structure in stretcher-bond, using Imperial bricks. Note how simple fractions of a yard are obtained at every stage in three dimensions. The fractions can also be expressed easily in *feet*, particularly the height, which is readily-estimated on site at four courses to the foot. A builder told that a wall rises n feet from the DPC knows that $4n$ courses of bricks will be needed; a similar simplicity obtaining for horizontal dimensions gives a whole number of yards or feet for every four bricks in stretcher bond.

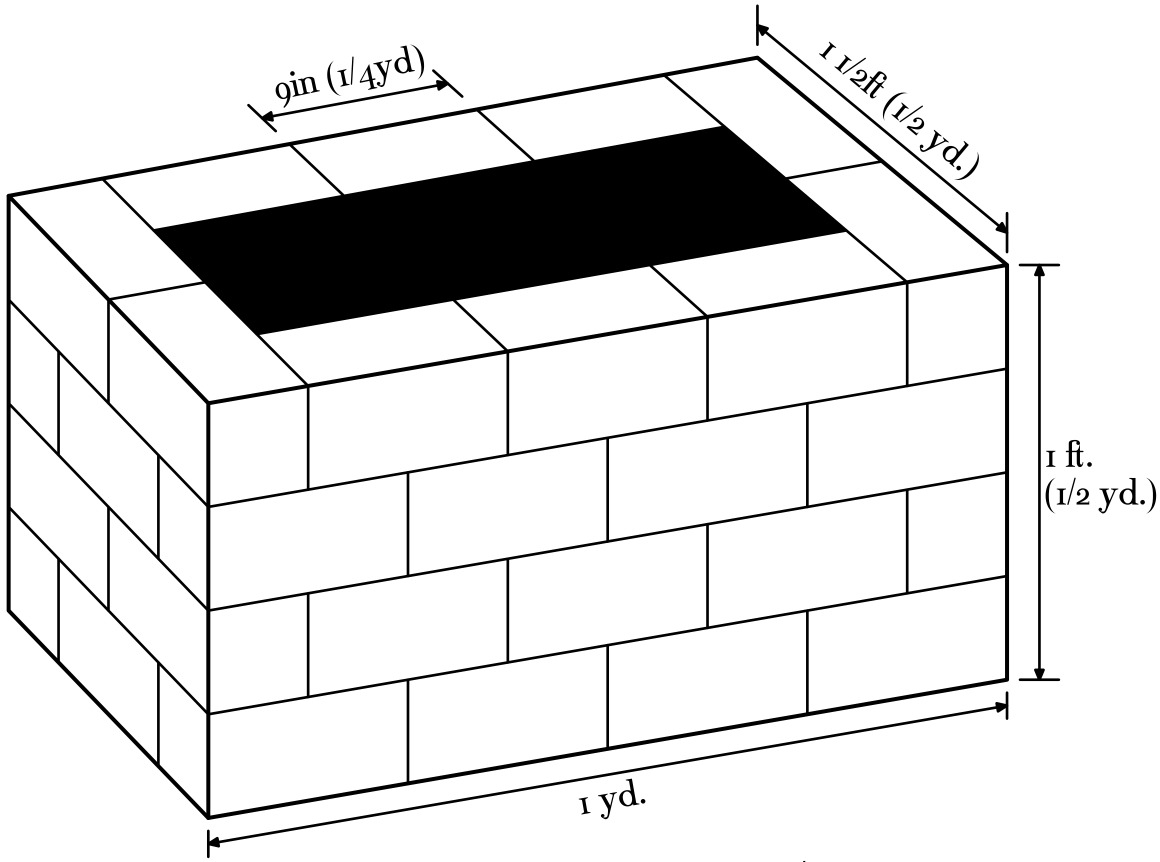
A glance a[t] figure B.2, which is the same structure made from 'metric' bricks reveals that these *will not fit a metre*, either lengthwise or coursewise. The desirable ratio — in lowest terms — of brick dimensions is 6 : 3 : 2, and this ratio cannot be obtained with metric units (try it!); the pathetic result, therefore, of the so-called 'metrication' process in the building industry, which was undertaken for political, not ergonomic, reasons has been the invention of the 'metric inch' of 25mm, the 'metric foot' of 300mm and the 'metric yard' of 900mm (not that anyone is officially allowed to say so). The 'metric standard' brick, laid in mortar, is thus given dimensions of: thickness 75mm (3 metric inches), width 112.5mm (4½ metric inches) and length 225mm (9 metric inches); thus sized, these bricks can be laid four courses to a metric foot and four lengths to a metric yard.

Hence, the price paid for 'metricating' the housebrick is abandonment of the metre itself: the *primary* unit, the Emperor of the metric system in his grand decimal raiment, has arrived at the builder's Yard and tripped over a brick. . . .

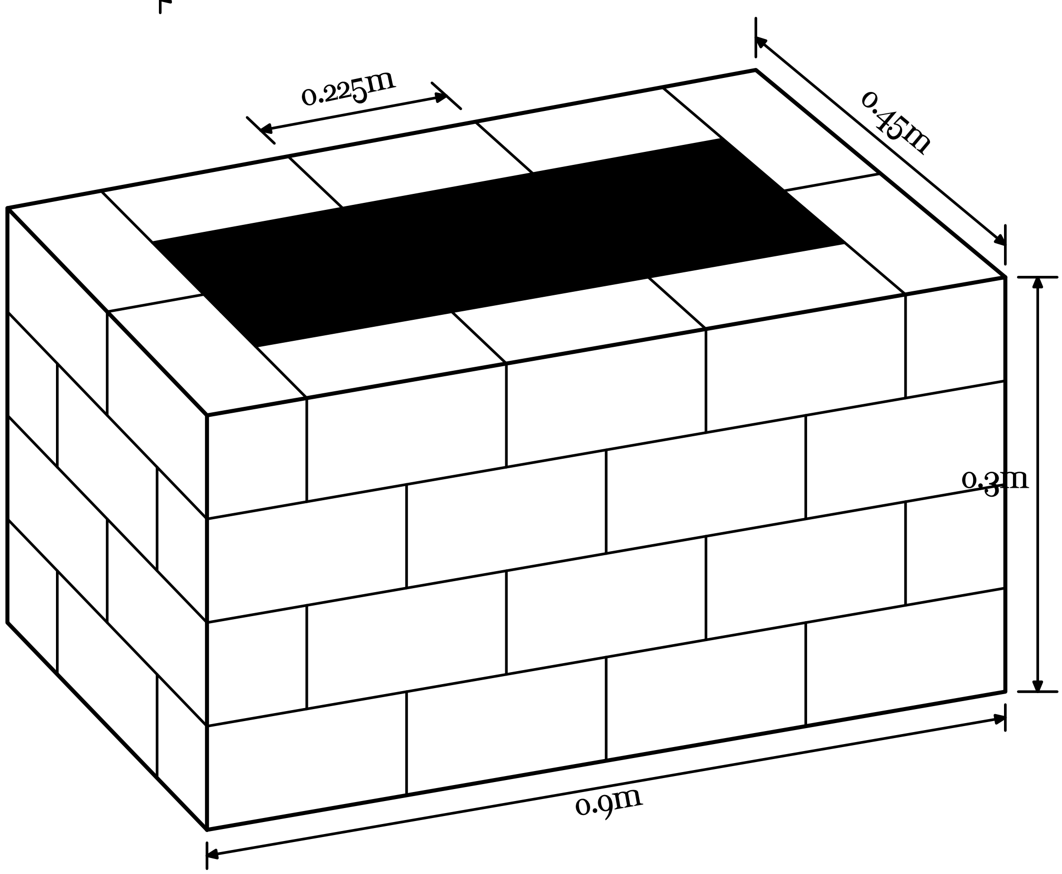
(No; this is not just a British reaction: the French themselves do not use the metre as a building module.)

This 'metric' brick is very close in actual size to the Imperial. It is a little smaller ($\frac{1}{8}''$ shorter and will lay to the yard and foot; so if you want to lay bricks stay with your folding yard and avoid wasting money on a folding metre that will not fit the work. (What a spiteful little change this is!).

Again we see that the criterion for efficient measuring units is the ready accomodation of *ratios* suitable for the work. The fabric of reality is tough and trying to cut patterns in it with blunt decimal tools is a self-defeating exercise.



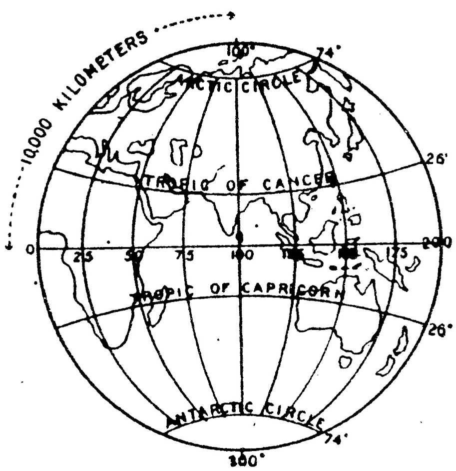
B.1



B.2

Choosing the Right Angle

The metre has been referred-to as the 'Emperor' of the metric system, which it is; but Emperors do not spring from nowhere: they result from some or other method of selection. Most who have taken any interest in these matters know that the metre is — or was originally supposed to be — one ten-millionth of a quadrant of the Earth from Pole to Equator.



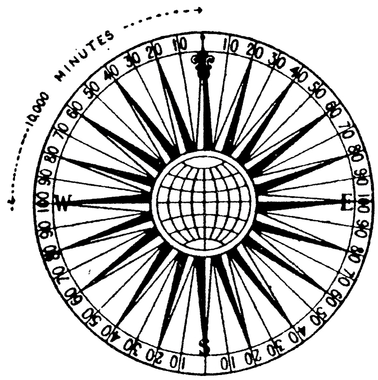
So, in an act of breathtaking contrariness, the very first and fundamental decimal-metric operation was the *denary division of the quadrant.* Now, numerous proposals have been made — by dozenists and others — regarding angular scales: most of these have been based on the circle or half-circle. Yet, as the French saw clearly, it is the right-angle, or quadrant, which really matters, for that is literally the corner of the three-dimensional world; and they divided the right-angle into one hundred Grades as the basis of a decimalized protractor. The length of arc at sea-level which subtends an angle of 1 Grade at the centre of the Earth was then found by direct measurement.[*] This distance was divided by one hundred thousand to give the metre.

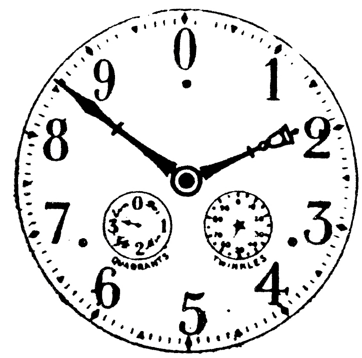
The *kilometre* was — and is — seen as a navigational unit: one hundred kilometres along a Great Circle is equivalent to one Grade on the denary protractor. Navigation, however, is not only a matter of angle, but also of time; the decimal clock is a necessary adjunct to the Grade protractor. The diagrams below were both taken from an article published in 1906 (112ᘔ), strongly advocating the system.[†]

It should be noted that children in State schools in Britain are being taught elementary navigational mathematics *exclusively* in terms of kilometres. Decimal-clock suggestions keep popping out of the woodwork and at least one Town Council (Leeds) has switched to decimalized time-sheets for its staff.

[*] The original measuring, of course, had to be done with existing units. These, ironically for us, were *double-toises*, each of twelve *pieds* (feet), each pied being of twelve *pouces* (inches) and each pouce being of twelve *lignes* (ligns). A completely dozenal system, in fact . . .

[†] See the following page. —Ed.





The Babylonians, developing Sumerian concepts, used sixty as a secondary counting-base. It seems probable that they arrived at the protractor we still use today by taking the natural *sextant* (one-sixth of a circle obtained by stepping-out the circumference with its own radius) and dividing it into sixty degrees. This automatically conferred ninety degrees on the right-angle: a very good number which caters well for the prime constructible angular divisions of the circle (halves, thirds and fifths). The scale itself, however, cannot be constructed in the plane and needs three-dimensional manufacturing methods.

By contrast, the Grade scale — also inconstructible in the plane — cannot accomodate thirds; under its regime the draughtsman's familiar and indispensable 'thirty-six' set-square, giving one-third and two-thirds of a right-angle, would have to become 'thirty-three point three recurring/sixty-six point six recurring' set-square. As Oliver Hardy would have said: 'Another fine mess!'

The centesimal Grade protractor can manage only six exact subdivisions of the right-angle if whole numbers of grades are used, whereas the Babylonian can give ten such with whole numbers of degrees; these include the thirds and sixths which are so imperative.

By blind insistence on powers of ten, the perpetrators of the Grade protractor threw away Rational notation for one-third and two-thirds of a right-angle; yet these are geometrically fundamental. Some dozenists, it must be said, have fallen into a similar trap by designing protractors based on powers of twelve: these provide excellent notation for halves, thirds, quarters and sixths, but fail (unlike the Babylonian scale) to accomodate fifths. As was explained at some length in an earlier article (Journal 8, p.11), five, while not important as a linear division, *is* significant in angular measure; hence, the Babylonian device is of better rationality than either pure decimal or pure dozenal versions.

The Babylonian protractor *can* be improved; the Rational protractor (Review No.30, p.ɛ) applies the classic sexagesimal scale to the *quadrant* instead of the sextant: this gives a scale of sixty or five dozen Rates (ϕ) to the right angle (thus allowing both decimal and dozenal notations to be numbered roundly), accommodates 2, 3 and 5 as factors and *can* be constructed on the plane. Both Babylonian a[n]d Rational protractors fit the existing clock; the Grade protractor does not, of course. Here is the comparative table of properties reproduced from Review 30.

Protractor	Division of Quadrant		Prime Factors 2	3	5	Compatible with Clock	Constructible in the Plane
Babylonian	*76	/90	•	•	•	•	
Grade	*84	/100	•		•		
TGM	*60	/72	•	•		•	
Rational	*50	/60	•	•	•	•	•

Measure for Measure

The lesson we should draw from these observations is surely one of discipilned flexibility: *we must recognize natural constraints and patterns*, simple proportions and efficient styles of measurement. We can see that different tasks may require different scales, and so should avoid falling into the doctrinaire trap of blindly imposing a single, rigid basis to all situations.

The decimal-metric system, being a product of revolutionary zeal (and zealots are notorious for their puritanism), permits no units which are not powers of ten; no secondary or auxiliary bases are allowed, even when mathematics itself demands them. The people of earlier times *counted* in tens, but were wise enough not to let that impede their mensuration: binary, ternary, duodenary and sexagenary scales were used where appropriate; no-one felt threatened by them. It was realized that powers of ten, though perhaps good enough for mere counting, raised unnecessary barriers to sensible working practices; and so such numbers were largely rejected for units of measurement. Decimal *currency*, even was abandoned c.130 BC.[*]

There is another irony here: because our forebears (not frightened of fractions) were happy with 8-pint gallons, 3-foot yards and so on, they were free of the stifling influence of the denary base (used solely for simple arithmetic) and so did not bother about changing it; *decimal numeration survived by being marginalized*. Had there been some sort of cosmic law which ordained a match between number-base and measures, we should have had a twelve-based numeration from time immemorial (especially once it was found that it made calculations easier, too!).

Yet…We all recognize the convenience afforded, particularly to the scientific world, by measuring-units which fit the number-base: a match between the two schemes, whereby successive units of measure correspond to successive powers of the radix, so permitting standard-form calculations and fraction-point transformations, is highly desirable to laboratory workers and accountants alike. It promises coherent systems and hence elimination of troublesome conversion-factors. It was this promise which seduced — and still seduces — academics and politicians (for different reasons) into uncritical acceptance of the decimal-metric idea.

* The denarius, as its name suggests, was originally ten As, but was made worth sixteen As at this time. Some assert that this was merely devaluation of the As; but in that case why choose sixteen?

They have been sold a pup. What looks so good on paper, with its elegant unit names and inspired series of power-prefixes, fails to accommodate natural ratios, often imposes problems where there were none before and has a marked propensity for expanding simple fractions into strings of decimal digits. Instead of grasping the nettle of decimal incompatibility with natural mensuration and arithmetic, L'Institut National shrank away from the chance of basing their system on the dozen and went for a quick denary fix.

Our dozenal base *is* amenable to true rationality: we have seen how a twelve-based weight system equals and sometimes betters the binary; how linear, areal and cubic measure, using feat-and-inches and the almost miraculous yard, are elegantly served. Accepting secondary bases where appropriate (so avoiding the disastrous rigidity of the metric system) we can have, for example, our inches divided-down dozenally in a power-of-twelve system, yet leave the other edge of the rule with the binary subdivisions which are so useful; we can have an even better protractor than we have now; we can leave the clock-dial alone (apart from the re-numbering that it always needed anyway); we can have a thermometric scale from 0 (freezing water) to ∗130 (boiling water) using Fahrenheit degrees; and — underlying it all — we can have the most efficient and (if I may use the expression) user-friendly arithmetic it is possible to devise.

This article was originally published in Number ↊ of The Dozenal Journal, which was a joint publication of the Dozenal Society of Great Britain and the Dozenal Society of America. Number ↊ was released in the spring of 11↊0 (1992.) in Denmead, Hampshire, England. Donald Hammond, a long-time stalwart of the DSGB, published this article (along with many other excellent pieces) under the pseudonym "Troy," a remarkably apt one given its topic.

The work has been completely retypeset using the LaTeX document preparation system, and is here set in the DRM font in 12×15. The figures depicting weights and bricks are all redesigned in the Metapost graphics description language; the photograph of the weights, as well as the globe, decimal clock, and compass, were scanned, clipped, purified in color, and then inserted. The few other alterations are marked in the text.

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