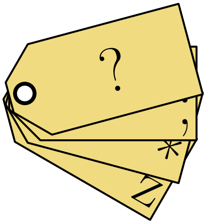
Base Annotation Schemes



John Volan

Bulletin Editor • Member 418_{z} (596_{d})

The Problem

The overwhelming majority of the population presumes all numbers are expressed in base ten, often with no awareness that there is even an alternative. To them, "100" means simply "a hundred"—ten times ten—and that's that.

Many advocates of base twelve fondly envision a world where dozenal is the presumed base. In such a world, ordinary folk would naturally read "100" as "a gross."

Of course, the reality is that supplanting decimal as the "civilizational" base has proven a stubbornly distant goal. This circumstance has persisted since the founding of the Dozenal Society of America nearly six dozen years ago—half a biquennium![1] No doubt this state of affairs will continue for the foreseeable future.

Consequently, the more sober advocates of dozenalism have long been reconciled to the need to be "bilingual" (perhaps a better term would be "binumeral") in our mathematical discourse. We recognize the need to be able to switch back and forth, as needed, between base ten and base twelve—and even other bases—preferably, in as neutral and equitable a manner as possible, favoring no base over any other. This is particularly important when introducing the subject to newcomers, a perennial task. ("Each One, Teach One" has been a motto of the DSA since its inception.[2])

Of course, admitting more than one base into the discussion renders any number longer than one digit ambiguous—unless care is taken to stipulate the base in use at any given moment. Various schemes to achieve this have been devised over the years.

An Early Expedient: Stylistic Marking

Largely at the behest of F. Emerson Andrews, co-founder of the DSA and author of the book *New Numbers*,[3] this publication in its earliest days established a convention of distinguishing dozenal numbers from decimal, by typesetting the former in italic style. A number typeset in normal style would simply default to a decimal interpretation.

$$\begin{aligned} \mathit{100} &= 144 \\ \mathit{16.9} &= 18.75 \end{aligned}$$

This convention persisted for over three unquennia.[1]

An advantage of this approach is that it is fairly non-intrusive, at least as far as conventions for mathematical notation are concerned. This means that readers

[1] See page 31_{z}.

[2] Ralph H. Beard, "Propagation", *Duodecimal Bulletin*, Vol. 1, No. 2, WN 1, Jun 1161_{z} (1945_{d}).

[3] F. Emerson Andrews, *New Numbers: How Acceptance of a Duodecimal Base Would Simplifiy Mathematics*, 1944_{d} (1160_{z}).

can take all their prior experience with how numbers work in decimal, and all their expectations about the "look and feel" of numbers, and simply transfer that to dozenal numbers, with minimal adjustment. Except for new symbols for digits ten and eleven, there is no other notation to learn. All other mathematical symbols and operators that people have been comfortable with for generations will largely look the same, and continue to behave in the same way.

However, a disadvantage is that this makes for a rather subtle distinction. Seeing italicized or non-italicized numerals in isolation affords the reader with no *positive* prompting about which base is being applied. If the italicization is not particularly strong, the intent may not be clear.

A stronger objection to this scheme is the fact that it interferes with other common usages of italic style. For instance, italics are generally used to show emphasis, or to set off foreign or quoted text. If there were ever an occasion to emphasize a decimal number, or to *not* emphasize a *dozenal* number, there would be no way to do that.

An even stronger objection is that this scheme is neither neutral nor equitable. It requires dozenal numbers to be marked in a particular—and peculiar—way, while requiring no marking or change at all for decimal numbers. This implies a favored status for decimal and relegates dozenal to an "also-ran" position.

This is problematic enough in typeset print. But consider what this requires of people writing by hand. Either they must go out of their way to artificially distinguish the degree of slant in their cursive, or they must represent italics via underscoring, which means branding every dozenal numeral as somehow out-of-place—a sore thumb, as it were. In an era before word-processing, when the typical mechanical typewriter provided one and only one font, this meant laboriously backspacing over a dozenal numeral and superimposing it with underscores. It is not surprising that aficionados of base twelve would seek out a more streamlined scheme for base annotation.

Humphrey's Radical Radix-Point

Very early in its history, one of the pioneers of the DSA, Herbert K. Humphrey, hit upon an idea: If a period is known as a "decimal" point, separating whole digits from fractional digits in decimal, then perhaps dozenal numbers need a radix point of their own too—a "dozenal" point, as it were. He began using a semicolon to that end:

$$16;9 \;=\; 18.75$$

The obvious advantage of this is that it allows us to mark a number as dozenal with no need for any change of font or style, nor any laborious backtracking and retyping. All it takes is the use of another key already available on the typewriter.

(Interestingly, this was not an entirely new idea. More than an unquennium prior the DSA's founding, Grover Cleveland Perry made a similar proposal, in his pamphlet "Mathamerica."[4] He suggested the colon, rather than the semicolon, for this purpose.)

Humphrey proposed this use of the semicolon in an early letter to the *Bulletin*,[5]

[4] Grover Cleveland Perry, "Mathamerica, or The American Dozen System of Mathematics", 1149_z (1929_d). Reprinted in *Bulletin*, Vol. 6, No. 3, WN 14_z (16_d), Dec 1166_z (1950_d).

[5] Herbert K. Humphrey, letter in "Mail Bag", *Duodecimal Bulletin*, Vol. 1, No. 3, WN 2, Oct 1161_z (1945_d).

and others were slowly influenced to adopt this practice. However, it was not until more than an unquennium later that it really took off, under the intensive and enthusiastic lobbying of Henry Clarence Churchman.[6] Churchman at that time had become editor of the *Bulletin* (and a prolific contributor), and even, for a time, DSA president.

The so-called "Humphrey point" did not, at first, supplant italics, despite its clear potential to do just that. This was not simply a case of inertia or incipient traditionalism.

Under the prevailing syntax rules for numbers, a "decimal" point can only appear as part of a number, if it is actually followed by fractional digits—i.e., "decimals" (meaning, "minuscule quantities in base ten"). In a pure integer, of course, no "decimal" point appears.

This rule ensures that a period only admits to an interpretation as a radix point, if it is embedded between digits (or at least, followed by one or more digits), without intervening whitespace. In any other context, it is interpreted as a terminator of a sentence. Another way of saying this is that a period is only interpreted as a radix point if it appears in "medial" position or "initial" position (in the middle or at the start of a numeral); in "terminal" position (at the end of a word or number), it is always interpreted as prose punctuation.

In conventional prose, a semicolon can only appear in terminal position, where it is only interpreted as punctuation (a separator between clauses in a sentence). However, it's certainly reasonable to consider using it in medial or initial position for some purpose, such as an alternate radix point, or for Internet jargon such as "tl;dr".

At first dozenalists limited themselves to using the semicolon as a "duodecimal" point in medial and initial positions only, where it would actually be followed by "duodecimals" ("miniscule quantities in base twelve"). They initially refrained from using it in terminal position, in deference to its role as prose punctuation.

So the italics were still needed to mark dozenal integers. In fact, for quite a few years, italics continued to be used for all dozenals, even while the non-integer dozenals began sporting Humphrey points:

$$\begin{aligned} \mathit{100} &= 144 \\ \mathit{16};\mathit{9} &= 18.75 \end{aligned}$$

However, in later issues Churchman and his followers became even more creative:

$$\begin{aligned} \mathit{100};\mathit{0} &= 144 \\ \mathit{16};\mathit{9} &= 18.75 \end{aligned}$$

In other words, they got into the habit of taking what otherwise would have been a pure integer, and appending a spurious 0 fractional digit, simply for the sake of embedding a "dozenal" marker—logic they never thought to apply to decimal integers.

In fact, it appears that Humphrey himself had always been sanguine about using a terminal semicolon to mark an integer as dozenal, without following it with any fractional digits at all. It took many years for other dozenalists to wear down their

[6] Henry Clarence Churchman, "A Dozenal Point Worth Making", *Duodecimal Bulletin*, Vol. 11_z (13_d), No. 1, WN 22_z (26_d), May 1171_z (1957_d).

inhibitions and accept this practice, but eventually it caught on. This was enough to abolish the former italic scheme:

$$\begin{aligned} 100; &= 144 \\ 16;9 &= 18.75 \end{aligned}$$

For an outsider looking in, this is a rather curious practice, with clear drawbacks. First, it falls short on the goal of being neutral and equitable. It requires a radical change to the syntax of numbers, but only for dozenal numbers, so they can be marked as such. Meanwhile, it imposes no change at all to the syntax of decimal numbers, leaving them essentially unmarked. This confers a privileged default status to decimal base, and relegates dozenal to secondary status, as surely as italicization did.

In an attempt to reclaim some neutrality, in the last few years we even see the period being appended onto decimal integers, so that it acts as a "decimal" base marker, even in terminal position:

$$\begin{aligned} 100; &= 144. \\ 16;9 &= 18.75 \end{aligned}$$

But this merely compounds the problem. Now the scheme is quite intrusive, interfering with the normal interpretation of key punctuation marks fundamental to commonly accepted prose style. For we can easily imagine a sentence such a this:

> A gross, in decimal, is 144; whereas in dozenal, it's 100.

Here, the semicolon ends a clause while following a *decimal* integer, and the period ends the sentence while following a *dozenal* integer. Yet, under the regime of "dozenal" and "decimal" points, that sentence would be impossible. Instead we'd need to write:

> A gross, in decimal, is 144.; whereas in dozenal, it's 100;.

If we simply wish to reverse the sentence, the result is even more unfortunate:

> A gross, in dozenal, is 100;; whereas in decimal, it's 144..

The circumstances where such statements would occur are quite ordinary. As awkward as these forms are, the circumlocutions necessary to avoid them are just as awkward.

Modularity in Design … and Its Lack

What this comes down to is that the Humphrey point is a classic example of a design which achieves very poor *modularity*. Modularity is the principle of good design that stipulates that, ideally, there should be a one-to-one correspondence between the functions that a system implements, and the specific features that implement them.

Features should implement their functions as independently of each other as possible. Proliferating new features that duplicate functions already implemented elsewhere—"reinventing the wheel"—should be avoided. Piggybacking multiple functions into a single feature can be tempting to naive designers, because it feels like "killing two birds with one stone." But when a single feature is overloaded trying to satisfy too many functions, it makes it difficult to adjust how one function is being handled, without interfering with other functions.

The Humphrey point is just such an example of a naive design. It attempts to implement more than one function at once: It tries to act both as a *base-indicator*, marking a number as dozenal, and at the same time as a *fraction-point*, marking the boundary between whole digits and fractional digits. As a fraction point, it unnecessarily duplicates the function already being adequately served by the medial-period. It interferes with the normal role of the terminal-semicolon, usurping its established function as a clause-separator, in order to overload it with a new function as a base-indicator. This leads logically and inevitably to interfering with the normal role of the terminal-period, co-opting its established function as a sentence-terminator, in order to make it into a decimal base-indicator to contrast with the Humphrey point.

The fact of the matter is that when Simon Stevin coined this usage of a medial-period, inventing the so-called "decimal point," the role he intended for that was simply to act as an indicator that subsequent digits are "miniscules," fractional powers of the base. It acquired the name "decimal point," simply because it was primarily applied to base ten, and in base ten, the miniscules are known as "decimals" (meaning, "divisions of ten").

By a different etymological route, the word "decimal" has also become a term for base ten itself, in contrast with other bases. But it was never Stevin's intention that the medial-period be *limited* to base ten. He meant for it to be a fraction-point, applicable to any base. Indeed, he is actually reputed to have considered applying it to dozenal. He most certainly never intended it as a base-*indicator*. It only acquired that connotation because of the unfortunate overloading of the term "decimal."

Mainstream mathematicians have studied *many* non-decimal bases, for biquennia now (at least as far back as Gottfried Leibniz's studies of binary base back in the Dozenth Biquennium). They have done so, and continue to do so, with apparently no idea that any particular base requires its own special punctuation to mark its fractional digits. Such a requirement is simply not scalable to all the bases we might like to employ. How many different punctuation marks can we co-opt?

Nearly four unquennia ago, Churchman himself discovered, much to his chagrin, just how untenable this program of punctuation-reassignment could become. In response to the burgeoning interest in hexadecimal base due to the rise of computing machinery, he wrote an article in the *Bulletin* entitled "Welcome, Hexadecimalists!"[7] In it, he proposed using the exclamation mark as the "hexadecimal identification point" (or "HIP" for short). This would then let us say:

$$\begin{aligned} \text{90!} &= \text{100;} = 144 \\ \text{12!C} &= \text{16;9} = 18.75 \end{aligned}$$

The very next issue saw letters to the editor, from correspondents in both England and the U.S., objecting to how this proposal would usurp the role of the exclamation mark as the symbol for the factorial operator![8] The HIP was never heard from again.

[7] Henry Clarence Churchman, "Welcome, Hexadecimalists!" *Duodecimal Bulletin*, Vol. $1\mathcal{E}_z$ (23_d), No. 1, WN 36_z (42_d), Sep 1180_z (1968_d).

[8] Letters from Shaun Ferguson, Stan Bumpus, *Duodecimal Bulletin*, Vol. $1\mathcal{E}_z$ (23_d), No. 2, WN 37_z (43_d), Dec 1180_z (1968_d).

Apparently, this misadventure had been inspired the year before by Tom Pendlebury, a member of the Dozenal Society of Great Britain, and the creator of the Tim-Grafut-Maz measurement system.[9] In a short editorial note, Churchman enthusiastically relates Pendlebury's suggestion to call the Humphrey point the "Dozenal Identification Tag," or "DIT" for short.[↊] Bestowing such a convenient handle upon it, with a concise pronunciation counterpointing the "dot" for the period, seems to have helped cement the Humphrey point's dubious appeal.

If the "DIT" had truly been nothing more than a "tag" indicating a base, there would be nothing to object to. But its role as a fraction point, overloaded onto its established role as punctuation, make it problematic.

Honourable (?) Mentions

Meanwhile, as the "DIT" was insinuating itself into the consciousness of most dozenalists, some members of the DSGB (including Pendlebury) had gotten into the habit of marking some dozenal numbers with an asterisk prefix. On face value, this potentially could have been a somewhat more modular solution than the Humphrey point, if it had been applied both to integers and to fractionals:

$$\begin{aligned} *100 &= 144 \\ *16.9 &= 18.75 \end{aligned}$$

This would neatly avoid any interference with the normal syntax of integers as well as the normal radix point of fractionals.

On the other hand, it does risk clashing with the use of the asterisk as a multiplication operator, and it rather gets in the way of prefixing a minus sign to make a negative number. But the chief disadvantage of this is that it would have been no more equitable or neutral than the Humphrey point. Once again, dozenal would be given the sole burden of carrying the special marking, while decimal would retain the privileged position of being able to remain unmarked.

However, what asterisk proponents actually suggested was the following:

$$\begin{aligned} *100 &= 144 \\ 16;9 &= 18.75 \end{aligned}$$

In other words, they made use of two completely different base-indicators for dozenal: the asterisk prefix for dozenal integers, and the Humphrey point for dozenal fractionals. This makes for worse modularity, because this proliferates multiple features implementing the same function of marking dozenal numbers—while still providing no feature to mark decimal numbers.

For a counterpoint to the preceding, let us go back more than a century (eight unquennia) prior to this. Sir Isaac Pitman, the Englishman who invented shorthand, was promoting both spelling reform (a phonetic alphabet for English) and "reckoning

[9] T. Pendlebury/D. Goodman, *TGM: A Coherent Dozenal Metrology*, 11Ɛ8$_z$ (2012$_d$)

[↊] Henry Clarence Churchman, editorial note relating "DIT" suggestion from Tom Pendlebury, bottom of p. 4, *Duodecimal Bulletin*, Vol. 1↊$_z$ (22$_d$), No. 0, WN 35$_z$ (41$_d$), Sep 117Ɛ$_z$ (1967$_d$).

reform” (adoption of base twelve).[↋] He advocated a system of base annotation where *decimal* numbers would be marked, but dozenal numbers would be left unmarked:

$$\begin{aligned} 100 &= \phi 144 \phi \\ 16.9 &= \phi 18.75 \phi \end{aligned}$$

The express purpose of these awkward-looking parentheses was to mark “obsolescent” numbers. Pitman’s clear intent was to declare dozenal the superior base, and to stipulate that decimal was henceforth deemed obsolete. While this approach was certainly modular, it was also clearly inequitable—although in this instance, on the opposite extreme from the cases we have considered so far. It appears this approach did not persuade many of Pitman’s Victorian-era countrymen to abandon decimal.

Bottom line, we shall see how all of these infelicities could have been avoided in the first place—once we examine how folks in the mainstream annotate their bases today.

The Mainstream Solution

Mainstream mathematicians and textbooks on mathematics actually have a fairly straightforward approach for annotating the base of a number, an approach that has been in existence for unquennia (perhaps biquennia): They simply suffix the number with a subscript expressing the base. Usually this is itself a numeral:

$$\begin{aligned} 90_{16} &= 100_{12} = 144_{10} = 220_8 = 400_6 = 1001{,}0000_2 \\ 12.\mathrm{C}_{16} &= 16.9_{12} = 18.75_{10} = 22.6_8 = 30.43_6 = 1{,}0010.11_2 \end{aligned}$$

One advantage of this scheme is that it is *comprehensive*: This syntax lets us express a number in any base we please. This assumes, of course, that we have sufficient digit characters to support that base. In fact, the convention is to use the letters of the standard Latin 1 alphabet (the English letters A through Z) as transdecimal digits ten through two dozen eleven, thereby supporting up to base three dozen. This convention is promoted both by the educational community and, to varying degrees, by several modern computer programming languages. The letters A through F are well-known as the transdecimal digits for hexadecimal.

Another advantage of this scheme is that it is highly modular. It augments the syntax of numbers with an additional feature, which serves *only* to identify the base of the number. It does this, while neither participating in, nor interfering with, any function of any other feature. Whether the number is an integer, or has a radix point and a fractional part; whether it is a positive number, or a negative one; whether it is expressed in scientific notation, or otherwise; and so forth—none of these have any bearing upon, nor are they perturbed by, this additional subscript annotation. A long and complex expression can be couched in parentheses, and such a subscript can be applied to the whole. Readers can take all their prior experience with how decimal numbers work, and transfer that to numbers in *any* other base. There is no need to reinterpret existing punctuation, nor to learn any new operators or symbols, other

[↋] Sir Isaac Pitman, “A New and Improved System of Numeration”, *The Phonetics Journal*, London, 9 Feb. $10↊8_z$ (1856_d), http://www.dozenal.org/drupal/sites/default/files/DSA_pitman_collected.pdf.

than any additional digits the new base requires; all other mathematical symbols and operators that people are familiar with continue to behave the same way.

In terms of how based number values are formatted, this scheme is entirely equitable. All bases are treated the same; none is favored over any other. If we decide, within a given context, to designate one particular base as the assumed default, then we can simply make a blanket statement about that, and then omit the subscripts from numbers of that base, without changing any other aspect of their syntax. We can do this equivalently, no matter which base we choose to favor. Once the annotation feature has been removed from those selected numbers, no lingering trace remains that it was ever there.

The main disadvantage of this convention is that it begs the question: What base is the annotation itself expressed in? The conventional answer, of course, is simply to assume decimal. But this grants decimal favored status, at least within the subscripts. If dozenal were ever to become the preferred base, would these subscripts be recast?

$$\begin{aligned} 90_{14} &= 100_{10} = 144_{\text{Ɀ}} \\ 12.C_{14} &= 16.9_{10} = 18.75_{\text{Ɀ}} \end{aligned}$$

Ultimately, this does not eliminate the ambiguity, it merely pushes it into the subscripts. We need some way to express the annotations themselves that is neutral to any base.

One way to mitigate this is to spell out the subscripts as words:

$$\begin{aligned} 90_{\text{sixteen}} &= 100_{\text{twelve}} = 144_{\text{ten}} = 220_{\text{eight}} = 400_{\text{six}} = 1001{,}0000_{\text{two}} \\ 12.C_{\text{sixteen}} &= 16.9_{\text{twelve}} = 18.75_{\text{ten}} = 22.6_{\text{eight}} = 30.43_{\text{six}} = 1{,}0010.11_{\text{two}} \end{aligned}$$

School textbooks teaching alternate bases will often use this style. (Indeed, even as Churchman was promoting the semicolon, Shaun Ferguson of the DSGB ably demonstrated this spelled-out technique in correspondence to the *Bulletin*.[10,11])

An obvious disadvantage of using spelled-out base names, is that they make rather unwieldy subscripts. They are fine enough for isolated demonstrations of fundamental principles in a textbook setting. As tools for everyday handling of numbers, where switching between competing bases may become a frequent occurrence, such long words become tedious to write, as well as read.

Interestingly, in a letter to the *Bulletin*, published in its very second issue,[12] William S. Crosby, then a U.S. Army private in World War II, suggested the following:

$$\begin{aligned} 100_{\text{unc}} &= 144_{\text{dec}} \\ 16.9_{\text{unc}} &= 18.75_{\text{dec}} \end{aligned}$$

where "dec" is short for "decimal," and "unc" is short for "uncial" (Crosby's preferred term for base twelve). Here we have the germ of an idea: To annotate a based number, use an *abbreviation* for the name of its base. How far might we abbreviate these annotations? We will revisit this question shortly.

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[10] Shaun Ferguson, "Number Base Oddments," *Duodecimal Bulletin*, Vol. 1Ɛ$_z$ (23$_d$), No. 2, WN 37$_z$ (43$_d$), Dec 1180$_z$ (1968$_d$).

[11] Shaun Ferguson, letter, *Bulletin*, Vol. 20$_z$ (24$_d$), No. 0, WN 38$_z$ (44$_d$), Apr 1181$_z$ (1969$_d$).

[12] William S. Crosby, "Uncial Jottings of a Harried Infantryman," *Duodecimal Bulletin*, Vol. 1, No. 2, WN 1, Jun 1161$_z$ (1945$_d$). Entire letter reprinted in full on page 29$_z$.

Approaches from Programming Languages

Even as the Humphrey point was rising to prominence within the dozenalist societies, the rise of computing machines led to a different sort of prominence for the semicolon: In numerous programming languages, the semicolon became the marker for the end of an "executable statement" of code. This makes it perhaps *the* premiere character of punctuation in most software.

If we deemed the Humphrey point to be an indispensible feature of dozenal numbers, we would run the risk of branding them incompatible with the design of most programming languages. Yet this is demonstrably unnecessary. While the dozenalist societies have been focused for generations on the rather narrow problem of how to distinguish numbers of just two bases, decimal versus dozenal, programming languages tend to support several bases besides decimal. Usually there is at least support for octal and hexadecimal, and often binary as well, and in some cases, many other bases, including dozenal.

For example the Ada programming language has built-in support for all bases between binary and hexadecimal:

$$\begin{array}{llllll} \texttt{16\#90\#} & = \texttt{12\#100\#} & = \texttt{10\#144\#} & = \texttt{8\#220\#} & = \texttt{6\#400\#} & = \texttt{2\#1001\_0000\#} \\ \texttt{16\#12.C\#} & = \texttt{12\#16.9\#} & = \texttt{10\#18.75\#} & = \texttt{8\#22.6\#} & = \texttt{6\#30.43\#} & = \texttt{2\#1\_0010.11\#} \end{array}$$

In this syntax, a based number (integer or real) is flanked by number-sign characters and prefixed with the base. The base itself must be expressed as a decimal number between 2 and 16, so Ada's syntax exhibits the same decimal bias as the mainstream subscript solution. It also is rather verbose and heavy-weight.

Other programming languages favor a more terse, streamlined style of annotation. For instance, languages such as C, C++, and Java, allow the following:

$$\texttt{0x90} = \texttt{144} = \texttt{0220} = \texttt{0b10010000}$$

In other words, a numeric literal always starts with a digit, but if the initial digit is 0, it is a signal that the base is non-decimal. If the zero is followed only by digits, then the literal is interpreted as octal base. If, however, the initial zero is followed by an "x", then the literal is hexadecimal. If it is followed by a "b", then the literal is binary.

Thus, these C-style languages have managed to reduce base annotations down to one or two alphanumeric characters, without resorting to any radical redefinition of punctuation. The downside is they provide only a limited repertoire of alternate bases, and once again, they single out decimal for special status as the unmarked base.

Gene Zirkel's "Unambiguous Notation"

The better part of three unquennia ago, our very own Gene Zirkel (Member 67_z (79_d), a past *Bulletin* editor and president of the DSA, today a member of its board) observed the base annotation syntaxes demonstrated in these and other programming languages. He was inspired to write an article for the *Bulletin* titled "Unambiguous Notation for Number Bases."[13] In it, he raised the issue of the ambiguity of the mainstream

[13] Gene Zirkel, "Unambiguous Notation for Number Bases," *Duodecimal Bulletin*, Vol. 28_z (32_d), No. 3, WN 48_z (56_d), Fall 1193_z (1983_d).

subscript notation. He proposed an alternative: assign each base a unique single-letter abbreviation, and use that as an annotation. In Zirkel's formulation, the annotation would be a prefix, with the value set off by bracketing apostrophes:

$$\begin{array}{l}
\texttt{x'90'} = \texttt{z'100'} = \texttt{d'144'} = \texttt{o'220'} = \texttt{h'400'} = \texttt{b'1001,0000'} \\
\texttt{x'12.C'} = \texttt{z'16.9'} = \texttt{d'18.75'} = \texttt{o'22.6'} = \texttt{h'30.43'} = \texttt{b'1,0010.11'}
\end{array}$$

Such a scheme is comprehensive, because it can accommodate a good number of bases. It is equitable, because all bases are treated the same, with none singled out for special consideration. It is relatively lightweight, because the annotation makes use of characters readily available on the keyboard, and does not require any additional fancy typesetting—although on the downside, couching every number in apostrophes does add a bit of weight. It is also a very modular solution, because the annotations only focus on specifying the base; within the bracketing apostrophes, the existing syntax for numbers can reside, unaffected by the annotation. Finally, this notation is unambiguous, because each annotation is a single letter uniquely associated with a particular base, without itself requiring any interpretation as a numeral in some base.

(The choice of base abbreviations shown above will be explained in a moment. They are slightly different than those which Zirkel selected in his original article. Nevertheless, they demonstrate the principles that Zirkel was promoting.)

A New/Old Solution

Let's revisit the mainstream subscript annotation solution. But instead of using decimal numerals in the subscripts, suppose we substitute single-letter abbreviations similar to those from Zirkel's notation:

$$\begin{array}{rcrcrcrcrcr}
90_{\mathrm{x}} & = & 100_{\mathrm{z}} & = & 144_{\mathrm{d}} & = & 220_{\mathrm{o}} & = & 400_{\mathrm{h}} & = & 1001{,}0000_{\mathrm{b}} \\
12.\mathrm{C}_{\mathrm{x}} & = & 16.9_{\mathrm{z}} & = & 18.75_{\mathrm{d}} & = & 22.6_{\mathrm{o}} & = & 30.43_{\mathrm{h}} & = & 1{,}0010.11_{\mathrm{b}}
\end{array}$$

This seems to make for an ideal solution. It shares with Zirkel's notation the traits of being comprehensive, neutral, equitable, and unambiguous. It is light-weight and modular: Subscripts such as these are fairly unobtrusive, interfering little with any other aspect of numeric syntax, nor with any surrounding punctuation. We can demonstrate this with our previous example sentences:

> A gross, in decimal, is 144_{d}; whereas in dozenal, it's 100_{z}.
> A gross, in dozenal, is 100_{z}; whereas in decimal, it's 144_{d}.

The subscript suffix position also avoids clashing with important unary functions, such as negation (additive inverse, or the "minus" sign), which by convention are prefixes:

$$100_{\mathrm{z}} - 100_{\mathrm{x}} = -112_{\mathrm{d}} = -94_{\mathrm{z}} = -70_{\mathrm{x}}$$

Subscripts do require a bit of formatting effort. However, modern word processors, typesetting software such as LaTeX, as well as software supporting on-line blogs, wikis, and forums, all readily provide the capability to do subscripts and superscripts.[14]

[14] This option is even available to people posting on the DozensOnline Forum. This author has been using this convention there for months.

Annotations		Base Names			
Nominal	Digital	SDN	Classical	English	Dozenal English
b	2	binal	binary	two	two
t	3	trinal	ternary	three	three
q	4	quadral	quaternary	four	four
p	5	pental	quinary	five	five
h	6	hexal	senary	six	six
s	7	septal	septenary	seven	seven
o	8	octal	octal	eight	eight
e	9	enneal	nonary	nine	nine
d	A	decial	decimal	ten	ten
ℓ	B	levial	undecimal	eleven	eleven
z	C	unqual	duodecimal	twelve	one dozen, dozenal
	D	ununial	tridecimal	thirteen	one dozen one
	E	unbinal	tetradecimal	fourteen	one dozen two
	F	untrinal	pentadecimal	fifteen	one dozen three
x	G	unquadral	hexadecimal	sixteen	one dozen four
	H	unpental	heptadecimal	seventeen	one dozen five
	I	unhexal	octadecimal	eighteen	one dozen six
	J	unseptal	nonadecimal	nineteen	one dozen seven
v	K	unoctal	vigesimal	twenty	one dozen eight
	L	unenneal	unvigesimal	twenty-one	one dozen nine
	M	undecial	duovigesimal	twenty-two	one dozen ten
	N	unlevial	trivigesimal	twenty-three	one dozen eleven
	O	binilial	tetravigesimal	twenty-four	two dozen
	P	biunial	pentavigesimal	twenty-five	two dozen one
	Q	bibinal	hexavigesimal	twenty-six	two dozen two
	R	bitrinal	septavigesimal	twenty-seven	two dozen three
	S	biquadral	octavigesimal	twenty-eight	two dozen four
	T	bipental	nonavigesimal	twenty-nine	two dozen five
	U	bihexal	trigesimal	thirty	two dozen six
	V	biseptal	untrigesimal	thirty-one	two dozen seven
	W	bioctal	duotrigesimal	thirty-two	two dozen eight
	X	bienneal	tritrigesimal	thirty-three	two dozen nine
	Y	bidecial	tetratrigesimal	thirty-four	two dozen ten
	Z	bilevial	pentatrigesimal	thirty-five	two dozen eleven
	Ω	trinilial	hexatrigesimal	thirty-six	three dozen

Table 1: “Nominal” and “Digital” Base Annotations

The best aspect of this scheme, however, may be its *familiarity*. It is a relatively minor twist on a notation that mainstream mathematicians, along with many reasonably educated people, are already quite familiar with. People not necessarily invested in dozenalism might find it easier to accept and adopt this syntax.

“Nominal” and “Digital” Annotations

All that is needed is to settle on a suitable convention for single-letter abbreviations for the bases. The first column in Table 1 specifies one possible standard, supporting the previous examples. These are termed “nominal” base annotations, because these single-letter abbreviations derive from names used for the bases.

For bases under one dozen, the abbreviations from Systematic Dozenal Nomenclature[15] are apropos, since the SDN digit roots were expressly designed to start with unique letters that would be amenable to single-letter abbreviations. These include “d” for decimal. The “z” for dozenal can be rationalized based on the fact that “zen,” as a contraction for “dozen,” was historically favored both by F. Emerson Andrews and by Tom Pendlebury. It can also be seen as a reference to the astrological Zodiac, the dozen constellations along the ecliptic. The “x” for hexadecimal reflects the existing convention in programming languages. The “v” for vigesimal is straightforward.

[15] John Volan, “Systematic Dozenal Nomenclature,” *Duodecimal Bulletin*, Vol. 51_z (61_d), No. 1, WN $ᘔ1_z$ (121_d), $11Ɛ9_z$ (2013_d). See also SDN Summary in this issue on page 31_z.

The second column specifies another possible standard, supporting the following:

$$\begin{array}{rcrcrcrcrcr} 90_{\mathrm{G}} & = & 100_{\mathrm{C}} & = & 144_{\mathrm{A}} & = & 220_{8} & = & 400_{6} & = & 1001{,}0000_{2} \\ 12.\mathrm{C}_{\mathrm{G}} & = & 16.9_{\mathrm{C}} & = & 18.75_{\mathrm{A}} & = & 22.6_{8} & = & 30.43_{6} & = & 1{,}0010.11_{2} \end{array}$$

These are termed "digital" base annotations, because they systematically exploit the character assignments for transdecimal digits typically used in modern programming languages for (digital) computers. For any given base, the numbers and/or letters up to but not including the base letter can act as the digits of that base. The base letter is always one greater than its largest digit. For bases two through nine, the actual digit characters suffice as base annotations, since they are not ambiguous in isolation. Bases ten through two dozen eleven are represented by the letters A through Z of the Latin 1 (English) alphabet. The Greek letter omega is included to represent base three dozen, rounding out the set. That base must utilize all ten decimal numerals and all two dozen two Latin 1 letters, in order to represent its digits.

As specified, the "nominal" annotations all use lowercase letters, while the "digital" annotations all use uppercase. This contrast allows both types of annotation to coexist without conflict. Users may employ whichever standard best suits their needs. The lowercase nominal forms are a bit more pleasant on the eye, and more suggestive of the names of the bases, so they might be good for frequent everyday usage. Whereas the digital annotations, being more exhaustively comprehensive, might be better suited to technical analyses about multiple number bases.

To Subscript or Not to Subscript

Subscripting might be problematic in certain disadvantaged environments, such as when writing by hand, or in email or other impoverished forms of text communication. In that case, a suitable inline syntax, utilizing the same annotation abbreviations, might be able to substitute for subscript notation.

One possibility would look at how mainstream mathematicians have inlined subscripts in other contexts. For instance, when a variable represents an array or set of quantities, or a vector quantity, mathematicians often use a subscript as an index referring to a specific element of the array, set, or vector. When subscripting is not available, the substitute is often to suffix the variable with the index in brackets:

$$a_0 = \mathsf{a}[0], \quad a_1 = \mathsf{a}[1], \quad a_2 = \mathsf{a}[2], \quad \text{etc...}$$

This syntax might also work as an inline substitute for base annotation subscripts:

$$\begin{array}{rcrcrcrcrcr} \mathtt{90[x]} & = & \mathtt{100[z]} & = & \mathtt{144[d]} & = & \mathtt{220[o]} & = & \mathtt{400[h]} & = & \mathtt{1001,0000[b]} \\ \mathtt{12.C[x]} & = & \mathtt{16.9[z]} & = & \mathtt{18.75[d]} & = & \mathtt{22.6[o]} & = & \mathtt{30.43[h]} & = & \mathtt{1,0010.11[b]} \end{array}$$

One way to rationalize this is to view these inlined annotations as parenthetical remarks about the preceding values. Indeed, we could read these numerals off as follows: "nine-zero (hexadecimal) equals one-zero-zero (dozenal) equals one-four-four (decimal) ..." In fact, such a reading might be just as applicable to the fully typeset subscript annotations. The suffix-subscript position is pretty much the "oh-by-the-way" position in mathematical notation.

The First Few Squares											
N			N^2			N			N^2		
[d]	[z]	[x]	[d]	[z]	[x]	[d]	[z]	[x]	[d]	[z]	[x]
1	1	1	1	1	1	13	11	D	169	121	A9
2	2	2	4	4	4	14	12	E	196	144	C4
3	3	3	9	9	9	15	13	F	225	169	E1
4	4	4	16	14	10	16	14	10	256	194	100
5	5	5	25	21	19	17	15	11	289	201	121
6	6	6	36	30	24	18	16	12	324	230	144
7	7	7	49	41	31	19	17	13	361	261	169
8	8	8	64	54	40	20	18	14	400	294	190
9	9	9	81	69	51	21	19	15	441	309	1B9
10	↊	A	100	84	64	22	1↊	16	484	344	1E4
11	↋	B	121	↊1	79	23	1↋	17	529	381	211
12	10	C	144	100	90	24	20	18	576	400	240

Table 2: Example table with blanket column-wise base annotations

Inline bracketed suffixes manage to remain about as unobtrusive as suffixed subscripts. For instance, they avoid interfering with unary operators in prefix position:

$$\texttt{100[z]} - \texttt{100[x]} = -\texttt{112[d]} = -\texttt{94[z]} = -\texttt{70[x]}$$

Moreover, bracketing the base abbreviations in this way might also make for convenient stand-alone tags useful as blanket annotations for whole regions of text. For instance, we could use them in table headers to annotate the bases for entire rows or columns of a table. This would allow us to avoid having to annotate each cell individually, making the table less cluttered overall, yet not shirking the obligation to explicitly specify the base in use at every point. Table 2 provides an example demonstrating this.

International Neutrality

Thus far, we have been presuming the Anglo-American convention for punctuating numbers, in which the period is used as the fraction point, and the comma is used as a grouping separator in long numbers:

$$\left(2^{36}+2^{-12}\right)_{\mathrm{d}} = 68{,}719{,}476{,}736.000\ 244\ 140\ 625_{\mathrm{d}}$$
$$\left(2^{30}+2^{-10}\right)_{\mathrm{z}} = 11{,}39↊{,}01↋{,}854.000\ 509_{\mathrm{z}}$$

On the continent of Europe, and elsewhere, the convention is the exact opposite:

$$\left(2^{36}+2^{-12}\right)_{\mathrm{d}} = 68.719.476.736{,}000\ 244\ 140\ 625_{\mathrm{d}}$$
$$\left(2^{30}+2^{-10}\right)_{\mathrm{z}} = 11.39↊.01↋.854{,}000\ 509_{\mathrm{z}}$$

Since the subscripted annotations proposed here provide a modular solution, they are completely independent of these considerations. So Continentals could readily adopt the same base annotations, while retaining their preferred punctuation.

A solution such as this, compatible with the local standards of other nations regarding number format, is much more likely to gain international acceptance than one that usurps their preferences. Even though the Humphrey point disrupts American/British standards as much as it does Continental standards, nevertheless there can be a *perception* that it constitutes a veiled attempt to impose Anglophile cultural hegemony. Base annotation should simply be a question of what is most practical. We should prefer a solution that avoids seeming political.

"Why Change?"[16]

Dozenalists are people who wish to bring the use of base twelve into the mainstream, because it is demonstrably a better base than decimal. As such, it would behoove us to do as much as possible to demonstrate how *normal* base twelve can be, how *little* people really need to change in order to make use of it.

It is therefore a great irony to see the earliest proponents of dozenalism in this country actually accepting—indeed, vigorously embracing—practices better geared to emphasize decimal as the "normal" or "default" base, and dozenal as a base set apart as "marked" and "different" and "peculiar"—and by implication, "second-rate".

In this author's opinion, the Humphrey point was a chief culprit. Yet today, it has become something of a cherished tradition within the dozenal societies, with roots spanning more than a human lifespan. Perhaps the foregoing has persuaded the reader to reconsider whether this was really a good thing. The Humphrey point should not persist merely for the sake of nostalgia.

The alternative set forth in these pages also has roots that go back at least as far, if not further. Its elements have been present since the founding of the DSA, and aspects of it have been touched at by contributors to this publication, at various times throughout its history.

The DSA has made major changes in the past, notably the adoption of the "Bell" characters as transdecimal digits, and later the abandoning of these to return to the Dwiggins characters. So it is not impossible to decide to change something seemingly fundamental, upon better judgment.

Going into a new biquennium, we should opt for a solution for base annotation that is more neutral, equitable, modular, and versatile, than the Humphrey point. We need a technique that marks all bases equally, without clashing with mainstream standards of mathematical notation and prose style—indeed, one that derives from, and extends upon, mainstream practices. A convention assigning single-character alphanumeric abbreviations to bases, with handy, and generally-familiar, places to position these, can satisfy these goals.

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[16] Title of an editorial essay by Ralph H. Beard, first editor of the *Bulletin*. First published in *Duodecimal Bulletin*, Vol. 4, No. 1, WN $Ɛ_z$ (11_d), Dec 1164_z (1948_d). Remastered in $11Ɛ7_z$ (2011_d) by Michael T. De Vlieger as `http://www.dozenal.org/drupal/sites/default/files/db043r2_0.pdf`. Quote: "Then, shouldn't we change? No! No change should be made and we urge no change. All the world uses decimals. But people of understanding should learn to use duodecimals to facilitate their thinking, and to ease the valuative processes of their minds. Duodecimals should be man's second mathematical language. They should be taught in all the schools. In any operation, that base should be used which is most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity. But no change should be made. Perhaps by the year 2000, or maybe by *1200*, which is *14* years later, duodecimals may be the more popular base. But then no change need be made, because people will already be using the better base." (Original italic marking of dozenal numbers retained.)