# DOZENS VERSUS TENS

OR

## THE OUNCE, THE INCH, AND THE PENNY

CONSIDERED AS

## STANDARDS

OF

## WEIGHT, MEASURE, AND MONEY

AND WITH REFERENCE TO A

## *DUODECIMAL NOTATION*

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The Dozenal Society of America is a voluntary, nonprofit educational corporation, organized for the conduct of research and education of the public in the use of dozenal (also called duodecimal or base twelve) in calculations, mathematics, weights and measures, and other branches of pure and applied science.

This little book is quite probably the earliest modern full-length exposition of the dozenal system ever produced. The volume currently before the reader is a faithful reproduction of the original work, with only a few minor changes. Some tables were superficially reformatted; pagination is done in dozenal, using the Pitman numerals (ↂ and ↈ) rather than Mr. Leech's "t" and "e"; and the thermometry page in the appendix was omitted. It is remastered and republished in the hopes that it might prove beneficial to the world.

# Contents

# Dozens versus Tens

The advantages to be derived from the adoption of a system of coinage and of weights and measures which should correspond with our system of notation, which is decimal or 'tennish,' are so obvious, that it is a matter of surprise that so much delay has taken place in carrying out the many and weighty recommendations in its favour. But, as has been observed, 'all change is in itself a disadvantage, and a great change is a great disadvantage;'* therefore, before we make a change, or a 'great change' such as this, it behoves us to see that the advantages overbalance the disadvantages. Now the advantages of a decimal system are—'the total abolition of "compound" arithmetic; a great saving of time in teaching arithmetic, which time might be applied to the teaching of other things; and a further great saving of time in the counting-house and places of business, and—as time is money—an increase of the wealth of the nation.' These are the advantages, as generally set forth in books and pamphlets on this subject. The disadvantages are—the change itself; the abolition of, or at any rate the interference with, the binary system, or system of halves and quarters, which is said to be, and indeed is, natural and convenient; the abolition of the penny, and consequent alteration of the rates of postage, and of tolls and railway fares.

Now, I am not going to consider the relative value of these advantages and disadvantages: this has already been done by competent authorities, and the 'change' will take place sooner or later, in spite of anything that could now be urged. But the fact that *any* objection can be raised to the adoption of a system which appears so very simple, leads one to consider whether—it being admitted that the *notation* and money system and weights and measures system should accord—whether the *notation* might not be adapted to our present money system, &c.; that is to say, whether a duodecimal for 'twelvish' notation might not be introduced, in which the digits or figures should increase by *twelves* or *dozens*, instead of tens, from right to left; in which, in fact, the sequences 10 and 100 should represent respectively a dozen and a gross, and in which there should be a 'carriage' of dozens instead of tens. I need hardly say that I do not write with a view to any practical

*Professor De Morgan, in Preface to *Proceedings of Decimal Association.*

result—at least not any *immediate* practical result—for one need not always write for the hour, but may occasionally enjoy the luxury of contributing one's mite to the payment of what Mr. Mill calls the debt due to posterity—but rather to indulge in an arithmetical recreation.

Now the 'change' here proposed would not be so violent as at first sight might appear: for instance, ALL *our present coins could be retained with their values unaltered*; this I shall prove as I go along.

The overwhelming advantages which the present system of arithmetic, in which the figures receive their value from their *position* in the series, has over the cumbrous old Roman system, are so very obvious that nothing need be said on the subject. How an 'ancient Roman' could ever have worked a sum in Practice or the Rule of Three, must ever remain a puzzle. But we are too apt to consider present advantages with reference to past disadvantages; and, just as a beggar considers 5*l*. a fortune, or a dweller in a common lodging-house considers a semi-detached cottage a palace, we look upon our present system of what I may call positional arithmetic as one of absolute perfection.

In books of arithmetic which touch upon this subject at all, we are generally told that man has ten fingers, and that 'uncivilized man' reckoned upon these fingers, and so came about the decimal system in the most natural manner possible. That man *has* ten *fingers* is a proposition open to question, but that civilized man in the latter half of the 19th century,—to say nothing of the 20th, 30th, and 50th centuries—should continue a particular system, because certain remote and savage ancestors reckoned on their 'pickers and stealers,' is not very intelligible, to say the least of it. Professor De Morgan* thinks that an educated community having to begin would at once adopt the decimal system; Dr. Colenso,† on the other hand, thinks a duodecimal notation would be preferable. What 'an educated community going to begin' would or would not do, it is not necessary to stop to enquire; what we have to do is to consider what is most convenient, and best suited to the ordinary purposes of life.

The most natural and *practically useful* divisions are binary and ternary—that is to say, divisions by 2, 3, and 4, or halves, thirds, and quarters; and that system of notation is the best in which these fractions ($\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$) can be most easily expressed with reference to the *base*. In the decimal or tennish system there are $\frac{1}{2} = \cdot 5$, $\frac{1}{4} = \cdot 25$, and $\frac{1}{3}$ cannot be *exactly* expressed, though

* *Proceedings of the Decimal Association.*
† *Arithmetic.*

the interminable decimal ·3333333 &c., is usually expressed by the symbol ·3̇: where it will be seen that out of the three natural and particularly useful fractions, one requires two figures to express it, and one is an interminable or circulating decimal. In the duodecimal system these fractions would be expressed respectively as follows—·6, ·4, ·3, all terminable, and none of more than one figure.

The number 10 (ten), moreover, is an unnatural number, notwithstanding we have so long been accustomed to it. Everything in Nature points to twos and threes, and not to twos and fives. The number 12 (twelve), to which the numbers 2 and 3 bear a simple relation, is incorporated, so to speak, in our present system, to express divisions of time, space, money, and weight.

If we must have analogies to support our position, it would not be difficult to find them. The very 'finger' theory must be given up, as we have not ten 'fingers,' but eight *plus* two thumbs, the element five not appearing at all.* Our limbs are set two and two, and so are all the organs of our body. Of the two classes into which our flowering plants are divided, one is entirely composed of plants whose parts are regulated by the number 3 or 2×3; and of the other, a large part is made up of those which are regulated by the number 4; and of those which appear at first sight to present an exception to what we have laid down, in which the number 5 appears (*Ranunculaceæ*, *Rosaceæ*, &c.) a very great number are (botanically. speaking) 'irregular,'—their petals being in threes and twos, or fours and ones, or all different.

Notwithstanding our decimal notation, the dozen (with its subdivisions, 2, 3, 4, 6, &c.) has great prominence given to it. We have twelve tribes, twelve apostles, four gospels; twelve months, twelve hours, six working days; twelve ounces to the pound, twelve inches to the foot, twelve pence to the shilling. But where we meet with ten we are sure to find something amiss: for instance, of the ten lepers cleansed, nine were ungrateful; of the ten virgins, half were foolish. And then, again, what a sad mess Dr. Colenso has got into through meddling with the Pentateuch† ($\frac{10}{2}$)! One might go on like this by the mile, but we must return to the practical part of the subject. If we adopt

*We never heard that the Maltese family whose members had an extra 'digit' on each hand, reckoned duodecimally.

†With respect to the Decalogue, the Roman Church makes the first two commandments (English) to coalesce, and the English Church unites the ninth and tenth of the Roman. Now if both sides are half right and half wrong—that is, suppose we allow the coalescence but not the division—we get 9 for the proper number, which is a good duodecimal number, being $\frac{3}{4}$ of a dozen.

12 (twelve) for the base of notation, we must invent two new digits, and, for our present purposes, the initial letters of the numbers they represent will suffice—viz., t = ten, e = eleven; the present digits representing the same numbers as now, thus:—

1, 2, 3, 4, 5, 6, 7, 8, 9, t, e, 0.

As the value of these digits will increase or decrease twelvefold instead of tenfold, according to their position in the series, it will be well to give a few examples. Thus 10 = twelve, or a dozen; 100 = a gross (one hundred and forty-four); 1000 = twelve gross (1728 decimal notation), for which a name will have to be invented. I suggest the word 'cubion' provisionally, it being the *cube of the base*. It will be well, however, to put in a tabular form a few numbers expressed both in the decimal and duodecimal system, with the value of the latter in words; for as we are to reckon by 'dozens' instead of 'tens,' a new nomenclature will have to be adopted. I by no means wish that I have used to be considered the best possible; but if 'cubion' be accepted for that of the *cube of the base* (1000), we should not want a new name until we arrived at its square (1,000,000). Let 'cubion' be represented by $c$; then we should, in fact, only want new names for $c^2$, $c^4$, $c^8$, &c., corresponding (in number of cyphers, but not in value) to our million, billion, trillion, &c.

This need not present much difficulty, for if we had no names at all they would not be much sought for; as even with the present notation we seldom use the names of such high numbers, and in the duodecimal system their want would be still less felt, *as by it high numbers are expressed tn fewer figures than in the decimal system.* Where numbers are frequently repeated, the words hundreds, thousands, &c., are generally left out. Thus 2356 would be given thus,—two, three, five, six; 17827—seventeen, eight, two, seven.

Subjoined are a few numbers expressed in both notations.*

In this table it is unnecessary to give in words the value according to the decimal system, this being generally known.

It may be well now to comparea few vulgar fractions with their equivalent expressions in the decimal and duodecimal scales, and for this purpose those fractions having the simple digits and the bases for their denominators will be taken:—†

On glancing over the first table, what one notices is the fact that the 'round' number in the one column is represented by numbers of mixed figures

*See page 9, to which it has been moved for typesetting purposes. —Ed., DSA, 11Ɛ9.
†See page ƻ, to which it has been moved for typesetting purposes. —Ed., DSA, 11Ɛ9.

| Decimal Tennish | Duodecimal Twelvish | |
|---:|---:|---|
| 10 = | t = | ten |
| 11 = | e = | eleven |
| 12 = | 10 = | dozen |
| 13 = | 11 = | dozen and one |
| 14 = | 12 = | dozen and two |
| 15 = | 13 = | dozen and three |
| 16 = | 14 = | dozen and four |
| 17 = | 15 = | dozen and five |
| 18 = | 16 = | dozen and six |
| 19 = | 17 = | dozen and seven |
| 20 = | 18 = | dozen and eight |
| 21 = | 19 = | dozen and nine |
| 22 = | 1t = | dozen and ten |
| 23 = | 1e = | dozen and eleven |
| 24 = | 20 = | two dozen |
| 30 = | 26 = | two dozen and six |
| 33 = | 29 = | two dozen and nine |
| 36 = | 30 = | three dozen |
| 35 = | 2e = | two dozen and eleven |
| 40 = | 34 = | three dozen and four |
| 45 = | 39 = | three dozen and nine |
| 48 = | 40 = | four dozen |
| 50 = | 42 = | four dozen and two |
| 54 = | 46 = | four dozen and six |
| 60 = | 50 = | five dozen |
| 100 = | 84 = | eight dozen and four |
| 144 = | 100 = | one gross |
| 1,000 = | 6e4 = | six gross, eleven dozen and four |
| 1,728 = | 1,000 = | one cubion (?) |
| 10,000 = | 5,954 = | five cubions, nine gross, five dozen and four |
| 20,736 = | 10,000 = | twelve cubions, or one dozen cubions |
| 1,000,000 = | 402,854 = | four dozen and two cubions, eight gross, five dozen and four |
| 2,985,984 = | 1,000,000 = | cubion squared (?) nomenclature |

| Vulgar fractions | | Decimal | Duodecimal |
|---|---|---|---|
| $\frac{1}{2}$ | = | ·5 | ·6 |
| $\frac{1}{3}$ | = | ·3333, &c., or ·$\dot{3}$ | ·4 |
| $\frac{1}{4}$ | = | ·25 | ·3 |
| $\frac{1}{5}$ | = | ·2 | ·2$\dot{4}$9$\dot{7}$ |
| $\frac{1}{6}$ | = | ·1666, &c., or ·1$\dot{6}$ | ·2 |
| $\frac{1}{7}$ | = | ·$\dot{1}$42857, &c. | ·$\dot{1}$86t3$\dot{5}$ |
| $\frac{1}{8}$ | = | ·125 | ·16 |
| $\frac{1}{9}$ | = | ·11111, &c., or ·$\dot{1}$ | ·14 |
| $\frac{1}{10}$ ($\frac{1}{\mathrm{t}}$) | = | ·1 | ·1$\dot{2}$49$\dot{7}$ |
| $\frac{1}{11}$ ($\frac{1}{\mathrm{e}}$) | = | ·0909, &c., or ·$\dot{0}\dot{9}$ | ·$\dot{1}$ or ·111, &c. |
| $\frac{1}{12}$ ($\frac{1}{10}$) | = | ·08$\dot{3}\dot{3}$ | ·1 |
| to which we may add | | | |
| $\frac{1}{13}$ ($\frac{1}{11}$) | = | ·$\dot{0}$76923 | ·0e0e, &c., or ·$\dot{0}\dot{\mathrm{e}}$ |

in the other. Now it is of course a great advantage to have 'round' numbers when we can. So much is this the case, that we should in most instances prefer to work with a round number, consisting say of five cyphers and a digit, rather than with a number consisting of only three figures if these were mixed; and, moreover, it will be evident that a system of notation with a large number of digits will have, on the whole, fewer 'round' numbers than a system with fewer digits; and, consequently, that if *sequences* of numbers up to the same point be taken from the decimal and duodecimal scales, the latter would be found to have the fewest 'round' numbers. But what we have to consider is, not what would happen when drawing numbers out of a lottery-bag, but what would happen in the actual business of life—the interchange of commodities, money, &c. Now we know from common experience that the numbers 2, 3, 4, 6, 8, 9, are much more used than any other of the numbers represented by single digits—5 for instance (I leave out the figure 7, as that is equally inconvenient in both systems)—and these numbers are much more simply related to the base *twelve* than to the base *ten*.

If we look at the second table, we find that of the first nine set down, only four can be expressed by terminating decimal fractions, while of the same nine fractions six can be expressed by terminating duodecimals. If we take eleven vulgar fractions (corresponding with the number of digits in the duodecimal scale), we still find a balance in favour of the duodecimal in the

proportion of 6 to 5, and if we take the whole thirteen we have a balance in the proportion of 7 to 5.

To put it in another way. Of the nine digits in the decimal scale, only two (2 and 5) are exact aliquots of the base ten; but of these same nine digits, there are four (2, 3, 4, 6) which are exact aliquots of the base twelve; so that for every *three*[*] operations in the decimal scale we should have fifteen[†] in the duodecimal scale, the results of which would be a simple relation to the base. And in practice, it must be again observed that the numbers 2, 3, 4, &c. are much more likely to be used than the number 5.

Again, with regard to 'round' numbers. No one supposes that the various statistics found in State Papers—such as population, customs, post-offices, &c.—give us the exact numbers ; at least, the numbers quoted in newspapers and in Parliament can scarcely be so conveniently 'round' as they appear. The pence and shillings are generally left out, and generally (if dealing with large figures) the hundreds and thousands. Take, for example, the following: Our trade with France in 1859 is represented by the figures £26,431,000, in 1864 by £49,797,000; showing an increase, said Mr. Gladstone, of £23,366,000, or *nearly* 90 per cent. What the *exact* figures would be I have no means at hand for ascertaining, but I infer that the 'hundreds' have been neglected. Now, if it were desirable to make the calculation still less exact, and to neglect the 'hundreds of thousands,' we should then say that in the one year the amount was 49 millions, and in the other 26 millions, and we might then say that the increase was nearly 100 per cent., or *nearly* doubled. All depends in these cases on the degree of accuracy required, and much on the temperament of the calculator, and the object he has to prove. These numbers, stated in the duodecimal scale of notation, would be as follows: 1859, £8,t27,874; 1864, £14,815,860, giving an increase of £7,9t9,et8, or '*nearly*' doubled as before. But as we are to retain the present shilling and penny as *coins*, and the present *florin* as coin *as well as in account* (as will be explained further on), the new 'pound,' or Victoria, will have a greater value than the present pound in the proportion of 6 to 5, since the new coin will contain twelve florins to the old one's ten. The following will be the representatives of the foregoing values in duodecimal notation and money:—

[*] $C_2 + C_1 = 2^2 - 1 = 3$.

[†] $C_4 + C_3 + C_2 + C_1 = 2^4 - 1 = 15$.

| | | | |
|---|---|---|---|
| £23,361,000 | = | 6,631,596 | New pounds, or Victorias, of twelve florins each |
| 26,431,000 | = | 7,463,490 | |
| 49,797,000 | = | 1,1t92,910 | |

Substituting cyphers for the three figures on the right, which has no doubt been done in the decimal values quoted, we can of course get 'round' numbers to suit our purpose. We thus see at a glance how much fewer figures we have to deal with, and how much easier to compare large numbers in the duodecimal than in the decimal scale.*

When speaking of the advantages and disadvantages of the two systems, I said that among the latter was the abolition of the penny, taking it for granted that that plan would be adopted by which the penny (and farthing) would be abolished, and I believe it is pretty certain that that plan will be carried out.

It may be well to recall attention to the two plans of decimal coinage which have been proposed. The one proposes to retain the *present pound*, and divide it into 1000 new farthings or 'mils,' as they are to be called; the other to retain the *present farthing*, and to coin a *new pound* which should contain, or be equal in value to, 1000 farthings. The advantages of this would be the retention of the farthing and penny, and also of the present rates of postage, toll, &c. The disadvantages would be the increased value

*To convert numbers in the decimal scale to the equivalent numbers in the duodecimal scale, divide by 12; the remainder will be the units figure in the scales; divide the quotient again by twelve, and the remainder will be the dozens figure, and so on. Thus:—

12)23366000
12)1947166 + 8
12)162263 + t
12)13521 + e
12)1126 + 9
12)93 + t
7 + 9 *Ans.* 70t9et8

To convert numbers in the duodecimal scale to equivalent decimal numbers, divide by t (ten) in a similar manner, recollecting to 'carry' *dozens* instead of tens:—

t)79t9et8
t)948248 + 0
t)e3278 + 0
t)11632 + 0
t)1428 + 6
t)175 + 6
t)1e + 3
t)2 + 3 *Ans.* 23,366,000, as above.

of the pound, and the supposed 'difficulty' of comparing large sums which happened to be expressed according to different systems. Now the increase, as before stated, would be in the proportion of 1000 to 960, so that £1 of the new coinage would be equal to £1 0*s.* 10*d.* of the present. This does not seem to present any very great difficulty: the addition of 10*d.* to the pound brings the new coinage into its equivalent in the present coinage, or £1 new (1000 farthings) = £$1\frac{1}{24}$ old, and £1 old (960 farthings) = £$\frac{24}{25}$ new. It is gravely stated, however, that the simple calculation necessary for expressing sums given in one coinage in terms of the other would be too much for tradesmen, merchants, bankers, Members of Parliament, and Chancellors of the Exchequer!

I will not dispute the fact of there being a vast amount of ignorance of arithmetic among all classes of society, and I can quite believe Mr. Gladstone's statement to be correct, that not one in ten of the members of the House of Commons could work the sum in Reduction and Division which he proposed;* but if this ignorance exists in high places, how much more so in low places! And would not a dispute about 'farthings' and 'mils' and '4 per cent.' be more likely to be settled by a mill proper than by an appeal to Cooker or Colenso?

I shall now give a few examples in the first four rules, to enable the reader to follow me in what succeeds:—

| Addition | Subtraction | Multiplication | Division |
|---|---|---|---|
| 3478 | e234 | 4263 | 7)25579 |
| 2459 | 812t | 7 | 2463 |
| 32e1 | 3106 | 25579 | |
| 9016 | | | |

The duodecimal or *twelvish* 'money-table' will be as follows:—

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 twelfthing or mite = $\frac{1}{6}d.$, or 3 twelfthings | | | | | = | $\frac{1}{2}d.$ |
| 10 (twelve twelfthings or mites) | | = | 1 doit | | = | $2d.$ |
| 100 twelfthings | = | 10 doits | = | 1 florin | = | $2s.$ |
| 1000 twelfthings | = | 10 florins | = | 1 victoria | = | $24s.$ |

*Viz., to find how often £2 13*s.* 8*d.* is contained in £1330 17*s.* 6*d.* This is what Mr. Gladstone *meant*. It is not a little surprising, however, that *he* should have spoken of dividing one concrete quantity by another!

There will be no 'compound' arithmetic. Questions relating to money will be worked in precisely the same manner as in the above examples, 'carrying' *twelves*, be it again observed. Let it be required to find how much money will pay 29 (*i.e.* two *dozen* and nine) men 2 victorias, 3 florins, 4 doits, 6 twelfthings (or mites) each. This sum may be worked with or without the points marking each denomination, thus:—

| | V | fl. | dt. | twg. |
|---|---|---|---|---|
| 2346 | 2 | 3 | 4 | 6 |
| 29 | | | | 29 |
| 18646 | 18 | 6 | 4 | 6 |
| 4690 | 46 | 9 | 0 | |
| V63,346 | V63 | 3 | 4 | 6 |

Reversing the process, which will serve also as a 'proof' of the foregoing, let it be required to divide 63 victorias, 3 florins, 4 doits, 6 twelfthings (or mites) among 29 men. Here:

29)63346 = 2346 or 2v. 3fl. 4dts. 6tw.
56
93
83
104
e0
146
146
...

As the lowest denomination of the duodecimal scale is $\frac{1}{6}$th of a penny of the present coinage and notation, it is easy to convert sums expressed according to one scale into equivalent sums in the other.

To convert sums in the dudoecimal scale to their equivalents in the decimal: express the gross sum according to the decimal *notation*; then divide successively by 6, 12, 20:—

t)2346 6)3942
t)28t.2 12)657
t)33.4 2,0)5,4.9
3.9 £2.14.9

whence it appears that 2 victorias, 3 florins, 4 doits, 6 twelfthings = £2 14*s*. 9*d*.—To convert sums expressed in the decimal notation to the equivalents in the duodecimal: express the whole in pence in the usual manner, multiply by 6, and express the result in the duodecimal scale. Thus—

| £ | *s*. | *d*. | |
|---|---|---|---|
| 2 | 14 | 9 | |
| 20 | | | |
| 54 | | | |
| 12 | | | |
| 657 | | | |
| 6 | | | |
| 12)3942 | | | |
| 12)328 | 6 | | |
| 12)27 | 4 | | |
| 2 | 3 | | *Ans.* V2. 3fl. 4dts. 6tws., as before |

But for large sums, where we only have to take care of the pounds, leaving the pence to take care of themselves, the calculation is not so complicated, as it resolves itself practically into merely changing the notation. For by adding a cypher (multiplying by 10 or 12) in either system we obtain an equivalent expression in *florins*. Thus £1,000 (decimal): 10,000 florins:

| | |
|---|---|
| 12)1000 | |
| 12)12 | |
| 12)833.4 | |
| 12)69.5 | |
| 5.9 | |
| 0 | 5,954fl. = V595. 4fl. |

Reversing the process, we have 595 victorias 4 florins = 5,954 florins, which, expressed in the decimal s[c]ale, gives:

| | |
|---|---|
| t)5054 | |
| t)12 | |
| t)6e4.0 | |
| t)84.0 | |
| t.0 | or 10,000 florins = £1,000. |

It will now only be fair to give an example, in which the duodecimal system should have the advantage of 'round' numbers. Let it be required to convert 1000 victorias (new) into equivalent pounds sterling. Here 1000 victorias = 10,000 florins:

t)10,000
t)1249.6
t)153.3
t)18.7
2.0 or 20,7366 (decimal) = £2,073 12*s*.

Reversing the process, we have—

12)23736
12)1728.0
12)144.0
12)12.0
1.0 or 10,0006 = V1,000*s*.

It may be well, perhaps, for the purpose of comparison, to give a list of a few small sums expressed in both notations*:—

It will be observed that the units figures have been doubled in the latter part of the table to express shillings. Now surely these calculations are not too complicated for ordinary accountants, and persons likely to refer to bygone State Papers. Tables could easily be made for purposes of reference. *All the coinage remains the same as now*, with the introduction of three new coins—viz., the new pound or 'Victoria,' 'doit,' and the 'twelfthing,' or whatever names may be given them. As I said before, I am not particular about the mere name. The twelfthing would be an exceedingly useful coin, $\frac{1}{6}d.$ being rather larger than the half-farthing, which one never sees, and would enable the poor man to derive the benefits of alteration in prices to a greater extent than he now can; and as three of them would go to the halfpenny, the doit would be equal to $2d.$ But with regard to this and the 'Victoria,' it may be observed that it would not be absolutely necessary to have them at all *as coins*, though they would be retained as money of *account*.† For a long

*This table was moved to page 15 for typesetting purposes. —Ed.

†See on this point—as to money of account and coinage—*Proceedings of the Decimal Association*.

| Decimal | | | | Duodecimal | |
|---|---|---|---|---|---|
| £ | | florins | | Vic. | fl. |
| 5 | = | 50 | = | 4 | 2 |
| 6 | = | 60 | = | 5 | 0 |
| 10 | = | 100 | = | 8 | 4 |
| 15 | = | 150 | = | 10 | 6 |
| 20 | = | 200 | = | 14 | 8 |
| 25 | = | 250 | = | 18 | t |
| 50 | = | 500 | = | 35 | 8 |

| Duodecimal | | | Decimal | | | |
|---|---|---|---|---|---|---|
| Vic. | | florins | florins | | £ | *s* |
| 5 | = | 50 | 60 | = | 6 | 0 |
| 6 | = | 60 | 72 | = | 7 | 4 |
| 10 | = | 100 | 144 | = | 14 | 8 |
| 16 | = | 160 | 196 | = | 19 | 12 |
| 20 | = | 200 | 288 | = | 28 | 16 |
| 26 | = | 260 | 360 | = | 36 | 0 |
| 50 | = | 500 | 720 | = | 72 | 0 |

time the pound sterling did not exist as a *coin*, and I am not aware that any great difficulty was experienced from the want of it. Still, if it should be necessary to have the new coin, it would be found to be a very handy and convenient one, slightly thicker and larger than the present sovereign. Its half, too (making a fourth new coin, however), would also be of a convenient size.

The following would be the relations of the present to the new moneys:—

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Farthing | = | $1\frac{1}{2}$ | twelfthings or mites | | | | | |
| Halfpenny | = | 3 | ” | | | | | |
| Penny | = | 6 | ” | | | | | |
| Threepenny piece | = | 18 | ” | = | $1\frac{1}{2}$ | doits | | |
| Fourpenny piece | = | 24 | ” | = | 2 | ” | | |
| Sixpence | = | 36 | ” | = | 3 | doits | | |
| Shilling | = | 72 | ” | = | 6 | ” | | |
| Two-shilling piece | = | 144 | ” | = | 12 | ” | = | 1 florin |
| Halfcrown | = | 180 | ” | = | 15 | ” | = | $1\frac{1}{4}$ ” |
| Crown | = | 360 | ” | = | 30 | ” | = | $2\frac{1}{2}$ ” |
| Half-sovereign | = | 720 | ” | = | 60 | ” | = | 5 ” |
| Sovereign | = | 1440 | ” | = | 120 | ” | = | 10 ” |

Or, expressing these relations in duodecimal notation, as follows:—

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Farthing | = | $1\frac{1}{2}$ | twelfthings | | | | | |
| Halfpenny | = | 3 | ” | | | | | |
| Penny | = | 6 | ” | | | | | |
| Threepenny piece | = | 16 | ” | = | $1\frac{1}{2}$ | doit | | |
| Fourpenny piece | = | 20 | ” | = | 2 | ” | | |
| Sixpence | = | 30 | ” | = | 3 | ” | | |
| Shilling | = | 60 | ” | = | 6 | ” | | |
| Two-shilling piece | = | 100 | ” | = | 10 | ” | = | 1 florin |
| Halfcrown | = | 130 | ” | = | 13 | ” | = | $1\frac{1}{4}$ ” |
| Crown | = | 260 | ” | = | 26 | ” | = | $2\frac{1}{2}$ ” |
| Half-sovereign | = | 500 | ” | = | 50 | ” | = | 5 ” |
| Sovereign | = | t00 | ” | = | t0 | ” | = | t ” |

Showing a large balance of ‘round’ numbers. The halfcrown and crown will probably gradually disappear altogether, and the sovereign and half-sovereign be gradually replaced by the victoria and half-Victoria.

I will now pass on to the consideration of measures of length, and will just remark, in passing, that there is no *necessary connection* between the *decimal* system and the *metrical* system, though these are often confounded. The latter, which is the system used in France and in many parts of the Continent, takes for a standard the metre, which is the $\frac{1}{40,000,000}$ of the circumference of the earth, taken at right angles to the equator—or, in other words, $\frac{1}{10,000,000}$ of an arc of the meridian. This *looks* very 'scientific,' but in truth there is nothing particularly scientific about it; there being no earthly reason for adopting such a standard, any more than the mere distance between the earth and the moon,* or the tail of Encke's comet, or the length of a cow's tail, or any other standard which an indulger in crotchets might suggest. I know I shall be told that we must be uniform; but, with all deference, I do not see the necessity, as the French judge said to the criminal when he said he must live. If uniformity be desirable, let those who think so adapt their system to ours. I dare say it will be quite as 'scientific' as the French metric system. When the recruit was told by his comrade that he was 'out of step,' he quietly replied, 'Change your'n then;' so, if the Frenchman tells us he does not like two systems, let us reply, 'Change your'n then.' And be it observed that *not one* of our present measures (whether of length, surface, or capacity), not one of our present weights, can be compared with those of the metric system without the help of interminable decimals. On the duodecimal scale *all* the present measures may be retained, and of the new ones introduced *all* have a very simple relation to the old. In considering a duodecimal system of measures of length, it will be well to take note of the relative merits of three standards or units of measurement which present themselves—viz., 1st, the inch; 2nd, the yard; 3rd, the mile.

1. THE INCH AS A STANDARD.—This gives us all we can desire for short lengths—for 'cloth measure,' for instance; but for long distances, land measure, &c., in the ascending scale, we get either too little or too much, for comparing readily, distances expressed in miles. Thus 10,000 (duodecimal) inches = 1000 feet (1728 decimal), which is too little, while the next number, 100,000 inches = 10,000 feet (or 20,736 decimal), is too much; the mile being 3,070 duodecimal, or 5,280 decimal feet. But this difficulty is in a great measure got over by taking the foot for unit, still preserving the inch.

2. THE YARD AS A STANDARD.—The nearest number in the ascending

*See Sir John Herschel, *On the Yard, the Pendulum, and the Metre.* Also, Professor Piazzi Smyth's *Our Inheritance in the Great Pyramid.*

scale with the yard for unit gives 1000 (= 1728 decimal) yards, which is certainly nearer than those before mentioned (1760 yards decimal being a mile); but then the subdivisions of the yard would be inconveniently small for practice, those in the descending scale immediately following the yard being respectively 3 inches, ·3in. (= $\frac{1}{4}$in.) ·03 in. (= $\frac{1}{48}$in). It would be inconvenient to invent names for, and to use, these short lengths; and so, notwithstanding they are simply related to the base, we must give up the yard as the *sole* standard.

3. THE MILE AS A STANDARD.—Of course it is not intended that a 'certain standard rod 1760 yards in length' is to be placed under cover in a 'certain place' for reference, but that the subdivision of the mile should he duodecimally made. This would, in fact, be giving up the principle we go upon—viz., the preservation of the inch and foot; but as such great point has been made about the pound sterling, there might be the same about the mile; and it may be worth considering whether it would be necessary for surveying purposes to preserve the mile for measuring distances, and to retain the foot and inch for short lengths, such as in cloth measure, carpenter's work, &c. However, I will just set down a few of the facts and figures which come under this head, and leave the reader to judge. The mile (1760 yards decimal), divided into 1000 (duodecimal) parts, (or 1728 decimal), would give us $\frac{1760}{1728} + 1\frac{1}{54}$ =1·0185 (decimal) or 1·028 yards (duodecimal):—

| | Duodecimal Notation | Land-yards | |
|---|---|---|---|
| 1760 yards | 1028· yards | 1000· | 1 mile |
| $146\frac{2}{3}$ | 102·2 | 100· | ? |
| $12\frac{3}{9}$ | 10·28 | 10· | |
| $1\frac{1}{34}$ | 1·028 | 1· | 1 land-yard |

I have not ventured to give names to these new lengths, except to the last in the table, which I have provisionally called a 'land-yard,' to distinguish it from the present yard of 36 inches. It may be observed, in passing, that 216 (decimal) or 160 (duodecimal) land-yards = 1 furlong; we, however, lose the acre, as will be seen from the following table of surface measures based on the square 'land-yard':—

| Square land-yards | Equivalents in present denominations | | (decimal notation) |
|---|---|---|---|
| | acres | square yards | decimal fractions |
| 10,000 | 4 | 2,249 | ·4528 |
| | | or 21,509 | ·4628 |
| 1,000 | ” | 1,792 | ·4544 |
| 100 | ” | 149 | ·3712 |
| 10 | ” | 12 | ·4475 |
| 1 | ” | 1 | ·0373 |

If, now, we retain the present acre, which is equal to 2,900 land-yards,* we lose the use of Gunter's chain in surveying, for 290 is not a perfect square. 17·e is the number, the square of which, 290·81 , comes nearest to the twelfth of an acre. Now, supposing we had a chain 17·e 'land-yards' long (= 20·2854 yards present measure and notation), and divided into 100 links, we should be obliged to deduct ·81 square land-yard for every square land-yard, before cutting off the cypher in figures to bring the result into acres. And this would be too troublesome for practice. Or, supposing we took an 'area' of, say, 2,940 square land-yards, and a chain of 18 land-yards (= 20 decimal), we should then have,—

| | | |
|---|---|---|
| 1 square chain | = | 294 square land-yards |
| 10 ” | = | 2940 ” or 1 'area.' |

But as this bears no relation to the decimal, the metric, or the present system, it would present no advantages. But engineers and surveyors are very well able to take care of themselves; and if carpenters, bricklayers, &c., can use duodecimal arithmetic in their trade, in spite of decimal notation, the surveyors might even use the decimal notation in their trade where it should be found convenient. If the acre, however, is given up, then the duodecimal notation presents no obstacle to the use of the chain, as will shortly be seen:—

*Duodecimal notation. In the following pages this notation must be understood to be used unless the contrary is expressed.

*Measures of Length (A).*

| Duodecimal denominations and values | Equivalents in present denominations and notation | | | | | | |
|---|---|---|---|---|---|---|---|
| | feet | miles | yards | feet | inches | ’ | ” |
| $\alpha$ | ·10,000 | 3 | 1632 | | | | |
| | ·10,000 | or | 6912 | | | | |
| $\beta$ | 1,000 | | 576 | | | | |
| $\gamma$ | 100 | | 48 | | | | |
| Length (?) | 10 | | 4 | | | | |
| Foot | 1 | | | 1 | | | |
| Inch | $\frac{1}{10}$ | | | | 1 | | |
| Second or Line | $\frac{1}{100}$ | | | | | $1'$ | |
| Third | $\frac{1}{0100}$ | | | | | | $1''$ |

*Measures of Surface (B).*

| Duodecimal denominations | square feet | Equivalents in present denominations acres, square yards, &c. |
|---|---|---|
| | $5 \cdot 3448$ | |
| 1 ‘Square’ | 100,000 | or 27648 |
| | 10,000 | 2304 |
| 1 ‘Area’ | 1,000 | 192 |
| | 100 | 16 |
| | 1 | 1 square foot |

If we take the foot as the unit, we get the preceding tables. I have not given names to those highest in the scale, but would suggest ‘length’ for 10 feet (= 2 fathoms).

It is observable at a glance how simple is the relationship between the duodecimal and the present decimal (not metric) expressions. Let us now compare this with the metric table put forth in the ‘Permissive Act,’ passed 29th July 1864:—

*Measures of Length (C).*

| Metric denominations and values | Equivalents in British denominations | | | | |
|---|---|---|---|---|---|
| | metres | miles | yards | feet | inches | decimals |
| Myriametre | 10,000 | 6 | 376 | 0 | 11 | ·9 |
| | | | 10,936 | 0 | 11 | ·9 |
| Kilometre | 1,000 | | 1003 | 1 | 10 | ·79 |
| Hecometre | 100 | | 109 | 1 | 1 | ·079 |
| Deckametre | 10 | | 10 | 2 | 9 | ·7079 |
| Metre | 1 | | 1 | 0 | 3 | ·3708 |
| Decimetre | $\frac{1}{10}$ | | | | 3 | ·9371 |
| Centimetre | $\frac{1}{100}$ | | | | 0 | ·3937 |
| Millimetre | $\frac{1}{1000}$ | | | | 0 | ·0394 |

*Measures of Surface (D).*

| Metric denominations and values | Equivalents in British denominations | | | |
|---|---|---|---|---|
| | square metres | acres | sq. yards | decimals |
| Hectare, *i.e.* 100 lires | 10,000 | 2 | 2,280 | ·3326 |
| | | | 11,960 | ·3326 |
| Deckare, *i.e.* 10 lires | 1,000 | | 1,196 | ·0333 |
| Are | 100 | | 119 | ·6033 |
| Centiare, *i.e.*, $\frac{1}{100}$ lire | 1 | | 1 | ·1960 |

*Measures of Length (E).*

| Duodecimal denominations | | Equivalents in present denominations | |
|---|---|---|---|
| | yards | miles | yards |
| $\alpha'$ | 10,000 | 11 | ·1376 |
| | | | 20736 |
| $\beta'$ | 1,000 | | 1728 |
| $\gamma'$ | 100 | | 144 |
| 1 'length' (?) | 10 | | 12 |
| 1 yard | 1 | | 1 |

*Measures of Surface (F).*

| Duodecimal denominations | Equivalents in present denominations | | |
|---|---|---|---|
| | square yards | acres | yards, &c. |
| 1 square (?) | 10,000 | 4 | ·1376 |
| | | | 20736 |
| | 1,000 | | 1728 |
| 1 'area' (?) | 100 | | 144 |
| = 1 sq. 'length' | | | |
| 1 square yard | 1 | | 1 |

Let us suppose it to be required to express a given number of metres in yards, feet, &c. Table C will show that we must first multiply by 39·3768, then divide by 12, 3, and, if the given number be large, by 1760 for miles. With the duodecimal scale, however, we have no 'reduction,' but simply to change the notation. This for small numbers may be done by simple addition, without the trouble of going through the process of division—thus: Required the equivalent decimal expression for 1279 feet. Here $1{=}12^3$, $2{=}2{\times}12^2$, $7{=}7{\times}l2$; then—

$$\begin{array}{r} 1728 \\ 288 \\ 84 \\ 9 \\ \hline 3)\overline{2109}\text{ feet} \\ \hline 703\text{ yards} \end{array}$$

The latter part of Table A would be used by opticians, drapers, painters, carpenters, and glaziers; and it is a matter of indifference whether they use the term 'length,' or restrict themselves to feet, miles, &c. The old term yard might very well be used in purchasing by retail, but the *account* would be kept in feet or 'lengths.' The terms feet, inches, 'lines' or 'seconds,' and 'thirds,' *are already in use.* The latter ($= \frac{1}{144}$ of an inch) is a very convenient and useful quantity for optical and horological purposes.

For the purpose of illustration, I will now work a few examples, combining duodecimal money with duodecimal measures:—

(Ex. l.) What is the value of $69\frac{3}{4}$ feet of velvet, at 7 florins 3 doits per foot?

69·9 feet
7·3 ”
1853
3e83
414·83

*Ans.* 41 Vict. 4fl. 8dt. $\frac{1}{4}$tw.

In the decimal system this question would have presented itself in the following form:—What is the value of $23\frac{1}{4}$ yards of velvet, at 2*l*. 3*s*. 6*d*. per yard?

(Ex. 2.) What is the value of $6\frac{1}{4}$ ‘lengths’ (Table A) of calico, at 1 doit 9 twelfthings per foot?

$6\frac{1}{4}$ lengths = 63 feet
1·9
483
63
te·3

*Ans.* t florins, e doits, 3 twelfthings
te doits, 3 twelfthings
t·e3 florins

And this question, put decimally, would be—What is the value of 25 yards of calico, at $10\frac{1}{2}d$. per yard?

(Ex. 3.) How many feet of carpet that is 2 feet 8 inches wide will cover a floor which is $37\frac{3}{4}$ feet wide by $43\frac{1}{2}$ feet long? And what would be the cost, at 1 florin 3 doits per foot?

379 in.
436 in.
19t6
te3
1270
28)137916(5t4e$\frac{1}{14}$in.
114
239
228
·111
t8
256
254
2 neglecting the $\frac{1}{14}$

·5t4·e
1·3
14t29
574e
*Ans.* 703·19
or
70V. 3fl 1dt. 9tw.

(Ex. 4.) What is the surface of a round table whose diameter is 4 feet 2 inches?

| | | | |
|---|---|---|---|
| | r | = | 21 inches |
| Here | $(r^2)$ | = | 441 " |
| | $\pi$ | = | 3·1848 &c. |

3·1848
441
31848
106968
106968
1177·5e08 square inches, or
11 sq. feet 77 sq. inches nearly.

The relations between these figures and what would have been introduced under the present system is simple enough; but under the metric system the calculation would be much longer, and at the end would not convey such a *correct notion* of the size of the table as the above.

For 4 feet 2 inches = 50 inches decimal notation, and 1 inch= 25·3 millimetres (metric system), whence the diameter comes out 1·265 metres = 12·65 decimetres = 126·5 centimetres = 1265 millimetres, none of which convey to the mind the same sort of notion as the good old-fashioned expression in inches. The reader may, however, take his choice; he has the 'permission' granted him by the 27th and 28th Vict. cap. 117. Doubtless, in practice, the table in question would be ordered of $1\frac{1}{4}$ metres diameter, or 1·25 metres. If the 'quarter' is admissible, that would lessen the awkwardness of the expression; but no cabinetmaker will ever make use of the expression '*one decimal twenty-five*,' instead of 'one-and-a-quarter.'

We must now turn our attention to the method of expressing distances and large areas. And here it will be found necessary to make an apparent, but only an apparent, deviation from the plan laid down for increasing and decreasing by twelves, and twelves only. In Table A it will be seen that the length $\beta$ expressed with three cyphers gives as its value 576 (decimal notation) yards; in Table C the corresponding number (kilo) is 1093 yards; and in Table E the corresponding number ($\beta'$) is 1728 yards, which is the 'nearest by far' to the present mile. The expressions with four cyphers in all the tables are too large to be taken as units.

Now, if we make the yard the unit in the table for land measure, we have all we can desire for expressing distances, and also for land-surveying

purposes; and as the yard and the foot are so simply related, values given in tens of Table A may be readily expressed in tens of Table E, or *vice versâ*, when required. This would not often happen, except as arithmetical recreations and school exercises, where the intervention of a break in the shape of a new multiplier would not be objectionable. In practice it is not required to express long distances in inches, nor short lengths (of ribbon for instance) in miles. We do, indeed, see occasionally in print curious calculations, which must be made by men whose minds are so happily constituted as to be totally indifferent to the 'disgust,' and careless of the 'errors' which Laplace says are 'inseparable' from long calculations. Nevertheless, let us not despise these curiosities of calculation: it may be necessary for some minds to be told how many barleycorn-lengths there are between the earth and the moon, or how many seconds have elapsed since the creation of the world; for it seems to be necessary to state the 'united ages' of all the brothers and cousins of a man before the fact becomes apparent that he is old, when within a year or two of a hundred.

But to return: whichever Tables (whether A and B, or E and F) are used, care must be taken as to the use of the 'length,' which in the former stands for 10 *feet*, in the latter for 10 *yards*; it will therefore be necessary, if the term be adopted at all, to restrict it to a certain sense.

In the following observations, if there is occasion to use the term length, I mean it to stand for 10 yards. With regard to the linear measure (Table E), littleneed be said. We may, however, compare some known distances expressed in terms of the present measures, and in those of the metrical and duodecimal systems. The distance from London to Brighton is about

| Duodecimal Notation | Decimal Notation | |
|---|---|---|
| 44 | 52 | miles |
| 6e | 83 | kilometres (C). |
| 45 | 53 | $\beta'$ (E). |

Whatever may be the name we give to the measure marked $\beta'$, it will be a very convenient one, and certainly has many advantages over the 'kilo.' As the actual operations with the numbers in Table E are the same as the others, we will pass on to Table F. Here it will be seen that what I have ventured to call the 'square' = 4 acres 1376 square yards (decimal notation), while the hectare in Table D = 2 acres 2280·3326 square yards. And it will also be noted that the equivalents of the metric system require more figures for their

expression, while the equivalents of the duodecimal notation are mostly in whole numbers (they are, in fact, simply a translation from one notation to the other), and afford a much more ready means of *comparing* one system with the other than exists between the metric and the present system. One example will do more to show the applicability of the table than pages of writing, and an example from land-surveying will do as well as any:—

Let us have a 'chain,' then, of 20 yards, two yards longer than the present Gunter's chain, and let it be divided into 100 links; then—

| | | |
|---|---|---|
| 400 square yards | = | 1 square chain. |
| 30 square chains | = | 1 square. |

Hence, to find the number of 'squares' in any number of square chains, we divide by 30, which, since $\frac{1}{30} = \cdot 04$, resolves itself into cutting off two cyphers or figures to the right and multiplying by 4.

The links are to be treated in a manner precisely analogous to that adopted under the present system with Gunter's chain. With these data we are enabled to answer the following question:—What is the area of a field which measures 15 chains 34 links one way, and 3 chains 6 links the other?

$$\begin{array}{r} 1534 \text{ links} \\ 306 \text{ "} \\ \hline 8780 \\ 43\text{t}00 \\ \hline 446780 \text{ square links} \end{array}$$

In order to express this result in acres and in decimal notation for the purposes of comparison, divide 15,626 by 2,974 (= 4,840 dec.); or, converting the former into decimal notation, we have 30,270 duodecimal (= 15,656) to be divided by 4,840 dec. The latter plan would be the best, as there are likely to be odd poles and roods, &c. In this particular instance the equivalent comes out 6 acres, 1230 square yards, 6 square feet; or 6 acres, 1 rood, 0 poles, 20 square yards, 6 square feet,—a calculation which is easily made, notwithstanding the change of notation; but in the metric system (Table D), the equivalent, in square yards, of the present system, all contain four places of decimals, so that any result coming out in 'hectares' must be multiplied by 11,960·3326, in order to get the equivalent expression in square yards! If the result be in 'ares,' we have 119·6033 to multiply by for a like purpose.

It has been made a great point of, that we must have ready means of comparing sums of money of the past and present, and therefore it is insisted

upon that 'we must stick to the pound.' It is surely hardly less necessary that we should be able readily to compare the dimensions of estates and tracts of land, and this the duodecimal system enables us to do far more readily than the metric. With respect to the other part of the process, working with a 'chain,' it would be exactly the same as in that given above for the duodecimal chain: taking 20 (decimal) metres for a chain, we get 400 (decimal) square metres = 1 square chain, and 30 square chains = 1 hectare—precisely the same *figures* as in the duodecimal, though of course of different value.

Having considered the PENNY and the INCH, we will now proceed to the consideration of the OUNCE. We have now no less than three different 'tables' of weights—viz., apothecaries', troy, and avoirdupois. The apothecaries' pound, ounce, and grain are the same as the troy pound, ounce, and grain; but there is a 'dram' with nothing corresponding in the troy table, and there is another 'dram' in the avoirdupois, which, however, is much less—it being the $\frac{1}{16}$ of an ounce, which is smaller than that of which the apothecaries' drams is—$\frac{1}{8}$th. Then, again, the avoirdupois table contains a *large* pound (7,000 grains) and *small* ounce ($437\frac{1}{2}$ grains); while the troy table contains a *small* pound (5,760 grains), and a *large* ounce (480 grains)! Anything that would get us out of this mess must be thankfully accepted. Subjoined are the metric and the duodecimal tables for comparison:—

*Duodecimal Weights (G).*

| Duodecimal denomination | Equivalents in avoirdupois weight | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Pounds | Tons | cwts. | qrs. | lbs. | decls. | | Grains |
| | 10,0000 | 7 | 12 | 1 | 10 | ·7657 | = | 119,439,360 |
| Gross- | 1,000 | | 12 | 2 | 21 | ·8971 | = | 9,953,280 |
| weight | 100 | | 1 | 0 | 6 | ·4914 | = | 829,440 |
| | 10 | | | | 9 | ·8742 | = | 69.120 |
| Pound | 1 | | | | | | | 5,760 |
| Ounce | $\frac{1}{10}$ | | | | | | | 480 |
| *a* | $\frac{1}{100}$ | (2 scruples) | | | | | | 40 |
| *b* | $\frac{1}{1000}$ | | | | | | | 3·3 |
| *c* | $\frac{1}{10000}$ | | | | | | | 0·277 |

*Metric Weights (H)*

| Metric denominations and values | Equivalents in present denominations | | | | | |
|---|---|---|---|---|---|---|
| | Grains | Cwts | stones | lbs | ounces | drams |
| Millier | 1,000,000 | 19 | 5 | 6 | 9 | 15·04 |
| Quintal | 100,000 | 1 | 7 | 10 | 7 | 6·304 |
| Myriagram | 10,000 | | 1 | 8 | 0 | 11·8304 |
| Kilogram | 1,000 | | | 2 | 3 | 4·3830 |
| Hectogram | 100 | | | | 3 | 8·4383 |
| Dekagram | 10 | | | | | 5·6438 |
| Gram | 1 | (15·4323 grains) | | | | 0·5643 |
| Decigram | $\frac{1}{10}$ | | | | | 0·056438 |
| Centigram | $\frac{1}{100}$ | | | | | 0·005643 |
| Milligram | $\frac{1}{1000}$ | | | | | 0·000564 |

It must be confessed that neither of these tables affords us ready means of comparison with the present tables; but this is the fault of the old system, not of the new. As we have three systems of weight in use, it would be necessary, for the purpose of ready-reckoning, to have three tables. I have chosen the troy* pound, as it is already duodecimally divided; moreover, as will be seen hereafter, it will enable us to obtain a measure of capacity which is very nearly equal to our present gallon.

The 'gross-weight' or, for short, the 'gross,' it will be observed, is a little more than our hundredweight. If we ignore for the present the l0lbs. weight, and keep our accounts in grosses, pounds, and ounces (for large quantities, that is, in ounces, and duodecimals of an ounce for smaller), we must place a cypher to the left of any figure representing pounds (for 1lb. is $\frac{1}{100}$ of a gross), in the same manner as the French do with their francs and centimes.

(Ex. 1.) Let it be required to add together 25grs. 3lbs. 8oz., and l3grs. 23lbs. 2oz.:—

| | grs. | lbs. | oz. |
|---|---|---|---|
| | 25„ | 03„ | 8 |
| | 13„ | 23„ | 2 |
| *Ans.* | 38„ | 26„ | t |

(Ex. 2) Multiply.

| | grs. | lbs. | oz. |
|---|---|---|---|
| | 3„ | 82„ | 6 by 4. |
| | | | 4 |
| *Ans.* | 12„ | 8t„ | 0 |

*See *Our Inheritance in the Great Pyramid*, by Professor PIazzi Smyth, as to the origin of the words 'Troy,' 'Overpose,' &c.

Of course in all cases the dots or points may be left out altogether, or shifted during the operation, and placed afterwards, according as we wish to call them ounces, or pounds, or grosses.

(Ex. 3.) What is the value of 33grs. 08lbs. 6oz., at $\frac{3}{4}$ florin per. lb.?

$\frac{1}{4} = \cdot 9$

3306·6 lbs.
·9
2536·46 florins = 253 victorias, 6 florins, $4\frac{1}{2}$ doits

(Ex. 4.) If $3\frac{1}{2}$ gross cost $77\frac{1}{2}$ florins, what must be given for 14grs. 11lbs. 9oz.?

3·6 : 14·119 :: 77·6 : *Ans.*

77·6
806t6
94803
94803 florins
3·6)t28·8t16(2e0·947 *Ans.*
70
328
326
28t
276
141
120
216
206
10

where the small fractions may be neglected. By cancelling, the above would be done in a neater manner:—

~~3·6~~ : 14·49 :: ~~77·6~~ : *Ans.*
0·7 : 13·3 :: 13·3
40353
40353
14119
·7)185·5583
2e0·094743 florins, or V2e. 0fl 9dt. 4tw.,
neglecting the last three figures.

Now it is evident that a similar operation, with cwts., qrs., and lbs. for the weight, and pounds, shillings, and pence for the price or cost, would

be more complicated than these examples. The great advantage that the duodecimal system possesses, in common with the metric, is the ease with which the ordinary process of 'reduction' is effected; while it does not possess the disadvantage which belongs to the metric, of introducing new weights, &c., so different to those already in use.

Where small quantities are in question, the duodecimal division of the ounce would be found convenient. I have not ventured to give their names, but perhaps the apothecaries' weight had better remain as it is: it is, however, very simply related to the duodecimal scale.

We may now turn our attention to measures of capacity. Of the fifteen or sixteen different measures of which our dry and liquid measure systems are made up, there is only one (the chaldron) which is duodecimally divided: this, however, is obsolete, or nearly so, and may be passed over. All the others, however (with the exception of the 'load,' which is made up of five quarters,* and the sack, which is three bushels), are divided into halves, quarters, and eighths. Leaving the load out then, the rest are easily adapted to the duodecimal system. I will now only make a few remarks and give a few figures on this point.

The pound avoirdupois = 7,000grs. 10lbs. = 70,000 grs. (decimal notation), which is the weight of 1 gallon of distilled water at 62° Fahr., and the bulk of this weight of water is 277·274 cubic inches. The pound troy = 5,760 grs., and 12 lbs. = 69,120 grs. (decimal), and the bulk of this weight is 273·784, showing a difference of only 3·490. Now if this bulk and weight of water be called a gallon, we may put the latter part of this statement into duodecimal language, thus:—One pound = 3,400grs., and l0lbs. = 34,000 grains = 100 ounces = 2,000 scruples, which is the weight of one gallon of distilled water; and at 62° Fahr. the bulk of this weight of water is 1t9·95 cubic inches.

We thus see that while we have changed the size of the gallon, we have still round numbers to express its weight in three different denominations; and of course, by giving names to the subdivisions marked *a*, *b*, and *c* (in G), we should have respectively 1 gallon = l0lbs. or 100 ounces, or 1,000 *a*, or 10,000 *b*, or 100,000 *c*. In the metric system the cubic metre is taken as the unit, the litre being $\frac{1}{1000}$th part of this; but the quantity in the duodecimal system analogous to the cubic metre, 1000 yards (1728 decimal), gives too

*The reader is again referred to Professor Piazzi Smyth's work for the origin of 'Quarter'—quarter of *what?* Is it the *quarter of the trough* in the 'king's chamber'?

large a quantity for subdivision. Taking solid measurement as a basis, we get—

$$\left.\begin{array}{lcr} 1 \text{ cubic foot} & = & 180{,}536 \\ \frac{1}{10} \text{ "} & = & 18{,}053 \\ \frac{1}{100} \text{ "} & = & 1{,}805 \\ \frac{1}{1000} = 1 \text{ cubic inch} & = & 185 \end{array}\right\} \text{grains nearly}$$

but these figures present nothing worth noticing. The $\frac{1}{10}$th cubic foot comes rather near the half-gallon, but not sufficiently so to make it available.

I have now gone over the subject proposed—the Ounce, the Inch, and the Penny; and if the reader is unable to endorse all that has been said, I hope he will feel that he has at least picked up some useful information.

In the Appendix will be found some tabular matter, with explanations.

# Appendix

QUESTIONS in Interest are easily worked by the duodecimal system.

What is the Interest on 3347 victorias 4 florins 2 doits 9 twelfthings, at 8 *per gross* per annum ?

| | | | |
|---|---|---|---|
| 3347 | 4 | 2 | 9 |
| | | | 8 |
| 2230t | 9 | t | 0 |

Then arranging the point to express division by 100, we get 223 victorias 0 florins t doits 9t twelfthings.

And this is equivalent to solving the following question:—

Required the Interest on £6805 12*s*. $5\frac{1}{4}d.$, at $5\frac{5}{9}$ *per cent.*, according to present system of notation.

The following table will be found useful for comparing the rates of Interest as expressed in both systems:—

*Comparative Tables of Rates of Interest expressed in Decimal and Duodecimal Notation.*

| Decimal | | Duodecimal | Duodecimal | | Decimal |
|---|---|---|---|---|---|
| 1 per cent. | = | $1\frac{1}{21}$ per gross. | 1 per gross. | = | $\frac{25}{36}$ per cent. |
| $1\ \frac{9}{16}$ | = | $2\frac{1}{4}$ | 2 | = | $1\frac{7}{18}$ |
| 2 | = | $2\frac{1}{21}$ | 3 | = | $2\frac{1}{12}$ |
| 3 | = | $4\frac{8}{21}$ | 4 | = | $2\frac{7}{9}$ |
| $3\frac{1}{8}$ | = | $4\frac{1}{2}$ | 5 | = | $3\frac{17}{36}$ |
| 4 | = | $5\frac{17}{21}$ | 6 | = | $4\frac{1}{6}$ |
| 5 | = | $7\frac{1}{5}$ | 7 | = | $4\frac{31}{36}$ |
| 6 | = | $8\frac{14}{21}$ | 8 | = | $5\frac{5}{9}$ |
| 7 | = | $\mathrm{t}\frac{2}{21}$ | 9 | = | $6\frac{1}{4}$ |
| 8 | = | $\mathrm{e}\frac{11}{21}$ | t | = | $6\frac{17}{18}$ |
| 9 | = | $10\frac{20}{21}$ | e | = | $7\frac{23}{26}$ |
| 10 | = | $12\frac{2}{5}$ | 10 | = | $8\frac{1}{3}$ |

There is, however, an awkwardness, arising from the nature of the bases of the decimal and duodecimal systems, which prevents rates being very readily compared. It may be remarked, however, that the rate *per gross* expresses the number of 'doits' per 'victoria,' just as in the proposed decimal system the rate *per cent.* expresses so many tenths of a florin, or so many times ten mils per pound sterling. Thus, the *mere figures* will show this for either system. 1 per cent. or 1 per gross = $\frac{1}{100}$ (*i.e.*, one-hundredth in the decimal—or one-hundred-and-forty-fourth for the duodecimal). Thus taking 1000 'mils' or 'twelfthings,' cutting two cyphers from the right, we get 10 'mils' or 10 (twelve) twelfthings (= 1 doit = 2 pence), according to which system we are working with. So that 1 per gross = 1 doit (2*d.*) in the victoria (twenty-four shillings).

For Compound Interest logarithms would be useful. As with decimal notation, of course a complete table cannot here be given. Three short tables are here given:—1st, of numbers from 1 to 10, and of prime numbers from 10 to 100; 2nd, of numbers from 187e0 (35700) to 1881e (35735); 3rd, of numbers from eee00 (248688) to 100000 (248832).

They need no remark. I will just mention that—

$$\begin{aligned} \mathrm{M} &= \cdot 49\mathrm{e}494944399 \\ \mathrm{e} &= 2\cdot 8752360694 \end{aligned}$$

Log. 10 (*i.e.* log. twelve to base twelve) is of course 1.

*Table of Logarithms from 1 to 10, and of all prime numbers from 10 to 100, to base 10 duodecimal notation.*

| No. | Logarithm | No. | Logarithm |
|---|---|---|---|
| 2. | ·34201e20 | 45. | 1·720e268t |
| 3. | ·537e817t | 49. | 1·763645e1 |
| 4. | ·68403t41 | 4e. | l·78361t16 |
| 5. | ·79324t5l | 51. | 1·7t283973 |
| 6. | ·879et091 | 57. | 1·837e29t7 |
| 7. | ·9492238t | 5e. | 1·87031219 |
| 8. | ·t0605962 | 61. | 1·8876e367 |
| 9. | ·t73e4338 | 67. | 1·91260e75 |

| | | | |
|---|---|---|---|
| t. | ·e1526972 | 6e. | 1·950t2852 |
| e. | ·e6e5tt99 | 75. | 1·98148251 |
| 10. | 1·00000000 | 81. | 1·t112e5e3 |
| 11. | 1·0477e322 | 85. | 1·t3531389 |
| 15. | 1·18226620 | 87. | 1·t45e965t |
| 17. | 1·22768432 | 8e. | 1·t6959951 |
| 1e. | 1·31850146 | 91. | 1·t7t4325t |
| 25. | 1·43174161 | 95. | 1·t9e50l99 |
| 27. | 1·46eet0e5 | t7. | 1·e4878157 |
| 31. | 1·55303970 | te. | 1·e661t188 |
| 35. | 1·5e24te79 | e5. | 1·e9141e01 |
| 37. | 1·61e647e4 | e7. | 1·e9e5124t |
| 3e. | 1·67148754 | | |

*Table of Logarithms from 18730 to 1881e (35700 to 35795 decimal).*

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | t | e | D | Pro. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | | | | 1 2t |
| | | | | | | | | | | | | | | 2 57 |
| | | | | | | | | | | | | | | 3 85 |
| 187e | 2759669 | 9945 | t020 | 5298 | t553 | t82t | te06 | e1t2 | e47t | 3756 | et32 | 0109 | | 4 e2 |
| | | | | | | | | | | | | | | 5 120 |
| | | | | | | | | | | | | | | 6 147 |
| 1880 | 27603t5 | 0681 | 0959 | 1034 | 1310 | 15t8 | 1884 | 1e5e | 2237 | 2554 | 2830 | 2e08 | 297 | 7 177 |
| | | | | | | | | | | | | | | 8 1t5 |
| | | | | | | | | | | | | | | 9 212 |
| | | | | | | | | | | | | | | t 234 |
| 1881 | 31t3 | 347e | 3757 | 3t34 | 4109 | 43t5 | 4680 | 4957 | 5033 | 530t | 55t6 | 5881 | | e 240 |

*Table of Logarithms from eee00 to 100000 (248688 to 248832 decimal)*

| N | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | t | e | D | Pro. | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| eee0 | ·eee7203 | 251 | 29e | 329 | 377 | 405 | 453 | 4t1 | 52e | 579 | 607 | 655 | 4t | | |
| 1 | 76t3 | 731 | 77e | 809 | 857 | 8t5 | 933 | 981 | t0e | t59 | tt7 | e35 | | 1 | 5 |
| 2 | 7e83 | 8011 | 05e | 0t9 | 137 | 185 | 212 | 261 | 2te | 339 | 387 | 415 | | 2 | t |
| 3 | 8463 | 4e1 | 53e | 589 | 617 | 665 | 6e3 | 741 | 783 | 819 | 867 | 8e5 | | 3 | 13 |
| 4 | 8943 | 991 | t1e | t69 | te7 | e45 | e93 | 9021 | 06e | 0e9 | 147 | 195 | | 4 | 17 |
| 5 | 9223 | 271 | 2ee | 349 | 397 | 425 | 473 | 501 | 54e | 599 | 627 | 675 | | 5 | 20 |
| 6 | 9703 | 750 | 79t | 828 | 876 | 904 | 952 | 9t0 | t2t | t78 | e06 | e54 | | 6 | 25 |
| 7 | 9et3 | t031 | 07e | 109 | 157 | 1t5 | 233 | 281 | t6t | 359 | 5t7 | 435 | | 7 | 2t |
| 8 | t482 | 510 | 55t | 5t8 | 636 | 684 | 712 | 760 | 30e | 838 | 886 | 914 | | 8 | 33 |
| 9 | t962 | 9e0 | t39 | t88 | e16 | e64 | ee2 | e040 | 7tt | 118 | 166 | 1e4 | | 9 | 38 |
| t | e242 | 290 | 31t | 368 | 3e6 | 444 | 492 | 520 | 08t | 5e8 | 646 | 695 | | t | 40 |
| e | e720 | 76t | 7e8 | 646 | 894 | 922 | 990 | 9et | 56t | t96 | e24 | e72 | | e | 45 |

## Per 'Cent.' and per 'Gross'.

It should be observed that all ordinary percentages, scientific and commercial, should be expressed as *per gross*, and that we must *translate* the expression so as to make it in accordance with the duodecimal system, and not merely change the notation. Thus, 50 per cent. will not become 42 (four dozen and two) per 84 (eight dozen and four); but 60 per gross, or six dozen per gross. And the composition of water, for instance, would be expressed thus:—

| | |
|---|---|
| Hydrogen . . . . . | 14 |
| Oxygen . . . . . | t8 |
| | 100 |

from which the equivalent H = 1, O = 16, may be deduced in the same way as from the decimal percentages usually given:

| | |
|---|---|
| Hydrogen . . . . . | 11·11 |
| Oxygen . . . . . | 88·88 |
| | 99·99 |

To convert numbers, whole and fractional, from one notation to the other. This subject has already been alluded to at p. 10 with reference to whole-numbers. It may be well, however, to give another example or two:—

*Rule for whole numbers.*—Divide the given number by the base of the notation to which it is required to convert it; the remainder, if any (cypher if there be no remainder), will be the units figure of the new number. Divide the quotient obtained by the same base, and the remainder will be the tens or dozens figure of the new number; and so on, the next remainder being the hundreds or gross figure, as the case may be.

*Example.*—From decimal to duodecimal.

12)1000
12)83 + 4
6 + e

From duodecimal to decimal.

$$\begin{array}{r} \text{t})\underline{1000} \\ \text{t})\underline{124} + 8 \\ \text{t})\underline{15} + 2 \\ 1 + 7 \end{array}$$

6e4 (duodecimal) = 1000 (decimal); 1728 (decimal) = 1000 (duodecimal).

For decimal and duodecimal fractions—*multiply* the given fraction by the base of notation to which it is required to convert it. Mark off as many places in the product as there are in the original fraction; the gure to the left of the point will be the first on the right in the new fraction. Multiply the figures point off, and again point off as before in the product; the figure on the left will be the second on the right of the new fraction.

*Example* 1.—Reduce ·1 (decimal) to a duodecimal fraction

$$\begin{array}{r} \cdot 1 \\ \underline{\quad 12} \\ 1\cdot 2 \\ \underline{\quad 12} \\ 2\cdot 4 \\ \underline{\quad 12} \\ 4\cdot 8 \\ \underline{\quad 12} \\ 9\cdot 6 \\ \underline{\quad 12} \\ 7\cdot 2 \end{array}$$

Here we find that ·1 (decimal) = $\cdot 1\dot{2}49\dot{7}$, which is a circulator, the figures 2497 being repeated.

*Example* 2.—Reduce ·0625 (decimal) to a duodecimal.

$$\begin{array}{r} \cdot 0625 \\ \underline{\quad\quad 12} \\ 0\cdot 7500 \\ \underline{\quad\quad 12} \\ 9\cdot 9900 \end{array}$$

Here we find that ·0625 (decimal) = ·09 duodecimal. Reversing the process, we have—

| | |
|---|---|
| ·09 | |
| t | |
| 0·76 | |
| t | |
| 6·30 | |
| t | |
| 2·60 | |
| t | |
| 5·00 | ·0625 is reproduced. |

Again—

| | |
|---|---|
| ·1̇2̇4̇9̇7 | |
| t | |
| 0·eeeet | |
| t | |
| 9·eeet4 | |
| t | |
| 9·eet74 | |
| t | |
| 9·ett14 | |
| t | |
| 9·e0514 | |
| t | |
| 9·24314 | ·099999&c. (decimal) |

The '&c.' does not in this instance, however, stand for a series of *nines*, for by continuing the process we obtain a long mixed circulator of upwards of thirty figures. Again, reducing ·1 duodecimal to a decimal, we get ·08$\dot{3}$, where the 3 is repeated, and on reconverting this we also get a long mixed circulator:—

| | |
|---|---|
| ·1 | ·08$\dot{3}$ |
| t | 12 |
| 0·t | 0·996 |
| t | 12 |
| 8·4 | e·952 |
| t | 12 |
| 3·4 | e·424 |

The best plan in such cases is to turn the decimal or duodecimal as the case may be, into a vulgar fraction: thus, taking ·1 (duodecimal) = $\frac{1}{10}$ =

$\frac{1}{12}$ (decimal). Or applying the rule, which is analogous to that in decimals, we may take the mixed duodecimal circulator $\cdot 1\dot{2}49\dot{7}$: subtracting the non-circulating part from the whole, we get 12496 for the numerator, and taking as many *elevens* as there are circulating places, and annexing as many cyphers as there are non-circulating places (in this case *one*), we get eeee0 for the numerator; and the vulgar fraction will be $\frac{12496}{eeee0}$, of which the numerator happens to be the greatest common measure.

12496)eeee0(t
eeee0
. . . . .

and the fraction reduced to its lowest terms is $\frac{1}{t} = \frac{1}{10}$ (decimal).