The Dozenal Society of America

De Arithmetica

quæ per periodos duodenarias recurreret

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Egimus heri de Arithmeticâ Duodenariâ, quam tibi placere testabaris, & debere in Europam introduci asserebas: hodie autem postulas, ut mandem scriptis, quæ dicebam. Ego autem tanti tua vota facio, ut nesciam negare, quæ petis. Ab aliis ergo negotiis, & curis supersedens, sumo calamum, &, ut tibi serviam, sic scribo.

CXLVI. Numerorum periodi, ut vidimus, per decades, apud omnes populos, & gentes fluunt: nec enim aliter Indi, & aliter numerant Europæi: nam, quæ de Paraguayensibus suspicamur, ut ex *Art. 3 Procæm. Disputationis* constant, non sunt certa. Hinc ab aliquibus hæc dispositio creditur esse naturalis: nam natura apud omnes est eadem: & varia sunt, quæ ab hominum placito pendent. Interim, tametsi homines linguis differant, haberi potest loco miraculi, aut prodigii Arithmeticæ revolutionis concordia: est enim æquè, ac idioma ad placitum; & sicut varii populi labio differunt, etiam possent differre Arithmeticâ. Habemus decem numeros, quibus numeramus unitates: posteã numeramus decades: posteã decadum decades, seu centenaria, &c. & cur non potuerunt ponere septem notas, & facere periodos per heptades? Et cur non potuerunt alium ponere numerum? Puto primis Arithmeticæ Inventoribus placuisse Denarium: & discipulos magistris suis sine ulteriore examine, ut fieri solet, subscripsisse.

CXLVII. Sanẽ numerus sexagenarius, qui multis Astronomis placet, nimis est magnus, & duplici nos implicat Arithmeticâ: nam per decades usque ad finem sexagenæ venitur, & posteã fit altera revolutio per sexagenas. Accedit, quõd, si hic numerus haberi deberet, totus in sexaginta Signa Circulus, & non in duodecim esset consequentiâ doctrinæ dividendus: & fingendi essent simplices characteres usque ad 60. (aut etiam nomina) ut revolutio ordine convenienti procederet. Novam igitur, & brevem Arithmeticam in Astronomorum gratiam instituamus. Revolutio per δοδεκαδες (per duodenas) fiat nam divisionibus Astronomicis ΔΩΔΕΚΑΣ ΔΕΚΑΔΙ multõ est commodior. Divisa enim

Yesterday, we did *Dozenal Arithmetic*, about which we bore witness to please you, and which you stated ought to be introduced in Europe. But today you have requested that I commit to writing what I said. And I do this according to your wish, because I do not know how to deny what you ask. Therefore, laying aside other tasks, and placing this above other cares, I take up my pen and write, so that I might serve you.

CXLVI. The cycles of numbers, as we have seen, go by tens among all peoples and nations; nor do the Indians count one way, and the Europeans another. We suppose the same of the Paraguayans, as is apparent from Article 3 of the *Procæm. Disputationis*, but this is not certain. Because of this, this arrangement is believed to be natural; for nature is the same for everyone, and those things which are at the pleasure of man vary. Nevertheless, it is like a miracle that we have agreement on the cycles of Arithmetic even though men differ in language; for it is, like language, a matter of pleasure. Just as the various peoples differ in tongue, so they might differ in Arithmetic. We have ten numbers, by which we count units; afterwards we count tens; then we count tens of tens, or hundreds, and so forth. And why could they not use seven symbols, and count by groups of sevens? And why could they not count by another number? I think that Decimal pleased the first inventors of Arithmetic, and students followed their masters without further examination, as often happens.

CXLVII. Certainly the number sixty, which pleases many astronomers, is too large, and it involves us in Arithmetic in two ways: for it proceeds in tens to sixty, and afterwards comes another cycle of sixty. The whole circle constitutes sixty signs, and because of this teaching ought not to be divided into twelve, if this number should be used. Simple characters and names must be created all the way to sixty, so the cycle may proceed in good order. We ought to establish, therefore, a new and short Arithmetic for the sake of astronomers. Let it be a cycle of δοδεκαδες (twelves), for a *twelve-decade* is much more convenient in astronomical divisions. Dividing by

[0] Hoc opus de Ioannis Caramuelis (Juan Caramuel y Lobkowitz), communiter "Lobkowitz" vocatus, extractum est ex eius opus magnus *Mathesis Biceps, Vetus et Nova*, imprimitum in Campania anno Domini ɛ72 (1670.). Hoc opus mundo datus est a Google Books (http://books.google.com); gratias eo propter liberalitatem eius.

per	2	3	4	5	6
ΔΕΚΑΣ dat	5	*	*	2	*
ΔΩΔΕΚΑΣ dat	6	4	3	*	2

	2	3	4	5	6
in tens gives	5	*	*	2	*
in twelves gives	6	4	3	*	2

Multiplicatio per 12 in communi Arithmeticâ est facilis: *Nam scribitur numerus multiplicandus, & postscribitur bis: & Summa dat quasitum numerum.* Pono exemplum.

Multiplication by twelve in common Arithmetic is easy: For the number to be multiplied is written, and then written again twice: the sum gives the desired number. I give an example:

12	2985984 . A
12	1492992 . B
12	746496 . C
144	248832 . AD
144	124416 . B
144	62208 . C
1728	20736 . AD
1728	10368 . B
1728	5184 . C
20736	1728 . AD
20736	864 . B
20736	432 . C
248832	144 . AD
248832	72 . B
248832	36 . C
2985984	12 . D

Habes duas numerorum columnas: prior multiplicat per 12. & dividit per 12. posterior. Multiplicat postscribendo bis. Dividit bipertiendo bis, & semel tripertiendo: nam ex A dat B. & ex B dat C bisectio: & ex C dat D trifectio.

Interim adlaborandum est, ut ex antiquâ Arithmeticâ, quantum fieri possit, nomina, & figuras notarum retineamus. Considera sequentes characteres.

You have two columns of numbers; the first multiplies by 12 and the second divides by 12. We multiply by writing below twice. We divide by bisecting twice, and trisecting once, for bisection from A gives B and bisection from B gives C, and trisection of C gives D.

Furthermore, we must make a special effort to retain from the old Arithmetic as much as possible of the names and figures of notation. Consider the following characters:

Unitates.	*Duodenarii Duodena.*	*Duodenarii Duodenar. Duodena.*
1	1d	1dd
2	2d	2dd
3	3d	3dd
4	4d	4dd
5	5d	5dd
6	6d	6dd
7	7d	7dd
8	8d	8dd
9	9d	9dd
p	pd	pdd
n	nd	ndd
1d	1dd	1ddd.

Units.	*Dozens.*	*Dozens of Dozens.*
1	1d	1dd
2	2d	2dd
3	3d	3dd
4	4d	4dd
5	5d	5dd
6	6d	6dd
7	7d	7dd
8	8d	8dd
9	9d	9dd
p	pd	pdd
n	nd	ndd
1d	1dd	1ddd.

Decem vulgõ scribunt 10. duobus, nimirũm, characteribus: nunc est numerus simplex, &, ut cognoscatur, postulat unicum characterem: notetur ergo sic p, utrunque characterem uniendo. Idem faciemus, ut undecim exprimamus. Vulgo scribunt sic 11. per duas notas, nos illas uniamus, & scribamus n.

Commonly, we certainly write ten "10" with two characters; now it is a simple number, and, as you can see, is written with one character. It is written, therefore, as "p," both characters united into one. We do the same in order to write eleven. Commonly we write it thus: "11," with two figures. We join them, and write "n."

CXLVIII. Circulus dividitur in duodecim Signa: Doctrinæ igitur consequentia postulare videtur, ut Signum in duodecim partes primas una prima in duodecim secundas: una secunda in duodecim tertias dividatur. &c. Utemur genera fœminino, ut sciant, qui audiunt, nos non minuta per sexagenas, sed particulas per ΔΩΔΕΚΑΔΑΣ dinumerare.

Cooptando communes numeros cum duodenariis hanc Tabellam formamus.

Numeri Duodenar.		*Numeri communes.* Gr.	′	″	‴	′v	v	v′
Circulus.		360	0	0	0			
Signum.		30	0	0	0			
I.	Pars.	2	30	0	0			
II.	Pars.	0	12	30	0			
III.	Pars.	0	1	2	30			
IV.	Pars.	0	0	5	12	30		
V.	Pars.	0	0	0	26	2	30	
VI.	Pars.	0	0	0	2	10	12	30

Hinc patet sufficere, si in observationibus præter signum, ut utamur trium figurarum numeris: & in supputatione numeris figurarum quatuor: nam tertius character serẽ coïncidit cum minutis: & quartus parum quinque secunda excedit.

Hæ Revolutiones Duodenariæ logarithmis facillimo negotio exprimuntur: nam, si numero artificiali 0.00000̃. addideris continuõ 1.07918. duodenas per duodenas multiplicaberis. Considera Notas sequentes.

Logarithmi.	
0.00000.	*Unitas.*
1.07918.	*Duodena.*
2.15836.	*Duod. Duodenarum.*
3.23754.	*Duod. Duod. Duod.*
4.31672.	*Duod. Duod. Duod. Duod.*
5.39590.	*Duod. Duod. Duod. Duod. Duod.*

Sed perfectiores essent logarithmi, si hoc modo numeris naturalibus cooptarentur.

	Logarithmi.
1	0.00000.
12	1.00000.
144	2.00000.
1728	3.00000.
20736	4.00000.
248832	5.00000.
2985984	6.00000.

Tunc enim essent puri, & revolutiones numerarent educique ex Vlacquianis, & Briggianis logarithmis possent: si 10.000. aut 100.000. in 12. partes divideremus: haberemusque in totiente 833⅓, aut 8333⅓. Sed hæc omnia meliũs intelligentur, quando Logarithmicam edisseremus: modõ periodos Duodenarias promoveamus. Tabulam sequentem contempleris.

CXLVIII. The circle is divided into twelve signs; the consequence of this teaching seems to be, that a sign is divided into twelve *primes* and a prime into twelve *seconds*, one second into twelve *thirds*, and so forth. We use feminine forms, so that those who hear may know that these are not sexagesimal minutes, but parts numbered in dozens.

We write some common numbers with dozenals in this table.

Dozenal Numbers		*Common Numbers* Deg.	′	″	‴	′v	v	v′
Circle.		360	0	0	0			
Sign.		30	0	0	0			
I.	Part.	2	30	0	0			
II.	Part.	0	12	30	0			
III.	Part.	0	1	2	30			
IV.	Part.	0	0	5	12	30		
V.	Part.	0	0	0	26	2	30	
VI.	Part.	0	0	0	2	10	12	30

From this it is clear that, in matters finer than a sign, we should use numbers of three figures in observations and four figures in computation. For the third character coincides with minutes; and the fourth is slightly larger than five seconds.

These dozenal cycles are very easily expressed in logarithms. For, if we continually add 1.07918 to the artificial number 0.00000, we are multiplying dozens by dozens. Consider the following figures.

Logarithms	
0.00000.	*Unity.*
1.07918.	*Dozen.*
2.15836.	*Dozen dozen.*
3.23754.	*Dozen dozen dozen.*
4.31672.	*Doz. doz. doz. doz.*
5.39590.	*Doz. doz. doz. doz. doz.*

But logarithms are more perfect, if we use natural numbers in this way.

	Logarithms
1	0.00000.
12	1.00000.
144	2.00000.
1728	3.00000.
20736	4.00000.
248832	5.00000.
2985984	6.00000.

For then they are pure, and the cycles can be deduced from the logarithms of Vlacquian and Briggian: if we divide 10.000 or 100.000 into 12 parts, we have a total of 833⅓, or 8333⅓. But these are all understood better when we set forth the logarithms; but only if we proceed in periods of dozens. Think about the following table.

TABVLA I.
Ad reducendum partes Duodedenarias
ad gradus, & minuta communia.

Partes	A. Primæ. Gr. ′		B. Primæ. Gr. ′ ″			C. Primæ. ′ ″ ‴			D. Primæ. ″ ‴ ′v		
1	2	30	0	12	30	1	2	30	5	12	30
2	5	0	0	25	0	2	5	0	10	25	0
3	7	30	0	37	30	3	7	30	15	37	30
4	10	0	0	50	0	4	10	0	20	50	0
5	12	30	1	2	30	5	12	30	26	2	30
6	15	0	1	15	0	6	15	0	31	15	0
7	17	30	1	27	30	7	17	30	36	27	30
8	20	0	1	40	0	8	20	0	41	40	0
9	22	30	1	52	30	9	22	30	46	52	30
p	25	0	2	5	0	10	25	0	52	5	0
n	27	30	2	17	30	11	27	30	57	17	30
ɪd	30	0	2	30	0	12	30	0	62	30	0

CXLIX. Vsus Tabellæ precedentis hic est. In primâ columnâ, quæ *Partes* inscribitur, Vnitates duodecim ã capite ad calcem descendunt. In columnâ sequenti, quæ A inscribitur, Signum integrum, hoc est, Gradus 30. in duodecim partes dividitur. Vna ex his partibus Grad. 2.30′ exæquat: & vocatur *Prima*. Vna prima (hoc est, Gr. 2.30′) in duodecim partes secundas dividitur in columnâ B. Vna secunda continet Gr. 0. 12.30″. Et hic similiter arcus in duodecim æquales partes, quæ *Tertiæ* vocantur, in columnâ sequenti (nempe C.) subdividitur, & sic deinceps. Pono exemplum. *Dicitur ⊙ esse in* p42n♎. Et tu vis hos duodenarios characteres ad communes reduci. Respondet computus.

	G.	′	″	‴	′v
p	25	0	0		
4		50	0		
2		2	5	0	
n			57	17	30
Summa.	25	53	2	17	30

Primum numerum dedit columna A, secundum B, tertium C, & quartum D.

CL. Particulæ quartæ, quæ videntur satis esse minutæ, sunt in circulo ɪddddd. (hoc est, 248832) in uno signo sunt ɪdddd. (20736) in duodecimâ parte unius signi (in gr. 2. 30′) sunt ɪddd. (1728) in duodecimâ unîus duodecimæ (in grad. 0.12′.30″) sunt ɪdd. (144) & in duodecimâ eius parte sunt ɪd. (12) & istæ ultimæ δοδεκαδες dividuntur in particulas simplices 12.

Circulus, quando dividitur in Signa, seu duodenas primarias, non exigit necessariõ primum numerum, idem est enim dicere 36ddd. ac *Post* ♊ 6ddd. hoc est. *Post Sign. 3 grad. 15.* aut *Post* ♊ *grad.* 15. [Nota illud *Post*; nam Signa completa numerantur.]

CLI. Cũm calamus quatuor numeros scribit, non

TABLE I.
For Reducing Dozenal Parts
to Common Degrees and Minutes

Parts	A. Prime Deg. ′		B. Prime Deg. ′ ″			C. Prime ′ ″ ‴			D. Prime ″ ‴ ′v		
1	2	30	0	12	30	1	2	30	5	12	30
2	5	0	0	25	0	2	5	0	10	25	0
3	7	30	0	37	30	3	7	30	15	37	30
4	10	0	0	50	0	4	10	0	20	50	0
5	12	30	1	2	30	5	12	30	26	2	30
6	15	0	1	15	0	6	15	0	31	15	0
7	17	30	1	27	30	7	17	30	36	27	30
8	20	0	1	40	0	8	20	0	41	40	0
9	22	30	1	52	30	9	22	30	46	52	30
p	25	0	2	5	0	10	25	0	52	5	0
n	27	30	2	17	30	11	27	30	57	17	30
ɪd	30	0	2	30	0	12	30	0	62	30	0

The use of the preceding table is as follows. In the first column, which is labelled *Parts*, the twelve units are listed from top to bottom. In the following column, which is labelled "A," is the whole Sign; that is, 30° divided into twelve parts. One of the these parts is equal to 2°30′, and is called a *Prime*. One prime (that is, 2°30′) is divided into twelve *second parts* in column B. One second contains 0°12′30″. And so, in the same way, this arc is subdivided into twelve equal parts, which are called *thirds*, in the next column, C, and so in the next. I will give an example. It is said that ⊙ is in p42n♎, and you wish to reduce these dozenal characters to common ones. This calculation does it.

	Deg.	′	″	‴	′v
p	25	0	0		
4		50	0		
2		2	5	0	
n			57	17	30
Sum	25	53	2	17	30

Column A gives the first number, B the second, C the third, and D the fourth.

CL. The fourth division seems to be small enough; there are ɪddddd (that is, 248832) in the circle, ɪdddd (20736) in the Sign, ɪddd (in degrees, 2°30″) in the twelfth part of a sign, ɪddd (1728) in a twelfth of a twelfth (in degrees, 0°12′30″), and ɪdd (144) in a twelfth of that part. The last *twelve-decade* is likewise divided into twelve simple parts.

The circle, when it is divided into Signs, or primary dozens, does not necessarily eliminate the first number, for saying "36ddd" and "*After* ♊ 6ddd" is the same. *After* 3°15′ or *After* ♊ 15 degrees. (Note that "After"; for [only] completed Signs are counted.)

CLI. When we write four numbers, it is not simply to

TABULA II. Dodecades ad Decades, & ẽ contrã reducens.									
Vnitates.		Duodenæ. I.		δοδεκαδες. II.		III.		IV.	
1	1	1d	12	1dd	144	1ddd	1728	1dddd	20736
2	2	2d	24	2dd	288	2ddd	3456	2dddd	41472
3	3	3d	36	3dd	432	3ddd	5184	3dddd	62208
4	4	4d	48	4dd	576	4ddd	6912	4dddd	82944
5	5	5d	60	5dd	720	5ddd	8640	5dddd	103680
6	6	6d	72	6dd	864	6ddd	10368	6dddd	124416
7	7	7d	84	7dd	1008	7ddd	12096	7dddd	145152
8	8	8d	96	8dd	1152	8ddd	13824	8dddd	165888
9	9	9d	108	9dd	1296	9ddd	15552	9dddd	186624
p	10	pd	120	pdd	1440	pddd	1728	pdddd	207360
n	11	nd	132	ndd	1584	nddd	19008	ndddd	228096
1d	12	1dd	144	1ddd	1728	1dddd	20736	1ddddd	248832

est, cur lingua aliud faciat, quãm simpliciter proferre quatuor notas. Cũm vulgõ Arithmeticus exprimit currentem annum, non scribit sic,

°	′	″	‴
1.	6.	4.	2.

sed sic simpliciter 1642. quia, decades centurarium v. g. ab aliis decadibus solus locus sufficienter distinguit. Ergo simĩliter non erit necessarium dodecades primas ã secundis apicum expressione distinguere; nam notæ sunt simplices, & illas locus satis clarẽ discernit. Sanẽ in vulgari Arithmeticâ religiosus est calamus, & lingua luxuriat. Cũm scribo 33333. myriades, millia, centurias, decades, & unitates calamus eodem modo conformat: non autem lingua eodem modo pronunciat. Sanẽ si oculis sufficit, hos characteres 33333. non distingui figurâ, sed loco: sufficeret auribus, si diceremus *Tria tria tria tria tria.* Ergo in hac Duodenariâ Arithmeticâ, scribat manus 7. 4381. & non pronunciet lingua, *Post Signum septimum, gradu quarto, minuto tertio, secundo octavo, tertio primo* sed dicat tantum, *Post finem Libræ quartâ tertiâ octavâ primâ.*

Mensam, quam vocant Pythagoricam, utiliter proponere solent, qui numeros deducunt per decades: ergo illos imitemur, & etiam conformemus abacum, qui per duodenas fluat.[1]

Præter cellulas primæ columnæ omnes aliæ habent duos characteres: quorum prior duodenas, & secundus unitates significant unde, si jubear multiplicare p per p (decem per decem) respondebo resultantem numerum esse 84. (octo duodenas, & unitat. quatuor.)

CLII. His præmissis Regulas nonnullas expediamus, quæ supputationibus Astronomicis sint convenientes.

[1] Tabula mota est ad paginam 6.—Ed.

write four figures, because in speech we do something else. Generally, when the Arithmetician writes the current year, he does not write it thus:

°	′	″	‴
1.	6.	4.	2.

but simply "1642," because the place alone sufficiently distinguishes the decades and centuries from one another. In the same way, therefore, it will not be necessary to distinguish dozenal primes and seconds by a mark, for they are plain numbers, and the place is enough to clearly distinguish them. Certainly, the pen is very terse in common Arithmetic, and speech is more verbose. When I write 33333, the pen gives shape to units, tens, hundreds, thousands, ten thousands; but the tongue does not pronounce them in the same way. Clearly, if it's enough for the eyes that these characters (33333) are not distinguished by shape, but by place; it ought to suffice for the ears, if we were to say, "Three three three three three." Therefore, in this Dozenal Arithmetic, the hand writes 7.4381, and the tongue does not say, "After the seventh sign, four degrees, three minutes, eight seconds, and one third," but rather, "After the seven four three eight one."

They who count their numbers by tens are accustomed to use a table, which they call Pythagorean. Let us, then, imitate them, and also conform to a table, which proceeds from dozens.[1]

Besides the cells of the first column, all the rest have two characters, of which the first are the dozens, and the second are units. From this, if I am told to multiply p by p (ten by ten) I will answer that the resulting number is 84 (eight dozens and four units).

CLII. We will now prepare a few rules which will be convenient for astronomical computations.

[1] Table moved to page 6. —Ed.

	1	2	3	4	5	6	7	8	9	p	n	1d
1d	1d	2d	3d	4d	5d	6d	7d	8d	9d	pd	nd	1dd
n	n	1p	29	38	47	56	65	74	83	92	p1	
p	p	18	26	34	42	5d	5p	68	76	84		
9	9	16	23	3d	39	46	53	6d	69			
8	8	14	2d	28	34	4d	48	54				
7	7	12	19	24	2n	36	41					
6	6	1d	16	2d	26	3d						
5	5	p	13	18	21							
4	4	8	1d	14								
3	3	6	9									
2	2	4										
1	1											

Additio.

Prima Regula Numeros datos conjungit, & eodem modo, quo in Arithmeticâ communi expeditur, saluâ semper duodenariâ periodo. Sit Legum, ac Regularum loco hoc exemplum.

$$\begin{array}{r} 5\,3\,4\,9\,n\,7 \\ p\,4\,n\,6\,p\,8 \\ \hline 1\,3\,8\,4\,4\,p\,3 \end{array}$$

Incipio igitur ã dexterâ, & deorsum colligo characteres hoc modo. [7. & 8. sunt 1.3. hoc est, 1. duodena, & 3.] subscribo igitur 3. & duodenam ad sequentem columnam transporto: & sic inquam. [1.n. & p. sunt 1.p. hoc est, 1. duodena, & 10. unitates.] Subscribo igitur unitates, & duodenam transfero: & sic progredior. [1.9. & 6. sunt 1. 4. hoc est, 1. duodena, & 4.] Subscribo igitur 4. & duodenam ulteriũs transmitto: & sic pergo. [1.4. & n. sunt 1. 4. hoc est, 1. duod. & 4.] Subscribo illas 4. unitates, & illam duodenam in aliam columnam transmitto. & sic ajo. [1. 3. & 4. sunt 8.] Illa subscribo: & pergo dicens. [5. & p. sunt 1.3. hoc est, 1. duod. & 3.] & illa subnoto. Et pronuntio Numerum 5349n7. additum Numero p4n6p8. dare Numerum 13844p3.

Addition

The first rule joins together given numbers in the same way as is done in common Arithmetic, except always in groups of a dozen. Here is an example of the laws and rules of addition.

$$\begin{array}{r} 5\,3\,4\,9\,n\,7 \\ p\,4\,n\,6\,p\,8 \\ \hline 1\,3\,8\,4\,4\,p\,3 \end{array}$$

I begin, therefore, at the right, and I add together the digits in this way. 7 and 8 are 13; that is, 1 dozen and 3. So I write 3 underneath and move the dozen to the following column. Then I say 1, n, and p are 1p; that is, 1 dozen and 10 units. So I write the units underneath, and move over the dozen; and so I proceed. 1, 9, and 6 are 14; that is, 1 dozen and 4. So I write 4 underneath and move over the dozen to the last column; and so I proceed. 1, 4, and n are 14; that is, 1 dozen and 4. I write 4 units underneath, and move the dozen into the next column. And so I continue. 1, 3, and 4 are 8. I write that underneath; and I proceed. 5 and p are 13; that is, 1 dozen and 3. And I write that underneath. And I find that the number 5349n7 added to the number p4n6p8 gives the number 13844p3.

Abstractio.

Secunda Regula unum Numerum subducit ab altero: &, si fortẽ aliquando non possit, superiori characteri duodecim addit, & in sequenti columnâ unum inferiori characteri restituit. Verbi gratiâ.

$$\begin{array}{r} 1\,3\,8\,4\;\;4\,p\,3 \\ p\,4\,n\;\;6\,p\,8 \\ \hline 5\,3\,4\;\;9\,n\,7 \end{array}$$

Subtraction

The second Rule subtracts one number from another; and if perchance it cannot be done in one column, it adds twelve to the upper digit, and reduces by one the lower digit in the next column. An example.

$$\begin{array}{r} 1\,3\,8\,4\;\;4\,p\,3 \\ p\,4\,n\;\;6\,p\,8 \\ \hline 5\,3\,4\;\;9\,n\,7 \end{array}$$

CLIII. A dexterâ igitur initium sumo, & inferiorem Numerum ã superiori subducturus, sic inquam. [8. ex 3. auferri non possunt. Ergo aufero 8. ab 1. 3. hoc est, ab 1. duod. & 3. & habeo 7.] Hæc subscribo. Addo 1. ipsi inferiori p. & habeo n. & dico. [n. ex p. auferri non possunt. Ergo addo 1. duodenam p. superiori, & ajo: n ex 1p. dant n.] & illa subscribo. Et addo 1. ipsi 6. ut sint 7. & dico. [Auferri 7. ex 4. non possunt: ergo ex 1. 4. relinquunt 9.] Illa subnoto. Addo 1. ipsi n. & sunt 1d. & ajo. [1d. ex 4. subduci non possunt. Ergo ex 1. 4. relinquunt 4.] Quæ subscribo. Addo 1. ipsi 4. & dico. Si 5. auferantur ex 8. manent 3.] Et hæc subscribo. & dico. [p. ex 3. excidi nequeunt: si autem exscidantur ex 1. 3. manent 5.] & hæc subscribo. Ergo, si ã Numero 13844p3. Numerus p4n6p8. auferatur, manebit Numerus 5349n7. Ergo Additio Subductionem, & Subductio Additionem confirmat.

CLIII. I take up the first column on the right, and I must take a smaller number from a greater, as I said before. 8 cannot be taken from 3; therefore I take 8 from 13; that is, from 1 dozen and 3, and I have 7. I write this underneath. I add 1 to the lower digit, p, and I have n. n cannot be taken from p, so I add one dozen to the upper p. n from 1p gives n, and I write that underneath. And I add one to the 6, giving 7. 7 cannot be taken from 4; so I take it from 14, giving 9. I write that below. I add 1 to the n, which gives 1d. 1d cannot be subtracted from 4. So I subtract it from 14, leaving 4, which I write underneath. I add 1 to the 4. 5 can be taken from 8, leaving 3. So I write this underneath. p cannot be taken out of 3, so I take it out of 13, leaving 5, and write this underneath. Therefore, if the number p4n6p8 is subtracted from the number 13844p3, the number 5349n7 is the answer. Therefore addition confirms subtraction, and subtraction addition.

Multiplicatio.

Tertia Regula unum Numerum per alium multiplicat: &, si Abacum feceris, reducetur ad primam. Nam, si 16n478. per 4pn5p. debeat multiplicari, majorem ad Abacum reducam: ut conspicis.

1	d16n478
2	d31p934
3	d48p1nd
4	d639668
5	d7p8n24
6	d9583pd
7	dnd7858
8	1d77114
9	122649d
p	1395p48
n	15453d4
1d	16n478d

Multiplication

The third rule multiplies one number by another; and, if you use an abacus, it is reduced to the first number. For, if 16n478 must be multiplied by 4pn5p, I will reduce the greater number to the abacus, as you may see.

4	d	6	3	9	6	6	8				
p		1	3	9	5	p	4	8			
n			1	5	4	5	3	d	4		
5				d	7	p	8	n	2	4	
p					1	3	9	5	p	4	8
Sum.		7	9	1	2	d	p	1	4	8	8

Abacum fuisse bene conformatum constat: quia in ultimo loco, ubi (1d) præscribitur Numerus multiplicandus. Ergo 16n478. ducta in 4pn5p. dant 7912dp1488.

We know that the abacus has done well, because in the last line, where 1d is placed, is the number to be multiplied. Therefore 16n478 multiplied by 4pn5p gives 7912dp1488.

Divisio prior.

Si Divisor fuerit simplex Numerus, Computus facili negotio expeditur. Nam, si Numerus 265689pn36. jubeatur per 5: sic poterit fieri operatio.

				1		4	3	2	2	
5 |	2	6	5	6	8	9	p	n	3	6
	d	6	1	1	4	1	n	9	5	6

Basic Division

If the divisor is a single digit, the computation is done with some easy work. For, if the number 265689pn36 is divided by 5, the operation can be done thus:

				1		4	3	2	2	
5 |	2	6	5	6	8	9	p	n	3	6
	d	6	1	1	4	1	n	9	5	6

CLIV. Incipio itaque sic. [5. in 2. non capitur.] Subscribo igitur ciphram, & pergo ulteriùs, dicens [5. in 2. duodenis, & 6. unitatibus, capiuntur sexies.] Subscribo igitur illud sexies, & procedo. [5. in 5. capiuntur semel.] Et subscribo 1. Et pergo, dicens [5. in 6. capiuntur se-

CLIV. I begin, therefore, thus. 5 cannot be divided from 2. So I write a zero, and I go on to the next. 5 can be divided from 2 dozen and 6 units, giving six. So I write six underneath that, and I proceed. 5 can be divided from 5 once, so I write 1 underneath. And I go mel, & superest 1.] Subscribo illud semel & superscribo illud unum. Et dico. [5. in 1. duod. & 8. unit. capiuntur quater.] Et subscribo illud quater: & dico. [5. in 9. capiuntur semel, & supersunt 4.] Subscribo illud semel, & superscribo illa 4. & ajo. [5. in 4. duod. & p. unit. capiuntur undecies, & supersunt tria.] Subscribo illud undecies, & superscribo illa tria. Et dico. [5. in 3. duod. & n. unitat. capiuntur novies, & supersunt 2.] Subscribo illud novies, & superscribo illa 2. Et subjungo. (5. in 2. duodenis, & 3. unitatib. capiuntur quinquies, & superest duo.) Subscribo illud quinquies, & superscribo illa 2. Et dico. (5. in 2. duod. & 6. unit. capiuntur 6.) & illa subscribo.

Stat igitur 265689pn36. divisa per 5. dare 61141n956.

Computum bene processisse Multiplicatio demonstrabit.

```
 6 1 1 4 1 n 9 5 6
                 5
------------------
2 6 5 6 8 9 p n 3 6
```

Et incipiendo ã fine, sic discurro. (5. per 6. dant 2. duod. & 6.) Subscribo hæc 6. & retineo 2. Dico. (5. per 5. dat 2. duod. & 1. Et addendo illa 2. quæ retinui, dabunt 2. duod. & 3.) Subscribo hæc 3. & retineo 2. Et pergo, dicens. (5. per 9. dant 3. duod. & 9.) Et additis illis 2. dabunt 3. duod. & n. Subscribo n. & retineo 3. Tunc sic. (5. per n. dant 4. duod. & 7. & additis 3. dabunt 4. duod. & p.) Subscribo p. & retineo 4. dicens. (5. per 1. dant 5. & 4. additis dant 9.) Illa subscribo. Pergo, & dico. (5. per 4. dant 1. & 8.) Subscribo hæc 8. & 1. retineo. & ajo. (5. per 1. dant 5. & cum illo 1. dabunt 6.) Et hæc 6. subscribo. Et tandem finem impono, dicens. (5. per 6. dant 2. duod. & 6.) & ipsa subscribo.

Ergo se mutuõ corroborant Multiplicatio, & Divisio.

Divisio posterior.

Si Divisor plures notas habeat, erit operatio intricatior: sed per analogiam ad doctrinam communem poterit expediri. Sit igitur numerus 7912dp1488. per 16n478. dividendus.

CLV. Serviet Abacus, quem in Multiplicatione posuimus.

Ipsa Operatio loco Regularum nos diriget.

Quæro in Abaco Numerum proximẽ minorem ipsi 79, quem posui A. & reperio B.[2] &, quia terminatur sub p. superpono quamcunque notam (puta M) & disco Quotientem habiturum esse notas quinque. Et, quia iste Nu-

[2] Hæc sententia difficilis est. Auctor significat quod divisor a numeris simplicibus multiplicandus est dum proventus erit minor parte dividendi quæ notas æquales habet, prima nota in sinistra essenda. Quia, in hac quæstione, 16n478 × 4 dant 639668, et 16n478 × 5 dant 7p8n24, quod est major quam 7912dp, numerus quem requirimus 4 est.

on. 5 can be taken from 6 once, and leaves 1. So I write one below and one above. And I proceed. 5 can be divided from 1 dozen and 8 units four times. So I write that four underneath, and move on. 5 can be divided from 9 once, and leaves 4. So I write that 1 underneath and that 4 above3, and continue. 5 can be divided from 4 dozen and p units eleven times, and leaves 3. I write that eleven underneath and that three above. And I move on. 5 can be divided from 3 dozen and n units nine times, leaving 2. I write that nine below and that 2 above. And I move on. 5 can be divided from 2 dozen and 3 units five times, and leaves two. I write the five below and the 2 above. And I move on. 5 can be divided from 2 dozen and units 6 times, and I write that below.

Therefore, 265689pn36 divided by 5 is 61141n956.

Multiplication shows that our computation went well.

```
 6 1 1 4 1 n 9 5 6
                 5
------------------
2 6 5 6 8 9 p n 3 6
```

Beginning from the end, we move forward thus. 5 times 6 gives 2 dozen and 6. I write that six underneath and keep the 2. 5 times 5 gives 2 dozen and 1. Adding the 2 which I kept before gives 2 dozen and 3. I write that 3 underneath and keep the 2. And I move on. 5 times 9 gives 3 dozen and 9. And adding to that the 2 gives 3 dozen and n. I write that n below and keep the 3. 5 times n gives 4 dozen and 7, and adding the 3 gives 4 dozen and p. I write the p below and keep 4. 5 times 1 gives 5, and adding the 4 gives 9. I write that below. I move on. 5 times 4 gives 1 and 8. I write the 8 below and keep the 1, and move on. 5 times 1 gives 5, and with the 1 gives 6. I write the six below. And I put the end on together: 5 times 6 gives 2 dozen and 6, and I write that below.

Therefore, division and multiplication confirm each other.

Advanced Division

If the divisor has multiple digits, the operation is more delicate; but it can be done by analogy to the general rule. Let, therefore, 7912dp1488 be divided by 16n478.

CLV. Let the abacus, which we discussed in Multiplication, serve us.

The operation itself directs us to the rules.

I seek on the abacus a number which is close to but lesser than 79, which I have placed at A, and I find B,[2] and because it ends under p, I place above it some random digit (here, M) and I learn that the quotient we're

[2] This statement is difficult. The author means that the divisor must be multiplied by simple numbers until the answer is greater than that part of the dividend which has the same number of digits, starting on the left. Because, in this problem, 16n478 × 4 is 639668, and 16n478 × 5 is 7p8n24, which is greater than 7912dp, the number we need is 4.

	M	
A	7912dp1488	
B	d639668	4
C	153762 1488	
D	1395p48	p
E	16179 5488	
F	15453d4	n
G	dd9265d88	
H	d7p8n24	5
I	1395p48	
K	1395p48	p

merus B. habet 4. in margine, etiam in margine 4. scribo. Aufero B. ab A, & adquiro C. Quæro in Abaco Numerum proximẽ minorem, & illum subscribo, &, quia marginalem p. habet in margine, etiam notam hanc in margine scribo. Aufero D. ã C, & adquiro E. & sic deinceps.

Sumo denique notas marginales, & ajo 4pn5p. esse Numerum, quem dant 7912dp1488. per 16n478. divisa.

Hæc est Synopsis Duodenariæ Arithmeticæ; quæ, si semel admitteretur Astronomicos computus ad summam facilitatem reduceret. Si dicat aliquis, esse faciliorem communem, quæ procedit per Decadas; respondebo, esse omninõ faciliora, quæ sciuntur, & difficiliora, quæ ignorantur: quam ob rem illi, qui adsueti sunt numerare per Decadas, sunt in novitate ipsâ aliqualem difficultatem reperturi, quousque adsuescant. Cæterũm, cũm hodie Circulus per 24. per 12. per 6. (seu 60.) per 3. (seu 30.) horas, signa, minuta, gradus, &c. dividatur, & has divisiones velint Astronomi manutenere, quia Numerus 12. has omnes divisiones subit, tertiâ, aut quartâ die Philomusus percipiet has duodenarias Numerorum revolutiones esse aptiores Arcubus mensurandis. Ergo, dum loca Planetarum observas, & Veterum observationes diligenter examinas, hac utere Arithmeticâ (Illustrissime Marchio) Authoremque honorare, & diligere perge. Lovanii 4. Maji 1640.

looking for has five digits. And, because this number B has 4 in the margin, I also write 4 in the margin. I take B from A, and get C. I look in the table for a lesser number nearby, and I write that, and, because it has a marginal p in the margin, I also write this digit in the margin. I take D away from C, and I get E, and so I continue.

I finally take the marginal notes, and I find that dividing 7912dp1488 by 16n478 gives the number 4pn5p.

This is a synopsis of Dozenal Arithmetic, which, if it were once admitted, would reduce astronomical calculation to the greatest ease. If anyone should say that common Arithmetic, which proceeds by tens, is easier, I will respond that those things which are known are easier, and those things which are not known are more difficult. Because of this fact, they who are accustomed to counting by tens are about to discover, in this new method, a difficulty to which before long they will also be accustomed. Furthermore, today the circle might be divided by 24, by 12, by 6 (or 60), by 3 (or 30) hours, signs, minutes, degrees, and so forth, and Astronomers may want to keep these divisions, because the number 12 absorbs them all. On the third or fourth day Philomusus saw that these twelve revolutions of numbers would be better for measuring arcs. Therefore, while you are observing the locations of the planets and carefully examining the observations of the ancients, use this Arithmetic, O most distinguished Marchio, and continue to honor and love the author. Louvaine, 4 May 1640.

Erratum: in tabula multiplicationis in pagina 6, corregi proventum celluli "n × n," perperam dicatum "p"; vere est "1p."

De auctore: Ioannis Caramuelis, vocatus in lingua sua "Juan Caramuel y Lobkowitz," natus 1↋ Maii ↋1↊ in Matritense, illustrissimus physicus, mathematicus, arithmeticus, ecclesiasticus, linguisticus, et astronomicus. In anno decimo ætatis suæ, iam tabulas astronomicas provulgavit. Fit præclarus scholasticus linguarum Asianarum, præsertim Serica. Ordinem Cisterciansem introit in Hispania, ordinatus est, et cursum suum extendit in Hispania, Austria, et Praga. Fit etiam miles cum Angermanniani Pragam oppugnavit in ↋54. Postea fit episcopus Konigratz, Otranti, et postremo Vigevani. Mortuus est in Vigevano 8 Septembris ↋82. Opus eius de Arithmetica est veterrimam expositionem arithmeticæ dozenalæ in mundo, in ↋48 scriptum.

Mistake: in the multiplication table on page 6, I corrected the answer in the cell for "n × n," which was wrongly stated as "p." Correctly, it is "1p."

About the author: Ioannis Caramuelis, known in his own language as Juan Caramuel y Lobkowitz, born on 1↋ May ↋1↊ in Madrid, was a brilliant scientist, mathematician, arithmetician, ecclesiastic, linguist, and astronomer. At ten years old he had already published astronomical tables. He became a widely-known scholar of Asian languages, particularly Chinese. He entered the Cistercian order in Spain, was ordained, and continued his career in Spain, Austria, and Prague. He even became a soldier when the Swedes attacked Prague in ↋54. Afterwards he became bishop of Konigratz, Otranto, and finally Vigevano. He died in Vigevano on 8 September ↋82. The work "On Arithmetic" is the world's oldest explanation of dozenal arithmetic, having been written in ↋48.