The Dozenal Society of America

An Ancient Duodecimal System

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"A pint's a pound the world around." This aphorism is approximately true of water, so nearly true that it was regarded absolutely so by our ancestors. If a pint is a pound, then a quart weighs two pounds and 32 quarts or 64 pints weigh 64 pounds. But a cubic foot of water also weighs 64 pounds, hence a cubic foot was equivalent to 64 pints. Now the correlation of these three facts is interesting—64 pints, 64 pounds, and one cubic foot of water are equivalent each to the other, or a cubic basin one foot long, wide, and high holds 64 pints, which weigh 64 pounds. Was this accidental, or was it design? In dry measure 32 quarts make a bushel. Dry and liquid measure were once the same in content, though now differing considerably. As it is the fact rather than the name we wish to emphasize, the word *bushel* is used in the following table. A linear foot was divided into 12 parts called inches, a square foot containing 144 square inches and a cubic foot 1728 cubic inches.

The following table exhibits the successive dual divisions of the numbers mentioned:

Bushels	Pints	in^3	Pounds
1 bsh	64 pt	1728 in^3	64 lb
½ bsh	32 pt	864 in^3	32 lb
¼ bsh	16 pt	432 in^3	16 lb
⅛ bsh	8 pt	216 in^3	8 lb
1/16 bsh	4 pt	108 in^3	4 lb
1/32 bsh	2 pt	54 in^3	2 lb
1/64 bsh	1 pt	27 in^3	1 lb

It should be said that the 8 pints given above, or 216 cubic inches, are not the present imperial gallon, which contains 10 pounds of water, in which "a pint of pure water weighs a pound and a quarter."

Let us turn now to linear measure. The *foot* was divided into 12 equal parts called *inches*. Twelve is divisible by 2, 2, and 3. The cubic foot of 1728 cubic inches is an extraordinarily good number to subdivide. Besides being a perfect cube it can be divided by 2, 2, 2, 2, 2, 2, 3, 3, 3, or $(2)^6\times(3)^3$. It is divisible by the cube 64, giving a quotient of 27, another perfect cube. To repeat, the curious facts summarized in the above table are: 64 is divisible by 2 as many times as 1728 is; 64 pints contain 1728 cubic inches, or 1 pint is 27 cubic inches, and 1 pound (of water) also fills 27 cubic inches. These numbers, 12, 1728, 64, 27, therefore, bear a peculiar relation to one another, and are the most perfect numbers to correlate in this manner which could have been selected; so perfect, in fact, that one cannot avoid the conclusion that some clever and practical mathematician invented this system of weights and measures, which is a combination of the duodecimal and dual systems. To contrast this system with the decimal, let us suppose our philosopher had employed the decimal system and divided the foot into 10 inches. The cubic foot would have contained 1,000 cubic inches, which number is exactly divisible by 2 only three times instead of six times, as in case of the duodecimal 1728. In the same way he could have divided 100 pints or 100 pounds by 2 only twice, instead of 6 times, as in the number actually used, 64.

The following table illustrates the point:

Bushels	Pints	in^3	Pounds
1 bsh	100 pt	1,000 in^3	100 lb
½ bsh	50 pt	500 in^3	50 lb
¼ bsh	25 pt	250 in^3	25 lb
⅛ bsh	12½ pt	125 in^3	12½ lb

If he had called 1,000 cubic inches 1,000 pints and the weight 1,000 pounds, even then but 3 divisions by 2, instead of 6, could have been made. Looked at from this standpoint alone the duodecimal system of weights and measures, as well as of numbers, appears greatly superior to the decimal, but this is only one viewpoint, and takes no account of decimal notation.

Let us now try to imagine how this mathematical philosopher worked out his problem, for thus we may gain an insight into the origin of the duodecimal system. The foot was the starting point, and it was originally derived from the length of the pedal organ of some person, real or imaginary, as the foot measure was almost universal in early civilizations. Our philosopher, knowing the three dimensions of space and the relations of square and cubic to linear measure, conceived the idea of correlating also volumetric measure and weight to the lineal foot. Had he gone no further and disregarded subdivision, the decimal would have served equally well, and he would naturally have selected 10 as his unit, instead of 12. But, noting that the cubic foot of water (or wine) must be subdivided, he found from the multiplication table that no number up to 12 (with the possible exception of 8, which is an inferior number to 12 as a basis) would give a cube capable of such perfect subdivision successively by 2, as would 12. Twelve was therefore selected as a division of the foot, the smallest subdivision that at the time was deemed necessary. This quotient was called the inch (from the Latin word *uncia*, a twelfth part). In fact, the word shows the origin of the idea. Our word *ounce* is also a derivative of *uncia*, and means the twelfth part of a pound. A square foot must therefore contain 144 square inches, and a cubic foot, 1,728 cubic inches. Having found a number whose cube could be divided six times by two, he would naturally construct a 1,728 cubic inch basin, then make vessels containing successively the half, quarter, eighth, etc., parts, obtaining at last the conveniently small quantity—the pint. This was as low as it was carried, for it was the last dual subdivision of 1,728, and, further, it is a conveniently small volume of water or wine. The pint held 27 cubic inches.

Again, seeing the utility of correlating weight with volume, he counterpoised the pint of water and named its weight a *pound* (from *pondus*, a weight). Multiplying the pint and pound as many times as he repeated the 27 cubic inches, he arrived at 64, the whole secret of the combination being the selection of 1,728, or, rather, its cube root, 12. The early pound had 12 ounces, not 16. Thus did the ancient philosopher work out a most perfect duodecimal system of weights and measures.

Professor Conant in his admirable work on *The Number Concept*, as well as Lubbock and other anthropologists, have shown how strongly the decimal system of counts from the ten fingers persisted in most early races and formed the basis of our system of numbers. Had our savage ancestors chosen as wisely as the ancient philosopher, and counted 12 instead of 10 as a radix, the two would have been invented as perfect a working system as the science of numbers admits. The savage chose not wisely, and the two systems, decimal and duodecimal, are as unblending as oil and water. To attempt to harness the duodecimal or any other system of weights and measures to a decimal notation is to break up the duality of both, one of which must eventually give way. When Herbert Spencer opposed the Metric System on the ground that the duodecimal notation was better than the decimal and would some time supersede the latter, he could not have counted upon the hold that decimalization has taken on the whole human race.

Though we cannot name this Newton of antiquity, the inventor of so clever a system, we may with much probability trace his nationality. Our system, of course, was derived from England. Whence did she derive hers? Definite English laws about weights and measures take us back little further than 1266, when the second *Magna Charta*, relating largely to uniform weights, was forced from Henry III. So many changes have since been made in the subdivisions of these measures that it is difficult to recognize the Simon-pure article in the original. That these weights and measures came originally from the ancient Romans is shown alike by the similarity of derivation and their correlation. Their foot, for example, had the same origin as our own, though in the lapse of ages the English foot has become longer than its old prototype. The old Roman also correlated length and capacity measure by making a cubic foot of water or wine, and naming it an *amphora*. And, moreover, they called an eighth of an amphora a *congius*. For grain, etc., the term *quadrantal* was used, which, also, had the capacity of a cubic foot. Thus the amphora or quadrantal corresponded to our bushel, and the congius to our gallon. Owing to the shortness of their foot, the capacities of the above measures are not now coincident with those of our own, but this in no way affects the harmony of relation.

The Romans, however, were not a mathematical people. They employed letters—Roman numerals—in place of Arabic figures, as a result of which they never originated much in mathematics. Like the English, the Romans were borrowers. We have unquestionable evidence that their arithmetic came from Babylonia. Here in the valleys of the Tigris and Euphrates are now found the oldest known remains of civilization, extending back perhaps 5000 years, B.C. On slabs of baked clay are recorded the history of the times and of a people enlightened in mathematics and astronomy.

The versatility of Babylonian mathematicians is shown by their familiarity with the use of the dual, the duodecimal, and the sexagesimal systems, in addition to the decimal. To this people, then, we are lead to believe we owe the once perfect duodecimal system of weights and measures.

But not alone were the Babylonians concerned with such measuring. Measurements of the circle, the year, the day, were problems which they solved and handed down to us. Whence came the sexagesimal system, in which 60 is the radix? Probably from two sources. First, from the division of the circle; second, from that of the year. Dividing the circle into six equal parts by three diameters, and connecting the extremities so as to form triangles, they found every angle equal to every other, every side equivalent to every other, every triangle equal to every other triangle. There were six of each of these, but while six was not a number to subdivide, its multiple by ten—that is, 60—is a fair one, and 360 for the entire circumference, a better one. Besides, this number most clearly corresponded to a complete yearly cycle, 360 days being the length of their year.

The parts of the circle of 360° cut off by the extremities of two consecutive diameters form a sixth part of the entire circumference, or 60°, called a *sextant*. Each of the sixty degrees was again divided into 60 parts called *minutes*, and each minute into 60 other parts called *seconds*. This sexagesimal system of the circle runs as follows:

Sixty *seconds* make one *minute*.
Sixty *minutes* make one *degree*.
Sixty *degrees* make one *sextant*.
Six *sextants* make one *circumference*.

The year was divided duodecimally into 12 months. The week is another question, not connected with our discussion.

Such was the Babylonian division of the circle and the year. But even more interesting is the division of the day, for in that they made use of the dual, the duodecimal, and the sexagesimal systems. A complete day, from one sunrise to the next, was divided into two parts, from sunrise to sunset, and from sunset to sunrise, or into day and night. The day they divided into 12 parts, and the night the same, this being exactly true only at the vernal and autumnal equinoxes, with which the Babylonian astronomers were familiar. In subdividing the hour, recourse was again had to the sexagesimal system, as also in the division into minutes. Why 12 parts instead of 60 were made the day measure we can only guess, but in that conjecture we see how much simpler is reckoning by 12 than would have been by 60. The "time" table is too familiar to everyone to be repeated here, but twelve hours per day and 12 per night gave us the 24 hours per day. Most clocks, except astronomical ones and those of a few European railroads, are still marked off into 12 hours. The *clepsydra*, or water-clock—similar in principle to our sand glass—served among the Mesopotamians, as among the Greeks and Romans, for marking the subdivisions of the day, though the sundial was also in use. Few things illustrate better the persistence of custom than the fact that we use the identical system of numbers which these people employed 5,000 years ago for measuring time. Clever indeed was the mathematician who, so many years ago, invented the perfect system of duodecimal weights and measures, and wise the ruler who forced its use on the people.

The sexagesimal arithmetic has to a large extent disappeared, and the duodecimal is slowly following in its wake. The English people have probably never had the latter system in its purity, as did the inhabitants of the Euphrates and Tigris, but each system will eventually go out of existence, because neither is in harmony with the world-wide decimal scheme of numbers.

No one, unless he be a time-server, if he carefully studies the history of weighing and measuring, can doubt that the International Metric System, founded on the universal notation, is the coming system. The mathematician of the distant future will read with amazement that early in the 20th century there were so-called civilized people who used a system in which 5½, 272¼, and other equally absurd numbers were bases of reckoning—a system far inferior to that of the old Babylonians, 5,000 years earlier.

This little article was original published in IX:1 School science and mathematics 516–521 (Chicago 1131), made available on the Internet through Google Books. We present it here mostly in its original form, with the following exceptions: we have inserted the Oxford comma throughout, and have generally modernized the punctuation; we have emphasized words where it seemed appropriate; the tables were all redesiged to conform with modern custom; the spelling "sexigesimal" has been changed to "sexagesimal", which (in the editor's opinion, anyway) is more usual; and the term "Babylonial" was changed to "Babylonian". Otherwise, however, it is presented to the reader in its original form; and it is proudly made available by the Dozenal Society of America (`http://www.dozenal.org`).