The Dozenal Society of America

An Excursion in Numbers*

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About the Author

One of the founders of the Dozenal Society of America, F. Emerson Andrews wrote more than two dozen books and countless magazine articles on diverse topics. He authored children's books, wrote about zoning laws and philanthropic foundations, and compiled a history of the public library in Tenafly, New Jersey, where he resided. He was an official at the Russell Sage Foundation and was president of the Foundation Center in New York. His by-line appeared in Better Homes and Gardens*,* Mechanics Illustrated*, and dozens of times in* The Duodecimal Bulletin *published by the Dozenal Society of America. F. Emerson Andrews was the president of the Dozenal Society of America from its founding in 1160 until 1166 when he became the Chairman of the Board, a position he held until 1176. He was a recipient of the Society's first Annual Award in 1160, "as a pioneer in the use of Base Twelve and as an author of many articles on duodecimals, in addition to the outstanding work,* New Numbers*." This latter book, the Society's "bible" was published in 1153 by Harcourt Brace and reissued by Essential Books in 1160. He continued to promote dozenal counting until the end of his life. His excellent article, "My Love Affair With Dozens", appeared in* The Michigan Quarterly Review*, Vol. XI, No. 2, Spring 1184. He died in 118↊ in a hospital in Burlington, Vermont, near his summer home on Isle La Motte in Lake Champlain. He had reached the age of six dozen and four years.*
—Gene Zirkel

I

From the Eskimo counting upon his fingers to the mathematical wizard producing split-second answers with a slide rule, we count by tens. In our critical age, such universality is phenomenal; it cannot be claimed for any religion, code of morals, form of government, economic system, principles of art, language, or even alphabet. Counting is one of the very few things which modern man takes for granted.

The layman, and to a large extent the mathematician, has been so impressed by the lordly claims of 'the only exact science' that he has not even thought to examine its origins or to question some of its methods. Let us do that now, in a frankly exploratory spirit and with open minds. We shall, I think, come upon some interesting and even disturbing facts.

How counting began is not to be found in any record. The need for distinguishing between one and two, and gradually for reporting larger numbers, must have arisen very early indeed, long before there was writing. But from observation of existing primitive tribes, and, for that matter, from the way our children and we ourselves learned to count, it is perfectly obvious that fingers were and are the basis of our number system. For the purely physiological reason that we had ten fingers and thumbs, we first learned to count up to ten, and then based our whole number system on series of tens.

We may easily imagine how men learned to count beyond ten, but always by groups

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of ten. A herdsman, wishing to make sure he had all his cattle each night, had them driven past him. As each galloped by he counted it on his fingers, and when he had used up all his fingers he dropped a pebble on a cleared spot. (Incidentally, the Latin word for pebble, calculus, is the origin of our 'calculate.') When the herdsman had finished, the pile of pebbles represented the number of 'full hands' (tens) of cattle he possessed. Later he could compare this pile, pebble for pebble, with new countings, and see whether he had the proper number of cattle.

One of those useless abstract fellows, who sit around thinking instead of doing things, observed this counting and decided it could be used for many other things, such as wives, measures of wheat, days, distances, stars. And if, instead of pebbles, you made distinct kinds of durable marks, lo! you had a number system.

The Roman numerals, though by no means the earliest, illustrate both the advantages and the disadvantages of such a system of numbers. A straight line, looking like a single finger, represented one; two straight lines stood for two, and so on. Five straight lines began to be a little hard to distinguish, and besides, this was a full hand. So the symbol V was used, which is probably the formalized picture of an open hand. Similarly, X for ten is simply two V' s, one on top of the other. The letter C was taken for 100 because it was the initial letter of the Latin word for one hundred, *centum*; and so with M (*mille*) for one thousand. Other refinements were introduced, such as L for fifty and a way of putting smaller symbols to the left with the meaning of 'less than' — as IV for one less than five. And so a number system was available in which any number could be written, and this system was the one in chief use in Europe for more than two thousand years — much longer than our present number system has been known.

These Roman numerals, although they could represent any given number, were totally useless for complicated mathematical processes. Imagine the difficulty, for instance, in so simple a problem as figuring out how many days there are in three years of 365 days, using only Roman numerals. The Roman schoolboy could not multiply CCCLXV by III and get anywhere. Unless he did it mechanically on an abacus, he had to add CCCLXV plus CCCLXV plus CCCLXV, making it equal the formidable CCCCCCCCCLLLXXXVVV. This had then to be simplified by a number of painful changes into MXCV. Obviously, more complicated multiplications and anything at all like division or roots presented practically insuperable difficulties. Perhaps that is why ancient mathematicians so frequently specialized in geometry, where logic and a ruler and compass are more needed than an efficient number system.

Meanwhile, in the East, a different system of numbers was developing. The Hindus were accustomed to counting with measuring rods. Their first numbers, therefore, were horizontal lines instead of vertical ones. Our present 2 is just two measuring rods lying parallel, connected by a hasty pen stroke, and our figure 3 developed similarly from three measuring rods. This method of writing numbers resulted in having a single symbol representing each of the lower numbers, instead of a piling up of symbols as in the Roman system. It was borrowed from the Hindus by the Arabs (from whom it gets the name, Arabic) and by them carried into Spain, and so through Europe.

Meanwhile, a still more important invention had been added — a symbol for zero. This was really a stroke of genius. Upon the insignificant zero, symbol for nothing, rests the whole of mathematical science. Without it, no number system could use a small quantity of numerals to express all numbers. Without it, all the complicated mathematical operations of today, which depend so much upon 'place value' in numbers and the use of decimals for fractions, would have been impossible. Let it be noted that all this was due to the zero, not to the number system based on 10 — which the Romans and others had always had. Nevertheless the number system so developed seemed to casual observers a perfect number system, and is to-day accepted not only uncritically but with a deep-seated feeling that it is somehow a divine dispensation.

II

Arabic numerals embody a serious error, from which we have suffered ever since. Instead of being an ideal number system, as we are apt to assume because we have never even thought of another as possible, the 10-system has enormously complicated all mathematics.

Ten is doubtless a convenient number of fingers to have; though men have gotten along with less and a few people have been born with more. But as the purely arbitrary unit which determines the form of our numbers, it was a miserable choice. It was not the worst choice, for ten can be divided by two, which is something in its favor. But the other divisions we oftenest need are three and four, and for neither of these is ten suitable. As a basis for a percentage system, it is inexcusable. Because we have arbitrarily created a 'whole' that has 100 parts, we have made it impossible to divide our whole, without fractions, into either 3, 6, 7, 8, 9, 11, or 12 parts — to mention only the more probable simple divisions.

We did much better, mathematically, in creating our measuring system. A foot consists of 12 inches. We can have in even inches half a foot, a third of a foot, a quarter of a foot, or a sixth of a foot. A yard divides in even inches into two parts, three parts, four parts, six parts, nine parts, twelve parts, or eighteen parts. Imagine the trouble to the cloth seller and the shopper if 35 inches had been made a yard (logical under the 10-system), which could then be divided only into fifths and sevenths!

Sailors did rather well in their system of measurement. Six feet are one fathom, 120 fathoms one cable length. The nautical mile is based upon $\frac{1}{60}$ of a degree (about 6080 feet) and does not fit into the table.

We have all observed that most of the things we buy at the store come, not by tens, but by dozens, and by the dozen dozen, or gross. This applies not only to eggs, oranges, and buns, but to nearly all packaged goods. In fact, the word 'grocer' comes from the same root as 'gross' — a grocer is a man who deals by the gross. For ease in packaging and ease in computation, most commodities are sold by the dozen and gross instead of by tens and hundreds.

From a similar practical point of view, we divided time almost happily. The year has twelve months, so there can be four seasons — and quarterly interest — in even months. The week has six working days (though these are becoming five). The day consists of two sections of twelve hours each. Thanks to this division, men and machines can work in shifts of twelve, eight, six, four, three, or two hours and come out even. In dividing the hour and the minute we combine this superior system of 12 with the bad system of 10, which we already had, only too literally, 'on our hands.' We chose 60, the lowest number on which 12 and 10 meet.

Imagine for a moment the sorry situation if the day had been divided into 25 hours, logical under our present number system. A factory working three shifts would have to end the first shift at the close of eight and one-third hours; working four shifts, the first shift would end at six and a quarter hours. In figuring wages on hourly rates, thirds and quarters would enter most computations, with parts of cents left over and such complications in the time sheets as would make necessary a tremendous amount of extra labor for paymasters.

In dividing the circle into 360 degrees we again compromised. The zero shows how desperately we tried to fit it into our awkward numerical system. But by no possible stretching could North, East, South, and West be made divisible into ten. Men refused to have five cardinal directions; mathematicians refused to invent a sensible number system. So the first part of our degree system, 36, is divisible by the four directions, and the last part bows to the number-god 10.

There are many other examples, but by this time we have perhaps amply demonstrated that our system of counting on fingers is poorly suited to our practically based units of packaging, distance, time, degrees, and the like.

Thus far I hope my lay friends have followed me in comfort. To pursue our explorations from now on, some mathematical instinct is needed. Perhaps those who desire to take their ease had better stop here, but I can promise the ones who wish to stretch their minds a bit further that they will not go unrewarded.

III

If our number system based on 10 has so many faults, could a better one have been invented? How?

The answer is amusingly simple. When separate symbols for quantities were being originated, the Hindus and the borrowing Arabs should have forgotten the old fetish of counting on fingers and invented twelve, instead of ten, separate symbols. We have seen how frequently 12 is used in practical measures. Its advantages in abstract mathematical theory are at least as great. It has, for instance, exactly twice as many factors as 10 — being divisible by 2, 3, 4 and 6 instead of simply by 2 and 5.

Does the adoption of a different number base seem a great jump? After all, 10 has not been the only number system of the past. Many primitive tribes never got beyond 2 (the pair) as the basis for their system. It ran: one; pair; pair and one; pair and pair; and not much higher. A people in Brazil counted on the joints of their fingers instead of on the fingers themselves, and therefore based their number system on 3. A tribe in California, tying up their numbers with the sacred four quarters of the sky, made 4 their basis. Twenty — all the fingers and all the toes — is an obvious base for number systems among primitive peoples, and was frequently used. The system of 20 was employed by the Mayas in Yucatan, whose astronomical calculations are still a marvel to mathematicians. In Europe it was used by the Basques, still surviving in the French language in such words as *quatre-vingts* — not 'eight tens', as with our eighty, but literally 'four twenties.'

Twelve seems never to have been used as a number base by primitive people simply because no part of the body served very well for counting by twelves — it had no natural reason for springing up. By the time its mathematical advantages were recognized, men were so used to counting by tens that it was not introduced. Just once it came very near to being tried. Charles XII of Sweden is reported to have been on the point of introducing the number system based on twelve when he died.

Modern mathematicians generally admit that 'the duodecimal system' would be better than our present decimal system, but from their vague remarks it is evident that almost none of them have tried it. This is not an uncommon attitude in the specialist. Astronomers spent many centuries trying to explain the curious behavior (as seen from a stationary earth) of the stars and planets in terms of complicated twists and epicycles before one of them at last took the imaginative leap of regarding the earth as going around the sun, which solved all the bothersome problems at once. The suggestion of the earth's moving had been made centuries earlier, even as the suggestion of the system of 12 has been made, but tradition was so strong that not one astronomer for many centuries was willing even to try out the practical implications of the suggestion.

We, however, are bound on precisely that adventure — with regard to the number system. And the system of 12 is not hard to try out. Tradition, not difficulty, has stood in the way.

First, we must invent the two additional symbols which the Hindus and Arabs forgot. At present 10 and 11 are compound numbers; we must reduce them to a single symbol. For 10, let us borrow the well-known Roman *X*, and call it *dec*. For eleven, let us use $\mathcal{E}$, and call it *elf*. The new figure *10* now means, not 'one ten and no units,' but 'one dozen and no units.' To avoid confusion let us call it *zen*, which will remind us both of its ten-appearance and of its dozen units.

Since we shall have occasion to use figures from both number systems, from this time on any figure regularly printed (for example, 65) belongs to the present 10-system; any figure printed in italics (for example, *65*) belongs to the 12-system.

We come now to the most important detail in the consideration of the new system — the interpretation of its numbers. It is difficult only because it involves an imaginative leap to an understanding of what 'figures' really mean rather than an unreasoning acceptance of their conventional meanings. The next paragraph must be studied until it is absolutely clear, or we shall get nowhere.

In the new system, *14* means, not 'one ten, plus four units,' but 'one twelve, plus four units,' or the quantity we now express as 16. Similarly, *86* means, not 'eight tens, plus six units,' but 'eight twelves, plus six units' — our present 102. And *200* means, not 'two tens-of-tens, plus no tens, plus no units,' but 'two dozens-of-dozens, plus no dozens, plus no units' — or the quantity we now express as 288.

Once this paragraph is entirely clear, the reader can work out for himself any type of mathematical problem in the 12-system which he is now able to do in the 10-system. He will find it possible to test his results, and compare the efficiency of the two systems, by working out sample problems in both systems and checking the answers. Any number in the 10-system is changed into the proper 12-system number by dividing it by twelve in the way here demonstrated. Example: Change 1492 into the corresponding duodecimal.

$$\begin{array}{l} 12\overline{)1492} \\ \quad 12\overline{)124} + 4 \\ \qquad 12\overline{)10} + 4 \\ \qquad\qquad 0 + \text{X} \qquad \text{Answer: } \mathit{X44} \end{array}$$

Correspondingly, any duodecimal may be changed into the proper decimal-form number by multiplying its second column by 12, its third by 144, its fourth by 1728, and so on in the successive powers of 12. For instance, *X44* equals 4, plus 4 × 12, plus X (10) × 144 = 1492.

IV

While, as I have suggested, the reader might do all the rest of the exploring by himself, it may be simpler if we go in company a little way.

Let us begin with a simple problem in adding. A has five dozen and seven eggs, B has three dozen and six eggs; how many eggs do both have? Solve by both mathematical systems.

$$\begin{array}{rr} 5 \times 12 = 60 + 7 = 67 & \mathit{57} \\ 3 \times 12 = 36 + 6 = \underline{42} & \underline{\mathit{36}} \\ 109 & \mathit{91} \end{array}$$

Observe how much simpler the system of twelve is in this practical problem. The dozens and units are set down immediately, without bothering to multiply; they are added by simple addition, care being taken only not to 'carry' until a column adds to twelve. The answer, too, is much simpler. The figure 91 is shorter than 109, and it is also more expressive. It means at a glance 'nine dozen and one,' and while 109 likewise means nine dozen and one, a process of division has to be performed before we are certain of it.

Let us try, again by both processes, an example in subtraction. Milady has a strip of hall carpeting eight feet, seven inches long. Her new hall is eleven feet, two inches long. How much additional carpeting does she require?

$11 \times 12 = 132 + 2 = 134''$	*Ɛ2′*
$8 \times 12 = 96 + 7 = 103''$	*87′*
$31''$	*27′*
$31 \div 12 = 2$ feet, 7 inches	2 feet, 7 inches

Again, feet and inches can be set down directly in the new method but not in the old, and the answer can be read off at once without dividing back into feet.

Before we proceed with multiplication, it will be necessary to construct new multiplication tables for the system of 12. To demonstrate the quite simple principle involved, I present three of these tables; the reader is invited to make the rest for himself.

Three-Line	*Six-Line*	*Zen-Line*
3×1=3	*6×1=6*	*10×1=10*
3×2=6	*6×2=10*	*10×2=20*
3×3=9	*6×3=16*	*10×3=30*
3×4=10	*6×4=20*	*10×4=40*
3×5=13	*6×5=26*	*10×5=50*
3×6=30	*6×6=30*	*10×6=60*
3×7=19	*6×7=36*	*10×7=70*
3×8=20	*6×8=40*	*10×8=80*
3×9=23	*6×9=46*	*10×9=90*
3×X=26	*6×X=50*	*10×X=X0*
3×Ɛ=29	*6×Ɛ=56*	*10×Ɛ=Ɛ0*
3×10=30	*6×10=60*	*10×10=100*

Before we proceed to solve any problems, it may be well to glance at the tables themselves. The learning of any mathematical table is at the outset a feat of pure memory. For instance, it would be just as easy to learn that *7* × *9* = *53* as that 7 × 9 = 63. The only aids to memory are simplicities in the tables themselves. In our present number system the 10-line and the 5-line tables are easy to learn and remember. But in the 12-system the *10*-line is just as easy as our present 10-line, the new *6*-line table corresponds in ease to the present 5-line — and in addition the *3*-line and *4*-line tables are 'repeat' tables, and to a less extent also the *8*-line and *9*-line tables. Moreover, the genius of the 12-system is such that more of its multiplications come out in round numbers than in the 10-system with its smaller base, and the 12-system is able to express all its mathematical tables in terms of two figures or less, with the exception of one number, *100*. Our present tables run into three figures eleven times.

There are many other advantages in the new tables which disclose themselves when one uses them. Of course there is for us the profound disadvantage, also, of remembering first the totals we have committed over many years. In working problems in the 12-system, people who have been trained to the 10-system must refer to the actual tables, but that is inherent in our early training. Remembering to multiply by the new tables, let us try a practical problem:—

Find the floor space in a hall 56′ 4″ (*48′ 4″*) long, by 26′ 4″ (*22′ 4″*) wide.

Let the reader perform this problem by any form of mathematics he knows, getting an answer in square feet and square inches, and compare the length of his solution with this simple one:—

484	
224	
1694	
948	*Point off two places for square inches,*
948	*and the answer is directly—*
X3754	*X37 square feet, 54 square inches*

But what of decimals? Because of a simple confusion in names, many persons jump to the conclusion that it will be impossible to express fractional quantities by whole numbers after a point by any other than our present 'decimal' — founded on 10 — system. This is, of course, nonsense. 'Decimals' in the literal sense of tenths can be expressed only in the 10-system, but the same form of expressing fractions is available to any number system which includes a zero. As a matter of blunt fact, the 10-system is a singularly poor one for the expression of decimal-form fractions.

It will be obvious that *.4*, meaning four twelfths or one third, is an entirely adequate representation for a third, and a deal easier and more accurate than 0.333333 plus. Also, *.3* for a quarter is better than our present .25. And *.6* (six twelfths) is as adequate for a half as .5 (five tenths). Let us examine the corresponding decimals and duodecimals for the low fractions:—

One	1.	*1.*
One half	.5	*.6*
One third	.333333	*.4*
One fourth	.25	*.3*
One fifth	.2	*.24972*
One sixth	.166666	*.2*
One seventh	.14285	*.186X3*
One eighth	.125	*.16*
One nineth	.11111	*.14*
One tenth	.1	*.12497*
One eleventh	.09090	*.11111*
One twelfth	.08333	*.1*

A glance at this table reveals that the 10-system has 50 per cent more (in this sample) of endlessly repeating numbers which cannot be accurately expressed without fractions. Moreover, in the case of both one fourth and one eighth, it requires an additional figure to express the same fraction.

Briefly, the 12-system is capable of expressing more fractions as whole numbers after a (duo)decimal point than the 10-system. It is particularly efficient with the smaller fractions which are most frequently used. Possibly most important of all, it is much more accurate for the same number of figures. For example, an ordinary decimal carried out to two places expresses a quantity to the nearest hundredth part; in the 12-system it expresses it to the much finer 144th part. Carried out to four places, the duodecimal has more than twice the probability of accuracy of the corresponding decimal. Therefore duodecimals are usually more accurate when carried out to the same number of places, and not infrequently an operation may be simplified by using a duodecimal one figure shorter than the necessary decimal. For instance, the square root of 2 can be expressed in simple twelfths (*1.5*) more accurately than in tenths or even any possible hundredths.

What has been said of decimals can be repeated with still more pertinence for percentages. We have already spoken of the inexcusable shortsightedness of creating a whole with 100 parts. In the 12-system the whole has the same visual and computational advantages of being expressed as *100* per cent, but it has 144 parts. This means that percentages are not only more accurate (by nearly a half), but, because 144 is a splendidly factorable number (which 100 is not), a percentage system based on 144 parts can express with complete accuracy a vastly greater number of much-used fractions than our present system.

Imagine the magical ease of working with a system where a third is an even *40* per cent and two thirds *80* per cent instead of being inexpressible in whole per cents, as at present; where a quarter is *30* per cent and three quarters *90* per cent; where even a sixteenth (a fraction frequently needed, especially in stock calculations) is precisely *9* per cent instead of the present miserable 6.25 percent!

I shall present no more figures, for fear of mathematical indigestion. But I trust our adventures along the first mile of a new trail have sufficiently pointed out the way. The interested reader may follow the trail, as I have done elsewhere, much further, and in any field of mathematics that specially interests him. Logarithms are much more accurate than in the 10-system. The search for prime numbers

(numbers not divisible by any whole number except 1) is narrowed. All perfect squares in the 12-system must end in either *0*, *1*, *4*, or *9*. Factoring is so simple in the 12-system that long division might become nearly obsolete. There are many other discoveries, some of which my own limited investigations have doubtless not hit upon.

V

And now where are we? Some of us have not yet clarified this new idea, with the result that the 12-system seems rather to confuse than to simplify mathematical calculations. Certain others (and many trained mathematicians may be in this group) have accepted and worked with the traditional system for so long that the mere idea of sailing to the Indies of the proper answer by another route is at once discarded and laughed at.

A few of us, I trust, have confirmed our earlier opinion that the system of 10 is a survival from barbaric finger-counting, unsuited to a civilized science of mathematics. We know that our present Arabic numerals were first introduced into Europe in 976, less than a thousand years ago; that number systems have changed repeatedly in history, and may change again.

Let us consider just for a moment the highly improbable chance of the present adoption of the duodecimal system. Colossal adjustments would be needed — but no greater than Turkey underwent quite recently when its new alphabet was adopted, no greater than the whole civilized world underwent in the tenth and eleventh centuries in changing from Roman to Arabic numerals. The present generation would have a most awkward time, chiefly in unlearning the old multiplication tables; but children for all future generations would find mathematics made vastly easier by the present sacrifice.

And would it be so tremendous a sacrifice? For the common man, getting used to the system of 12 would be very little more troublesome than getting used to measuring by meters and weighing by grams and kilograms. Most of Europe has already managed that. But if we in America are going to make any change of that formidable sort, instead of changing our measures to fit a bad number system would it not be more intelligent to adopt an ideal number system which would also fit our present rather efficient measures?

We already have many units based on twelve, with all of which startling mathematical short cuts would be instantly available with the adoption of the 12-system. The English have a 12-based monetary system. As for ours, its decimal-form advantages need not be changed. Let the Brain Trust play with this idea: under the number system of 12, every dime is immediately worth 12 cents, and every dollar twelve dimes — 144 cents. Would that not accomplish at one stroke the currency inflation we may have forced upon us by more dangerous methods — accomplish it without altering at all the relative possessions of people or the adequacy of the gold reserve behind existing currency?

The unhappy fact remains that man is ruled more by habit than by reason. We shall continue counting on our fingers in the logically silly system of 10 to the end of our days. Nevertheless this excursion beyond tradition has, I hope, vividly introduced an idea that may be new to some of us; has given us all a clearer conception of what quantity really is and the various means we have or might invent for setting it down; and may even help arouse the needed intelligent interest in what promises to be mathematics' next great step forward — the adoption of an efficient number system.

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