The Dozenal Society of America

Twelves and Tens*

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Many people will be wondering why a mathematician should argue against the proposed change-over to decimal currency and the metric system of weights and measures. Surely, they will think, everyone is agreed that the decimal system is the most practicable system in which to carry out both small calculations and large ones? I wish to point out certain reasons why this is not so, and to show that there exists a better system, which in Britain we have in part already, and which, unwisely in my opinion, we are preparing to throw away.

The Evolution of Arithmetic

Arithmetic, like any other scientific technique, is the outcome of a long evolution, which has by no means reached the final stage. In Europe at least, until about the time of the invention of printing in the fifteenth century, arithmetic was carried out with Roman numerals; and most cumbrous it must have been. But in India and in the Moslem world the numerals—which we are accustomed to call Arabic—but which we ought more properly to call Indo-Arabic, had been in use for several centuries before this time. Towards the end of the twelfth century a remarkable man, an Italian, Leonardo Pisano, while traveling round the coasts of the Levant on his father's commercial business, had occasion over and over again to see for himself how much superior this Indo-Arabic system was to the Roman. In 1202, on his return to Italy, he embodied it with other mathematical matter in a famous book, *Liber Abaci*, the 'book of the abacus'. However, his advocacy of Arabic arithmetic fell upon stony ground; so that long after his death the use of Arabic numerals was forbidden by law, and only gradually, and at first surreptitiously, did the new system make its way. We are able to trace its movement across Europe in those account books, or those calendars, or in dates in Arabic on rare tombstones. It appears last ofr all in England, where the earliest coin bearing Arabic figures is of date 1551; in Scotland the date is 1539.

However, once the usage of Arabic numerals took root, it spread rapidly. Already by about 1500, in Italy at least, it is fairly common. A century later John Napier of Merchiston calculated his famous logarithms using Arabic numerals, publishing these in 1614. In 1617, the year of his death, he went further by inventing the decimal point. Now, surely with this invention finality has been reached? Surely a system in which, multiplying or dividing by so many tens, or hundreds, or thousands and so on, one has only to move the decimal point to the right or left is a complete and coherent system?

Petrification by the French Revolutionaries

When in the history of science any such apparent culmination is reached it is well to adopt

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a Cartesian standpoint, to hold belief in suspense for a little and to go back to first principles. If in this spirit we re-examine the claims of the number ten to be a basis of numeration, we find them wanting, at any rate when weighed against the claims of the number twelve. I have never known of anyone who could sustain the case for ten. Unfortunately, however, the French, in their enthusiasm born of the French Revolution, in their determination to 'change all that', foreclosed the issue in 1799 for the world at large (with a few exclusions) by fastening upon it the metric system, a premature and regrettable thing to do, since it detrimentally affected the future of arithmetic by petrifying it at that point. Every other science in the world is in a state of perpetual development; why not common arithmetic also? During the same period the French introduced decimal currency; but here—and this is less well known—the Americans, who had had their dollars and cents since 1786, preceded the French by a few years.

One can hardly doubt that in the American case, as in the French, it was ardour of liberation, the determination to be independent, that led them to decimalize their coinage, a step which both countries must then have thought progressive, but which, when viewed in retrospect from the probable future of a few centuries hence, will appear not as a terminus but as a mere station on the way. I feel exactly the same regarding the recent adoption of decimal coinage by South Africa, itself, one may hardly doubt, a gesture of independence, but none the less retrogressive for that. And now, at long last, Britain is being urged to decimalize by many who do not do their own arithmetic, or whose knowledge of arithmetic and its history is fragmentary—who for example seem to think that the decimal system is the only one that possesses the 'shifting-point' property, that is, the property under which multiplication and division by the base leaves the digits unchanged. (This property is equally possessed by the duodecimal scale.) By such persons we are being urged to surrender our traditional adherence to the dozen and to tail in very late in the decimal queue.

The number ten was used among primitive races for counting—on the fingers of course. This was natural, and almost universal; and if counting had been the only operation of arithmetic this would have been well enough; it would have mattered little whether one counted by fives or sixes, by tens or dozens or scores. But counting, the most primitive of operations, hardly qualifies to be called arithmetic. The fundamental operations are adding, subtracting, multiplying, and dividing; and we have constantly to remember that arithmetic, in the business of life, is not done, as it were, in a vacuum; it is performed in reference to numberless practical applications, e.g. buying, selling, packaging. In packaging, for example, a dozen cups can be packed six by two, or three by four, or three by two by two; in short, in reference to every kind of exchange and measurement.

Subdivisions and Successive Halving

For instance, consider an inch, divided into twelfths; look at those subdivisions and you will see a half, a third, a quarter, and a sixth, as well as a twelfth. By contrast, begin making a systematic list of decimal fractions: first of all, a half, 0.5, that is well enough; but next, one third, the second of all fractions, 0.333333...? We are faced at once—to the perplexity of countless millions of school children at so early a stage—with a non-terminating decimal, as we likewise are with a sixth, five-sixths, a twelfth, and all those common, useful, and indeed indispensable fractions. In the decimalistic enthusiasm following the French Revolution attempts were actually made to have a year of ten months,

an hour of one hundred minutes, a clock-face of ten hours, a right angle of one hundred degrees. These attempts at innovation fell stillborn on the printed page; simply because the dozen and the gross have so many more divisors than ten or a hundred have.

Let us go on to successive halving; one soon comes, as any carpenter or any assistant in a tool-shop knows, to one sixty-fourth and its various multiples. In many tool-shops one will see, hanging up on the wall, a table of the decimal equivalents of these. A sixty-fourth is 0.015625; and, for example, twety-seven sixty-fourths is 0.421875. Does anyone like those six decimals? The duodecimal fractions are so much easier. Thus, a half is 0;6 (because the half of twelve is six), a third is 0;4, a quarter is 0;3, a sixth is 0;2, and a twelfth is 0;1. Could anything be simpler than that? Even a sixty-fourth is 0;023, two significant digits only, as against the decimal 0.015625. With whole numbers it is equally easy. Purely provisionally, let me adopt the following convention: when I wish to refer to so many dozens, so many units, or it may be so many gross, let me prefix an asterisk to the digits, but meanwhile call it 'star'. So then, five eightpences are 3s.4d. (3 shillings four pence, with twelve pence to the shilling); by which you will understand me when I say 'five times eight is "star" 34, or again, that five times 0;8 is 3;4'. This is almost the notation in which we see our accounts rendered every month.

What we see in any account rendered in shillings and pence, or in any statement by feet and inches, is just elementary duodecimal arithmetic; and all those persons who know, as countless persons do by habit, by serving customers, by working at carpenters' benches, and so on, the first few entries in any ready reckoner, such as that sevenpence plus eightpence is 1s.3d., or that seven times eightpence is 4s.8d., or the similar results in feet and inches—such persons are well on their way to duodecimal versatility, and with little trouble and some slight improvement in notation could extend their proficiency. This proficiency they have acquired by years of patient application; and it is being proposed to sacrifice the fruits of this to the decimal chimera.

What is wrong with the British system of currency? The real disadvantage is that it is hybrid; there are twelve pence to the shilling, but twenty shillings to the pound. Here we see the typical British compromise—though, in fairness, the French also had it before 1795. Twelve, because of its excellent divisibility, is so useful; on the other hand, we used to count by the score; and so we keep both, and end up by having a mixture of dozen and score. It is true that the outcome, 240 pence to the pound, is a number of outstanding divisibility. Even so, the system is hybrid; and it is this that confuses foreigners, who find it illogical.

A "Royal" of Twelve Shillings

I would rectify this—and the suggestion has been made before—by simply having a pound of a dozen shillings. I will call it a 'royal'—for that has the proper sound and connotation, and, besides, a stag of twelve points is a 'royal'; and so R.s.d. (Royals, shillings, pence) replaces £.s.d. (Pounds, shillings, pence). Then those simple tables of addition and multiplication in terms of dozens can serve as the basis of everything; millions of people know them already. If this simple modification of currency were adopted, requiring no more than this 'royal' of a dozen shillings, taking the place of our present pound and also ten-shilling note, with no necesssity whatever for new minting, we should have by daily usage, in shop and school, by adults and children alike, the first easy and natural steps in a phased process of education out of the palpably inferior decimal system into the provably superior duodecimal one. And this at a saving, I suppose of, £150,000,000, which is 250,000,000 royals.

Naturally there are later stages in the education, of which I could, and will, give the simplest blue-print.

In the newspapers now the talk is mostly about currency. But currency is only the thin end of the wedge: the ultimate intention—and it would be illogical in the promoters to stop short of it—is the total transfer to the metric system; and the implementation of this will be vastly more costly and disruptive than that of the change-over to new currency. Something should herefore be said concerning our British system of weights and measures: I am sure that no one could defend them. They have reputable antecedents, but they look wholly irrational. We should long ago have taken in hand that medley of inches, feet, yards, and so on up to miles, as likewise ounces, pounds, stones, and so on up to tons, and we should have subjected these to a radical uniformization. I will only remark that duodecimal societies in America and Britain, and in France M. Jean Essig, an Inspector-General of Finances, in a noteworthy book, have gone deeply into this whole matter and have independently evolved duodecimal systems which yet have much common ground. Meanwhile, I should myself be content to begin modestly with a duodecimal currency—that is, the existing one with only a slight modification. Familiarity with it would inevitably lead to versatility in duodecimal arithmetic generally; an educational momentum would be generated. Ideally, these are matters of universal education and should be considered supra nationally, and a suitable body for such a talk would be UNESCO, in assocation with all scientific societies; but I should be content to leave the names of units, of whatever kind, to the individual genius of each language. For example, one can have nothing but admiration for the felicitous names chosen by the French for the various units of their metric system.

Something more must be said about notation. If you look at the shillings and pence only in any account rendered, you will see almost a duodecimal notation; you will see 0.6 (or 0;6) for a half (just think of a shilling), 0.4 (or 0;4) for a third, 0.8 (or 0;8) for two thirds, 0.9 (or 0;9) for three-quarters. If we had a 'royal', comprising a dozenal shillings [sic], or a gross of pence, these equivalents of useful fractions would leap to the eye even more: for example, a sixteenth of a 'royal' is ninepence, and 0.09 (or 0;09) is duodecimal for a sixteenth of anything—and surely this, by the way, is preferable to 0.0625? However, the full availability of duodecimal arithmetic we need two extra symbols, a single symbol of ten and another for eleven. For many years I have myself wished, even in ordinary decimal arithmetic, that we had alternative symbols available for ten, one being the familiar two-digit symbol 10, the other thing being a single symbol such as the Romans had in their X. I could show all kinds of situation in which the use of such an alternative symbol would shorten ordinary calculations—and is it such a strenuous thing to ask, for, after all, the Hindus invented all ten of their digits.

Arithmetic of the Future

Many people will say that the arithmetic of the future will be done by calculating machine, whether mechanical, electrical, or electronic; and so rapidly that it will not matter what base we use for numeration. I do not believe this; it will be a long time, I fancy, before the stationer's shop at the corner uses an electrical machine. Cash registers will be used—and, by the way, the duodecimal one will prove superior—but by no means universally; there will still be ample scope for pen-and-paper or even mental arithmetic. I am thinking of the great mass of ordinary people, in every country, of whom I am one. I am not thinking of electronic computers; they will be used in large undertakings; they can look after them-

selves; and I will make this point about them in relation to decimal arithmetic: the scale of ten is alien to the electronic computer; what it uses is the binary scale, in which the only digits are zero and one. For compact recording and storing of binary numbers the most convenient scale is the scale of eight. Indeed the development of computers was retarded for some years by the misguided attempt to work them in the decimal scale. Thus the future of electronic computation resides in the binary and octonary scales; the decimal scale will have vanished from the scene.

I will conclude by bringing forward my strongest personal reason for asserting that the duodecimal system is superior. Comparative experiments are the criterion. I have carried out thousands myself, not in great and heavy calculations, though even there I am certain that duodecimal arithmetic would show to advantage, but in the multifarious types of calculation which are the staple of ordinary life. The duodecimal tables are easy to master, easier than the decimal ones; and in elementary teaching they would be so much more interesting, since young children would find more fascinating things to do with twelve rods or blocks than with ten. Anyone having these tables at command will do these calculations more than one-and-a-half times as fast in the duodecimal scale as in the decimal. This is my experience; I am certain that even more so it would be the experience of others. If therefore we consider the millions of man-hours that would thus be saved every day of our lives, we must, unless we are deliberately blind, see what a prodigious release of time and energy, of human potential, this would continuously ensure for the better and wider ends of mankind.

Twenty-five Years out of Date

Nothing inefficient, even relatively inefficient, will last indefinitely. Sooner or later some nation, strong in will-power, untrammeled by obsolescent tradition, will see this. There have been revolutions in arithmetic already; there will be others. It is truly extraordinary that arithmetic should have been hindered so long by a vestigial remnant from anatomy, by the irrelevant circumstance that, except for rare and favoured individuals, we are born with four fingers and a thumb on each hand. It is regrettable to have read that Britain is considering moving over to the decimal camp, which at the moment is about twenty-five years out of date; and since, if she does so, she will have taken more than 150 years to make up her mind, it is to be presumed that she thinks she will still be in that camp at least 150 years hence. Is it perhaps possible that in the remote future annals of mathematics some paragraph like this may appear:

> Britain, toward the end of what was then called the second millennium, for reasons owing nothing to arithmetic but everything to political and economic necessity, extirpated the admirable number twelve from her system of numeration and metric, being the last major nation to do so; and at such and such a later date, when all the other nations had adopted duodecimalism, Britain was again the last to change.

If, after Britain has spent £150,000,000 some other nation, the Russian, but even more probably the Chinese, outstrips her by becoming duodecimal, then that will have been the greatest sum ever paid for a copyright not acquired but actually surrendered and there, if you like, is economics seen through the Looking-Glass!
