Manual of The Dozen System

THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

ERRATA

MANUAL OF THE DOZEN SYSTEM

Page 8. In table *THE NUMBER SERIES* change first line, next-to-last column from ƐƐ to Ɛ.

Page 15. Next to bottom line change 18 to 16 and 16 to 14.

Page 21. Second line of Table A, change 6X8 to Ɛ6X8; in Table B change 60 to 5 in next-to-last column.

Page 22. Table D, third column, change .020893 to .002893.

Page 23. Below the line "K Scale of cubes" add (4)LL Scales, Log Log.

Page 28. Change value of $.2^c$ from $1/\sqrt{3}$ to $\sqrt{3}$.

On cover and next two pages, cross out 20 Carlton Place Staten Island 4, N.Y. and enter

Secretary, 11561 Candy Lane,
Garden Grove, CA 92640

$\pi = 3;184\ 809\ 493\ Ɛ91\ 866$ (Terry)

MANUAL OF THE DOZEN SYSTEM

A collation of material from many sources, presenting the number system, the arithmetic and the measures of the twelve-base, and the current practices for using that base most conveniently.

DUODECIMAL SOCIETY OF AMERICA, INC.
20 Carlton Place Staten Island 4, New York

ALL FIGURES IN ITALICS ARE DUODECIMAL

This Manual is gratefully dedicated

to the memory of

F. HOWARD SEELY

whose manuscript "Dipping Into Dozenals"

has been largely incorporated into its text.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

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PREFACE

The current activity in duodecimals dates from the publication of an article by F. Emerson Andrews in the Atlantic Monthly for October, 1934, titled "An Excursion in Numbers." It earned a surprising response, and led to the publication in 1935, of his book, "New Numbers," which is still the basic text of duodecimals. It led, also, to the publication by George S. Terry of his monumental work on the mathematical tables of duodecimals, "Duodecimal Arithmetic," and to the brochure condensed from its text, "The Dozen System."

These books are, today, the landmarks of duodecimals. But the idea of counting by dozens, coupled with the use of a duodecimal notation, antedates them by several centuries.

First, a word about the system of notation we use for our numbers might be helpful. It is called the "positional notation," because each successive "place" in our numbers is in the next power of the number base. On the ten-base, the number 365 represents 5 units (10^0), 6 tens (10^1), and 3 hundreds (10^2). On base-twelve, the same figures would represent 5 units, *6* dozens, and *3* gross, a much larger quantity.

This system of positional notation uses the numerals called Hindu-Arabic, from the region where they originated, and includes the zero. It was first brought to Europe by the Moors, and was introduced by Gerbert of Aurillac (Pope Sylvester II), about 1000 A.D.

About 600 years later, Simon Stevin was the first to realize that the series of successive lower powers ("places"), could easily accommodate the statement of systemic fractions on the same base if continued into the negative powers. He originated the "decimal point" to separate the fractionals from the whole number. (The terms "decimal" and "decimal point", are properly used only with the ten-base. The term "fractionals" can be used for systemic fractions on any base, and the term "unit-point" (or simply "the point"), is also proper for any base).

John Wallis of Oxford credited Simon Stevin with the origin of the concept of the number base (or "radix"), and it was Simon Stevin who first (1585) suggested that the twelve-base offered superiorities to the base of ten.

From that beginning, the record sparkles with famous and exciting names; like that of Thomas Hariot, who was surveyor and explorer to Sir Walter Raleigh in the establishment of the Virginia colony, and who discovered the binary base a hundred years before Leibniz.

More recently, in England, there have been Isaac Pitman, pioneer of shorthand, whose Phonetic Society advocated duodecimals in its magazine, Thomas Leech, author of "Dozens vs Tens", and Herbert Spencer, who provided funds in his will to oppose compulsory adoption of the Metric System. It is his eloquent appeal for duodecimals that is so often quoted, that "since a better system would facilitate both the thoughts and actions of men, and in so far diminish the frictions of life throughout the future, the task of establishing it should be undertaken." And early in this century, rear Admiral G. Elbrow, R.N., strongly advocated the use of the twelve-base in his pamphlet, "The New English System of Money, Weights and Measures, and of Arithmetic."

In America, in the same period, were: Henry Martin Parkhurst, also a pioneer in shorthand and stenography, whose duodecimal log tables have earned high praise; John W. Nystrom, author of "Elements of Mechanics", and of a duodecimal metric proposal, Robert Morris Pierce, and - more recently - Grover Cleveland Perry, who called his duodecimal proposal, "Mathamerica".

But, prior to the works of Mr. Andrews and Mr. Terry, few people in America were acquainted with duodecimals. However, the active response to their books led to the incorporation of the Duodecimal Society of America in 1944, as a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science. Its official organ, *The Duodecimal Bulletin*, was first published in 1945.

The establishment and wide adoption of the French decimal metric system has proved that the system of weights and measures is an integral part of the number system, and must necessarily conform to the same base. But it is due in large measure to the work of the Duodecimal Society, that the faults and limitations of the decimal system are now widely recognized. And it is true that the decimal system is limited in the range of its application. The ten-month year and the ten-hour day have been decisively rejected. The decimal system has made little progress in adoption for measures of the circle, in navigation, and in angular measure.

When, in 1790, at the instigation of Talleyrand, the National Assembly of France formed the Commission of the French Academy of Sciences which designed the metric system, consideration was given to the advantages offered by duodecimals, but the decimal base was selected as more acceptable to the public at that time.

Ten is awkward to subdivide even when the number notation is based on ten, and it is this relative unfactorability that limits its application. Twelve, then appears to remain as the ultimate choice as the base of the number system. It can efficiently and comfortably accommodate the entire field of measurement to obvious advantage.

The problem of effecting the change in the accepted radix is as formidable as ever. Yet we are more accustomed to change. The amazing scientific developments of today have led to an intense scrutiny of our educational system. Education of the public in the use of the twelve-base may receive added impetus because of this revaluation.

Because such education is the primary purpose of the Society, it is giving special attention to the development of the necessary tools for the application of dozenal standards in laboratory, workshop, and factory.

INTRODUCTION

What do the words "duodecimal" and "dozenal" mean? When a Roman wanted to speak of **twelve** he had to say "duodecim", which was too long a name for so popular a number; so it was gradually modified and cut down, eventually reaching us as dozen. Now, we who are interested in a system of counting based on twelve find that the word **duodecimal** is frequently unwieldly; so we resort to **dozenal** which means the same thing.

Why was the decimal system adopted in the first place? It was a biological accident. We happen to have ten fingers, hence primitive man could count on his fingers up to ten. Anything beyond that would have been an abstraction, of which he was not capable. There were exceptions. For example, the Mayas, and several other ancient peoples, by using their toes, counted up to twenty. It must have been inconvenient to pick up a foot every time they wanted to go above ten, and what did they do when they had to pick up both feet? In some languages, traces of the use of the twenty-base still exist; but the world was saddled with the ten-base and has never escaped from it, regardless of there being better bases.

What is wrong with ten as a base? For one thing it lacks factors. It is literally **un-satis-factory** as it has **not-enough-factors**. It can be divided evenly only by 2 and 5, not by the useful 3 and 4. Also, ten articles of unifrom size do not pack conveniently.

Is twelve any better? Very much so, for it can be divided by 2, 3, 4 and 6. It is the most factorable number for its size. Twelve articles can be packed in any one of several convenient ways. Take ten lumps of sugar and see how limited you are in putting them into a compact arrangement. Now take twelve and note the difference.

Look at the top layer of a case of canned goods. Twelve cans in 3 rows of 4 each, making a suitable shape for a convenient box. Now take away two of the cans and try to put the remaining ten into compact form. Two rows of 5 each is the only way, and that makes a very inconvenient case. It can't be done, and rather than have empty spaces in the box, you may as well put the two cans back.

Eggs and bottles and a multitude of other goods are packed and priced by the dozen because of this convenience, and because, when dozens or grosses are to be broken, so many useful fractions come out evenly in whole numbers. No matter how customary it is to count by tens, we will not forego the convenience of twelves, so we use dozens wherever possible.

Are the anticipated advantages worth the trouble? Perhaps not, if we were not going anywhere. But we advance at an accelerating rate. Science plays an increasing part in the day of the common man. Therefore man's education in science must be improved and eased. This requires that his numbers and his measures agree. It was thought that the French Decimal Metric System would answer this requirement. But we now know that its decimal base has inherent limits that make it unacceptable. Ultimately, the change to the numbers and measures of the dozen base becomes a necessity rather than a choice.

There can be little doubt that twelve is the best possible base. Within convenient size, supreme factorability is the determining characteristic. Louis P. d'Autremont demonstrated the superiority of the dozen, by a comparison of the ratios of base-numbers to the number of their divisors, in his paper, "The Rank of Numbers," (*Duodecimal Bulletin*, Vol. 6, No. 3, December, 1950.)

Is the use of the zero the same as in the decimal system? Yes. All systems today use the device of place-value, and the zero to represent the empty column. We now count up to 9, and instead of having a symbol for 10, we enter a zero in this column, move over one column, and enter 1. The Arabs introduced this innovation during the Middle Ages, but it took Europe 500 years to adopt it, and in the meanwhile people went on doing the best they could with the Roman numerals. Place value is used equally well with any base. In dozenals, we count up to *Ɛ* and instead of having a symbol for *10* we enter a zero in this column, move over one column, and enter *1*.

Is the twelve-base a new idea? Far from it. The decimal system has been criticized for a good many years, and the twelve-system has had many adherents. Simon Stevin first suggested it in 1585. When the metric system was introduced in France, Napoleon, objecting to it, said, "Twelve has always been preferred to ten as a divisor. I can understand the twelfth part of an inch, but not the thousandth part of a metre." Late in the seventeen hundreds the naturalist Buffon proposed universal adoption of the duodecimal system. John Quincy Adams wrote in 1821: "Decimal arithmetic is a contrivance of man for computing numbers, and is not a property of time, space, or matter. Nature has no partiality for the number ten." Pitman, the inventor of the well-known system of shorthand, discussed in 1857 the advantages of the twelve-base. Many great minds have been exponents of the system.

Have any other bases been considered? Yes, several; notably 2, 8 and 16 have had their proponents, the chief argument being that they accommodate halving well. But as they do not contain the 3 factor, they do not handle thirds any better than the decimal. They are not as factorable as twelve.

How does the dozen system operate? The place value is changed from ten to twelve. Numbers are expressed in successive powers of twelve, as dozens, grosses, great-grosses, etc., instead of tens, hundreds, thousands, etc. New symbols are used for ten and eleven. *10* then represents the dozen, *100* the gross, and *1000* the great gross. We move over one column whenever we reach twelve.

What symbols are used for ten and eleven? The symbol *X* (called **dek**) is used for ten, and *Ɛ* (called **el**) for eleven. *10* representing the dozen, is called **do**. The system of names used, represents a consensus derived by consent. Many different names and symbols have been proposed, but none has approached these in general acceptance.

How do we count in dozens? As follows:

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*X*	*Ɛ*	*10*
one	two	three	four	five	six	seven	eight	nine	dek	el	do

A more complete list of the names of numbers is on page 8.

Is it hard to change from one system to the other? All change involves disturbing our habitual practices. We have been brought up on the decimal system until it is almost a part of our nature. In learning the dozen system it will be necessary for a time to translate the dozenal quantities into decimal statements in order to make them intelligible. Just as one eventually attains the ability to think in a new language, so one can, in a much shorter time, begin to think in dozens. The addition and multiplication tables of the dozen are much easier than the tables of the decimal system. Presently you begin to think in dozens rather than translate. For you, it is easy to learn. For the world to change to thė new base is a different problem.

Remember that no compulsory change can be considered. When enough people are familiar with dozenal figuring by personal daily use to prefer it to decimal, the change will be well on its way. When most people prefer it, the change will have already been accomplished. It is a problem in public education and time.

How can we tell when figures are dozenals? To identify duodecimals in printed text, these figures are shown in italics. For manual script and typewriting, the semicolon can be used as the point, in place of the dot. As a further safeguard, duodecimals can be underlined when this seems necessary.

COUNTING IN DOZENS

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*X*	*Ɛ*	*10*
one	two	three	four	five	six	seven	eight	nine	dek	el	do

The series beyond the first dozen runs: do one, do two, do three, do four, do five, do six, do seven, do eight, do nine, do dek, do el, two do, etc. Of course, for clarity, we will sometimes say, one do five, etc., but the brief form above is standard.

And the series continues up through the nine dos, *99*, *9X*, *9Ɛ*, and then *X0*, and its series, and *Ɛ0*, and its series before we reach *100*, gro; then *101*, gro one, etc.

THE NUMBER SERIES

1	2	3	4	5	6	7	8	9	X	ƐƐ	10
11	12	13	14	15	16	17	18	19	1X	1Ɛ	20
21	22	23	24	25	26	27	28	29	2X	2Ɛ	30
31	32	33	34	35	36	37	38	39	3X	3Ɛ	40
41	42	43	44	45	46	47	48	49	4X	4Ɛ	50
51	52	53	54	55	56	57	58	59	5X	5Ɛ	60
61	62	63	64	65	66	67	68	69	6X	6Ɛ	70
71	72	73	74	75	76	77	78	79	7X	7Ɛ	80
81	82	83	84	85	86	87	88	89	8X	8Ɛ	90
91	92	93	94	95	96	97	98	99	9X	9Ɛ	X0
X1	X2	X3	X4	X5	X6	X7	X8	X9	XX	XƐ	Ɛ0
Ɛ1	Ɛ2	Ɛ3	Ɛ4	Ɛ5	Ɛ6	Ɛ7	Ɛ8	Ɛ9	ƐX	ƐƐ	100

THE NUMERICAL PROGRESSION

1	One		
10	Do	*.1*	Edo
100	Gro	*.01*	Egro
1 000	Mo	*.001*	Emo
10 000	Do-mo	*.000 1*	Edo-mo
100 000	Gro-mo	*.000 01*	Egro-mo
1 000 000	Bi-mo	*.000 001*	Ebi-mo
1 000 000 000	Tri-mo	and so on	

Names for fractionals are formed by including the prefix "e" in the name of the last place; for instance .2 is 2 edo, .006 is 6 emo, and .425 is 4 gro 2 do 5 emo, or, more simply, point four two five. Frequently, this simple practice of naming the figures in their sequence will be preferred, as: 408.X75 could be called "four oh eight point dek seven five."

It is common practice in scientific work to state a value in the form 5.88×10^{12}, using three significant figures as coefficients of some power of ten. The definite order of names in duodecimals permits a simpler practice. The above figure is a value for the length of the light-year in miles. In dozenals (using the 3-figure group of the mo), we would state this value as 7Ɛ0 M^3, or 7 gro Ɛ do tri-mo miles. The three dozenal figures are more significant, the name is exact, and the concept is clearer. As proof, just try to name the decimal value in millions, or billions, or trillions of miles.

BASIC OPERATIONS

ADDITION in dozenals is the same as in decimals except that we carry 1 when we reach a dozen, instead of when we reach ten. Rapid ease in addition comes with continued practice in grouping the pairs that make 10. They are:

1	2	3	4	5	6	7	8	9	X	Ɛ
Ɛ	X	9	8	7	6	5	4	3	2	1

To aid in avoiding error, use surscripts to indicate the carry.

Examples:

```
62        1
43       89      9 + 7 = 14,
--       57      1 + 8 + 5 = 12.
X5      ---
        124

  1 1
 9Ɛ046      6 + 2 = 8,    4 + 9 = 11,
 37892      1 + 8 = 9,    Ɛ + 7 = 16,
------      1 + 9 + 3 = 11.
116918

  1212
 30528   8 + 6 + Ɛ = 21;             put down 1 and carry 2;
 9X430   2 + 2 + 3 + 9 = 14;         put down 4 and carry 1;
 61Ɛ96   1 + 5 + 4 + Ɛ + 8 = 25;     put down 5 and carry 2;
 7580Ɛ   2 + X + 1 + 5 = 16;         put down 6 and carry 1;
------
226541   1 + 3 + 9 + 6 + 7 = 22.
```


The equivalents of the first two dozen numbers are:

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	2	3	4	5	6	7	8	9	X	Ɛ	10	11	12	13	14	15	16	17	18	19	1X	1Ɛ	20

Exercises:

```
(1)  203     (2)  914     (3)  2Ɛ76     (4)  13 981     (5)  672 346
     688          60X          4X31          X2 476               9ƐX
     ---          497          9867          60 74X            49 270
                  ---          36Ɛ0           1 Ɛ85           107 8X7
                               ----          44 288             6 413
                                             ------           398 0XƐ
                                                              -------

(6)            (7)              (8)              (9)        (X)     (Ɛ)
1 234 567      24 68X 024       857 326 4Ɛ0      ƐƐ ƐƐƐ     289     3 448
2 345 678      35 79Ɛ 125        39 280 X65      XX XXX     376     3 157
3 456 789      46 8X0 246         6 5Ɛ2 103      99 999     929       280
4 567 89X      57 9Ɛ1 357           476 487      88 888     Ɛ41     5 X76
5 678 9XƐ      68 X02 468            90 ƐX3      77 777     508     4 9Ɛ7
6 789 XƐ0      79 Ɛ13 579             Ɛ 439      66 666     X53     8 765
---------      ----------               276      55 555     467     Ɛ 620
                                         13      44 444     709     1 736
(10)  8 9ƐX                               Ɛ      33 333     X32     X 980
      8 9ƐX                     -----------      22 222     Ɛ45     2 039
      8 9ƐX                                      11 111     Ɛ89     1 776
      8 9ƐX                                      ------     760     1 492
      8 9ƐX                                                 ---     -----
      8 9ƐX
      -----
```

ADDITION AND SUBTRACTION TABLE

0	1	2	3	4	5	6	7	8	9	X	Ɛ
1	2	3	4	5	6	7	8	9	X	Ɛ	10
2	3	4	5	6	7	8	9	X	Ɛ	10	11
3	4	5	6	7	8	9	X	Ɛ	10	11	12
4	5	6	7	8	9	X	Ɛ	10	11	12	13
5	6	7	8	9	X	Ɛ	10	11	12	13	14
6	7	8	9	X	Ɛ	10	11	12	13	14	15
7	8	9	X	Ɛ	10	11	12	13	14	15	16
8	9	X	Ɛ	10	11	12	13	14	15	16	17
9	X	Ɛ	10	11	12	13	14	15	16	17	18
X	Ɛ	10	11	12	13	14	15	16	17	18	19
Ɛ	10	11	12	13	14	15	16	17	18	19	1X

SUBTRACTION is performed in dozenals in the same way as in decimals, except that we borrow a dozen instead of ten. The use of a surscript can give a clear indication of the borrow.

Examples:

67X	427	5136
329	31Ɛ	2368
	1	111
351	108	298X

Exercises:

(1)	697 384	(2)	X39 846	(3)	1 000 000 Ɛ ƐƐƐ	(4)	1 000 000 1
(5)	27 832 014 17 914 Ɛ03	(6)	X 643 X85 9 754 Ɛ97	(7)	149 380 000 49 479 38X		
(8)	2 222 1 333	(9)	3X 0X0 X0X 20 Ɛ0Ɛ 0Ɛ1	(X)	36 437 905 7 837 X16		
(Ɛ)	157 6Ɛ	(10)	5 896 45Ɛ 246 4 907 693 23X				

Problems in Addition and Subtraction:

(1) When John went out at recess he had 36 marbles. It was against the rules to play for keeps, but he did it anyway. When the bell rang he had only 19. How many did he lose?

(2) Bill was the boy who won John's marbles. He started out with 13. Besides the ones he took from John, he won 6 from Carl and 4 from Clarence. How many did he end up with?

(3) Carl had 23 left and Clarence 8. How many marbles were there altogether?

(4) Nellie gave a party, inviting 1Ɛ little girls. Her mother supplied 20 paper cups of ice cream. 5 of the girls had colds and stayed away. When refreshments were served, 6 said they couldn't eat ice cream because it made them fat. However, 9 who didn't care about that ate two cups each. How many cups were used? How many were left for Nellie to eat later?

(5) Jerry was buying an automobile. Of all the cars on the lot, his choice was between a five-year-old Ford that he could have just as it stood for $84, and a two-year-old Nash that had a guarantee, and was priced at $210. He took the Ford. On the way home his clutch failed and it cost him $18 to have it fixed. Within a week he needed new brake linings for $21, and two new tires for $8 each. On the road later his steering column fell out, costing him $X for a tow and $42 for repairs. Adjusting transmission and aligning wheels cost $26 more, and he had to get a new battery for $10. Did the Ford cost more or less than the Nash would have cost, and how much?

(6) The sun is not at the center of the earth's orbit. When the earth is nearest the sun its distance is approximately 26 988 8X8 miles. When farthest from the sun the distance is around 27 592 194 miles. What is the difference between these distances?

(7) Farmer Brown owned a quarter-section of land (114 acres). He sold 34 acres of orchard for $1560 and bought 68 acres of pasture for $920. He then bought a tractor for $358. (a) How much land did he own after the transaction? (b) How much money did he have left?

(8) John and Harry started to paddle down the river 26 miles to visit their grandparents. The canoe tipped over, and they hung on and floated 11 miles before they were rescued. Then they found that they were 7 miles from their destination. How far had they paddled before the canoe capsized?

(9) Mr. McIntosh had a well 20 feet deep, which supplied his house with water. It went dry, and he dug it 18 feet deeper. That answered for a while, but the water table kept getting lower, and he had it bored 67 feet more, which answered for several years. However, it dried up again and he had a pipe driven 84 additional feet. Now he gets plenty of water. How deep is his well?

(X) A certain regiment has 724 enlisted men and 42 officers. 87 of the men are in hospital and 11 officers are on leave. What is the effective strength of the regiment?

(Ɛ) A rectangular city block is *280* feet long on two sides, and *200* feet long on the other two. (a) How far is it around the block? An alley, *10* feet wide, cuts through the center of the block, parallel with the longer sides. (b) How far is it around if you cut through the alley?

(*10*) Take your age; add *10*; add *14*; subtract *26*; double it and subtract your age less *4*. What have you left?

TWELVE-TIMES TABLE

1 x 12 = 12	which is *1* do		7 x 12 = 84	which is *7* do
2 x 12 = 24	*2* do		8 x 12 = 96	*8* do
3 x 12 = 36	*3* do		9 x 12 = 108	*9* do
4 x 12 = 48	*4* do		10 x 12 = 120	*X* do
5 x 12 = 60	*5* do		11 x 12 = 132	*Ɛ* do
6 x 12 = 72	*6* do		12 x 12 = 144	*1* gro

MULTIPLICATION. It has long been the custom to teach the multiplication tables up to twelve times twelve. This is because the dozen is such an important quantity that familiarity with it and its multiples is considered a necessity of everyday life. Hence the student learns 144 combinations. The dozenal tables up to *10* x *10* also contain *100* combinations. However, the dozenal tables are far the easier to learn. Because of the larger number of factors, more tables repeat terminal figures in a regular pattern. This makes them as easy to learn as the 5 table in the decimal system.

DOZENAL MULTIPLICATION TABLE

1	2	3	4	5	6	7	8	9	X	Ɛ	10
2	4	6	8	X	10	12	14	16	18	1X	20
3	6	9	10	13	16	19	20	23	26	29	30
4	8	10	14	18	20	24	28	30	34	38	40
5	X	13	18	21	26	2Ɛ	34	39	42	47	50
6	10	16	20	26	30	36	40	46	50	56	60
7	12	19	24	2Ɛ	36	41	48	53	5X	65	70
8	14	20	28	34	40	48	54	60	68	74	80
9	16	23	30	39	46	53	60	69	76	83	90
X	18	26	34	42	50	5X	68	76	84	92	X0
Ɛ	1X	29	38	47	56	65	74	83	92	X1	Ɛ0
10	20	30	40	50	60	70	80	90	X0	Ɛ0	100

Forty of the products end in zero, compared with thirty-three for the decimal table. Also, more than fifty other products have end figures that repeat one of their multipliers; as *8* x *X* = *68*. These also memorize easily.

After all, you will not have to learn the dozenal multiplication table unless you wish to. Since you already know the twelve-times table, you can mentally convert the products into dozens and set them down. For example, 7 x 9 is 63, "**which is**" 5 dozen and *3*; so set down *53*.. Using this "**which is**" step, you will be able to multiply and divide dozenal numbers without referring to the table. With use, the dozenal products become familiar and the "**which is**" step is omitted.

In writing down your products, as well as in summing them up, the use of surscripts will reduce error.

Examples:

```
   57823            470Ɛ98              48769X1
       9               3Ɛ7                 Ɛ9X8
   ─────           ───────            ─────────
  5 6 1 2         4    5 4           5 5 4 6 6
  429183           2816X78           31906688
                  6    8 7          7 6 5 8 8
                  425X9X4           3Ɛ23824X
                      2 2           6  5 7 7
                  1192Ɛ50           36581469
                 ─────────          7 6 6 9 9
                  1 1 1 1 1         43XƐ302Ɛ
                  1624Ɛ98Ɛ8        ──────────
                                    1 2 1 1 1 2 1 1
                                    4797324 2868
```

Exercises:

(*1*) 934 x 6 (*2*) 14X0 x 9 (*3*) XƐ978 x X

(*4*) 870X1 x 5 (*5*) 543210 x 54 (*6*) ƐX9876 x ƐX9 (*7*) 6922 x 6922

(*8*) X98 x 89X (*9*) 27X6894 x 3Ɛ105 (*X*) 989898 x ƐXƐXƐX (*Ɛ*) 462598 x 97Ɛ46

(*10*) 2468X x 13579

Problems:

(*1*) A laborer working at the rate of $*1* an hour for a day of *8* hours gets how much? How much does he earn in a week of 5½ days? In *4* weeks? In *43* weeks?

(2) He is paying $13 a month on his radio, and will pay it out in 9 months. How much will it cost him?

(3) A lunar month is 25½ days. How many days in 10 lunar months?

(4) How many days in 44 weeks?

(5) A room with 8 corners had a cat in each corner; 7 cats before each cat, and a cat on every cat's tail. How many cats altogether?

(6) I have 2 parents, 4 grandparents, 8 great-grandparents, and so on. How many progenitors have I altogether to and including the 6th generation?

(7) One of my relatives had 4 children. Each of them had 3, each of them had 2, each of them had 1, and they didn't have any. How many children altogether?

(8) It is 340 feet around the promenade deck of a ship. A passenger taking his constitutional walked around it Ɛ times. How many feet did he walk?

(9) A streamliner travels X hours at a constant speed of 55 miles an hour. How far does it go?

(X) Take the average number of kernels on an ear of corn as 340. There are 36Ɛ4 ears in a bin. How many kernels.

(Ɛ) An average page in the telephone book contains 312 names, and the book has 638 pages. How many names?

(10) A building (quite a tall one) has 6Ɛ0 stairs to the top floor, with 7 inch risers. How many feet is it? (In dozenals, inches are changed to feet by pointing off one place.)

DIVISION. The same methods are used in dozenals as are used in decimals. Short Division is the form best used when the products and remainders are easily handled. Remember that the carry is in dozens, and use the "**which is**" procedure. For example, 87 is 8 dozen 7, "**which is**" 103.

Examples:

```
2) 458      Ɛ) 8752      6) 427
   22X         94X          85.2
```


Exercises: (1) 8) 4768 (2) 3) 532X6 (3) X) 4229Ɛ6

(4) Ɛ) X9ƐƐX4 (5) 7) 67X04 (6) 5) 40790X5

Problems:

(7) There were 1X guests at Mrs. Peters' afternoon party. She had 4 lemon custard pies with meringue on top. Into how many pieces would she have to divide each pie so that each person present could have a slice and there would be a slice for Mr. Peters when he got home?

(8) How many weeks in a year of 265 days?

(9) Call the distance from New York to San Francisco 1900 miles. Bill drives it in 9 days. What is his daily mileage?

(X) The Streamliner makes it in 3 days. What is its daily run?

(Ɛ) In Ɛ kegs there are 37Ɛ10 ten-penny nails. How many to a keg?

(10) Farmer Johansen put a hog-tight fence around a quarter section, with posts X feet apart. (a) How many posts did he use? (Note: A quarter-section is one-half U.S. mile square. This is an instance where the decimal notation would be simplest. But, if Mr. Johansen used dozenals, he would set his posts 10 feet apart. (b) How many posts would he save?

Long Division is an extension of the same process into a series of steps to handle larger divisors. At any step, we estimate by inspection what multiple of the divisor may apply. If our estimate proves wrong, we correct it and proceed.

Examples:

```
      21            2Ɛ1              184.4
14) 294      52Ɛ) 133Ɛ57      469) 78X93
    28             X5X             469
     14             4X15           3219
     14             4981           3060
                     547            1793
                     52Ɛ            1630
                      18             1630
                                     1630
```

Exercises:

(1) 84) X46£70 (2) 729) 207986139

(3) £6028) 345564680 (4) 65074) 300303514

(5) 255) 24590 (6) 199X£) 20000000

Problems:

(7) At a certain latitude the circumference of the earth is *9314* miles. How long will it take an airplane, traveling at low altitude and a speed of *294* miles per hour, to make the circuit?

(*8*) The speed of light is *8£ 978* miles per second. The sun is distant approximately *27 128 268* miles. How long does it take the sun's light to reach the earth? (Count *50* seconds to the minute, and give the answer in minutes and seconds.)

(*9*) From the same field as in Problem *7*, two men started flying in opposite directions. A flew west at *294* miles per hour, B flew east at *84* m.p.h. (a) How many miles did each fly before they met, and (b) how many hours did it take?

(*X*) Henry D. Rogers died at the age of *64*, leaving his estate, valued at $*15 440*, to his four sons, the provision being that Henry was to get twice as much as William, William twice as much as Edward, and Edward twice as much as John. How much did each get?

Division is slower and more awkward than multiplication, and is to be avoided when possible. Sometimes, in decimals, we multiply by 2 and point off an extra place, to escape dividing by 5. Similarly in dozenals, we can use this substitution of method between 2 and *6*, and between *3* and *4*, pointing off an extra place. We can use the same method between *8* and *18*, and between *9* and *16*, by pointing off two extra places.

CLEAR FACTORS AND DIVISIBILITY

We are familiar with the recognition of odd and even numbers in the decimal numeration, and with the presence of the 5 factor in numbers ending in 0 or 5. Dozenal numbers are more liberal in offering information about their factors and divisibility by their end figures.

The odd and even numbers are equally evident in dozenals, and further information is much more abundant.

(a) Every number ending in *0* is divisible by *2*, *3*, *4*, and *6*.

(b) Numbers which end in *2*, *3*, *4*, or *6*, are divisible respectively by those factors.

(c) All multiples of *3* and of *9* end in *3*, *6*, *9*, or *0*.

(d) All multiples of *4* and of *8* end in *4*, *8*, or *0*.

(e) All multiples of *6* end in *6* or *0*.

Powers of Numbers are more readily recognized in dozenals than in decimals.

(a) All squares end in *0*, *1*, *4* or *9*, and so do all of the even powers.

(b) All powers of *0*, *1*, *4*, and *9* end respectively in the same figure; i.e., 9^n ends in *9*.

(c) All powers of *6* end in *0*, and all powers of *X* in *4*.

(d) All odd powers of *3*, *5*, *7*, *8*, and *£* end respectively in the same figure; i.e., $7^{(2n-1)}$ ends in *7*.

Extensive material on factors, powers, roots, and primes may be found in George S. Terry's compendious "Duodecimal Arithmetic", and in his "The Dozen System".

It should be noted that no power of any number ends in *2*, *6*, or *X*.

Powers and roots of numbers in any desired degree are easily determined by the use of logarithms.

ROOTS and POWERS of the FIRST DOZEN

Cube Roots	Square Roots	Numbers	Squares	Cubes
1.31 519	1.4Ɛ 792	2	4	8
1.53 826	1.89 4ƐX	3	9	23
1.70 704	2.	4	14	54
1.86 2X1	2.29 ƐƐ1	5	21	X5
1.99 7ƐX	2.54 887	6	30	160
1.XƐ 566	2.78 ƐX4	7	41	247
2.	2.9 363	8	54	368
2.0Ɛ 647	3.	9	69	509
2.1X 2X4	3.1Ɛ 450	X	84	6Ɛ4
2.28 305	3.39 716	Ɛ	X1	92Ɛ
2.35 817	3.56 9Ɛ7	10	100	1000

The many excellent abbreviations are underlined; i.e., for the square root of 2, use 1.5 .

Prime Numbers are dozenally confined within four possible endings, 1, 5, 7, and Ɛ, (with the exclusion of the primes 2, and 3). They are of the form of (10n ± 1) or (10n ± 5). These four possible cases are variants of the formulas (4n ± 1) and (6n ± 1). But each of the four dozenal classifications has its special characteristics. As an illustration, consider the prime number 7, which is of the form (10n − 5). If we divide 1 by 7, we get as a reciprocal, a circulating duodecimal of 6 figures. The number of places in such reciprocals is always one less than the prime, (P − 1), or a submultiple of this. But all of the primes whose reciprocals extend to the full period,,(P − 1), occur in the groups (10n ± 5). It is to be noted that the 10n formulas refine the 4n formula by eliminating numbers ending in 3 or 9 as composites, although they number one-third of the possible 4n cases. This is a good illustration of the special refinements which dozenals offer in number analysis.

PRIMES BELOW 600

giving the Periods and Submultiples of the circulating duodecimals of their reciprocals. (Primes 2, and 3, excluded.)

P.	Per.	S.M.	P.	Per.	S.M.	P.	Per.	S.M.	P.	Per.	S.M.
11	2	6	5	4		7	6		Ɛ	1	X
31	9	4	15	14		17	6	3	1Ɛ	Ɛ	2
51	13	4	25	4	7	27	26		3Ɛ	1Ɛ	2
61	30	2	35	34		37	36		4Ɛ	25	2
81	14	6	45	44		57	56		5Ɛ	2Ɛ	2
91	46	2	75	8	Ɛ	67	22	3	6Ɛ	35	2
			85	84		87	86		8Ɛ	45	2
			95	94		X7	X6		XƐ	55	2
			Ɛ5	Ɛ4		Ɛ7	Ɛ6				
111	3	44	105	104		107	106		11Ɛ	6Ɛ	2
131	76	2	125	124		117	116		12Ɛ	75	2
141	20	8	145	144		147	56	3	13Ɛ	7Ɛ	2
171	96	2	175	8	25	157	12	13	16Ɛ	95	2
181	0	2	195	194		167	166		17Ɛ	9Ɛ	2
1Ɛ1	Ɛ6	2	1X5	1X4		1X7	46	5	18Ɛ	X5	2
			1Ɛ5	1Ɛ4		1Ɛ7	1Ɛ6		19Ɛ	XƐ	2
221	66	4	205	204		217	86	3	21Ɛ	10Ɛ	2
241	120	2	225	224		237	92	3	24Ɛ	125	2
251	73	4	255	254		267	266		25Ɛ	12Ɛ	2
271	27	10	285	284		277	276		27Ɛ	13Ɛ	2
291	83	4	295	294					2XƐ	155	2
2X1	150	2							2ƐƐ	37	X
2Ɛ1	26	12									
301	90	4	315	314		307	102	3	30Ɛ	165	2
321	170	2	325	324		327	10X	3	32Ɛ	175	2
391	1X6	2	365	364		347	46	9	33Ɛ	17Ɛ	2
			375	34	11	357	11X	3	34Ɛ	2Ɛ	12
			3X5	3X4		377	376		35Ɛ	18Ɛ	2
			3Ɛ5	3Ɛ4		397	396		3XƐ	1Ɛ5	2
						3Ɛ7	3Ɛ6				
401	100	4	415	414		427	426		40Ɛ	205	2
421	63	8	435	434		437	436		41Ɛ	20Ɛ	2
431	109	4	455	454		447	446		45Ɛ	22Ɛ	2
471	13	38	465	464		457	15X	3	46Ɛ	7	7X
481	24	20	485	44	11	497	496		48Ɛ	245	2
4Ɛ1	9X	6	4X5	4X4					4ƐƐ	25Ɛ	2
511	266	2	535	534		507	182	3	51Ɛ	45	12
531	276	2	545	544		517	516		58Ɛ	2X5	2
541	54	10	565	564		527	526		59Ɛ	2XƐ	2
591	3X	16	575	574		557	556		5ƐƐ	2ƐƐ	2
5Ɛ1	159	4	585	584		577	116	5			
			5Ɛ5	5Ɛ4		587	1XX	3			
						5Ɛ7	66	Ɛ			

from The Dozen System by George S. Terry

Common or **Vulgar Fractions**, such as 7/16, occur regardless of the number base. On the twelve base, they are somewhat more easily reduced to their lowest terms because common factors are more evident. But it has become customary now to express them as fractionals, instead. Thus we would represent 7/16 by the decimal .4375; and its dozenal equivalent, *7/14*, equals the duodecimal *.53*. In fractionals, the advantages of the dozen show clearly.

Fractions	Fractionals Decimal	Fractionals Duodecimal
one-half	.5	*.6*
third	.333333	*.4*
fourth	.25	*.3*
fifth	.2	*.249724*
sixth	.166666	*.2*
seventh	.142857	*.186X35*
eighth	.125	*.16*
ninth	.111111	*.14*
tenth	.1	*.124972*
eleventh	.090909	*.111111*
twelfth	.083333	*.1*

Half again as many endless repeaters develop for the ten base in this example as for the twelve. It is also true that each place in a duodecimal is a more refined statement than in decimals because its denominator is larger, as follows:

Fractional	Denominators Decimal	Denominators Duodecimal
one-place	10	12
two-place	100	144
three-place	1000	1728
four-place	10000	20736
five-place	100000	248832
six-place	1000000	2985984

Percentage and **Egrossage**. Since these are only other terms for two-place fractionals, the same dozenal superiorities we have cited for them apply. When we say 3% this is equivalent to saying .03, and the % symbol refers to the hundred. In dozenals, our symbol, e/g, refers to the gross, and we find things a little simpler because more parts come out even.

As a basis for the convenient statement of ratios, percentage was a noticeably poor choice. The gross is far more flexible, with relatively more factors, and nearly twice as many divisors. This means that nearly twice as many ratios in common use come out exactly in whole numbers.

Suppose your city wishes to borrow $100,000 for one month to meet the municipal pay roll, at an annual interest rate of 3 1/8% - the monthly charge would be 1/12 of .03125 times $100,000 or $260.42.

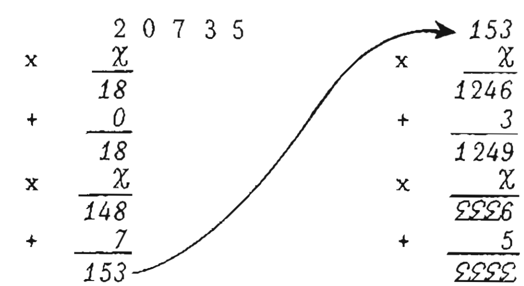
Putting this into dozenal terms, this would be a loan of *$49,X54* at an annual interest rate of *4'.6* e/g, *(.046)*, a monthly rate of *.0046*, which would make the monthly charge *$198.60*. This demonstrates the advantage of having the monthly rates be *.1* (one twelfth) of the yearly rates, since we can change from the one to the other by merely moving the point, and even though we are working with decimal dollars, each with 100 cents, the advantage still exists.

If we were commercially using duodecimal numbers and dollars, the problem would be further simplified. Keeping the figures within the same general range, we would negotiate a loan of *Ɖ50,000* at an annual interest rate of *.046*, a monthly rate of *.0046* times *Ɖ50,000*, or *Ɖ1X6.00*.

CONVERSION

In general, any whole number may be converted from one base to any other by a procedure originally suggested by Robert Morris Pierce (1898), and recently recommended by Nelson B. Gray: Starting at the left, multiply the first figure by the original base, expressing the result in the notation of the new base. Then add the next digit and multiply the sum by the original base as before. Continue this process until the last digit has been added. As an example:

Convert 20735 from base-ten to base-twelve.



This process can be somewhat lengthy, and many will prefer to use the conversion tables herewith. However, the computation of conversions is quite simple, usually requiring separate operations for whole numbers and fractionals.

Conversion of whole numbers. (A) Decimal to dozenal: Divide successively by twelve. The successive remainders, stated in reverse order, are the dozenal number.

Example: Change 13579 to base-twelve.

12)13579		Ans. *7X37*	Proof: *7*	=	7
12)1131	*7*		*30*	=	36
12) 94	*3*		*X00*	=	1440
7	*X*		*7000*	=	12096
			7X37	=	13579

(B) Dozenal to decimal: Divide successively by dek. The successive remainders, in reverse order, are the decimal number.

Example: Change *2XƐ7X5* to base-ten.

X)2XƐ7X5		Ans. 725165	Proof: 5	*5*
X)35Ɛ70	5		60	*50*
X)4243	6		100	*84*
X)505	1		5000	*2X88*
X)60	5		20000	*Ɛ6X8*
7	2		700000	*299114*
			725165	*2XƐ7X5*

Conversion of fractionals. (C) Decimals to duodecimals: Multiply successively by twelve. Successive final carries are the duodecimal.

Example: Convert .02468 to duodecimal.

	.02468	Ans. *.03679*	Proof: *.03*	=	.020833
	12		*.006*	=	.003472
0	29616		*.0007*	=	.000338
	12		*.00009*	=	.000036
3	55392		*.03679*	=	.024679
	12				
6	64704				
	12				
7	76448				
	12				
9	17376				

Conversion of fractionals. (D) Duodecimals to decimals: Multiply successively by dek. The successive final carries are the decimal.

Example: Convert *.3579Ɛ* to decimal.

	.3579Ɛ	Ans. .28925	Proof: .2	=	*.249725*
	X		.08	=	*.0Ɛ62X7*
2	*X8632*		.009	=	*.013676*
	X		.0002	=	*.000419*
8	*Ɛ1278*		.00005	=	*.000105*
	X		.28925	=	*.3579X8*
9	*30248*				
	X				
2	*61ƐX8*				
	X				
5	*17XX8*				

F. Howard Seely developed the following method of converting mixed numbers, to facilitate conversions by machines. Treat the mixed number as a whole number (disregarding the point). Convert as in (A) or (B), and then divide by dek (if decimal to dozenal, or by twelve if dozenal to decimal), as many times as there are fractional places.

Example: Convert 34.567 to base-twelve.

12)34567		*X)18007.000*	Proof: *2X*	=	34
12)2880	*7*	*X)2000.84X*			
12)240	*0*	*X)249.806*			
12)20	*0*	Ans. *2X.698*	*.6*	=	.5
1	*8*		*.09*	=	.0625
			.008	=	.00463
			2X.698	=	34.56713

TABLE A. CONVERSION OF WHOLE NUMBERS, Decimal to Dozenal

	θ 000 000 000	θ00 000 000	θ0 000 000	θ 000 000	θ00 000	θ0 000	θ 000	θ00	θ0	θ	
1	23X X93 854	29 5X6 454	3 423 054	402 854	49 X54	5 954	6Ɛ4	84	X	1	1
2	479 967 4X8	56 Ɛ90 8X8	6 846 0X8	805 4X8	97 8X8	6X8	1 1X8	148	18	2	2
3	6Ɛ8 83Ɛ 140	84 577 140	X 069 140	1 008 140	125 740	15 440	1 8X0	210	26	3	3
4	937 712 994	Ɛ1 Ɛ61 594	11 490 194	1 40X 994	173 594	1Ɛ 194	2 394	294	34	4	4
5	Ɛ76 5X6 628	11Ɛ 547 X28	14 8Ɛ3 228	1 811 628	201 428	24 Ɛ28	2 X88	358	42	5	5
6	1 1Ɛ5 47X 280	148 Ɛ32 280	18 116 280	2 014 280	24Ɛ 280	2X 880	3 580	420	50	6	6
7	1 434 351 Ɛ14	176 518 714	1Ɛ 539 314	2 416 Ɛ14	299 114	34 614	4 074	4X4	5X	7	7
8	1 673 225 768	1X3 Ɛ02 Ɛ68	22 960 368	2 819 768	326 Ɛ68	3X 368	4 768	568	68	8	8
9	1 8Ɛ2 0Ɛ9 400	211 4X9 400	26 183 400	3 020 400	374 X00	44 100	5 260	630	76	9	9

1 Ɛ30 Ɛ91 054 = 10 000 000 000

TABLE B. CONVERSION OF WHOLE NUMBERS, Dozenal to Decimal

	θ00 000 000	θ0 000 000	θ 000 000	θ00 000	θ0 000	θ 000	θ00	θ0	θ	
1	429 981 696	35 831 808	2 985 984	248 832	20 736	1 728	144	12	1	1
2	859 963 392	71 663 616	5 971 968	497 664	41 472	3 456	288	24	2	2
3	1 289 945 088	107 495 424	8 957 952	746 496	62 208	5 184	432	36	3	3
4	1 719 926 784	143 327 232	11 943 936	995 328	82 944	6 912	576	48	4	4
5	2 149 908 480	179 159 040	14 929 920	1 244 160	103 680	8 640	720	60	60	5
6	2 579 890 176	214 990 848	17 915 904	1 492 992	124 416	10 368	864	72	6	6
7	3 009 871 872	250 822 656	20 901 888	1 741 824	145 152	12 096	1 008	84	7	7
8	3 439 853 568	286 654 464	23 887 872	1 990 656	165 888	13 824	1 152	96	8	8
9	3 869 835 264	322 486 272	26 873 856	2 239 488	186 624	15 552	1 296	108	9	9
X	4 299 816 960	358 318 080	29 839 840	2 488 320	207 360	17 280	1 440	120	10	X
Ɛ	4 729 798 656	394 149 888	32 845 824	2 737 152	228 096	19 008	1 584	132	11	Ɛ

5 159 780 352 = 1 000 000 000

TABLE C. CONVERSION OF FRACTIONALS, Decimal to Duodecimal

	.θ	.0θ	.00θ	.000 θ	.000 0θ	.000 00θ	
1	.124 972	.015 344	.001 88X	.000 20Ɛ	.000 026	.000 003	1
2	.249 725	.02X 688	.003 558	.000 419	.000 050	.000 006	2
3	.372 497	.043 X10	.005 226	.000 628	.000 076	.000 009	3
4	.497 24X	.059 153	.006 XƐ4	.000 836	.000 09Ɛ	.000 010	4
5	.600 000	.072 497	.008 782	.000 X45	.000 105	.000 013	5
6	.724 972	.087 81Ɛ	.00X 450	.001 054	.000 12Ɛ	.000 016	6
7	.849 725	.0X0 Ɛ63	.010 11X	.001 262	.000 155	.000 019	7
8	.972 497	.0Ɛ6 2X7	.011 9X8	.001 471	.000 17Ɛ	.000 020	8
9	.X97 24X	.10Ɛ 62Ɛ	.013 676	.001 67Ɛ	.000 1X5	.000 023	9

TABLE D. CONVERSION OF FRACTIONALS, Duodecimal to Decimal

	.θ	.0θ	.00θ	.000 θ	.000 0θ	.000 00θ	
1	.083 333	.006 944	.000 579	.000 048	.000 004	.000 000	1
2	.166 667	.013 889	.001 157	.000 096	.000 008	.000 001	2
3	.250 000	.020 833	.001 736	.000 145	.000 012	.000 001	3
4	.333 333	.027 778	.002 315	.000 193	.000 016	.000 001	4
5	.416 667	.034 722	.022 893	.000 241	.000 020	.000 002	5
6	.500 000	.041 667	.003 472	.000 289	.000 024	.000 002	6
7	.583 333	.048 611	.004 051	.000 338	.000 028	.000 002	7
8	.666 667	.055 555	.004 630	.000 386	.000 032	.000 003	8
9	.750.000	.062 500	.005 208	.000 434	.000 036	.000 003	9
X	.833 333	.069 444	.005 787	.000 482	.000 040	.000 003	X
Ɛ	.916 667	.076 389	.006 366	.000 530	.000 044	.000 004	Ɛ

DOZENAL LOGARITHMS (BASE-TWELVE)

Log values up to *1000* are given to five places, in the following table. They embody more than twice the accuracy of similar five-place decimal tables. Direct proportion of their differences will afford four-place accuracy for interpolation.

Extended values of the dozenal logarithms may be found in **Duodecimal Arithmetic**, by George S. Terry. Extended values may also be derived from the natural logarithms of primes, which are given to *43* places in the Duodecimal Bulletin, Vol. *7*, No. *1*, 1951.

Dozenal logarithms may be developed from decimal logarithms by multiplying by .9266284 and converting the result to the dozenal notation. The reverse operation involves dividing the dozenal log by *.Ɛ1526X* and converting the result to the decimal notation.

THE DOZENAL SLIDE RULE

The circular Dozenal Slide Rule may be purchased through the Society. It has many unique advantages, and its scales include:

Face:
- C Scale for multiplication, division and ratios.
- CI Scale for reciprocals.
- L Scale for logarithm values.
- A Scale of squares.
- K Scale of cubes.

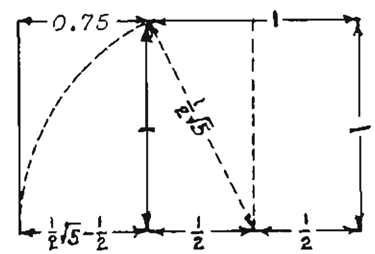
Back:
- Degree Quadrant, distended over 360° periphery.
- Dozenal Quadrant to correspond with 90 decimal degrees.
- Sine-Cosine and Tangent-Cotangent Scales supplying trigonometric values of the functions of angles.
- Log-log Scale, of four turns, affords decimal-dozenal conversions, and log-log values to 10^9.

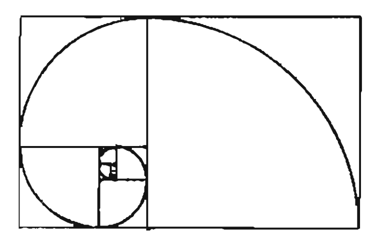
	0	1	2	3	4	5	6	7	8	9	X	Ɛ	
10	00000	00499	00971	01241	01708	01Ɛ8X	02448	02901	03172	0361Ɛ	03X83	04323	10
1	477Ɛ	5013	5463	58XƐ	6133	6572	69XX	7223	7653	7X80	82X5	8706	1
2	8Ɛ24	933Ɛ	9751	9Ɛ61	X369	X771	XƐ72	Ɛ370	Ɛ767	ƐƐ5X	1034X	10737	2
3	10Ɛ21	11304	116X4	11X80	12256	12629	129Ɛ8	13185	1354Ɛ	13913	4093	4451	3
4	4808	4Ɛ80	5331	56X0	5X48	61Ɛ2	6555	68Ɛ6	7054	73XƐ	7744	7X96	4
5	8227	8574	88ƐƐ	9044	9387	9707	9X45	X181	X4Ɛ6	X82X	XƐ5Ɛ	Ɛ28X	5
6	Ɛ5Ɛ6	Ɛ921	20045	20367	20688	209X6	21102	21418	21730	21X42	22152	22460	6
7	22768	22X73	3177	3479	377X	3X79	4175	4470	4766	4X59	514Ɛ	543Ɛ	7
8	5729	5X15	6100	63X5	6688	6969	7049	7328	7604	789Ɛ	7Ɛ75	8248	8
9	851X	87X0	8X7X	9148	9413	969X	9963	X026	X2X8	X568	X827	XXX5	9
X	Ɛ161	Ɛ417	Ɛ691	Ɛ944	ƐƐƐ7	30268	30517	30785	30X32	3109X	31344	31589	X
Ɛ	31850	31XƐ2	32153	323Ɛ3	32651	28XX	2Ɛ46	31X0	3435	3689	3920	3Ɛ72	Ɛ
20	4202	4451	469Ɛ	4928	4Ɛ73	51ƐX	5443	5687	590X	5Ɛ4Ɛ	6190	640Ɛ	20
1	664X	6887	5Ɛ03	713X	7374	75X9	7821	7X53	8085	82Ɛ6	8525	8754	1
2	8981	8ƐXX	9215	943Ɛ	9665	9889	9XƐ1	X113	X334	X555	X774	X993	2
3	XƐƐ0	Ɛ209	Ɛ425	Ɛ63Ɛ	Ɛ855	ƐX6X	40082	40295	404X7	406Ɛ8	40908	40Ɛ18	3
4	41126	41334	41540	41748	41953	41Ɛ59	2163	2367	256X	2771	2973	2Ɛ74	4
5	3174	3374	3572	3770	3969	3Ɛ65	4160	4356	4550	4745	4939	4Ɛ30	5
6	5123	5315	5506	56Ɛ6	58X5	5X94	6082	626Ɛ	6458	6644	682Ɛ	6X15	6
7	6ƐƐX	71X3	7387	756Ɛ	7751	7933	7Ɛ15	80Ɛ5	8295	8474	8653	8831	7
8	8X0X	8ƐX6	9182	9359	9533	9709	98X2	9X77	X04X	X222	X3Ɛ4	X586	8
9	X757	X928	XXƐ8	Ɛ087	Ɛ255	Ɛ423	Ɛ5Ɛ1	Ɛ77X	Ɛ946	ƐƐ41	50098	50263	9
X	50428	505Ɛ2	50776	5093X	50Ɛ01	51084	51246	51408	51589	51749	1909	1X88	X
Ɛ	2047	2205	2383	2540	26Ɛ8	2874	2X30	2ƐX6	3161	3316	348Ɛ	3644	Ɛ
30	37Ɛ8	3970	3Ɛ23	4095	4247	43Ɛ8	4569	471X	488X	4X39	4ƐX8	5156	30
1	5304	5471	561X	5786	5932	5X99	6044	61XX	6354	64Ɛ9	6662	6806	1
2	696X	6Ɛ12	7075	7217	7379	751X	767Ɛ	7820	7980	7Ɛ1Ɛ	807Ɛ	8219	2
3	8377	8515	8672	880Ɛ	8968	8Ɛ04	905Ɛ	91Ɛ6	9351	97X7	9641	9796	3
4	992Ɛ	9X83	X017	X163	X302	X454	X5X7	X738	X88X	XX1Ɛ	XƐ6Ɛ	Ɛ100	4
5	Ɛ24Ɛ	Ɛ39Ɛ	Ɛ52X	Ɛ678	Ɛ806	Ɛ954	ƐXX1	6002X	60176	60303	6044X	60595	5
6	60720	60867	609Ɛ1	60Ɛ37	61080	61205	61349	1492	1615	1759	18X0	1X22	6
7	1Ɛ65	20X7	2228	2369	24XX	262X	276X	28XX	2X29	2Ɛ68	30X7	3225	7
8	3363	34X0	3619	3756	3893	3X0Ɛ	3Ɛ46	4082	41Ɛ9	4333	446X	45X4	8
9	4719	4852	4987	4Ɛ00	5034	5168	52Ɛ0	5413	5546	5678	57XƐ	5920	9
X	5X52	5Ɛ83	60Ɛ4	6225	6355	6485	65Ɛ5	6724	6853	6982	6XƐ0	701X	X
Ɛ	7148	7275	73X2	750Ɛ	7637	7763	788Ɛ	79Ɛ7	7Ɛ22	8049	8174	829X	Ɛ
40	8404	852X	8653	8778	88X1	8X05	8Ɛ2X	9051	9175	9298	93ƐƐ	9522	40
1	9645	9767	9889	99XX	9Ɛ10	X031	X151	X272	X392	X4Ɛ2	X611	X731	1
2	X850	X96X	XX89	XƐX7	Ɛ105	Ɛ223	Ɛ340	Ɛ459	Ɛ576	Ɛ693	Ɛ7XƐ	Ɛ907	2
3	ƐX23	ƐƐ3X	70055	70170	70287	703X1	704Ɛ8	70612	70727	70841	70956	70X6Ɛ	3
4	70Ɛ83	71097	11Ɛ0	1303	1417	152X	1641	1754	1867	1979	1X8Ɛ	1ƐX1	4
5	20Ɛ3	2204	2315	2426	2536	2647	2757	2867	2976	2X86	2Ɛ95	30X4	5
6	31Ɛ2	3301	340Ɛ	3519	3627	3734	3841	394X	3X57	3Ɛ64	4070	4178	6
7	4284	438Ɛ	4497	45X2	46X9	47Ɛ4	48ƐX	4X04	4Ɛ0X	5014	511X	5223	7
8	5328	5431	5536	563X	5742	5846	594X	5X52	5Ɛ55	6058	615Ɛ	6262	8
9	6364	6467	6569	666Ɛ	6770	6872	6973	6X74	6Ɛ75	7075	7176	7276	9
X	7376	7476	7575	7675	7774	7873	7972	7X70	7Ɛ6X	8069	8167	8264	X
Ɛ	8362	845Ɛ	8558	8655	8752	884Ɛ	8947	8X43	8Ɛ3Ɛ	9037	9132	922X	Ɛ

	0	1	2	3	4	5	6	7	8	9	X	Ɛ	
50	79325	79420	79517	79611	79708	79802	798Ɛ8	799Ɛ2	79XX7	79ƐX1	7X096	7X18Ɛ	50
1	X284	X379	X471	X566	X65X	X752	X846	X939	XX31	XƐ24	Ɛ017	Ɛ10X	1
2	Ɛ200	Ɛ2Ɛ3	Ɛ3X5	Ɛ497	Ɛ589	Ɛ67Ɛ	Ɛ770	Ɛ862	Ɛ953	ƐX44	ƐƐ35	80026	2
3	80117	80207	802Ɛ7	803X7	80497	80587	80676	80766	80855	80944	80X32	0Ɛ21	3
4	1010	10ƐX	11X8	1296	1384	1471	155Ɛ	1648	1735	1822	190Ɛ	19Ɛ8	4
5	1XX4	1Ɛ90	2079	2165	2250	2338	2423	250Ɛ	25Ɛ6	26X1	2788	2872	5
6	2959	2X43	2Ɛ2X	3014	30ƐX	31X3	3289	3370	3457	3540	3625	370X	6
7	37Ɛ3	3897	3980	3X64	3Ɛ48	4030	4113	41Ɛ7	429X	4382	4465	4548	7
8	462X	4711	47Ɛ4	4896	4978	4X5X	4Ɛ40	5022	5103	51X5	5286	5367	8
9	5448	5529	560X	56XX	578Ɛ	586Ɛ	594Ɛ	5X2Ɛ	5Ɛ0Ɛ	5ƐXƐ	608X	616X	9
X	6249	6328	6407	64X6	6585	6663	6742	6820	68ƐX	6998	6X76	6Ɛ54	X
Ɛ	7032	710Ɛ	71X8	7286	7363	743Ɛ	7518	75Ɛ5	7691	776X	7846	7922	Ɛ
60	79ƐX	7X96	7Ɛ72	8049	8125	8200	8297	8372	8449	8524	86ƐX	8695	60
1	876Ɛ	8846	8920	89Ɛ6	8X90	8Ɛ65	903Ɛ	9114	91XX	9283	9358	9431	1
2	9506	959Ɛ	9673	9747	9820	98Ɛ4	9988	9X60	9Ɛ34	X008	X09Ɛ	X173	2
3	X246	X319	X3Ɛ0	X483	X556	X629	X6ƐƐ	X792	X864	X936	XX08	XX9X	3
4	XƐ70	Ɛ042	Ɛ114	Ɛ1X5	Ɛ276	Ɛ348	Ɛ419	Ɛ4XX	Ɛ57Ɛ	Ɛ650	Ɛ720	Ɛ7Ɛ1	4
5	Ɛ881	Ɛ952	ƐX22	ƐXƐ2	ƐƐ82	90052	90121	901Ɛ1	90280	90350	9041Ɛ	904XX	5
6	90579	90648	90717	907X6	90874	0943	0X11	0X9Ɛ	0Ɛ6X	1038	1106	1193	6
7	1261	132Ɛ	13Ɛ8	1485	1553	1620	16X9	1776	1842	190Ɛ	1998	1X64	7
8	1Ɛ31	1ƐƐ9	2085	2151	2219	22X5	2371	2438	2504	258Ɛ	2656	2721	8
9	27X9	2874	293X	2X05	2X90	2Ɛ56	3021	30X7	3171	3237	3302	3387	9
X	3451	3517	35X1	3666	372Ɛ	37Ɛ5	387X	3943	3X08	3X91	3Ɛ56	401X	X
Ɛ	40X3	4168	4230	42Ɛ4	4378	4440	4504	4588	4650	4714	4797	485Ɛ	Ɛ
70	4922	49X6	4X69	4Ɛ30	4ƐƐ3	5076	5139	51ƐƐ	5282	5344	5407	5489	70
1	554Ɛ	5612	5694	5755	5817	5899	595Ɛ	5X20	5XX2	5Ɛ63	6024	60X6	1
2	6167	6228	62X8	6369	642X	64XƐ	656Ɛ	6630	66Ɛ0	6770	6830	68Ɛ0	2
3	6970	6X30	6XƐ0	6Ɛ70	702Ɛ	70XƐ	716X	7229	72X9	7368	7427	74X6	3
4	7565	7623	76X2	7761	781Ɛ	789X	7958	7X16	7X94	7Ɛ53	8011	808X	4
5	8158	8206	8284	8341	83ƐƐ	8478	8535	85Ɛ2	8670	8729	87X5	8862	5
6	891Ɛ	8998	8X54	8Ɛ11	8Ɛ89	9046	9102	917X	9236	92Ɛ2	936X	9426	6
7	94X2	9559	9615	9690	9748	9803	987X	9935	99Ɛ1	9X68	9Ɛ22	9Ɛ99	7
8	X054	X10Ɛ	X185	X240	X2Ɛ6	X370	X427	X4X1	X557	X611	X687	X741	8
9	X7Ɛ7	X870	X926	X99Ɛ	XX55	XƐ0X	XƐ83	Ɛ039	Ɛ0Ɛ2	Ɛ167	Ɛ220	Ɛ295	9
X	Ɛ34X	Ɛ402	Ɛ477	Ɛ530	Ɛ5X4	Ɛ658	Ɛ711	Ɛ758	Ɛ839	Ɛ8Ɛ1	Ɛ965	ƐX19	X
Ɛ	ƐX91	ƐƐ45	ƐƐ99	X0070	X0124	X0197	X024Ɛ	X0302	X0375	X0429	X04X0	X0553	Ɛ
80	X0606	X0679	X072Ɛ	07X2	0855	0907	097X	0X30	0XX3	0Ɛ55	1007	1079	80
1	112Ɛ	11X1	1253	1305	1377	1429	149X	1550	1601	1673	1724	1795	1
2	1847	18Ɛ8	1969	1X1X	1X8Ɛ	1Ɛ3Ɛ	1ƐƐ0	2061	2111	2182	2232	22X3	2
3	2353	2403	2474	2524	2594	2644	26Ɛ4	2763	2813	2883	2933	29X2	3
4	2X52	2Ɛ01	2Ɛ70	3020	308Ɛ	313X	31X9	3258	3307	3376	3425	3493	4
5	3542	35Ɛ1	365Ɛ	370X	3778	3826	3895	3943	39Ɛ1	3X5Ɛ	3Ɛ09	3Ɛ77	5
6	4025	4092	4140	41XX	4257	4305	4372	4420	4489	4536	45X3	4650	6
7	46ƐX	4767	4813	4880	4929	4996	4X42	4XXƐ	4Ɛ58	5004	5070	5119	7
8	5185	5231	5299	5345	53Ɛ1	5459	5505	5571	5619	5685	5730	5798	8
9	5843	58XƐ	5956	5X01	5X69	5Ɛ14	5Ɛ7Ɛ	6026	6091	6138	61X3	624X	9
X	62Ɛ4	635Ɛ	6406	6470	6517	6581	6628	6692	6738	67X3	6849	68Ɛ3	X
Ɛ	6959	6X03	6X69	6Ɛ12	6Ɛ78	7022	7088	7131	7197	7240	72X6	734Ɛ	Ɛ

	0	1	2	3	4	5	6	7	8	9	X	Ɛ	
90	X73Ɛ4	X745X	X7503	X7568	X7611	X7676	X771Ɛ	X7784	X7829	X7891	X7936	X799Ɛ	90
1	7X43	7XX8	7Ɛ50	7ƐƐ5	8059	8101	8166	820X	8272	8316	837X	8422	1
2	8486	852X	8591	8635	8699	8740	87X4	8847	88XƐ	8952	89Ɛ5	8X59	2
3	8Ɛ00	8Ɛ63	9006	9069	9110	9173	9216	9279	9320	9382	9425	9488	3
4	952X	9591	9633	9695	9738	979X	9840	98X2	9944	99X6	9X48	9XXX	4
5	9Ɛ50	9ƐƐ2	X054	X0Ɛ5	X157	X1Ɛ9	X25X	X300	X361	X403	X464	X505	5
6	X566	X608	X669	X70X	X76Ɛ	X810	X871	X912	X972	XX13	XX74	XƐ14	6
7	XƐ75	Ɛ016	Ɛ076	Ɛ116	Ɛ177	Ɛ217	Ɛ277	Ɛ318	Ɛ378	Ɛ418	Ɛ478	Ɛ518	7
8	Ɛ578	Ɛ618	Ɛ678	Ɛ718	Ɛ777	Ɛ817	Ɛ877	Ɛ916	Ɛ976	ƐX15	ƐX75	ƐƐ14	8
9	ƐƐ74	Ɛ0013	Ɛ0072	Ɛ0111	Ɛ0170	Ɛ0210	Ɛ026Ɛ	Ɛ0310	Ɛ0369	Ɛ0407	Ɛ0466	Ɛ0505	9
X	Ɛ0564	0602	0661	0700	075X	0803	0857	08Ɛ6	0954	09Ɛ2	0X50	0XXƐ	X
Ɛ	0Ɛ49	0Ɛ47	1045	10X3	1141	119Ɛ	1239	1296	1334	1392	1430	1489	Ɛ
X0	1527	1584	1622	167Ɛ	1719	1776	1813	1870	190X	1967	1X04	1X61	X0
1	1XƐX	1Ɛ57	1ƐƐ4	2151	20X9	2146	21X3	223Ɛ	2298	2335	2391	242X	1
2	2486	2522	257Ɛ	2617	2673	270Ɛ	2768	2804	2860	28Ɛ8	2954	29Ɛ0	2
3	2X47	2XX3	2Ɛ3Ɛ	2Ɛ97	3032	208X	3126	3181	3219	3274	3310	3367	3
4	3402	345X	34Ɛ5	3550	35X7	3642	3699	3734	378Ɛ	3826	3881	3918	4
5	3973	3X09	3X64	3XƐƐ	3Ɛ55	3ƐƐ0	4046	40X1	4137	4192	4228	4282	5
6	4319	4373	4409	4463	44Ɛ9	4553	45X9	4643	4699	4733	4789	4822	6
7	4878	4912	4967	4X01	4X57	4XƐ0	4Ɛ46	4Ɛ9Ɛ	5034	508X	5123	5178	7
8	5212	5267	5300	5355	53XX	5443	5498	5531	5586	561Ɛ	5673	5708	8
9	5761	57Ɛ5	584X	58X3	5937	5990	5X24	5X79	5Ɛ11	5Ɛ65	5ƐƐX	6052	9
X	60X6	613X	6192	6226	627X	6313	6366	63ƐX	6452	64X6	653X	6592	X
Ɛ	6625	6679	6711	6764	67Ɛ8	684Ɛ	68X3	6936	698X	6X21	6X74	6Ɛ08	Ɛ
Ɛ0	6Ɛ5Ɛ	6ƐƐ2	7045	7098	712Ɛ	7183	7216	7268	72ƐƐ	7352	73X5	7438	Ɛ0
1	748Ɛ	7521	7574	7607	7659	76Ɛ0	7742	7795	7827	787X	7910	7962	1
2	79Ɛ5	7X47	7X99	7Ɛ2Ɛ	7Ɛ82	8014	8066	80Ɛ8	814X	81X0	8232	8283	2
3	8315	8367	83Ɛ9	844Ɛ	84X0	8532	8583	8615	8667	86Ɛ8	874X	879Ɛ	3
4	8830	8882	8913	8964	89Ɛ5	8X47	8X98	8Ɛ29	8Ɛ7X	900Ɛ	9060	90Ɛ1	4
5	9142	9193	9224	9274	9305	9356	93X7	9437	9488	9519	9569	95ƐX	5
6	964X	969Ɛ	972Ɛ	977Ɛ	9810	9860	98Ɛ0	9941	9991	9X21	9X71	9Ɛ01	6
7	9Ɛ51	9ƐX1	X031	X081	X111	X161	X1Ɛ1	X241	X290	X320	X370	X3ƐƐ	7
8	X44Ɛ	X49Ɛ	X52X	X57X	X609	X659	X6X8	X737	X787	X816	X865	X8Ɛ5	8
9	X944	X993	XX22	XX71	XƐ00	XƐ4Ɛ	XƐ9X	Ɛ029	Ɛ078	Ɛ107	Ɛ156	Ɛ1X5	9
X	Ɛ233	Ɛ282	Ɛ311	Ɛ360	Ɛ3XX	Ɛ439	Ɛ487	Ɛ516	Ɛ564	Ɛ5Ɛ3	Ɛ641	Ɛ690	X
Ɛ	Ɛ71X	Ɛ769	Ɛ7Ɛ7	Ɛ845	Ɛ893	Ɛ922	Ɛ970	Ɛ9ƐX	ƐX48	ƐX96	ƐƐ24	ƐƐ72	Ɛ

THE APPLICATION OF THE GOLDEN MEAN





The ratio of the Golden Mean approximates .75 to 1.00

WEIGHTS AND MEASURES

There is no completely adequate system of weights and measures in general use. The Anglo-American standards are entirely unsystemized, and lack integration with the number base. The French metric system is limited in its applicability because it is decimal. The ten-month year and the ten-hour day have been decisively repudiated. In navigation, in most measurements of time and angle, conformance to the ten-base has proved unacceptable. In some fields, as in music, it is useless.

This is one of the most important problems of world civilization. Duodecimals offer the one clear possibility for its solution in a comprehensive metric system capable of handling all measurement. For this very reason, it is essential that the selected architecture of this duodecimal metric system be suited to man's preferences as well as to the practical convenience of the scientists. Intense study is being given to its design.

For present applications, the use of the Do-Metric System is suggested. It adapts many of the conventional units into acceptable integration on the duodecimal base. Its inch and yard conform to the recent internationally standardized ratio of 25.4 millimeters to the inch.

THE DO-METRIC SYSTEM

There are two linear scales, related in the ratio of *3* to *1*.

The Mechanic's Scale

Twelve Points equal	*1* Line.	The Point equals	*.001* foot.
Twelve Lines equal	*1* Inch.	The Line equals	*.01* foot.
Twelve Inches equal	*1* Foot.	The Inch equals	*.1* foot.

The Basic Scale

Twelve Karls equal	*1* Quan.	The Karl equals	*.001* yard.
(The Karl is the quarter-line.)			
Twelve Quans equal	*1* Palm.	The Quan equals	*.01* yard.
(The Quan is the quarter-inch.)			
Twelve Palms equal	*1* Yard.	The Palm equals	*.1* yard.
(The Palm is *3* inches.)			

The square and cubic measures are directly derived from these.

Volumes and Weights

The Palm is *3* inches. The cubic palm (*23* cu. in.) is the Do-Metric Pint. It is 6-1/2% smaller than the present pint. The weight of this pint of water is the Do-Metric Pound. It is 2-1/2% lighter than the present avoirdupois pound.

Weight

Twelve Carats equal	*1* Gram.	The Carat equals	*.001* pound.
Twelve Grams equal	*1* Ounce.	The Gram equals	*.01* pound.
Twelve Ounces equal	*1* Pound.	The Ounce equals	*.1* pound.

Liquid Measure

Twelve Dribs equal	*1* Dram.	The Drib equals	*.001* pint.
Twelve Drams equal	*1* Founce.	The Dram equals	*.01* pint.
Twelve Founces equal	*1* Pint.	The Founce equals	*.1* pint.

(The Founce is the fl. oz.)

Palm, Pint, and Pound are correlatives. The cubic yard holds *1000* pints, or *1* Tun, which weighs *1000* pounds, or *1* Ton. Thus Yard, Tun, and Ton are correlatives. The Do-Metric Mile is *1000* yards.

THE DUODECIMAL CIRCLE AND TIME

In the measurement of time and angle, the greatest simplicity is attained by using the circle and the day as the fundamental units, and the lesser division as duodecimals of these. In this way no conversion is necessary between minutes of time and minutes of angle. Time and longitude are expressed by the same number. The surscript c can replace the degree symbol.

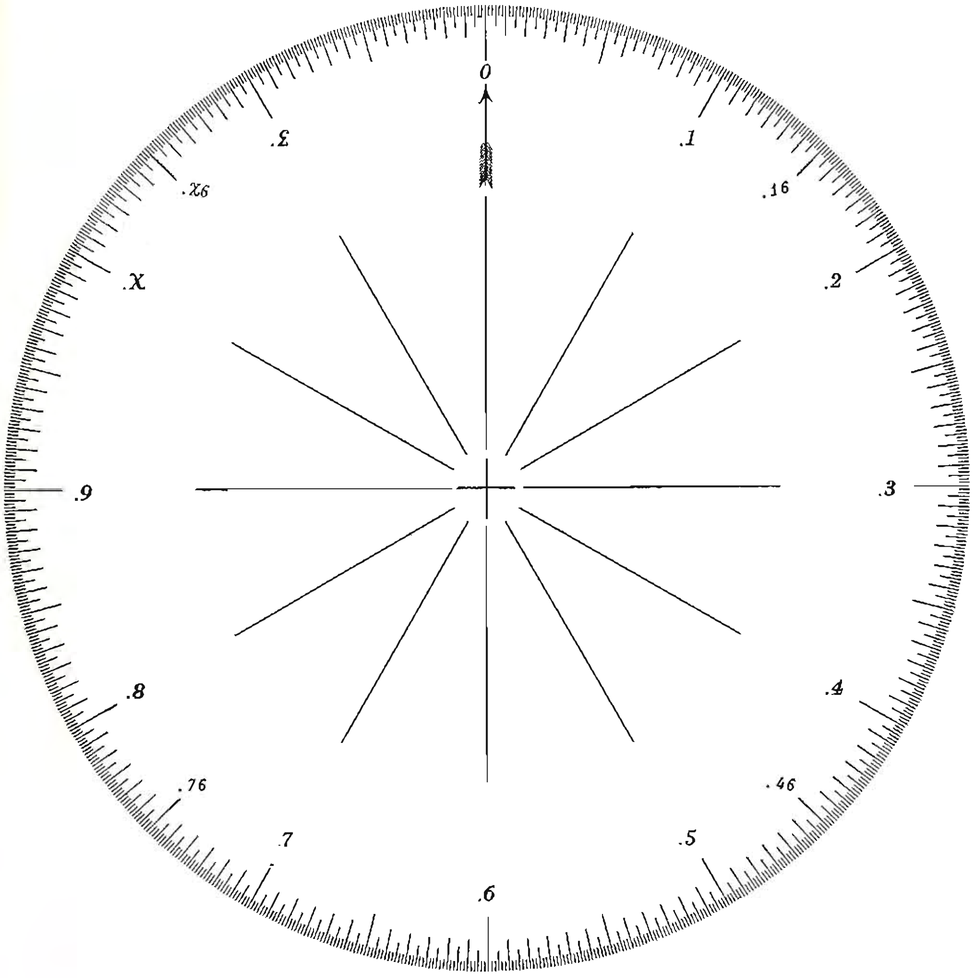
.1^c	is called the	duor	= 2 hours	or 30°
.01^c		temin	10 minutes	2° 30'
.001^c		minette	50 seconds	12' 30"
.0001^c		grovic	4.16 seconds	1' 2½"

angle	*.1*^{c}	*.16*^{c}	*.2*^{c}
sin	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$
cos	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$
tan	$1/\sqrt{3}$	1	$1/\sqrt{3}$

CONSTANTS

π	*3.1848*
$1/\pi$	*.39X06*
$\sqrt{\pi}$	*1.932X*
log π	*.5406*
e	*2.875236*
1/e	*.44Ɛ842*
$\sqrt{2}$	*1.5*
$\sqrt{3}$	*1.895*

THE DUODECIMAL CIRCLE



DUODECIMAL NATURAL SINES

Arguments are Temins ($.01^c$) and Minettes ($.001^c$), paralleled with Degrees and Minutes of Arc.

Degrees & Minutes	Temins	Minutes 0 / Minettes 0	12.5 / 1	25 / 2	37.5 / 3	50 / 4	62.5 / 5	75 / 6	87.5 / 7	100 / 8	112.5 / 9	125 / X	137.5 / Ɛ	150 / '0		
00	00	0000	0063	0107	016X	0212	0275	0318	0380	0423	0486	052X	0591	0634	2Ɛ	87 30
02 30	01	0634	0698	073Ɛ	07X2	0846	08X9	0950	09Ɛ3	0X57	0XƐX	0Ɛ61	1004	1067	2X	85
05	02	1067	110X	1171	1214	1278	131X	1381	1424	1487	152X	1591	1634	1697	29	82 30
07 30	03	1697	1739	17X0	1843	18X5	1948	19XX	1X51	1XƐ3	1Ɛ56	1ƐƐ8	205X	2101	28	80
10	04	2101	2163	2205	2267	2309	236Ɛ	2411	2473	2515	2577	2619	267X	2720	27	77 30
12 30	05	2720	2782	2823	2885	2926	2987	2X29	2X8X	2Ɛ2Ɛ	2Ɛ90	3031	3092	3133	26	75
15	06	3133	3194	3234	3295	3336	3396	3437	3497	3537	3597	3637	3697	3737	25	72 30
17 30	07	3737	3797	3837	3897	3936	3996	3X35	3X95	3Ɛ34	3Ɛ93	4032	4091	4130	24	70
20	08	4130	418Ɛ	422X	4288	4327	4385	4424	4482	4520	457X	4618	4676	4713	23	67 30
22 30	09	4713	4771	480X	4868	4905	4962	49ƐƐ	4X58	4XƐ5	4Ɛ52	4ƐXƐ	5047	50X3	22	65
25	0X	50X3	5140	5198	5234	5290	5328	5383	541Ɛ	5476	5512	5569	5604	565Ɛ	27	62 30
27 30	0Ɛ	565Ɛ	56Ɛ6	5750	57X7	5841	5898	5932	5988	5X22	5X77	5Ɛ11	5Ɛ67	6000	20	60
30	10	6000	6055	60XX	6143	6198	6231	6285	631X	6372	6406	645X	64Ɛ2	6545	1Ɛ	57 30
32 30	11	6545	6599	6630	6684	6717	676X	6800	6853	68X5	6938	698X	6X20	6X72	1X	55
35	12	6X72	6Ɛ03	6Ɛ55	6ƐX6	7037	7089	7119	716X	71ƐƐ	724Ɛ	729Ɛ	732Ɛ	737Ɛ	19	52 30
37 30	13	737Ɛ	740Ɛ	745Ɛ	74XX	7539	7588	7617	7666	76Ɛ4	7743	7791	781Ɛ	7869	18	50
40	14	7869	78Ɛ7	7944	7991	7X1X	7X67	7XƐ4	7Ɛ41	7Ɛ89	8015	8061	80X9	8135	17	47 30
42 30	15	8135	8181	8208	8253	829X	8325	836Ɛ	83Ɛ6	8440	8486	8510	8555	859Ɛ	16	45
45	16	859Ɛ	8624	8669	86Ɛ2	8736	877Ɛ	8803	8847	888Ɛ	8912	8956	8999	8X20	15	42 30
47 30	17	8X20	8X63	8XX6	8Ɛ28	8Ɛ6X	8ƐƐ0	9032	9074	90Ɛ5	9136	9177	91Ɛ8	9239	14	40
50	18	9239	9279	92Ɛ9	9339	9379	93Ɛ8	9438	9477	94Ɛ6	9534	9573	95Ɛ1	962Ɛ	13	37 30
52 30	19	962Ɛ	9669	96X6	9724	9761	979X	9816	9853	988Ɛ	9907	9943	997Ɛ	99Ɛ6	12	35
55	1X	99Ɛ6	9X31	9X68	9XX3	9Ɛ19	9Ɛ53	9Ɛ89	X003	X039	X072	X0X7	X120	X155	11	32 30
57 30	1Ɛ	X155	X189	X201	X235	X269	X2X0	X313	X346	X379	X3Ɛ0	X422	X454	X486	10	30
60	20	X486	X4Ɛ7	X529	X55	X58Ɛ	X5ƐƐ	X630	X660	X690	X700	X72Ɛ	X75X	X789	0Ɛ	27 30
62 30	21	X789	X7Ɛ8	X826	X854	X882	X8Ɛ0	X91X	X947	X974	X9X0	XX09	XX35	XX61	0X	25
65	22	XX61	XX89	XXƐ4	XƐ20	XƐ47	XƐ71	XƐ98	Ɛ002	Ɛ028	Ɛ052	Ɛ077	Ɛ0X1	Ɛ106	09	22 30
67 30	23	Ɛ106	Ɛ12X	Ɛ153	Ɛ177	Ɛ19Ɛ	Ɛ203	Ɛ226	Ɛ249	Ɛ270	Ɛ293	Ɛ2Ɛ5	Ɛ318	Ɛ339	08	20
70	24	Ɛ339	Ɛ35Ɛ	Ɛ381	Ɛ3X2	Ɛ403	Ɛ423	Ɛ444	Ɛ464	Ɛ483	Ɛ4X3	Ɛ502	Ɛ521	Ɛ540	07	17 30
72 30	25	Ɛ540	Ɛ55Ɛ	Ɛ579	Ɛ597	Ɛ5Ɛ5	Ɛ612	Ɛ630	Ɛ649	Ɛ665	Ɛ682	Ɛ69X	Ɛ6Ɛ6	Ɛ711	06	15
75	26	Ɛ711	Ɛ729	Ɛ744	Ɛ75Ɛ	Ɛ775	Ɛ790	Ɛ7X6	Ɛ800	Ɛ815	Ɛ82X	Ɛ843	Ɛ858	Ɛ870	05	12 30
77 30	27	Ɛ870	Ɛ885	Ɛ899	Ɛ8Ɛ0	Ɛ904	Ɛ917	Ɛ92X	Ɛ940	Ɛ952	Ɛ964	Ɛ976	Ɛ988	Ɛ999	04	10
80	28	Ɛ999	Ɛ9XX	Ɛ9ƐƐ	ƐX0Ɛ	ƐX1Ɛ	ƐX2Ɛ	ƐX3Ɛ	ƐX4X	ƐX59	ƐX68	ƐX76	ƐX85	ƐX93	03	07 30
82 30	29	ƐX93	ƐXX0	ƐXXX	ƐXƐ7	ƐƐ04	ƐƐ10	ƐƐ19	ƐƐ25	ƐƐ31	ƐƐ38	ƐƐ43	ƐƐ4X	ƐƐ55	02	05
85	2X	ƐƐ55	ƐƐ60	ƐƐ66	ƐƐ70	ƐƐ75	ƐƐ7Ɛ	ƐƐ84	ƐƐ88	ƐƐ91	ƐƐ95	ƐƐ99	ƐƐX1	ƐƐX4	01	02 30
87 30	2Ɛ	ƐƐX4	ƐƐX7	ƐƐXX	ƐƐƐ1	ƐƐƐ3	ƐƐƐ5	ƐƐƐ7	ƐƐƐ9	ƐƐƐX	ƐƐƐƐ	ƐƐƐƐ	10000	1.0000	00	00
Minettes		'0	Ɛ	X	9	8	7	6	5	4	3	2	1	0	Temins	Degrees & Minutes
Minutes		150	137.5	125	112.5	100	87.5	75	67.5	50	37.5	25	12.5	0		

DUODECIMAL NATURAL COSINES

DUODECIMAL NATURAL TANGENTS

Arguments are Temins (*.01*) and Minettes (*.001*), paralleled with Degrees and Minutes of Arc.

Degrees & Minutes	Temins	Minutes 0	12.5	25	37.5	50	62.5	75	87.5	100	112.5	125	137.5	150		
		Minettes *0*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*X*	*£*	*'0*		
00	*00*	*0000*	*0063*	*0107*	*016X*	*0212*	*0275*	*0318*	*0380*	*0423*	*0487*	*052X*	*0592*	*0635*	*2£*	87 30
02 30	*01*	*0635*	*0699*	*0740*	*07X4*	*0848*	*08X£*	*0953*	*09£7*	*0X5£*	*0£02*	*0£66*	*100X*	*1072*	*2X*	85
05	*02*	*1072*	*1116*	*117X*	*1222*	*1286*	*132£*	*1393*	*1437*	*14X0*	*1544*	*15X9*	*1651*	*16£6*	*29*	82 30
07 30	*03*	*16£6*	*175£*	*1804*	*1868*	*1911*	*1976*	*1X20*	*1X85*	*1£2X*	*1£94*	*2039*	*20X3*	*2148*	*28*	80
10	*04*	*2148*	*21£2*	*2258*	*2302*	*2368*	*2412*	*2479*	*2523*	*258X*	*2634*	*269£*	*2746*	*27£1*	*27*	77 30
12 30	*05*	*27£1*	*2858*	*2904*	*296£*	*2X17*	*2X82*	*2£2X*	*2£96*	*3042*	*30XX*	*3157*	*3203*	*3270*	*26*	75
15	*06*	*3270*	*3319*	*3386*	*3433*	*34X1*	*354X*	*35£8*	*3666*	*3714*	*3782*	*3831*	*389£*	*394X*	*25*	72 30
17 30	*07*	*394X*	*39£9*	*3X68*	*3£18*	*3£87*	*4037*	*40X7*	*4157*	*4208*	*4278*	*4329*	*439X*	*444£*	*24*	70
20	*08*	*444£*	*4501*	*4573*	*4624*	*4697*	*4749*	*4800*	*4873*	*4926*	*4999*	*4X51*	*4£05*	*4£79*	*23*	67 30
22 30	*09*	*4£79*	*5032*	*50X6*	*515£*	*5215*	*528X*	*5344*	*53£X*	*5475*	*552£*	*55X6*	*5662*	*7199*	*22*	65
25	*0X*	*5719*	*5795*	*5852*	*590X*	*5987*	*5X44*	*5£20*	*5£80*	*603X*	*60£9*	*6178*	*6237*	*62£6*	*21*	62 30
27 30	*0£*	*62£6*	*6376*	*6437*	*64£8*	*6579*	*663X*	*6700*	*6782*	*6845*	*6908*	*6990*	*6X54*	*6£18*	*20*	60
30	*10*	*6£18*	*6£X1*	*7066*	*712£*	*71£5*	*7280*	*7347*	*7412*	*749X*	*7566*	*7633*	*7701*	*778X*	*1£*	57 30
32 30	*11*	*778X*	*7859*	*7927*	*79£7*	*7X86*	*7£57*	*8027*	*80£9*	*818£*	*8261*	*8334*	*8407*	*84X0*	*1X*	55
35	*12*	*84X0*	*8574*	*8649*	*8723*	*87£X*	*8895*	*8970*	*8X49*	*8£25*	*9003*	*90X1*	*9180*	*925£*	*19*	52 30
37 30	*13*	*925£*	*933£*	*9420*	*9502*	*95X4*	*9687*	*976X*	*9853*	*9938*	*9X22*	*9£08*	*9££3*	*X0X0*	*18*	50
40	*14*	*X0X0*	*X188*	*X276*	*X365*	*X454*	*X544*	*X635*	*X727*	*X819*	*X911*	*XX05*	*XX££*	*X££5*	*17*	47 30
42 30	*15*	*X££5*	*£0£0*	*£1X8*	*£2X5*	*£3X3*	*£4X2*	*£5X2*	*£6X3*	*£7X5*	*£8X8*	*£9£1*	*£X£6*	*10000*	*16*	45
45	*16*	*1.0000*	*1.0107*	*1.0214*	*1.0321*	*1.0430*	*1.0540*	*1.0651*	*1.0763*	*1.0877*	*1.0990*	*1.0XX6*	*1.1001*	*1.1119*	*15*	42 30
47 30	*17*	*1.1119*	*1.1237*	*1.1356*	*1.1477*	*1.1599*	*1.1700*	*1.1825*	*1.194£*	*1.1X77*	*1.1£X4*	*1.2112*	*1.2243*	*1.2374*	*14*	40
50	*18*	*1.2374*	*1.24X7*	*1.2620*	*1.2757*	*1.2893*	*1.2X11*	*1.2£51*	*1.3092*	*1.3215*	*1.335X*	*1.34X5*	*1.3631*	*1.3780*	*13*	37 30
52 30	*19*	*1.3780*	*1.3910*	*1.3X62*	*1.3££7*	*1.4151*	*1.42XX*	*1.4448*	*1.45X9*	*1.4750*	*1.48£5*	*1.4X60*	*1.500X*	*1.517X*	*12*	35
55	*1X*	*1.517X*	*1.5330*	*1.54X5*	*1.5661*	*1.581X*	*1.599£*	*1.5£62*	*1.6127*	*1.62£4*	*1.6483*	*1.6655*	*1.6829*	*1.6X05*	*11*	32 30
57 30	*1£*	*1.6X05*	*1.6£X4*	*1.7185*	*1.736X*	*1.7556*	*1.7745*	*1.7938*	*1.7£32*	*1.812£*	*1.8330*	*1.8534*	*1.8740*	*1.8950*	*10*	30
60	*20*	*1.8950*	*1.8£63*	*1.917£*	*1.939X*	*1.9601*	*1.9829*	*1.9X59*	*1.X091*	*1.X309*	*1.X54X*	*1.X794*	*1.XX22*	*1.£076*	*0£*	27 30
62 30	*21*	*1.£075*	*1.£312*	*1.£573*	*1.£819*	*1.£X89*	*2.0142*	*2.0400*	*2.0685*	*2.0953*	*2.1027*	*2.1305*	*2.15XX*	*2.1899*	*0X*	25
65	*22*	*2.1898*	*2.1£92*	*2.2292*	*2.2599*	*2.28£0*	*2.3009*	*2.3332*	*2.3663*	*2.399£*	*2.4124*	*2.4475*	*2.4813*	*2.4£79*	*09*	22 30
67 30	*23*	*2.4£79*	*2.5331*	*2.56£1*	*2.5X80*	*2.6258*	*2.6642*	*2.6X37*	*2.723X*	*2.7650*	*2.7X72*	*2.82X4*	*2.8726*	*2.8£79*	*08*	20
70	*24*	*2.8£78*	*2.941£*	*2.9893*	*2.X159*	*2.X635*	*2.X£24*	*2.£426*	*2.£940*	*3.026X*	*3.07£0*	*3.1148*	*3.16£X*	*3.2086*	*07*	17 30
72 30	*25*	*3.2086*	*3.266X*	*3.3069*	*3.3685*	*3.40££*	*3.4753*	*3.5206*	*3.5899*	*3.6391*	*3.6XX7*	*3.7624*	*3.8185*	*3.8950*	*06*	15
75	*26*	*3.8950*	*3.9541*	*3.X15X*	*3.X9X4*	*3.£659*	*4.0342*	*4.1059*	*4.19X8*	*4.2770*	*4.3570*	*4.43X9*	*4.5267*	*4.6166*	*05*	12 30
77 30	*27*	*4.6166*	*4.70XX*	*4.807X*	*4.9099*	*4.X149*	*4.£251*	*5.03£3*	*5.15£5*	*5.285£*	*5.3£73*	*5.5339*	*5.677X*	*5.8080*	*04*	10
80	*28*	*5.8080*	*5.9649*	*5.£0X8*	*6.0824*	*6.2446*	*6.4159*	*6.5£70*	*6.7X91*	*6.9£10*	*7.0078*	*7.2366*	*7.47X7*	*7.7196*		07 30
82 30	*29*	*7.7196*	*7.9949*	*8.06X2*	*8.363X*	*8.6800*	*8.X012*	*9.16X3*	*9.5467*	*9.9560*	*X.1X08*	*X.6679*	*X.£790*	*£.51£2*	*02*	05
85	*2X*	*£.51£2*	*£.£1X2*	*10.5835*	*11.0X22*	*11.887X*	*12.5506*	*13.3102*	*14.1X74*	*15.2047*	*16.396X*	*17.7640*	*19.1813*	*1X.XX18*		02 30
87 30	*2£*	*1X.XX18*	*20.£X41*	*23.5X66*	*26.668£*	*2X.44£6*	*33.343£*	*39.9£51*	*46.££84*	*58.9002*	*77.8051*	*£5.610X*	*1X£.0280*		*00*	00
Minettes		*'0*	*£*	*X*	*9*	*8*	*7*	*6*	*5*	*4*	*3*	*2*	*1*	*0*	Temins	Degrees & Minutes
Minutes		150	137.5	125	112.5	100	87.5	75	62.5	50	37.5	25	12.5	0		
					DUODECIMAL NATURAL COTANGENTS											

Temperature

The Do-Metric System uses two scales of temperatures, both having degrees of the same value, with *100* degrees between the freezing and the boiling points of water. The Scientific Scale parallels the Kelvin Scale, with its *0* at Absolute Zero, while the Popular Scale has its zero at the freezing point of water and *100* degrees at its boiling point, and Absolute Zero at *-289.47*.

Dynamic Units

The dynamic units of the Do-Metric System have been stated in full detail in the Duodecimal Bulletin, Volumes *6* through *9*. They are directly derived from the foregoing basic units.

The ease with which duodecimals comprise all measurement is attested in many ways. The World Calendar is well fitted for duodecimal use, with its equal quarters, and twelve months, each with the same number of week days. Duodecimally, monthly charges and rates are *.1* of the annual charges. Velizar Godjevatz has proposed a new duodecimal musical notation without sharps or flats, for which George Bernard Shaw expressed high praise. Louis Loynes advocates a duodecimal system of color values in the Byraz Colour Notation. In short, duodecimals integrate all measurement into an ordered system on base-twelve, flexibly, conveniently, and comfortably.

The Work of the Duodecimal Society

is the education of the public in the use and application of duodecimals in numeration, mathematics, weights and measures, and other branches of pure and applied science. It seeks to serve as a center for information and consultation on duodecimals. In this work it has now been joined by the Duodecimal Society of Great Britain. Fullest cooperation will be extended to the formation of duodecimal groups elsewhere. Some of our literature is available in Esperanto.

You are invited to join in this rewarding venture, to support our effort to relieve man's thinking of much unnecessary friction, and to hasten the general use of the comprehensive and flexible duodecimal metric system that the world so sorely needs.

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