THE DUODECIMAL BULLETIN

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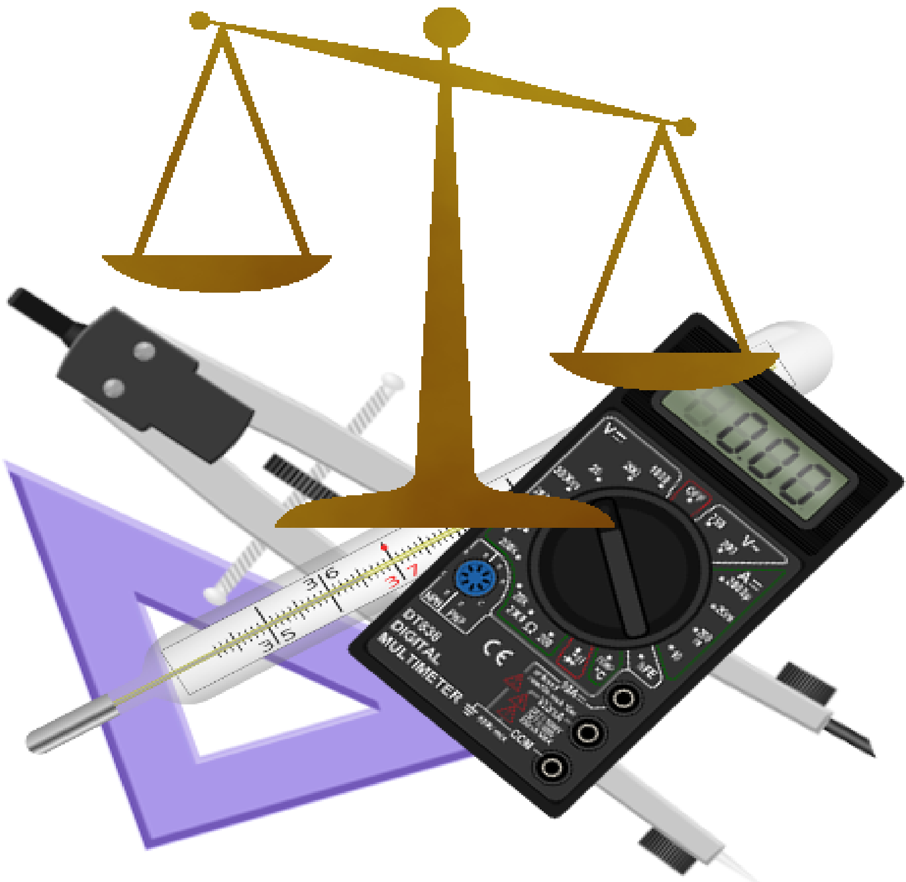

Metrologies

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The Dozenal Society of America

is a voluntary, nonprofit corporation, organized to conduct research and educate the public in the use of base twelve in calculations, mathematics, weights and measures, and other branches of pure and applied science.

Basic membership is free, but as a nonprofit we depend upon the generosity of our members to continue operating. An annual subscription of US\$$18.00_d$ (US\$$16.00_z$) entitles the member to delivery of paper copies of *The Duodecimal Bulletin.*

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The Duodecimal Bulletin, Volume Five Dozen Three, Number One, Whole Number Ten Dozen Three.

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The Duodecimal Bulletin

Volume Sixty Three (63$_d$) • Number 1 • Whole No. One Hundred Twenty Three (123$_d$)

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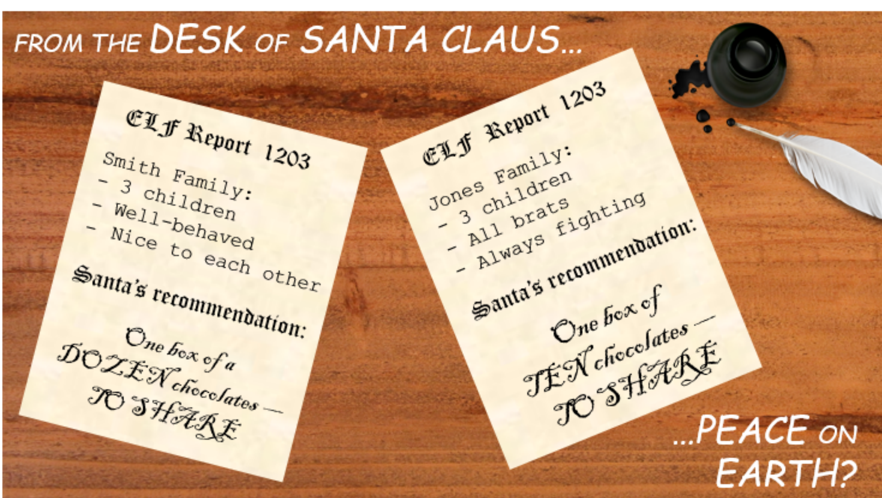

President's Message

Greetings to our membership! We've certainly had a profitable number of years, and I'm excited to share with you what we've accomplished in the past year, and in all the time since our last issue was published.

The Dozenal Society of America was a bit late catching up to the digital age, and for many years our primary outreach continued to be by our analog, paper publications, including our flagship publication, the *Bulletin*. However, as publishing for small organizations like ours increasingly moved to the Web, our paper publications continued to shrink, until in the last unquade or so we've had few or none. Several years ago we began publishing wall calendars and organizers (all, of course, in dozenal); however, other than this, everything we've published has been digital. This has allowed us to make a large volume of materials available to every Internet-connected person in the world; however, it has also encouraged us to let updates lapse, and to rest on our digital laurels.

This year, we began publishing again in earnest, uniting digital availability with the outreach capabilities of real, paper copies. The venerable *Manual of the Dozen System* saw its first update in nearly five dozen years; it's now available both digitally and in print, fully updated for the modern era. We also published *The Dozenal Primer*, a short, full-color explanation of dozenal counting in only a dozen pages. Finally, we've published an informational pamphlet *very* briefly explaining the dozenal system, and referring interested readers to more expansive materials. This one page (front and back) pamphlet is designed to be tri-folded for easy passing out at math clubs, conferences, and the like.

We've also made available a mathematics textbook for adult learners, *Basic Dozenal Arithmetic*. This goes through *all* of arithmetic, from reading numbers to counting to place notation to the four functions to logarithms and even a bit of basic algebra, all from the dozenal perspective. It's available digitally for free, and in a print version, as well. It has full exercises, a glossary, and a number of features that make it indispensible for understanding arithmetic in a way that our modern education all too often makes impossible.

We've further been continually drawing new members, with membership numbers now in the high seven-gross range. These new members come from all walks of life, from the venerable old mathematician to the young, up-and-coming scholar. We were regaled by a couple such young members at our 1201 Annual Meeting; one of them, now on our board, presented a dozenal version of Napier's bones that fascinated all of us. There is a great deal of promise in our new and old membership, which we hope our members will help us leverage in the future.

More and more of our members are getting involved, helping the Society to proceed into the future. With your help, the dozenal movement and the Dozenal Society of America will continue to be strong for many years to come.

Taking the Measure *of Measures*

This issue has been a long time coming, for which I humbly beg our membership's pardon. Since our last issue, the spare time to devote to what is essentially a volunteer activity has been hard to come by, what with the demands of career and family. But at last this issue is in your hands, and it turns out to be extra hefty. I suppose I could have edited it down, but your long-sustained patience deserves a reward of comparable magnitude.

The theme of this issue is "Metrologies" — systems of measure — in dozenal form. How do we go about building a metrology? How do we name all the units we need, and decide their sizes? Can we structure it as well as SI — or better? Can we out-metric Metric? Can we make the results sound organic rather than contrived? I believe the answer is "yes" — perhaps multiple flavors of "yes."

Learning from past experience of others is vital. This issue features not one but two articles "From the Archives." One digs back half a biquennium, to rediscover the first metrology proposed in this publication: Do-Metric. As quaint as it might seem today, it did demonstrate that the hodge-podge of customary units could be turned into something "dozenal-metric." The other article is a review by Tom Pendlebury himself, revealing his thought process in developing the Tim-Grafut-Maz (TGM) metrology. I have spiced these past articles up with a new twist, by imagining what they might have looked like had a current bit of dozenal nomenclature been available to the original authors.

My own article offers an introduction to Primel, a metrology I have been developing for several years. Inspired by but diverging from TGM, it aspires to be even more systematic, demonstrating generic techniques I've worked out that I hope others might capitalize on in designing their own metrologies.

Paul Rapoport's article showcases dozenal timekeeping in Primel and TGM, as well as his experience living immersively under a new dozenal calendar system he invented. We must commend Paul for volunteering as guinea pig for dozenal.

Even the "In the Media" column gets into the act, featuring a science fiction trilogy by Greg Egan about aliens in a different universe with different physical laws, who happen to count in dozens. Rather then spoil the plot for you, I focus on their units of measurement, which are strictly dozenal. Egan's protagonists use perfectly ordinary words for these, that sound entirely natural and prosaic — yet you never completely forget how truly alien this species is.

The parallels and contrasts in style and substance among dozenal metrologies, actual and fictional, past and present, are endlessly fascinating. You can look forward to a regular "Metrology" column in future issues.

New Members

Since the last issue, we've seen unprecedented growth in our membership. Our rolls have tripled! Two reasons may explain this: (1) It's easy to join electronically, at the DSA's website: dozenal.org. Just click the **Join Us!** button on the top right. (2) Membership is free. Optionally, for a donation of \$16$_z$ (\$18$_d$) per year, members can subscribe to receive hard copies of the *Bulletin* as they are published. (Subscribing members are highlighted in red below. The electronic version is free to all members.) The DSA Board would like to invite all of our members, new and old, to come to our annual meetings. We'd love to meet you all! If you can't attend, then feel free to email editor@dozenal.org your ideas for future *Bulletin* articles.

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$71↊_z$ (1030_d) Eshan Uniyal
$71↋_z$ (1031_d) John Watson
720_z (1032_d) Scott Draper
721_z (1033_d) Ahmad Harb
722_z (1034_d) Pini Klein
723_z (1035_d) John D Hansen Jr.
724_z (1036_d) Joe Conway
725_z (1037_d) Sean Richard Stroud
726_z (1038_d) Isaac Calvin Resnikoff
727_z (1039_d) Alireza Ebrahimi
728_z (1040_d) Timothy Paul MacGregor

729_z (1041_d) Kevin Lipinski
72↊_z (1042_d) Declan Michael Andrews
72↋_z (1043_d) Jarom Prestwich
730_z (1044_d) John Joseph Pudloski
731_z (1045_d) Simeon Zebina
732_z (1046_d) Martin Walter Rotter
733_z (1047_d) Kyle Freed
734_z (1048_d) Wendy Beck
735_z (1049_d) Mark Lentz
736_z (1050_d) Francisco A Lopez Frutos
737_z (1051_d) Harper Maiorino Chisari
738_z (1052_d) Fon Woolridge
739_z (1053_d) Valters Liberts Muzikants
73↊_z (1054_d) Alfredo B Bear-Lacis
73↋_z (1055_d) Alex M Hare
740_z (1056_d) Heather Weed
741_z (1057_d) Ariel Ortiz Beltran
742_z (1058_d) Alex Hare
743_z (1059_d) J Alex Mashett
744_z (1060_d) Alex Dzurick
745_z (1061_d) Charounson Saintilus
746_z (1062_d) Benjamin Jeffrey Mennenga
747_z (1063_d) Mason H Nakamura
748_z (1064_d) Thomas Lennix Joseph Faulkner
749_z (1065_d) Matt Goldman
74↊_z (1066_d) Andon Epp
74↋_z (1067_d) Olivia N Gunther
750_z (1068_d) Natalie Pendragon
751_z (1069_d) Raphael Seitz
752_z (1070_d) Joshua Dean Cannon
753_z (1071_d) Kevin S Barlow
754_z (1072_d) Seth Lutz
755_z (1073_d) Clayton P Allred
756_z (1074_d) Gaston Octavio Lacayo
757_z (1075_d) Bryan Shennum Dierking
758_z (1076_d) Amir Alizada
759_z (1077_d) Alessandro Gabriel Reinares
75↊_z (1078_d) Kevin James Tracy
75↋_z (1079_d) Lawrence M An
760_z (1080_d) Piper Keefer
761_z (1081_d) Joshua T Taylor
762_z (1082_d) Daniel Flaum
763_z (1083_d) Thomas Anthony Simbulan
764_z (1084_d) William Bruce Carter
765_z (1085_d) Alexandre de Spindler
766_z (1086_d) Kumar Nilesh
767_z (1087_d) Toby Bell
768_z (1088_d) Victor Czerniak
769_z (1089_d) Cillian Fintan Conlon
76↊_z (1090_d) Dan Stenger
76↋_z (1091_d) Kasie M Moon
770_z (1092_d) Wallace Klayton Brewer
771_z (1093_d) Rohan Bafna
772_z (1094_d) Worramait Kositpaiboon
773_z (1095_d) Lammert Jan Broekema
774_z (1096_d) Nathaniel Reid Zeiger
775_z (1097_d) Ryan Khang Truong
776_z (1098_d) Ian Syndergaard
777_z (1099_d) Gregory Kai
778_z (1100_d) Christopher E. Repetto
779_z (1101_d) Vincent Lenart
77↊_z (1102_d) Ulf Celion
77↋_z (1103_d) Elijah C Rodgers
781_z (1105_d) Ethan J Alvaré
782_z (1106_d) Cole Ethan Young
783_z (1107_d) Brian Lawrence Frye
784_z (1108_d) John David Bergmayer
785_z (1109_d) Eric A Larsen
786_z (1110_d) Manuel Ulliac
787_z (1111_d) Stephen Chrisomalis
788_z (1112_d) Miles Bradley Huff
789_z (1113_d) Stephen M Bell
78↊_z (1114_d) Nathan Yax
790_z (1116_d) Christopher Zahn
792_z (1118_d) Ashby Lowell Teegan
793_z (1119_d) spiro andritsis
794_z (1120_d) Chris Estes
795_z (1121_d) Matthew Dods
796_z (1122_d) Steve Robert Frandsen
797_z (1123_d) Gabel Harold
798_z (1124_d) Michael Ignacio Basulto
799_z (1125_d) Daniel Boemio
79↊_z (1126_d) Nicholas K Fox
79↋_z (1127_d) Christina I Gardner
$7\text{↊}0_z$ (1128_d) David A Ling
$7\text{↊}1_z$ (1129_d) Nancy L Ling
$7\text{↊}2_z$ (1130_d) Robert T Pero
$7\text{↊}3_z$ (1131_d) Nathan Nissen
$7\text{↊}4_z$ (1132_d) Xander Ultsch
$7\text{↊}5_z$ (1133_d) Angi Permana

In memory of

Gene Zirkel

Member No. 67_z

56_z Years Dedicated Service
Fellow of the Society
Past President
Beloved Teacher
Dear Friend

Our entire next issue will be a special tribute to Gene

Split-Promote-Discard

by Treisaran

To born decimalists (as I expect the majority of us are) perhaps the greatest shock of transitioning to dozenal is the extreme shift in the usability of the prime factor 5. In decimal, 5 occupies the most privileged position for a number in a given base, that of a divisor. In dozenal, it passes to the other extreme of not even being a neighbor of the base. Twelve is a member of the set $5n \pm 2$, which means that even the tricks of a neighbor relationship, like those used in decimal to deal with the prime factor 3, are unserviceable in dozenal.

That being said, I think dozenalists can grant a significant difference between dealing with 3 as a non-divisor as opposed to 5. Notwithstanding the extremity of the case of $5n \pm 2$ in dozenal, the prime factor 3 is so important that even the best case in decimal, that of dividing $r - 1$ (the omega totative, to use former DSA editor Michael De Vlieger's terminology), is not good enough. Thirds are such important fractions, that they may well be the single most compelling reason to favor dozenal over decimal; making their point-form fractions terminating is so critical, that not even the minimal recurrence of one digit in decimal ($0.\overline{3}_\text{d}$) is satisfactory. Not so for fifths. Decimal inflates their importance, due to the royal status 5 enjoys as a divisor of the base; in dozenal, they deflate to their true importance. There are uses for the prime factor 5, such as quintiles in statistical distributions, but fifths are nowhere near as frequently needed as thirds. The compromises necessary to make 5 usable in dozenal are much more acceptable than the workarounds for 3 in decimal. In this article, I will lay out a workable test for divisibility by 5 in dozenal.

Divisibility tests have long attracted my disordered interest, but it was De Vlieger's systematic work on number bases that has made me delve into them in earnest. De Vlieger categorized numbers in relations to the base as follows:

- Divisor: Divides the base (2 and 5 in base ↊; 2, 3, 4 and 6 in base 10_z)
- Semidivisor digit or regular number: Does not divide the base, but all its prime factors are shared with the base (4, 8, 16_d and 20_d in decimal; 8, 9, 14_z and 16_z in dozenal)
- Totative digit or coprime number: Has a prime factor not in the base (3, 7 and 11_d in decimal; 5, 7, ↋ and 11_z in dozenal)
- Semitotative digit or semi-coprime number: Has a mixture of prime factors, some shared and some not shared with the base (6 and 14_d in decimal; ↊, 12_z and 13_z in dozenal)

In addition to those natural categories, De Vlieger also added the helper categories of neighbor relationships:

- Omega totative: For any base r, this is $r - 1$, one less than the base (9 in decimal, ↋ in dozenal)
- Alpha totative: For any base r, this is $r + 1$, one more than the base (the number written "11" in any base)

Crucially, such neighbor coprimes are governed by inheritance: if the neighbor coprime is composite, then its rules apply to its factors. In decimal, therefore, the benefits of 9 as an omega totative also apply to its factor 3. For divisibility testing, the omega totative relationship means one can test for divisibility by the number by summing its digits until a short number immediately recognized as divisible or indivisible is attained. This is why the digit-sum test for divisibility by 3 works in decimal: it is actually the "decimal rule of 9"; it is because it is the "decimal rule of 9" that it is also the "decimal rule of 3," not the other way round. By the same token, the digit-sum test for 3 and 5 in unquadral (hexadecimal) works because $3 \cdot 5$ is F_x, the unquadral omega totative, while 9 is left without a workable divisibility test in unquadral.

So, for any base that is not a multiple of 3, we have either an $r - 1$ or an $r + 1$ relationship with 3, giving us the digit-sum test in the former case, or the alternating digits test[1] in the latter. There will always be a usable divisibility test for 3, although, because of the importance of this factor, especially its fractions, people will want better. We know this because we still use the Babylonian base 60_d, divisible by 3, for angles and time.

But what are we going to do about the case of $5n \pm 2$? The neighbor relationships are of no help in dozenal; its two neighbors are the high, unimportant primes ↋ and 11_z, and because they are primes, there are no factors inheriting them. Michael De Vlieger, in his DSA FAQs, expressed his frustration at that; indeed many dozenalists, including me, have thought it a veritable pity the way dozenal alienates the prime 5. We may do without 7, and certainly without primes higher than 7, but a little something for 5 would be desirable.

In the DSA FAQs, De Vlieger devised two tests for divisibility by 5 in dozenal based on modular arithmetic. Actually all divisibility tests have a basis in modular arithmetic, but the ones we use most—the divisor, regular and neighbor tests—are shortcuts that take away the complexity. De Vlieger's tests were based on the nuts and bolts of modular arithmetic, therefore not so easy to carry out. Still, I wanted to evaluate them; I saw no easy "dozenal rule of 5" forthcoming. Here is the summary of those tests:

- Split the last digit away from the number; multiply it by 3; add it to the number; repeat until you get a recognizable multiple of 5. ($441_z \rightarrow 44_z|1 \rightarrow 44_z + 3 = 47_z$, which divides by 5)
- Split the last digit away from the number; multiply it by 2; subtract it from the number; repeat until you get a recognizable multiple of 5. ($441_z \rightarrow 44_z|1 \rightarrow 44_z - 2 = 42_z$, which divides by 5)
- Split the last two digits away from the number; subtract it from the number; repeat until you get a recognizable multiple of 5. ($441_z \rightarrow 4|41_z \rightarrow 41_z - 4 = 39_z$, which divides by 5)

The first two tests are variants of a single test, called the "trim-right test"; it is probably the most general neighbor test, the father of all neighbor tests. The first variant is based on the fact that 2↋$_z$ $(5 \cdot 7)$ is one less than 3 times the base, and the second, on the fact that 21_z (5^2) is one more than 2 times the base. In other words, those tests are predicated on 5 being the inheritor of one less or one more than a

[1] For those interested, the divisibility test for "11" of the base and any of its factors goes as follows: Take the number to be checked, sum the digits in its odd positions, then sum the digits in its even positions, then subtract the two sums to see if you get a number you recognize as divisible. Example: 2↊3↋3$_z$ gives the sums 8 $(2 + 3 + 3)$ and 19_z (↊ + ↋), whose difference is 11_z, therefore passing the test.

multiple of the base. The main disadvantage of the trim-right or multiple-neighbor test, however, is that it is so slow, as well as error-prone; I used the short number 441_z in my examples, but make the tested number a little longer and the test becomes unbearably tedious.

The second test is based on the fact that 101_z, one more than the square of the base, is divisible by 5 ($5 \cdot 25_z$). Although I would have wished to avoid subtraction, at least multiplication is absent, and the test is much faster, disposing of two digits at a time. I had resigned myself to the fact that this would be as good as it could get for testing divisibility by 5 in dozenal, and started practicing it with longer numbers.

As it so happens in such efforts, I stumbled upon a shortcut that actually made this test easy—nearly as easy as the digit-sum test. Working at first with powers of numbers divisible by 5, I noticed that 4768_z (8000_d) left me with no work to do as it consisted of two consecutive two-digit multiples of 5. I then wished all numbers could be like that, wistfully. Soon enough, however, wistfulness turned into an idea: what if I made it so all numbers would be like that?

It was then that I came upon the missing piece of the puzzle: the "Promote" stage of the SPD method, where SPD stands for "Split, Promote, Discard." The complete test is carried out as follows:

1. Split the last two digits away from the number.
2. Promote those two right-hand digits to a two-digit multiple of 5 by addition or subtraction.
3. Add to or subtract from the left-hand number the same amount.
4. Discard the right-hand number.
5. Repeat until you get a recognizable number.

In order for the test to work, the set of all the two-digit multiples of 5 in dozenal need to be memorized. Part of this set should already be known from the dozenal multiplication table; putting the whole set into a neatly aligned form might help:

$$\left\{ \begin{array}{cccccccccccc} 00, & 05, & 0↊, & 13, & 18, & 21, & 26, & 2↋, & 34, & 39, & 42, & 47, \\ 50, & 55, & 5↊, & 63, & 68, & 71, & 76, & 7↋, & 84, & 89, & 92, & 97, \\ ↊0, & ↊5, & ↊↊, & ↋3, & ↋8 & & & & & & & \end{array} \right\}_z$$

Once this table is committed to memory, the salami-slice procedure of SPD should work smoothly even with long numbers; it is less error-prone than the digit-sum test, for one need never add or subtract more than 4 in the promotion stage. Here is a rundown of SPD at work with $23↊,↊93,854_z$ ($1,000,000,000_d$):

1. Split $23↊↊93854_z$ into $23↊↊938_z$ and 54_z.
2. Promote 54_z to 55_z. Synchronize $23↊↊938_z$ to $23↊↊939_z$.
3. Discard 55_z. Restart with $23↊↊939_z$.
4. Split $23↊↊939_z$ into $23↊↊9_z$ and 39_z.
5. Discard 39_z. Restart with $23↊↊9_z$.
6. Split $23↊↊9_z$ into $23↊_z$ and $↊9_z$.
7. Promote $↊9_z$ to $↊↊_z$. Synchronize $23↊_z$ to $23↋_z$.
8. Discard $↊↊_z$. Restart with $23↋_z$.
9. Split $23↋_z$ into 2 and $3↋_z$.

↊. Promote 3↋$_\text{z}$ to 39$_\text{z}$. Synchronize 2 to 0.

↋. Discard 39$_\text{z}$. The remaining 0 is a multiple of 5. So 23↊↊93854$_\text{z}$ passes the divisiblity test.

The procedure is probably clearer in an animated form; I've prepared an animated GIF for exactly that purpose, available in my profile at the DeviantArt website: http://treisaran.deviantart.com/art/SPD-Test-Guide-large-font-slow-version-310345233.

This, until an easier test is found, can be considered the "dozenal rule of 5"; a neighbor test based on 101$_\text{z}$, one more than the square (r^2+1, or square-alpha) of the base. At first I thought it a fortunate coincidence that the dozenal r^2+1 is a multiple of 5, but it turns out this will be true for any base $r = 5n \pm 2$:

$$\begin{aligned} r^2+1 &= (5n \pm 2)^2 + 1 \\ &= 5^2n^2 \pm 2 \cdot 5n \cdot 2 + 2^2 + 1 \\ &= 5^2n^2 \pm 4 \cdot 5n + 5 \\ &= 5\left(5n^2 \pm 4n + 1\right) \end{aligned}$$

To illustrate this, consider that, in decimal, all $5n$ end in 5 or 0, so all $5n \pm 2$ end in 2, 3, 7 or 8; all squares of such numbers end in 4, 9, 9 or 4 respectively, making them one less than some multiple of 5.

Among bases satisfying $r = 5n \pm 2$, dozenal is small enough to make its table of two-digit multiples of 5 sufficiently compact to memorize. This is also true for octal, and perhaps unhexal (base 16$_\text{z}$ (18$_\text{d}$)). In a larger such base, for instance 40$_\text{z}$ (48$_\text{d}$), the two-digit multiples of 5 would simply be too many to digest. It is really fortunate that dozenal is reasonably sized.

In the broader field of number theory, I think the discovery of SPD now introduces the power-neighbor as a new category to augment De Vlieger's original scheme. My own categorization of number/base relationships is as follows:

- Base-divisor: Divides the base itself (2, 3, 4 and 6 in dozenal).
- Power-divisor: Divides one of the powers ≥ 2 of the base (14$_\text{z}$, 16$_\text{z}$, 23$_\text{z}$, 28$_\text{z}$ in dozenal).
- Base-neighbor: Divides one less (omega) or one more (alpha) than the base itself (↋ and 11$_\text{z}$ in dozenal; 3, 5, F$_\text{x}$ and 11$_\text{x}$ in unquadral).
- Power-neighbor: Divides one less (omega) or one more (alpha) than one of the powers ≥ 2 of the base (5 and 25$_\text{z}$ in dozenal; 7 in both decimal and dozenal; 27$_\text{d}$ and 37$_\text{d}$ in decimal; 7, 9 and D$_\text{x}$ in unquadral).

The "101" of any base is the square-alpha, upon which SPD is based (if only the table is small enough). The "1001" of any base is the cube-alpha, and in some bases the cube-omega is helpful. No base is small enough that its cube-neighbor relationships can yield a memorizable table of three-digit multiples, but we can use the relationship to shorten the tested number by three digits each time (either by subtracting the last three digit from the rest as in the case of testing for 7 in dozenal or decimal, or summing triplets of digits as in the case for 7, 9 and D$_\text{x}$ in unquadral). Once we are left with a three-digit number, we can complete the test by trying to reformulate the number as a sum or difference of two multiples of the factor: for example, 554$_\text{z}$ is divisible by 7 because it is 530$_\text{z}$ + 24$_\text{z}$, a sum of two multiples of 7.

Those, of course, are neither complete nor easy divisibility tests, but primes 7 and higher are not in such demand. Many dozenalists have wished for something to deal with 5 in dozenal, and now we have that. It is not so straightforward a test like the digit-sum test, but it works well once you get the hang of it. In my imagination, in a dozenal civilization the standard dozenal multiplication table would be augmented by the two-digit multiples of 5, as well as the two-digit multiples of 14_z, thus covering a great swathe of divisibility tests: base-divisor tests (2, 3, 4 and 6), power-divisor tests (8, 9, 14_z and 28_z), the power-neighbor test for 5 and combinations for semi-coprimes like ↋ and 13_z. Dozenal thus has all the divisibility tests we need.

Why do mathematicians confuse Christmas and Halloween?

Because

Oct31 = Dec25

In octal, 31 indicates three units of eight and one unit of one. Three units of eight is two dozen (20), or in decimal 24; and one unit of one makes it two dozen and one (21), or in decimal 25.

In decimal, 25 indicates two units of ten and five units of one. Two units of ten is one dozen and eight (18), or in octal 24; and five units of one makes it two dozen and one (21), or in octal 31.

UP THE DOWN STAIR-CASE

Prof. Jay L. Schiffman & Michael De Vlieger

Dozenal Society of America Board of Directors

There is a popular base ten problem that students are commonly assigned in elementary number theory courses. The problem asks if there are any primes in the integer sequence 9_d, 98_d, 987_d, 9876_d, 98765_d, 987654_d, 9876543_d, 98765432_d, 987654321_d, 9876543219_d, 98765432198_d, 987654321987_d, etc., where one cycles around the clock using the nine non-zero digits in reverse order. One discovers that there are no primes in this sequence. In order to see this, observe that the first term 9 is divisible by 3, the terms 98_d, 9876_d, 987654_d, 98765432_d, etc. are even, and the term 98765_d, is divisible by 5. It is well known that any integer is divisible by 3 or 9 if and only if the integer obtained by forming the digital sum is divisible by 3 or 9. As a consequence, the terms 987654321_d and 9876543219_d are divisible by both 3 and 9. If one appends the digits 987_d, 9876_d, 987654_d, 9876543_d, 987654321_d, to the integer 987654321_d, then divisibility by 3 is preserved. Clearly 98765432198765_d is divisible by 5, and $9876543219876543 2_d$ is divisible by 2. Hence there are no primes in this sequence.

The base twelve analogue of this problem seeks to secure any primes in the integer sequence ↋, ↋↊$_z$, ↋↊9$_z$, ↋↊98$_z$, ↋↊987$_z$, ↋↊9876$_z$, ↋↊98765$_z$, ↋↊987654$_z$, ↋↊9876543$_z$, ↋↊98765432$_z$, ↋↊987654321↋$_z$, ↋↊987654321↋↊$_z$, ↋↊987654321↋↊9$_z$, etc. This problem was posed a number of years ago in the “Problem Corner” column in a previous issue of the *Duodecimal Bulletin*. (See reference 2). Charles S. Ashbacher, the fine editor of the *Journal of Recreational Mathematics*, wrote a computer program which discovered two solutions: the obvious ↋, as well as ↋↊98765$_z$. (See reference 3.)

The first author has done an intensive investigation of this using Wolfram Mathematica. The second author has written a neat Mathematica program addressing this as well. This work has uncovered two additional solutions in the range $\left[1, 10_z^{214}\right]$. We continue our discussion with some divisibility tests in our favorite number base.

For the base twelve integer sequence ↋, ↋↊$_z$, ↋↊9$_z$, ↋↊98$_z$, ↋↊987$_z$ ↋↊9876$_z$, ↋↊98765$_z$, ↋↊987654$_z$, ↋↊9876543$_z$, ↋↊98765432$_z$, ↋↊987654321$_z$, ↋↊987654321↋$_z$, ↋↊987654321↋↊$_z$, ↋↊987654321↋↊9$_z$, ..., one notes that an integer in base twelve is even if and only if its units digit is 0, 2, 4, 6, 8, or ↊. Hence ↋↊$_z$, ↋↊98$_z$, ↋↊9876$_z$, ↋↊987654$_z$, ↋↊98765432$_z$, and ↋↊987654321↋↊$_z$ are even. Moreover, an integer is divisible by 3 if and only if its units digit is 0, 3, 6, or 9. Hence, ↋↊9$_z$, ↋↊9876$_z$ (already divisible by 2), ↋↊9876543$_z$, and ↋↊987654321↋↊9$_z$ are divisible by 3. The only possible primes in the sequence involve integers whose units digits are 5, 7, or ↋.

We note that if we have all the digits in a grouping in consecutive descending order starting with ↋, then the integer is divisible by ↋, for we have all the digits $↋↊987654321_z$, and their sum is 56_z, which is divisible by ↋. In addition, in any positive integer base $b : b \geq 2$, an integer is divisible by $b-1$ if and only if the sum of the digits of the integer is divisible by $b-1$. Hence any integer in base twelve is divisible by eleven if and only if the sum of the digits of the integer is divisible by eleven. This takes care of the terms in the sequence such as $↋↊987654321_z$, $↋↊987654321↋↊987654321_z$, $↋↊987654321↋↊987654321↋↊987654321_z$, etc. We are thus appending full groupings to those groupings which are divisible by eleven.

Another divisibility test which is useful in passing is that in any positive integer base $b : b \geq 2$, an integer is divisible by $b+1$ if and only if the alternating sum of the digits of the integer starting from the right is divisible by $b+1$. Hence any integer in base twelve is divisible by one dozen one if and only if the alternating sum of the digits of the integer is divisible by one dozen one. For example, the integer $↋↊987654321↋↊987654321_z$ is divisible by one dozen one. To see this, we note that

$$11_z \mid ↋↊987654321↋↊987654321_z \iff$$

$$11_z \mid [1-2+3-4+5-6+7-8+9-↊+↋-1+2-3+4-5+6-7+8-9+↊-↋] \iff$$

$$11_z \mid [1+2+3+4+5+6+7+8+9+↊+↋-1-2-3-4-5-6-7-8-9-↊-↋] \iff$$

$$11_z \mid [56_z - 56_z] \iff 11_z \mid 0$$

One can prove these divisibility tests via congruences related to modular arithmetic. For example, we prove in base twelve that an integer is divisible by eleven if an only if the alternating sum of its digits is divisible by eleven. Let:

$$N = \sum_{i=0}^{n} a_i \cdot 10_z^i = a_n \cdot 10_z^n + a_{n-1} \cdot 10_z^{n-1} + a_{n-2} \cdot 10_z^{n-2} + \cdots + a_2 \cdot 10_z^2 + a_1 \cdot 10_z + a_0$$

Then the following congruences are true:

$$1 \equiv 1 \pmod{↋} \Rightarrow a_0 \equiv a_0 \pmod{↋}$$

$$10_z \equiv 1 \pmod{↋} \Rightarrow a_1 \cdot 10_z \equiv a_1 \pmod{↋}$$

$$10_z^2 \equiv 1^2 = 1 \pmod{↋} \Rightarrow a_2 \cdot 10_z^2 \equiv a_2 \cdot 1 = a_2 \pmod{↋}$$

$$\ldots$$

$$10_z^{n-2} \equiv 1^{n-2} = 1 \pmod{↋} \Rightarrow a_{n-2} \cdot 10_z^{n-2} \equiv a_{n-2} \cdot 1 = a_{n-2} \pmod{↋}$$

$$10_z^{n-1} \equiv 1^{n-1} = 1 \pmod{↋} \Rightarrow a_{n-1} \cdot 10_z^{n-1} \equiv a_{n-1} \cdot 1 = a_{n-1} \pmod{↋}$$

$$10_z^{n} \equiv 1^{n} = 1 \pmod{↋} \Rightarrow a_{n} \cdot 10_z^{n} \equiv a_{n} \cdot 1 = a_{n} \pmod{↋}$$

Hence

$$N = \sum_{i=0}^{n} a_i \cdot 10_z^i \equiv \left(\sum_{i=0}^{n} a_i \right) \pmod{↋}$$

Our divisibility test for division by eleven in base twelve is now established.

Using Wolfram Mathematica, we found four primes in the range $\left[1, 10_{\text{z}}^{214}\right]$:

1. ↋ (1 dozenal digit)
2. ↋↊98765$_{\text{z}}$ (7 dozenal digits)
3. ↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321 . . .
   ↋↊987654321↋↊987654321↋↊987654321↋↊987$_{\text{z}}$ (88$_{\text{z}}$ dozenal digits)
4. ↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321 . . .
   ↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321 . . .
   ↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321 . . .
   ↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊987654321 . . .
   ↋↊987654321↋↊987654321↋↊987654321↋↊987654321↋↊98765$_{\text{z}}$ (214$_{\text{z}}$ dozenal digits).

The Mathematica program, using the command `IntegerDigits`, enables one to convert from base ten to any other number base of one's choosing, while the command `FromDigits` converts any numeral in a different number base to base ten. An analysis of the commands is furnished in my article "Number Base Conversion with a Computer Algebra System" (see reference 4).

References

1. Mathematica 9.0, Wolfram Research Inc., Champaign IL, 11↋↋$_{\text{z}}$ (2015$_{\text{d}}$).
2. Ashbacher, Charles, "Solution to Problem Corner, Problem 3," *The Duodecimal Bulletin*, Whole Number 78$_{\text{z}}$, page 1↊$_{\text{z}}$, 11↊5$_{\text{z}}$ (1997$_{\text{d}}$). (https://tinyurl.com/yyghgaqv)
3. Schiffman, Jay L., "Problem Corner," *The Duodecimal Bulletin*, Whole Number 77$_{\text{z}}$, page 20$_{\text{z}}$, Problem 3, 11↊4$_{\text{d}}$ (1996$_{\text{z}}$). (https://tinyurl.com/y5xcnolw)
4. Schiffman, Jay L., "Number Base Conversion with a Computer Algebra System," *The Duodecimal Bulletin*, Whole Number 83$_{\text{z}}$, page 15$_{\text{z}}$-16$_{\text{z}}$, 11↊8$_{\text{z}}$ (2000$_{\text{d}}$). (https://tinyurl.com/y4wldbfl)

The Duodecimal Bulletin, Vol. 1, No. 2, July 1161_z (1945_d).

THE UNCIA·METRIC SYSTEM[1]

A Dozenal System of Weights and Measures

by Ralph E. Beard

> The ensuing article is submitted by Mr. Beard as a basis for discussion and consideration. It does not express the views of the Society, nor of its Committee on Weights and Measures,of which Mr. Beard is Chairman.

Progress in the organization of the world as an economic whole is forcing consideration of a system of weights and measures that shall be standard for the entire world. Daily, the need of a world standard becomes more apparent. Yet none of the present official standards seem acceptable for this purpose.

Duodecimals offer a solution of this problem that is amazingly simple. With minor modifications, the Anglo-American standards of weight and measure can be integrated and organized into an ideal unified duodecimal metric system. (The word, "metric," will be used in this article in its normal sense, as meaning "measuremental.") The importance of the problem today, emphasizes the necessity for serious consideration of this possibility.

As the only set of standards that can be properly called a system, primary consideration is given to the French Decimal Metric System. It has become the official system of many countries, and its use is permitted in nearly all the rest. Yet, where the Anglo-American measures are commonly used, the French Metric System has made little progress in supplanting them in the eighty years of competitive use.

The English and American standards have achieved wide recognition because of the preponderance of England and America in world production and world trade. The scales of their measures and the sizes of their units are convenient for practical use in measuring things. However, because these standards are relatively unsystematized, they are unsatisfactory for scientific applications.

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[1] *EDITOR'S NOTE:* This article is something of a what-if thought experiment. It was originally published under the title "The Do-Metric System," in the very second issue of the *Duodecimal Bulletin*. But what if, rather than their do-gro-mo scheme, the founders of the Dozenal Society of America had access to Systematic Dozenal Nomenclature (SDN)? (See the SDN Summary on page 31_z.) SDN features the Latin *uncia* as its first negative power prefix. Rather than call their system "Do-Metric," they might have called it "Uncia·Metric." I have redacted this article accordingly, coloring the text blue wherever it now diverges from the original. Also, the *Bulletin* during that era marked dozenal numbers by italicizing them, but did not mark decimal numbers in any way; I have replaced this with the default base annotation scheme described on the copyright page (but have not highlighted those redactions). Anyone wishing to read the article in its original unredacted form, may easily find it at https://tinyurl.com/ybpwz9tf.

The French Metric System offers two great advantages. Its components do constitute a *system*, in that the measures of area, length, capacity and weight are interrelated, permitting ready conversion between them without complicated mathematical operations. And, secondly, it is a *unified* metric system, in that the scales of its measures conform to the number system. Both use the base ten.

Yet the way in which these advantages are fitted into the French Metric System has created the obstacles which have blocked its progress into general use. The sizes selected for the basic units of the system are not well adapted for practical application in trade and commerce. Their sizes have not been determined by a long process of selective survival in practical competition, as is the case with the Anglo-American units. Moreover, the scale of ten is awkward in actual use in weighing and measuring things. It has too few factors. It is not flexible enough in subdivision.

Sidney A. Reeve has admirably summarized the metric controversy in a terse statement:

> The reasons for both the continued advocacy and the continued rejection of the metric system are plain. They are parallel and quite compatible.
>
> 1. The metric system is attractive because its measures are arranged on the same system as our numerical notation.
> 2. The metric system is cumbrous because it is decimal in its arrangement.

We are confronted, then, with this impasse. A world standard of weights and measures is necessary. This standard should constitute a unified metric system, whose units are convenient in practical use, whose scales accommodate ready subdivision into halves, thirds, and quarters, and whose components are precisely integrated. None of the present official standards meets these requirements. None shows any real promise of becoming the world's standard. Yet there is a fully adequate solution to this problem.

Today, there is a growing interest in the use of base twelve in numeration. It is generally recognized that counting by dozens offers many advantages over counting by tens. It is to be expected that the change from tens to twelves may take a long time, but, since the dozen base is better, ultimately the change is inevitable.

With this change, then, there is available to use a unified metric system whose units are accustomed, convenient and practical in size, whose scales facilitate easy subdivision, and whose elements are precisely integrated. This duodecimal metric system, termed "The Uncia·Metric System," offers excellent potentialities for adoption as the world standard.

In the interim, while we continue to count by ten, the same units of weight and measure form a simple measurement system that should prove advantageous and popular. All of its scales would be arranged in steps and subdivisions of twelve, but the numeration would be decimal.

For those reasons it seems justifiable to propose that these weights and measures be legalized as standards for permissive use, and be granted official recognition. In selected applications, these standards will be found of immediate advantage, and can begin to earn their way into popular favor. Thus the initial step toward establishment of a world standard can be accomplished.

BASIC DEFINITIONS

The *Yard* will be the base of the Uncia·Metric System. This is the familiar English and American yard, whose relation to the meter is established as 25.4_d millimeters to the inch, in accordance with standard manufacturing practice. The inch and the foot will be retained exactly as they are. Their subdivisions will acquire new names, but their present scale divisions will coincide with divisions of the new scales.

A new set of metric prefixes will be used, paralleling in form the prefixes of the French Metric System. Steps and scale divisions will be in twelfths and multiples of twelve.

The following will illustrate the new Uncia·Metric prefixes:

10_z	yards	equal	1	unqua·yard
10_z	unqua·yards	equal	1	biqua·yard
10_z	biqua·yards	equal	1	triqua·yard
0.1_z	yard	equals	1	uncia·yard
0.1_z	uncia·yard	equals	1	bicia·yard
0.1_z	bicia·yard	equals	1	tricia·yard

…

… Thus, 1 triqua·yard equals 1 mile (the Uncia·Metric mile being 1728_d (1000_z) yards instead of 1760_d (1028_z).)

LINEAR MEASURE

By basing the duodecimal measures on the yard, rather than on the foot, we are able to secure the advantages of a duodecimal relation with the measures of weight and capacity, the pint and the pound. This was first proposed, we believe, by Sidney A. Reeve, and later by Admiral Elbrow and George Terry.

The first ordinate subdivisor of the yard (uncia·yard) is the Palm, the familiar unit of 3 inches. The cubic palm is the new pint, being 23_z (27_d) cubic inches instead of 24.↊6$_z$ (28.875_d). This pint of water weighs the new pound, which is three percent lighter than the pound avoirdupois, being 3↋49$_z$ (6825_d) grains. Thus our correlatives are the Palm, Pint, and Pound.

It is important that the smallness of these changes be recognized and adequately evaluated. And instead of being new values, these changes restore to our accustomed measures the original orderliness. The cubic foot, or twelve-inch cube, was the old amphora, the six-inch cube was the gallon, and the three-inch cube was the pint, which weighs one pound. Considering the minor changes involved, it is surprising that these measures were not restored to their original sizes long ago.

Ordinate units are arranged in steps of 0.1_z (uncia·) or 10_z (unqua·). Basic units are arranged in steps of 0.001_z (tricia·) or 1000_z (triqua·). The ordinate subdivision of the palm is the Quan (uncia·palm), or quarter-inch. The quan is equal to 3 lines.

Originally, twelve points equaled one line, and twelve lines equaled one inch. The present typographical "point" is approximately double the original. The uncia·metric scale uses the original point in the subordinate duodecimal series of point, line, inch, and foot.

The ordinate subdivision of the quan (uncia·quan) is the Karl, or quarter-line. This is also one of the basic units, being a tricia·yard. It should also be noted that, using the SDN prefixes, alternate names are available for all these quantities.

```
The Palm is also the  uncia·yard or the  biqua·karl
The Quan is also the  bicia·yard or the  unqua·karl
The Karl is also the tricia·yard or the triqua·cad
```

Standards of length are nowadays defined as so many wavelengths of the red line of the cadmium spectrum. The basic subdivision of the karl (0.001_z karl) is approximately half ($0.685\mathcal{E}_z$) of this wave-length, and for this reason is termed the Cad.

It is important to realize that our customary subdivisions of the inch correspond exactly with scale divisions of the new measures:

1/2	inch equals	2_z	quans
1/4	inch equals	1_z	quan
1/8	inch equals	6_z	karls
$1/16_d$	inch equals	3_z	karls
$1/32_d$	inch equals	16_z	biqua·cads
$1/64_d$	inch equals	9_z	biqua·cads

and that machinist's decimal subdivisions of the inch are within practical tolerances of dozenal subdivisions.

7_z	unqua·cads equals	1.0127_d	milli·inch
1_z	uncia·cad equals	1.0047_d	micro·inch

The foot, the inch, the line, and the point, constitute an interior dozenal series which is intermediate to the dozenal ordinal units. In itself, this interior series affords the extra advantage of the ease of accustomed units which are still commensurate and interchangeable with the ordinate system.

LINEAR TABLE

Basic Units
Arranged vertically in steps of 1000_z

1000_z Cads	equal	1 Triqua·cad	or Karl
1000_z Karls	equal	1 Triqua·karl	or Yard
1000_z Yards	equal	1 Triqua·yard	or Mile

Ordinate Units
Arranged vertically in steps of 10_z

10_z Karls	equal	1 Quan
10_z Quans	equal	1 Palm
10_z Palms	equal	1 Yard

Intermediate Units
Arranged vertically in steps of 10_z

4 Biqua·cads	equal 1 Point,	and 3 Points	equal 1 Karl
4 Karls	equal 1 Line,	and 3 Lines	equal 1 Quan
4 Quans	equal 1 Inch,	and 3 Inches	equal 1 Palm
4 Palms	equal 1 Foot,	and 3 Feet	equal 1 Yard

Each ordinate linear unit represents a “place” in dozenal figures. For instance:

1.894_z yard means 1 yard, 8 palms, 9 quans, and 4 karls, and
1.483_z foot means 1 foot, 4 inches, 8 lines, and 3 points.

And note that conversions among these terms is accomplished by merely shifting the “uncial” point; the stated 1.894_z yard also means 18.94_z palms, or 189.4_z quans, or 1894_z karls; and 1.483_z foot also means 14.83_z inches, or 148.3_z lines, or 1483_z points.

The uncia·metric Acre is the area of the square whose side is 60_z yards. The present acre is not the square of anything.

The length of the atomic bond, as measured between atoms in the pure carbon of the diamond, is $0.↊56_z$ tricia·cad.

SQUARE MEASURE

Basic Units
Arranged vertically in steps of $1000^2{}_z$, or $1{,}000{,}000_z$

$1{,}000{,}000_z$ square Cads	equal	1 square Karl
$1{,}000{,}000_z$ square Karls	equal	1 square Yard
$1{,}000{,}000_z$ square Yards	equal	1 square Mile

Ordinate Units
Arranged vertically in steps of $10^2{}_z$, or 100_z

100_z square Karls	equal	1 square Quan
100_z square Quans	equal	1 square Palm
100_z square Palms	equal	1 square Yard

Intermediate Units
Arranged vertically in steps of $10^2{}_z$, or 100_z

14_z sq. Biqua·cads	equal 1 sq. Point,	and 9 sq. Points	equal 1 sq. Karl
14_z sq. Karls	equal 1 sq. Line,	and 9 sq. Lines	equal 1 sq. Quan
14_z sq. Quans	equal 1 sq. Inch,	and 9 sq. Inches	equal 1 sq. Palm
14_z sq. Palms	equal 1 sq. Foot,	and 9 sq. Feet	equal 1 sq. Yard

The area of the uncia·metric Acre is 30_z sq. unqua·yards, and equals the area of a square whose side is 6 unqua·yards. There are 400_z acres to the sq. mile.

CUBIC MEASURE

Basic Units
Arranged vertically in steps of $1000^3{}_z$, or $1{,}000{,}000{,}000_z$

$1{,}000{,}000{,}000_z$ cubic Cads	equal	1 cubic Karl
$1{,}000{,}000{,}000_z$ cubic Karls	equal	1 cubic Yard
$1{,}000{,}000{,}000_z$ cubic Yards	equal	1 cubic Mile

Ordinate Units
Arranged vertically in steps of $10^3{}_z$, or 1000_z

1000_z cubic Karls	equal	1 cubic Quan
1000_z cubic Quans	equal	1 cubic Palm
1000_z cubic Palms	equal	1 cubic Yard

Intermediate Units
Arranged vertically in steps of $10^3{}_z$, or 1000_z

54_z cu. Biqua·cads	equal 1 cu. Point,	and 23_z cu. Points	equal 1 cu. Karl
54_z cu. Karls	equal 1 cu. Line,	and 23_z cu. Lines	equal 1 cu. Quan
54_z cu. Quans	equal 1 cu. Inch,	and 23_z cu. Inches	equal 1 cu. Palm
54_z cu. Palms	equal 1 cu. Foot,	and 23_z cu. Feet	equal 1 cu. Yard

CAPACITY MEASURE

The unit of coordination for the uncia·metric measures is the cubic palm. A cubic palm of water, at the temperature of its maximum density, and normal barometric pressure, is the capacity of the uncia·metric Pint and the weight of the uncia·metric Pound.

10_z Dribs	equal 1 Dram
10_z Drams	equal 1 Founce (fluid – ounce)
10_z Founces	equal 1 Pint
10_z Pints	equal 1 Sigal (sesqui – gallon)
10_z Sigals	equal 1 Kin
10_z Kins	equal 1 Tun

Intermediate Units

3 Founces	equal	1 Gill
4 Gills	equal	1 Pint
2 Pints	equal	1 Quart
4 Quarts	equal	1 Gallon (216_d cu.in.)
6 Quarts	equal	1 Sigal
8 Gallons	equal	1 cu. Foot

Correspondence

The Drib	is the volume of 1 cu. Quan, and weighs 1 Carat
The Pint	is the volume of 1 cu. Palm, and weighs 1 Pound
The Tun	is the volume of 1 cu. Yard, and weighs 1 Ton

WEIGHT MEASURE

10_z Carats	equal	1 Gram
10_z Grams	equal	1 Ounce
10_z Ounces	equal	1 Pound
10_z Pounds	equal	1 Stone
10_z Stones	equal	1 Burden
10_z Burdens	equal	1 Ton

TIME AND THE CIRCLE

The use of separate standards for the measurement of time, of latitude and longitude, and of the circle, is not only unnecessary, but is excusable only on the grounds of habit and custom. The increasing use of measurements of time and angular motion, and of the time units in combination with other measures, requires a unified standard for such measurements. This was first proposed by George S. Terry.

The fundamental unit of the uncia·metric unified time and circular measure will be the Day, representing the mean solar day of twenty-four hours, and the 360°_d circle as well.

The first ordinate subdivision is the Duor. This unit of two hours, or 30°_d, is already used as a time unit in some oriental countries, where the complete rotation of the earth is divided into twelve *shí*. As a unit of angular measure it is very convenient, since the most frequently used angles are simple multiples and parts of this unit.

The duor is composed of twelve Temins. The temin is ten of our accustomed minutes, and is subdivided into twelve Minettes. Each Minette is fifty seconds of time, and the minette, being one tricia·day, is the second basic unit.

The third basic unit, the tricia·minette, is termed the Vic, because it is, very nearly, the vibration period of $C^\sharp{}_0$, of the standard diatonic scale.

The ordinate subdivisions between the minette and the vic, are the biqua·vic and the unqua·vic. The present nautical mile is one minute of circular measure, or of arc. The biqua·vic, being 1.04_d minutes of arc, will be the new nautical mile. The present nautical mile is 6080.2_d feet, or 1.15_d land miles. The new nautical mile is 6333.6_d feet, or 1.2_d land miles.

The unqua·vic is about 1/3 second (0.3472_d) of time, and will probably be the unit generally used for small time measurements.

Basic Units
Arranged vertically in steps of 1000_z

1000_z Vics	equal 1 Minette
1000_z Minettes	equal 1 Day

Ordinate Units
Arranged vertically in steps of 10_z

10_z Vics	equal 1 Unqua·vic
10_z Unqua·vics	equal 1 Biqua·vic
10_z Biqua·vics	equal 1 Minette
10_z Minettes	equal 1 Temin
10_z Temins	equal 1 Duor
10_z Duors	equal 1 Day

Tables of the natural functions of angles, of the log functions, and the numerical logs to 9 duodecimal places, may be found in George S. Terry's monumental work, "Duodecimal Arithmetic". Mr. Terry uses the duodecimal fraction of the circle for the arguments of his tables, but it should be noted that the "places" of duodecimal fractions also represent the units of the uncia·metric measure. For example: the angle $.87Ɛ,653_z$ is also 8 duors, 7 temins, Ɛ minettes, 6 biqua·vics, 5 unqua·vics, and 3 vics.

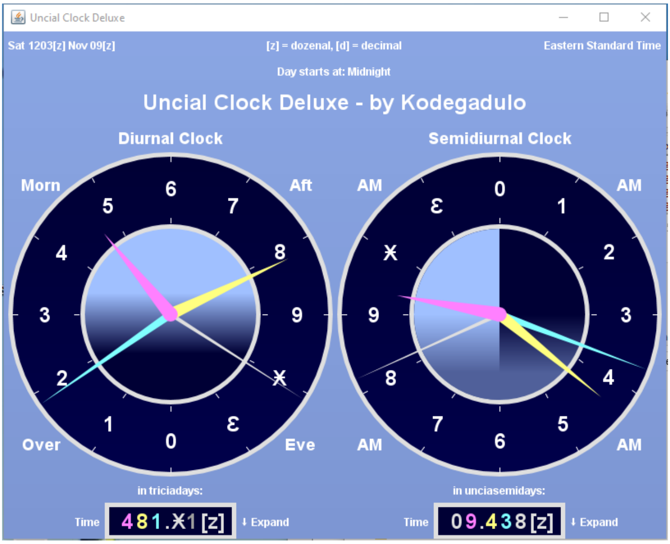
A word should be said about the 24_d hour uncia·metric watch dial. The bottom half of the dial might be given a darker color to indicate the night hours. Most probably the 0 would be located at the bottom of the dial, to indicate midnight, which is the beginning of the day. Noon would then be indicated by the hour hand pointing to 6 at the top of the dial, and the minute hand pointing to 0 at the bottom of the dial.[2] The names of the most important cities might be shown around the rim of the dial, affording a direct reading of their respective local civil times.

If one faced toward the south, the hour hand would move almost exactly with the sun, indicating that one could approximate time quite readily by the sun. Conversely, the watch could be used to some extent as a compass.

TEMPERATURE

The dozenal temperature scales provide 100°_z between the freezing point and the boiling point of water. There are two dozenal temperature scales:

[2] *EDITOR'S NOTE:* This very configuration and coloration scheme for a "Diurnal" clock is supported by my UncialClockDeluxe Java application, which you can download as an executable jar file from https://sourceforge.net/projects/uncialclock/files/UncialClockDeluxe-12000418.jar/download. See page 22_z for an illustration.



UncialClockDeluxe Java app displaying Diurnal time (for Uncia·Metric or Primel) and Semidiurnal time (for TGM). Diurnal dial configured with midnight 0 at bottom, noon 6 at top, darker coloration in bottom half indicating nighttime. All as suggested by Ralph Beard in 1161_{z} *(*1945_{d}*).*

The Popular scale, using 0° as the freezing point, and 100°_{z} as the boiling point; and the Scientific Scale, using Absolute Zero as 0°.

	Centigrade Scale	Fahrenheit Scale	Dozenal Scientific Scale	Dozenal Popular Scale
Absolute Zero	$-273.18^{\circ}_{\text{d}}$	$-459.72^{\circ}_{\text{d}}$	0.00°_{z}	$-289.46^{\circ}_{\text{z}}$
Water Freezes	0.00°_{d}	32.00°_{d}	$289.46^{\circ}_{\text{z}}$	0.00°_{z}
Normal	20.00°_{d}	68.00°_{d}	2Ɛ2.21$^{\circ}_{\text{z}}$	24.97°_{z}
Blood Heat	37.00°_{d}	98.60°_{d}	312.7↊$^{\circ}_{\text{z}}$	45.34°_{z}

To convert from Centigrade to the dozenal Popular Scale, use the same methods as you would to convert any decimal number to dozenal figures. The reverse is also valid. The same procedure applies for conversions between the Kelvin, or Absolute Scale, and the dozenal Scientific Scale.[3]

To convert from Fahrenheit to the dozenal Popular Scale, subtract 32°_{d} and decimally multiply the remainder by 0.8_{d}, then convert the result to dozenal

[3] *EDITOR'S NOTE:* The Celsius or Kelvin hectodegree is indeed identical to the Uncia·Metric biqua·degree, so converting between those is indeed just a matter of converting the base of the numeral. However, to convert between the respective *degrees*, we must shift the scale two orders of magnitude to the left, convert bases, then shift the scale two orders to the right again.

figures. To convert from the dozenal Popular Scale to Fahrenheit, convert from dozenal to decimal figures, mulitply decimally by 1.25_{d}, and add $32°_{\mathrm{d}}$ to the result.

…[4]

EPILOGUE

There are many measures derived from the fundaments of size, weight, time and temperature. For convenience in use, they are defined in a great variety of ways. The units of work, force, flow, energy and momentum, for instance, would fill an extensive index, and they differ widely in size among themselves.

In this summation of the uncia·metric measures, no proposal for these terms is included. The bases for their determination have been presented in the foregoing fundamental units. But, since they are derived units, and as their sizes will form an important element of the their practicability, it is felt that their definition should await practical application.

In designing the fundamental units of the Uncia·Metric System, many problems of nomenclature have presented themselves. It is beyond possibility that these have all been happily and adequately handled. Names are of secondary importance. But the units selected and defined seem in themselves relatively inescapable and ultimate.

Criticism of any of these proposals, as well as comment and suggestion, will be most welcomed. One could not work long with dozenals and preserve much of an attitude of omniscience. What is most desired is their practical use.

In all history there has been no people to whom a natural and flexible metric system possessed equal importance, no people to whom the implications of a world standard of weights and measures offered greater opportunities than to ourselves. There has been no time when the urgency of the requirement for a unified metric system was greater than today.

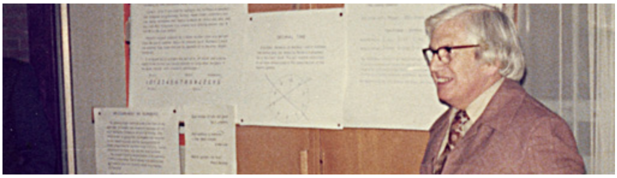
As the greatest makers of tools, and the greatest users of tools that the world has ever known, to us the perfection of our most important tool, our system of weights and measures, is of greatest importance.

Because of the importance of this problem, you should consider this proposal as addressed to you, yourself, personally. It is your comment and your criticism that will aid in eliminating the faults and omissions you may have observed.

[4] Section on CURRENCY omitted, as being too out-of-date.

FROM THE ARCHIVES:

The Duodecimal Review, Number 22_z, Year. Ɛ, No. 1, July 1181_z (1969_d).



"THE DOZEN, AND METROLOGY" by TOM PENDLEBURY[1]

This article attempts to analyse the situation regarding dozenal numeration and metrology, and reveal facts, in order to help the formation of policy.

1. The first question that comes to mind is: Should the dozenal movement be looking for one system and one only, or is there room for more than one?

1.1 A layman measures his room and finds it is twelve by sixteen feet. Is there anything wrong in his putting that dozenally as one-dozen by one-dozen-four *feet*? If he weighs ten stones (140_d lb), why should he not say he weighs eleven-dozen-eight ($Ɛ8_z$) pounds? Or that his battery is three-dozen-two *amp-hour* capacity? After all, these units exist, and are likely to for a long time to come.

1.2 On the other hand, an engineer trying to ascertain whether this battery is adequate to turn the starter motor has to express the volts, amps and hours in terms of power and starting torque, etc. volts × amps = watts = newton-metres per second. Now newtons are *kilo*grammes times 9.80665_d metres per second per second, and *kilo* means 'thousand', which in dozenal becomes $6Ɛ4_z$. There are sixty times sixty seconds in an hour which becomes five-dozen times five-dozen or 2100_z dozenally. This is just one example. Before such problems can be tackled in dozenal numeration a new metrology is required, co-ordinating the units on the dozenal base. It inevitably leads to the creation of some completely new units, but at the same time it makes

[1] *EDITOR'S NOTE:* Tom Pendlebury used a number of idioms and styles that were common during his era but which new readers of the *Bulletin* may find unfamiliar. Therefore I have taken the liberty to redact his text to use the idioms and styles visible elsewhere in the current publication. For instance, where Pendlebury annotated dozenal numbers with a preceding asterisk (*), and decimal numbers with a preceding slashed hyphen (⌿), I have replaced this with the default annotation scheme described on the copyright page. Where Pendlebury used the abbreviation "zen," I have replaced this with the full word "dozen." Pendlebury also invented his own somewhat odd set of dozenal-metric power prefixes; I have replaced these with Systematic Dozenal Nomenclature (see page 31_z), following the lead of Don Goodman III's very good book *TGM: A Coherent Dozenal Metrology* (http://dozenal.org/drupal/sites_bck/default/files/tgm_0.pdf). (As it stands, this article describes a somewhat early version of his prefixes, not reflecting the final forms he eventually developed.) I have also replaced Pendlebury's superscript/subscript abbreviation style with suggested SDN prefixes (see page 31_z). All text redacted from the original has been marked in blue. Pendlebury's original article can be found in its original form at http://www.dozenalsociety.org.uk/archives/DR/Review%4222.pdf.

possible a simpler system, by ironing out the snags and flaws of the decimal metric system (dms).

1.3 Just as the dms becomes impracticable in dozenal numeration, so a new metric system devised for dozenal numeration becomes virtually unworkable in decimal numeration. Such a system therefore can be of use only within the dozenal movement, until our cause is accepted by others.

1.4 On the other hand, since science and engineering cannot be handled in dozenal without such a system, it is essential to our cause. Otherwise the non-dozenal world has a very strong case against us.

1.5 As it is fact that dozenal metric systems have been evolved, the very strong case against us does *not* exist in reality.

1.6 Our layman now finds himself in a dilemma. Should he continue to use existing units and just dozenise the numbers, or should he convert all his data to one of the new systems, and if so, which? The old units are familiar to him, the new, though real in value, can as yet only be conceived in the abstract. Is it not making dozenal unnecessarily complicated to the layman to expect him to convert everything to a new system of unfamiliar units? – at least in the present stage of our development?

1.7 There is a case here for dividing dozenal metrology into two fields: *The Popular Field* and *the Technical Field*.

2. The Popular Field simply accepts *all* existing units and merely dozenises the numbers.

2.1 Locally derived units are possible. Feet can still be squared and cubed, so can miles, metres and kilometres.

2.2 The kilometre = 1000_d metres = $6Ɛ4_z$ metres, so to use the word kilometre to express 1000_z metres is ambiguous and wrong.

The statute mile = 1760_d yards = 1028_z yards, so to call 1000_z yards also a *mile* is equally wrong and misleading.

In short, *established words should convey established meanings*. Their use to express our *new dozenal meanings* is in fact *misuse*.

If we accept *all* existing units into the Popular Field including all the kilo-units, centi-, milli-, micro-, etc, units, new words or additional qualifying words are required to express the dozenal derivatives. A simple set of prefixes is the minimum requirement for this purpose. Examples of this method (using Systematic Dozenal Nomenclature prefixes):

1 *triqua·metre* = 1000_z metres (1 kilometre = 1000_d m = $6Ɛ4_z$ m)

1 *biqua·centimetre* (abbr. 1 b↑cm) = 100_z cm = 1.44_d m (1.54_z m)

To convert *statute miles* to *triqua·yards* (1000_z yd) multiply by 1.028_z.

To convert *statute miles* to *triqua·metres* multiply by 0.↋21$_{z}$

In such examples the dozenal part of the meaning is covered by the new prefixes, while the traditional part of the meaning is covered by the traditional word, with *exact value significance*.

2.3 Since these Popular schemes serve only to bide dozenal over the early stages, while data still pours in in the old units, and since all of them fail to form a comprehensive system for all science, further elaboration of them and vocabulary for them beyond that indicated above servers no useful purpose, and only adds to confusion.

3. The Technical Field has already divided into two channels: the *Great Circle* and the so-called *Gravitational*.

4. The Great Circle is based primarily on the circumference of the Earth. This obviously has advantages for geography and navigation. The question is: is it a suitable starting point for all the other sciences?

4.1 The length of the equator is 40,076,592$_{d}$ m (11,508,580$_{z}$ m). The length of the meridian circle is 40,009,152$_{d}$ m (11,495,540$_{z}$ m)

4.2 J. Essig started with the figure 40$_{d}$ million metres and divided dozenally to give a "metre-duodecimale" of 1.116$_{d}$ m. The circle of forty million metres has no real physical significance since it represents a subterranean circle lying about 3 km below sea level at the poles or ten km down at the equator.

His system is one of the most thoroughly worked out, going well into mechanical and electrical units. The link up between mechanical and electrical units was, however, not rationalized to finality. In justice we must add that he did not claim to have solved the problem completely.

4.3 H. C. Churchman rounded off his unit to make it equal to 3.8$_{z}$ feet, which gives for the sea-level equator 10,014,7↋2.6$_{z}$ unqua·metrons (a unit of twelve metrons of 3.8$_{z}$ inches each), and a meridian circle of ↋,↋↊4,↊3↋$_{z}$ unqua·metron. His Great Circle is an average sea-level circumference.

4.4 T. Pendlebury started with the equator (as given in 4.1 above), first dividing this into two-dozen parts (the others used one-dozen) to accord with the *hour* basis for longitude. Further dozenal division by hexqua· ($10^{6}{}_{z}$) comes to a little under 2 feet, from which he produced the Nafut (short for NAvigational Foot) which was an auxiliary unit close to the Grafut (GRAvity Foot) of his dynamic system. The Equator is 4 s↑Nf exactly. (1 Nf = 0.↋4212↊↊$_{z}$ Gf = 0.9173754$_{d}$ ft). (For satellite orbits T. Pendlebury uses the Grafut for measuring the *radius* from Earth centre. 4 septqua·Grafut radius is within 3 minutes of 1 day orbit).

5. The Gravitational systems start from the dynamic relationship between Force and Mass. Since this relationship is not just confined to gravity, a better name for them is *Dynamic Based Systems*. This term is especially applicable if the system also contains a simple relationship between mechanical force and the electrical units.

5.1 Any system which is to be used throughout science is involved in a Dynamic Network of relationships between its units. A table of this network is given at the end of this article.

5.2 Two of the main links in the network are the natural laws:
(1) Force = Mass × Acceleration (2) Force = Electric current × Magnetic Flux (at right angles to each other).

6. We measure Force and Mass by their effect on each other. When we buy a pound of butter, the weight *one pound* is used to measure the quantity of matter, that is, its Mass. When we hold out our hand to receive the butter, our hand would go down and the butter fall if we did not use a bit more strength in our arm and hand muscles. This 'bit more strength' is 1 lb of Force. If we let the butter fall, it accelerates. Its downward speed starts from 0 and the further it falls the greater the splodge when it lands. This acceleration is caused by gravity exerting 1 lb force, which we had to counteract when holding the butter.

6.1 This *acceleration due to gravity* (gravity itself is force) is called g by scientists, and is 9.80665_{d} metres per second per second, which is the same as 32.1741_{d} feet per second per second. There is a very slight variation, things being heavier at the Poles and lighter at the Equator, but it is so slight that the occasions on which it has to be taken into consideration are very rare. The figures given above are the average figures, and they have been *internationally agreed upon* as a basis for the defining of units of mass and force:
1 lbf (pound force) is that force which when applied to a mass of 1 lb gives it an acceleration of 1 g (as defined by the above figures);
1 kgf (kilogramme-force) is that force which when applied to a mass of 1 kg gives it an acceleration of 1 g.

6.2 The second is not a dozenal division of the day or the hour (and it is not decimal either). What does g come out to when we use say the pentcia·day (that is the mean solar day divided by dozen to the fifth) or the quadcia·hour (the hour divided by dozen to the fourth) as our unit of time?

$$g = \frac{9.80665_{\mathrm{d}}\ \mathrm{m}}{\mathrm{sec}^2} = \frac{32.1741_{\mathrm{d}}\ \mathrm{ft}}{\mathrm{sec}^2} = \frac{1.22307_{\mathrm{z}}\ \mathrm{m}}{\mathrm{p}{\downarrow}\mathrm{day}^2} = \frac{0.36692_{\mathrm{z}}\ \mathrm{m}}{\mathrm{q}{\downarrow}\mathrm{hr}^2}$$

1.22307$_{\mathrm{z}}$ m = 1.362389$_{\mathrm{z}}$ yd = 3.↊66↋3$_{\mathrm{z}}$ ft = 1.08410$_{\mathrm{z}}$ unqua·metron.
0.36692$_{\mathrm{z}}$ m = 0.3↊66↋2$_{\mathrm{z}}$ yd = 0.↋778↊$_{\mathrm{z}}$ ft = 0.32103$_{\mathrm{z}}$ unqua·metron.

The first of these is for the pentcia·day, the second for the quadcia·hour.

6.3 These two lengths are nobody's concoction, but facts that come into existence when one uses dozenal numeration to express ideas. The Great Circle systems must come to terms with them before they can evolve units of Force, Work, Energy, Power, Pressure, etc.

6.4 W.S.Crosby used the former of these as the unit of length for his 'uncial' system, calling it the *ell*.

T. Pendlebury used the latter, calling it the *Grafut* (short for *gra*vity foot).

In both these systems $g = 1$ unit of length per unit of time squared. The long numbers shown in 6.2 above therefore vanish.

Since 1 pentcia·day = 2 quadcia·hours, Crosby's and Pendlebury's systems are virtually the same.

6.5 Though the relationship between Force and Mass is defined by their 'weight' at Earth surface, this does not mean the relationship is Earth-bound. *Anywhere* in the Universe that 1 lb mass receives an acceleration of 9.80665_d metres per sec per sec, the force causing this acceleration is thereby measured as 1 lbf. Sea level on Earth is the physical datum where a known and experienced constant relationship between mass and force is taken as standard for comparison of phenomena elsewhere. And it is a *real* equilibrium of nature: where the force of gravity pulling the Earth together equates to the forces giving the Earth its bulk and size.

6.6 Systems where g is not equal to 1 unit of acceleration are apt to split into *two or more systems* at this point. Take the Anglo-American Foot-Pound-Second system; the number 32.1741_d can be attached to (a) the acceleration unit or (b) the mass unit or (c) the force unit, giving three systems:

a) force (lbf) = mass (lb) × 32.1741_d ft/sec^2;
b) force (poundal) (i.e. lbf divided by 32.1741_d) = mass (lb) × 1 ft/sec^2;
c) force (lbf) = mass (slug) (i.e., lb × 32.1741_d) × 1 ft/sec^2.

The dms also splits up:

d) force (dyne) (i.e., grammeforce/980.665_d) = mass (g) × 1 cm/s^2;
e) force (newton) (i.e., kgf/9.80665_d) = mass (kg) × 1 m/s^2;
f) force (kgf) = mass (kg) × 9.80665_d m/s^2;

The pressure of practical application in different fields will cause divisions also to occur in dozenal in those systems where $g \neq 1$ unit of length per unit of time^2. Here is a glorious opportunity for dozenal to put itself still one more jump ahead of decimal.

7. Electrical units are also defined nowadays by the force relationship between current and magnetic flux. Here is the present-day definition of the ampere:

> The ampere is that current which when maintained in each of two infinitely long parallel conductors situated in a vacuum and separated 1 metre between centres, produces between these conductors a force of $2\times10_{\mathrm{d}}^{-7}$ newton per metre length.

High-faluting, totally impossible to put into practics, yet it works! The point is that it *defines* the ampere under perfect conditions with all extraneous phenomena removed.

Where does the flux come from? Magnetic flux is a radiation phenomenon that occurs when electrons move. The current in one wire radiates a flux, that strikes upon the other wire, and vice versa.

7.1 Using a similar definition in dozenal, the metre and the newton of course are replaced by the corresponding units of the new system, and the number $2\times10_{\mathrm{d}}^{-7}$ must become $\mathrm{N}\times10_{\mathrm{z}}^{\mathrm{n}}$ where N and n are simple integers.

7.2 The 'metre apart' cancels out the 'per metre length' as regards the value of our new unit, and at first sight it appears that the unit of length has no bearing on the unit of current, for we have:

$$\text{Force varies as } \frac{ii' \cdot ll'}{r^2}$$

where i is the current in one conductor, i' in the other, and l and l' are the lengths considered of each conductor (one yard, one metre, or what have you), and r is the distance between centres: so

$$ll' = r^2$$

and we have:

Unit force varies as a unit of current squared.

7.3 But the unit of force is based on the unit of mass, and the unit of mass (in all systems I have so far encountered) is based on the unit of volume derived from the unit of length cubed. So now we have:

Unit length varies as the cube root of the current squared.

or, put the other way round:

Unit current varies as (Unit of length)$^{\frac{3}{2}}$.

7.4 Existing instruments measure current in amps, so a lot of trouble can be saved if the new unit is a simple ratio, or as near-as-dammit close to simple ratio to the amp.

Essig took the amp as existing, which gives him the coefficient $2.Ɛ744\times10_z^{-6}$, and suggests rounding this up to $3\times10_z^{-6}$, which of course would make his amp-duodecimale *not* equivalent to his amp decimale. (g in Essig's system = 32.17_z metres-duodecimale par seconde-duodecimale par seconde-duodecimale). (Note the long-windedness of using existing words for new meanings).

Pendlbury uses the coefficient $1\times10_z^{-9}$ and gets 1 KUR = 0.495722_d amp (about 1% under 1/2 amp) (and g = 1 unit).

7.5 Current × electrical pressure (voltage in dms) = Power. So the unit of electric pressure is found by dividing the power unit of the new system (force unit × length unit divided by time unit) by our new found unit of current.

In dms: 1 watt = 1 amp × 1 volt

In Essig: 1 watt_{dd} = 1 amp_{dd} × 1 volt_{dd} (dd=duodecimale)

In Pendlebury: 1 POV = 1 kur × 1 pel.

The watt-duodecimale = 0.1109_d watt (decimale) ($0.13Ɛ8_z$),

so the volt-duodecimale = 0.1109_d volt (decimale)

...

7.9 Dozenal systems other than those of Essig and Pendlebury have not (as far as the author is aware) developed their electrical units.

8. Beside the natural relationships such as the force-to-mass and force-to-current relationships, the powers-of-dozen relationship also deserves attention.

In the much vaunted dms, reputed to be so beautifully adapted to decimal numeration, we find that though the metre is the basic unit of length, being evolved into *centi*metres, *deci*meters, *kilo*meters, etc, the unit of capacity, the litre, was originally based on the cubic decimeter of *one thousand* cubic *centi*metres. Thus the gramme is based not on the litre but the *milli*litre.

Then the newton, joule, watt, amp, volt etc are all based on the *kilo*gramme. This hopping about on the powers of the base not infrequently leads to misplaced decimal points in calculation. Where to put the decimal point is often a difficult enough problem without having extra complications built in by the system itself.

8.1 Essig imitates the dms on this point (except for time). He also uses traditional prefixes but with dozenal value, e.g. kilo- for $1000_z\times$, centi- for $0.01_z\times$.

The Do-Metric system of the Duodecimal Society of America introduces prefixes: do-, gro- and mo- for multiplied-by 10_z, 100_z and 1000_z respectively, and edo-, egro- and emo- for divided-by them, but also uses many traditional names usually with values different to their traditional twins.

Churchman also uses the Do-metric prefixes, e.g. the dometron = 10_z

metron, the metron being equivalent to 3.8_z inches. His unit of capacity is the *jon*, the volume of 1 cubic metron.

Pendlebury uses one word only for the units of this kind, this word represents the basic unit, from which the larger and smaller are derived by a system of prefixes. … The basic units are all related by a 1:1 ratio except for the bridge from mechanical to electrical units. …

1 b↓Mag × 7 t↑Grafut = 7 u↑Werg

(bicia·mags × triqua·grafuts = unqua·wergs)

(Mag = force unit, Grafut = length unit, Werg = work or energy unit.)

9. To sum up let us try to formulate the requirement of a dynamic system:
(1) to be co-ordinated by dozenal arithmetic;
(2) to have 1:1 ratio as far as practicable between units of different kind;
(3) the up and down derived units of a like kind should be simply expressed and understood without having continually to refer back to tables;
(4) its vocabulary should be easy to memorize and unambiguous;
(5) it must conform to the natural liaison of physical laws (see table at the end of this article);
(6) it should give simple factors to as many natural "units" as possible;
(7) it should as far as is practicable preserve established dozenal units, e.g. inch-foot, clock, volts, either by (a) an exact ratio of 1:1, or, if that is impracticable (b) by simple ratio 2:1, 3:1, etc., or (c) close approximation to a simple ratio so that for most practical conversions and use of existing measuring equipment the difference could be ignored.
(8) It must at least cover provision of units for the following sciences: mechanics, chemistry, electricity, magnetism, electronics, astronomy, nucleonics, hydraulics, fluidics, pneumatics, light, heat, acoustics, and of course mensuration and geometry.

Definitions.
(9) Units to be defined in other units of the same system, *in dozenal*.
(Ɛ) Units to be defined accurately in terms of the existing dms in *decimal* numeration.
(Ɛ) Units of the present decimal metric system to be *accurately expressed* in units of the proposed system, in *dozenal*.

(Ɛ) and (Ɛ) are necessary for the conversion of data into the new system. Without them the system can never get started.
(10) The basic units to be defined against natural phenomena, e.g. the mean solar day, diameter or circumference of Earth, lightyear, wavelengths of light, velocity of light, etc. This makes the system independent of dms for all time.
(11) Other conversion information as opportune should be included, e.g. conversion factors (in *dozenal*) to other people's dozenal systems, handy bits, e.g. 2 mm is just about 1 bicia·Grafut, etc.

Comparison with dms.

(12) It should not lose any of the advantages found in dms except where absolutely unavoidable. Only a very limited number of such cases should be permitted, and only provided that:
(13) It should contain improvements on dms (in addition to the use of dozenal numeration). By *improvements* is meant more *facility* in application.
(14) It must be condemned as a failure if it does not achieve (12) and (13), for then it would offer no advantage for the dozenal cause.

<u>Conversion.</u> This is not part of the system, but an early-stage necessity.
(15) decimal-to-dozenal conversion and vice-versa consists of two stages: (a) transcription of the *quantity number* and (b) conversion by multiplication factor into *other units*. The order in which this is easiest depends on the case, e.g. if the conversion factor is 5, ↊, or 2.5_d obviously this is best done while the quantity number is in decimal form. But if 3 or 9 then do the multiplication in dozenal.
(16) Since the ratio of dozen to ten is $1.2_d : 1$, conversion factors are altered by a factor that is some power of 1.2_d for every change in the number order. This is very handy in that awkward conversion factors further up or down the scale of number orders can come out quite reasonable. It is up to the devisor of the system to find which orders convert most easily and quote them: to convert from metres to your units may not be easy, but to convert kilometres or millimetres may be quite simple.
(17) Use can be made of the Popular Systems as stepping stones, e.g. 1 statute mile = 1028_z yards = 3080_z ft; now to convert to dometrons we multiply by 3/↋, that is, 9200_z divided by ↋ = $↊00_z$ dometrons.

↊. This is a very severe terms-of-reference. Will anything be good enough?

EACH ONE

TEACH ONE

Motto of the DSA since its founding

[z] • Systematic Dozenal Nomenclature Summary • [z]						
N	Root	Abbr	Multiplier Prefix $N\times$	Reciprocal Prefix $\frac{1}{N}\times$	Power Prefixes: Positive $10^{+N}\times$	Power Prefixes: Negative $10^{-N}\times$
0	nil	n	nili·	nilinfra·	nilqua·	nilcia·
1	un	u	uni·	uninfra·	unqua·	uncia·
2	bi	b	bina·	bininfra·	biqua·	bicia·
3	tri	t	trina·	trininfra·	triqua·	tricia·
4	quad	q	quadra·	quadinfra·	quadqua·	quadcia·
5	pent	p	penta·	pentinfra·	pentqua·	pentcia·
6	hex	h	hexa·	hexinfra·	hexqua·	hexcia·
7	sept	s	septa·	septinfra·	septqua·	septcia·
8	oct	o	octa·	octinfra·	octqua·	octcia·
9	enn	e	ennea·	enninfra·	ennqua·	enncia·
↊	dec	d	deca·	decinfra·	decqua·	deccia·
↋	lev	L	leva·	levinfra·	levqua·	levcia·
10	unnil	un	unnili·	unnilinfra·	unnilqua·	unnilcia·
11	unun	uu	ununi·	ununinfra·	ununqua·	ununcia·
12	unbi	ub	unbina·	unbininfra·	unbiqua·	unbicia·
13	untri	ut	untrina·	untrininfra·	untriqua·	untricia·
14	unquad	uq	unquadra·	unquadinfra·	unquadqua·	unquadcia·
15	unpent	up	unpenta·	unpentinfra·	unpentqua·	unpentcia·
16	unhex	uh	unhexa·	unhexinfra·	unhexqua·	unhexcia·
17	unsept	us	unsepta·	unseptinfra·	unseptqua·	unseptcia·
18	unoct	uo	unocta·	unoctinfra·	unoctqua·	unoctcia·
19	unenn	ue	unennea·	unenninfra·	unennqua·	unenncia·
1↊	undec	ud	undeca·	undecinfra·	undecqua·	undeccia·
1↋	unlev	uL	unleva·	unlevinfra·	unlevqua·	unlevcia·
20	binil	bn	binili·	binilinfra·	binilqua·	binilcia·

etc...

uncia was Latin for *one twelfth* • retains same meaning • *inch* and *ounce* are English derivatives
Concatenating roots = positional place-value • Suggested pronunciation: -cia = /ʃə/ ("-sha")
Concatenating prefixes = multiplication • mix & match freely • Commutative Law applies
Prefer Unicode abbreviations where supported • ASCII abbreviations for email, text, etc.

SDN Form	Example Value [z]	Example SDN	Abbreviation: Unicode	Abbreviation: ASCII
Root Form	46	quadhex	qh	`qh`
Multiplier Prefix	46×	quadhexa·	qh•	`qh*`
With Fractional Part	4.6×	quad.dot.hexa·	q.h•	`q.h*`
Ordinal	46^{th}	quadhexal	qh′	`qh'`
Reciprocal Prefix	$\frac{1}{46}\times$	quadhexinfra·	qh\	`qh\`
Positive Power Prefix	$10^{+46}\times$	quadhexqua·	qh↑	`qh@`
Negative Power Prefix	$10^{-46}\times$	quadhexcia·	qh↓	`qh#`
Rational Number	$4\times\frac{1}{5}\times$	quadra·pentinfra·	q•p\	`q*p\`
Rational Number	$\frac{1}{5}\times 4\times$	pentinfra·quadra·	p\q•	`p\q*`
Scientific Notation	$4\times 10^{+6}\times$	quadra·hexqua·	q•h↑	`q*h@`
With Fractional Part	$4.5\times 10^{+6}\times$	quad.dot.penta·hexqua·	q.p•h↑	`q.p*h@`
Scientific Notation	$10^{+6}\times 4\times$	hexqua·quadra·	h↑q•	`h@q*`
With Fractional Part	$10^{+6}\times 4.5\times$	hexqua·quad.dot.penta·	h↑q.p•	`h@q.p*`
one dozen years	$10^{+1}\times$ year	unqua·year, unquennium	u↑yr	`u@yr`
one gross years	$10^{+2}\times$ year	biqua·year, biquennium	b↑yr	`b@yr`
one galore years	$10^{+3}\times$ year	triqua·year, triquennium	t↑yr	`t@yr`
two hours (a "dwell")	$10^{-1}\times$ day	uncia·day	u↓dy	`u#dy`
ten minutes (a "breather")	$10^{-2}\times$ day	bicia·day	b↓dy	`b#dy`
fifty seconds (a "trice")	$10^{-3}\times$ day	tricia·day	t↓dy	`t#dy`

For more info see:
Original article: `http://www.dozenal.org/drupal/sites_bck/default/files/DSA_kodegadulo_sdn.pdf`
Wiki page: `https://primelmetrology.atlassian.net/wiki/display/PM/Systematic+Numeric+Nomenclature%3A+Dozenal`
Forum: `https://www.tapatalk.com/groups/dozensonline/systematic-dozenal-nomenclature-f31/`
Original thread: `https://www.tapatalk.com/groups/dozensonline/systematic-dozenal-nomenclature-t463.html`

The Primel Metrology

by John Volan

Primel is a coherent, dozenal-metric, day-gravity-water-based metrology. I named it “Primel” because it would be the first (i.e., *prime*) metrology to make use of *quantitels*, a set of neologisms I invented to systematically provide generic names for all coherent units of measurement: e.g. ⊡**timel**, ⊡**lengthel**, ⊡**massel**, etc., where ⊡ (pronounced “prime”) is Primel’s “brand mark.”

I first began devising Primel back in 11Ɛ8$_{z}$ (2012$_{d}$). At the time, I had just learned about Tom Pendlebury’s Tim-Grafut-Maz (TGM) metrology,[1] and was very impressed with what he had accomplished with it. However, some of the specific choices Pendlebury had made seemed unsatisfying to me. I wanted to see what sort of system of measurement one could derive by applying many of the same principles embodied in TGM, but starting from a slightly different set of initial conditions.

This article provides a brief overview of the main Primel units for mechanics and temperature, with particular attention on the nomenclatures and stylistic features Primel uses. Future articles may go into greater depth about specific topics.

A Coherent Dozenal-Metric “DGW” System

Primel, like TGM, is a *dozenal-metric* system, in the same way that the International System of Units (SI) is a decimal-metric system. Primel regularizes its units around dozenal as its base of numeration, just as SI regularizes its units around decimal.

Primel is also like TGM and SI in strictly adhering to the principle of *coherence*[2] in measurement systems. That is, it strives to maintain simple one-to-one dimensional relationships between the coherent units it defines for different kinds of physical quantity, avoiding as much as possible any arbitrary factors between coherent units.

Finally, like TGM, Primel is a *day-gravity-water (DGW)* system. It derives its coherent units of measurement from what I like to call certain “mundane realities” of human life on Earth:

1. the mean solar day, a fraction of which becomes Primel’s coherent unit of time (⊡**timel**);
2. the acceleration due to Earth’s gravity, used as the coherent unit of acceleration (⊡**accelerel**), and then used to derive coherent units of velocity (⊡**velocitel**) and length (⊡**lengthel**), then area (⊡**areanel**) and volume (⊡**volumel**), and all the other units of kinematic mechanics;
3. the density of water, used as the coherent unit of density (⊡**densitel**), and then used to derive the coherent unit of mass (⊡**massel**), and from there coherent units of force (⊡**forcel**), energy (⊡**energel**), power (⊡**powerel**), and all the other units of dynamic mechanics;
4. the specific heat capacity (“massic heatability”) of water, used as a coherent unit itself (⊡**masselic·heatabilitel**), and then used to derive a coherent unit of temperature (⊡**temperaturel**), and then all the other units of thermodynamics;

[1] See *TGM: A Coherent Dozenal Metrology, based on the system and booklet by Tom Pendlebury, DSGB, updated and revised by Donald Goodman, USA* at https://tinyurl.com/y75t83h6.

[2] See https://en.wikipedia.org/wiki/Coherence_(units_of_measurement).

and so forth. The table on page 34_z shows a representative sample (by no means exhaustive) of Primel's coherent units and how they derive from the above selections.

Over the years, I have striven to consolidate the best ideas I could find from past dozenal metrologies, while also trying to prune out practices that I felt were contrived or pretentious or otherwise counter-productive, as well as to invent nomenclature and systematization where needed to enrich the metrology-building process, but with a flexible enough structure that people could inject their own favorite cultural elements into their own systems. I have helped other members of the DozensOnline forum[3] explore many variations on this style of measurement system, including regularizing around their own preferred bases other than decimal or dozenal.[4] My intent throughout was to make these elements available as generic reusable tools for the benefit of anyone wanting to experiment with new systems of measure.

Diverging from Pendlebury

Even though Primel follows a similar "DGW" derivation pattern as TGM, Primel diverges from TGM in some of its specific selections. My primary difficulty with TGM was that Pendlebury elected to divide the day in *half* first, before starting to divide it dozenally. This happens to yield the familiar customary hour as a primary unit, and then fractional dozenal powers of the hour, ultimately leading to the quadcia·hour (10_z^{-4} of an hour) as Pendlebury's coherent unit of time, the Tim (equivalent to 0.21_z or $0.1736\overline{1}_d$ seconds). When combined with Pendlebury's selected value for Earth's gravity, his Gee, this yields his coherent velocity unit, the Vlos, and then his coherent length unit, the Grafut, or "gravity-foot." This being a fair approximation of the customary foot of the United States Customary (USC) and British Imperial (BI) systems, it made TGM rather attractive to members of both the Dozenal Society of Great Britain (DSGB) and the Dozenal Society of America (DSA).

But if TGM is supposed to be a *dozenal*-metric system, on the assumption that dozenal is the "best" base, why would we want to inject a digit of *binary* base right at the beginning of its derivation? TGM ostensibly considers the mean solar day a "fundamental reality," yet the mean solar day itself is not a whole dozenal power of the Tim. Instead, the *hour* is. This seems an unnecessary sacrifice of principle just for the sake of keeping one familiar clock unit. It also means an awkward division by two when switching between time measured in days and time measured in Tims.

In contrast, Primel divides the mean solar day in a strictly dozenal-metric fashion, the way the founding members of the DSA did for their Do-Metric System.[5] Primel, in fact, selects the hexcia·day (10_z^{-6} of a day) to be its coherent ⊡ **timel**, equivalent to 0.042_z or $0.02893\overline{518}_d$ seconds, 6 times smaller than the Tim. Since the day is a dozenal power of the ⊡timel, the transition from counting times-of-day to counting whole days is a simple shift of the radix point.

The ⊡timel itself may seem to be a dauntingly small time unit to base a metrology on, being well beneath human perception. However, dozenal scalings of the ⊡timel provide more convenient units for everyday use, and there are certainly applications for precision timing down to the ⊡timel or even finer. (The table on page 35_z shows Primel's dozenal divisions of the day, which are all dozenal scalings of the ⊡timel.)

Next, Primel takes Earth's gravity as another "mundane reality," and uses a

[3] https://www.tapatalk.com/groups/dozensonline/index.php

[4] For examples, see "Day Gravity Water System" spreadsheet at https://tinyurl.com/y86kwnyh.

[5] Or, as I have fancifully re-imagined it, their "Uncia·Metric" system. See article on page 16_z.

Primel ⊡ Selected Mechanical and Thermodynamic Units

Quantity	Quantitel *Abbrev*	Colloquial *Abbrev*	Derivation	SI and USC Equivalents
Time	⊡timel ⊡*tmℓ*	⊡vibe·time ⊡*vb·tm*	hexcia·day	$0.042_z=0.02893\overline{518}_d$ s
Acceleration	⊡accelerel ⊡*accℓ*	⊡gravity ⊡*grv*	Earth gravity at $34°01'34.56''_d$	$9.79651584_d\ \frac{m}{s^2}$ $32.1408_d\ \frac{ft}{s^2}$
Velocity Speed	⊡velocitel ⊡*vcℓ* ⊡speedel ⊡*spdℓ*		⊡accℓ×⊡tmℓ	$0.283464_d\ \frac{m}{s}$ $1.0204704_d\ \frac{km}{h}$ $0.93_d\ \frac{ft}{s}$ $11.16_d\ \frac{in}{s}$
Length Height Width *etc...*	⊡lengthel ⊡*lgℓ* ⊡heightel ⊡*hgtℓ* ⊡widthel ⊡*wdℓ* *etc...*	⊡morsel·length ⊡*mo·lg* ⊡morsel·height ⊡*mo·hgt* ⊡morsel·width ⊡*mo·wd* *etc...*	⊡vcℓ×⊡tmℓ	$8.20208\overline{3}_d$ mm $0.3↊6_z=0.32291\overline{6}_d$ in
Area	⊡areanel ⊡*arℓ*	⊡morsel·area ⊡*mo·ar*	⊡lgℓ^2	$67.2741710069\overline{4}_d$ mm^2 $0.1042751736\overline{1}_d$ in^2
Volume	⊡volumel ⊡*vmℓ*	⊡morsel·volume ⊡*mo·vm*	⊡lgℓ^3	0.551788356779_d ml 0.111949104137_d tsp
Density	⊡densitel ⊡*dsℓ*	⊡water·density	water density at 4°C	999.972_d kg/m^3
Mass	⊡massel ⊡*msℓ*	⊡morsel·mass ⊡*mo·ms*	⊡dsℓ×⊡vmℓ	0.551772906706_d g 0.019463216516_d oz
Momentum	⊡mom~~entum~~el ⊡*mmℓ*	⊡morsel·momentum ⊡*mo·mm*	⊡msℓ×⊡vcℓ	$15.6407755_d\ \frac{g\cdot cm}{s}$ $1.56407755\times10_d^{-4}\ \frac{kg\cdot m}{s}$
Action	⊡actionel ⊡*actℓ*	⊡morsel·action ⊡*mo·act*	⊡mmℓ×⊡lgℓ	$12.8286944_d\ \frac{g\cdot cm^2}{s}$ $1.28286944\times10_d^{-6}\ \frac{kg\cdot m^2}{s}$
Force	⊡forcel ⊡*fcℓ* ⊡weightel ⊡*wtℓ*	⊡morsel·force ⊡*mo·fc* ⊡morsel·weight ⊡*mo·wt*	⊡msℓ×⊡accℓ	0.5512027063908_d g_f 5.40545202062705_d mN
Energy Work	⊡energel ⊡*ngℓ* ⊡workel ⊡*wkℓ*	⊡morsel·energy ⊡*mo·ng* ⊡morsel·work ⊡*mo·wk*	⊡fcℓ×⊡lgℓ	44.3359679275_d μJ
Power	⊡powerel ⊡*pwℓ*	⊡morsel·power ⊡*mo·pw*	⊡ngℓ÷⊡tmℓ	1.53225105157502_d mW
Tension	⊡tensionel ⊡*tsℓ*	⊡morsel·tension ⊡*mo·ts*	⊡fcℓ÷⊡lgℓ	$0.659034028422907_d\ \frac{N}{m}$
Pressure	⊡pressure ⊡*psℓ*	⊡morsel·pressure ⊡*mo·ps*	⊡fcℓ÷⊡arℓ	80.3495894444997_d Pa
Heat	⊡heatel ⊡*htℓ*	⊡morsel·heat ⊡*mo·ht*	⊡ngℓ	44.3359679275_d μJ
Massic Heatability	⊡masselic·heatab~~ilit~~el ⊡*msℓ\htbℓ*		slightly above water average	$4198.76286389748_d\ \frac{J}{kg\cdot K}$
Heatability	⊡heatab~~ilit~~el ⊡*htbℓ*	⊡morsel·heatability ⊡*mo·htb*	⊡msℓ\htbℓ ×⊡msℓ	$2.31676358998144_d\ \frac{J}{K}$
Temperature	⊡temp~~era~~turel ⊡*tpℓ*	⊡morsel·temp~~era~~ture ⊡*mo·tp*	⊡htℓ÷⊡htbℓ	19.1370272388791_d μK

Primel ⊡ Selected Powers of the ⊡ Timel			
Quantitel Form *Abbrev*	Colloquial *Abbrev* (Rationale)	Derivation *Abbrev*	SI and USC Equivs
⊡ hexqua·timel ⊡ $h{\uparrow}tm\ell = 10_z^{+6}$ ⊡ $tm\ell$	day *dy*	day *dy*	$42{,}000_z$=$86{,}400_d$ s ↊00$_z$=1440_d min
⊡ pentqua·timel ⊡ $p{\uparrow}tm\ell = 10_z^{+5}$ ⊡ $tm\ell$	⊡ dwell ⊡ *dw* (How long sun "dwells" in a "house," astrological term for uncia·turn of sky)	uncia·day $u{\downarrow}dy = 10_z^{-1}\ dy$	4200_z=7200_d s ↊0$_z$=120_d min 2 hr
⊡ quadqua·timel ⊡ $q{\uparrow}tm\ell = 10_z^{+4}$ ⊡ $tm\ell$	⊡ breather ⊡ *br* (Time enough for a short rest from work,"taking a breather")	bicia·day $b{\downarrow}dy = 10_z^{-2}\ dy$	420_z=600_d s ↊=10_d min
⊡ triqua·timel ⊡ $t{\uparrow}tm\ell = 10_z^{+3}$ ⊡ $tm\ell$	⊡ trice ⊡ *tr* (Archaic term for a short duration; pun on dividing the day "thrice")	tricia·day $t{\downarrow}dy = 10_z^{-3}\ dy$	42_z=50_d s
⊡ biqua·timel ⊡ $b{\uparrow}tm\ell = 10_z^{+2}$ ⊡ $tm\ell$	⊡ lull ⊡ *lu* (Time for long, embarrassing pause)	quadcia·day $q{\downarrow}dy = 10_z^{-4}\ dy$	4.2_z=$4.1\overline{6}_d$ s
⊡ unqua·timel ⊡ $u{\uparrow}tm\ell = 10_z^{+1}$ ⊡ $tm\ell$	⊡ twinkling ⊡ *tw* (Time for an eye blink)	pentcia·day $p{\downarrow}dy = 10_z^{-5}\ dy$	0.42_z=$0.347\overline{2}_d$ s
⊡ timel ⊡ $tm\ell$	⊡ vibe ⊡ *vb* (Short for "vibration." Period of note C♯$_1$, at threshold of audibility.)	hexcia·day $h{\downarrow}dy = 10_z^{-6}\ dy$	0.042_z s $0.0289\overline{3518}_d$ s

candidate value for that as its ⊡ **accelerel**. Multiplying this by the ⊡ timel yields the ⊡ **velocitel**. Multiplying that in turn by the ⊡ timel yields the ⊡ **lengthel**.

This explains the need for the ⊡ timel to be so small. In effect, for any DGW metrology, the lengthel is proportional to the square of the timel, with the accelerel as the proportionality constant. Earth's gravity makes for a relatively large accelerel, so in order to maintain coherence, either the timel must be small, or the lengthel, and further units derived from it, will be large. For Primel, I opted for the former.

Remarkably, the ⊡ **velocitel** is a fair approximation of a foot per second, as well as almost exactly 1 kilometer per hour.[6] People from metric countries may find Primel speedometers relatively easy to adapt to. (See table on page 36_z for a comparison of typical speedometer values.)

The ⊡ **lengthel** is about $\frac{1}{36}_d$ or $\frac{1}{30}_z$ of a Grafut, or about a dozenth of a decimeter, or a third of an inch. This may seem small, but it is on the order of a centimeter in size. Recall that for much of the Nineteenth Century the centimeter proved quite serviceable as the coherent unit of length in the centimeter-gram-second (CGS) system. Furthermore, a third of an inch was actually an archaic English unit of measure known as a "barleycorn." Interestingly, shoe sizes in the United States continue to use this measure for their denominations. So a unit of this size is not unprecedented.

To be more precise, I have carefully selected a very specific value for Earth's gravity, exactly 9.79651584_d meters per second per second, or 32.1408_d feet per second per second,[7] which is within the natural range but a bit lower than SI's gravity standard,

[6] It is also exactly one ⊡ morsel·length per ⊡ vibe, or one ⊡ hand·length per ⊡ twinkling, or one ⊡ ell·length per ⊡ lull, or one ⊡ habital·length per ⊡ trice, or one ⊡ stadial·length per ⊡ breather, or one ⊡ dromal·length per ⊡ dwell, or one ⊡ itineral·length per day. All of these are equally valid ways to describe one ⊡ velocitel, putting into question whether any one "length unit per time unit" formula should be preferred over simply calling it a "⊡ velocitel."

[7] Corresponding to a latitude of $34°01'34.56''_d$ or 11.73↊↋566↊$_z$ bicia·turns.

Primel ⊡ Speeds on the Road			
Primel Speed	Metric Speed	USC Speed	Possible Usage
10_z ⊡vcℓ	12.2456448_d km/h	$7.6\overline{09}_d$ mph	
20_z ⊡vcℓ	24.4912896_d km/h	$15.2\overline{18}_d$ mph	school zone speed limit
30_z ⊡vcℓ	36.7369344_d km/h	$22.8\overline{27}_d$ mph	
40_z ⊡vcℓ	48.9825792_d km/h	$30.4\overline{36}_d$ mph	residential speed limit
50_z ⊡vcℓ	61.2282240_d km/h	$38.0\overline{45}_d$ mph	
60_z ⊡vcℓ	73.4738688_d km/h	$45.6\overline{54}_d$ mph	urban arterial road speed limit
70_z ⊡vcℓ	85.7195136_d km/h	$53.2\overline{63}_d$ mph	
80_z ⊡vcℓ	97.9651584_d km/h	$60.8\overline{72}_d$ mph	urban expressway speed limit
90_z ⊡vcℓ	110.2108032_d km/h	$68.4\overline{81}_d$ mph	
↊0_z ⊡vcℓ	122.4564480_d km/h	$76.0\overline{90}_d$ mph	rural freeway speed limit
Ɛ0_z ⊡vcℓ	134.7020928_d km/h	83.700_d mph	
100_z ⊡vcℓ	146.9477376_d km/h	$91.3\overline{09}_d$ mph	autobahn speed limit

in order to make the ⊡lengthel come out to *exactly* 0.3↊6_z or $\frac{31}{96}_d$ USC inches. Since the USC inch has been defined as exactly 25.4_d millimeters, transitively this makes the ⊡lengthel *exactly* $8.2021\overline{6}_d$ millimeters. The main reason for this particular choice is that it allows for exact conversions between Primel lengths and both USC and SI lengths. The chief benefit of such exact conversions is that it makes it feasible to construct machine tools with relatively simple gear ratios that can then precisely manufacture machine parts measured in SI, USC, or Primel units. In an advanced modern industrial civilization, any proposed metrology that did not offer this capability would be at a severe disadvantage. Moreover, scaling up the ⊡lengthel by dozenal powers eventually results in units exactly equivalent to whole numbers of USC feet. (See table on page 37_z.)

Another advantage this confers is that the ⊡accelerel is closer to the theoretical average value for Earth's gravity integrated over the surface area of the Earth.[8] SI's standard gravity is *not* an "average" value for gravity on Earth. It is actually an *inaccurate* Nineteenth Century estimate of gravitational acceleration at *median latitude* (45°_d or 16_z bicia·turns, or 1 octant). But parallels of latitude subtend progressively more surface area approaching the equator, so the latitude of the *average* gravity is correspondingly lower. Furthermore, more people live closer to the equator than to median latitude, so using a lower gravity standard actually increases the chances the estimate will match the average human's experience.

In contrast, Pendlebury's Gee is even larger than SI's gravity standard, corresponding to an even higher latitude.[9] He chose that value in order to make a dozenal power of the Grafut exactly equal to ten times the polar diameter of Earth. This was simply in order to be able to precisely specify the Grafut in terms of something which could be measured with extreme accuracy using the technology available during Pendlebury's

[8] DozenOnline forum member Dan has calculated this to be 9.7975827196164_d m/s^2, corresponding to a latitude of $35°17'17.82''_d$ or 12.14731821_z bicia·turns.

[9] About $49°16'05.51''_d$ or 17.70727872_z bicia·turns.

PRIMEL ⊡ SELECTED POWERS OF THE ⊡ LENGTHEL

QUANTITEL FORM *Abbrev*	COLLOQUIAL *Abbrev* (RATIONALE)	SI AND USC EQUIVALENTS
⊡ septcia·lengthel ⊡ *s↓lgℓ* = 10_z^{-7} ⊡ *lgℓ*	⊡ atomical·length ⊡ *ato·lg* (Size of an atom.)	228.90509274_d pm
⊡ hexcia·lengthel ⊡ *h↓lgℓ* = 10_z^{-6} ⊡ *lgℓ*	⊡ polymeral·length ⊡ *pol·lg* (Size of large polymer molecule.)	2.7468611129_d nm
⊡ pentcia·lengthel ⊡ *p↓lgℓ* = 10_z^{-5} ⊡ *lgℓ*	⊡ somal·length ⊡ *som·lg* (Size of a ribosome.)	32.9623333548_d nm
⊡ quadcia·lengthel ⊡ *q↓lgℓ* = 10_z^{-4} ⊡ *lgℓ*	⊡ luminal·length ⊡ *lum·lg* (Wavelength range of visible light.)	395.54800025721_d nm
⊡ tricia·lengthel ⊡ *t↓lgℓ* = 10_z^{-3} ⊡ *lgℓ*	⊡ chondrial·length ⊡ *chn·lg* (Size of a mitochondrion.)	4.7465760031_d μm
⊡ bicia·lengthel ⊡ *b↓lgℓ* = 10_z^{-2} ⊡ *lgℓ*	⊡ cellular·length ⊡ *cel·lg* (Size of a eucaryotic cell.)	$56.958912\overline{037}_d$ μm
⊡ uncia·lengthel ⊡ *u↓lgℓ* = 10_z^{-1} ⊡ *lgℓ*	⊡ granular·length ⊡ *grn·lg* (Size of a grain of salt.)	0.03↊6$_z$ in = $26.097\overline{2}_d$ thou $683.5069\overline{4}_d$ μm
⊡ lengthel ⊡ *lgℓ*	⊡ morsel·length ⊡ *mo·lg* (Size of a small bite of food.)	$\frac{31}{96}_d$=0.3↊6$_z$=$0.32291\overline{6}_d$ in $8.20208\overline{3}_d$ mm
⊡ unqua·lengthel ⊡ *u↑lgℓ* = 10_z^{+1} ⊡ *lgℓ*	⊡ hand·length ⊡ *hd·lg* (Approximates customary 4-inch hand.)	$3\frac{7}{8}$=3.↊6$_z$=3.875_d in 0.98425_d dm
⊡ biqua·lengthel ⊡ *b↑lgℓ* = 10_z^{+2} ⊡ *lgℓ*	⊡ ell·length ⊡ *ℓ·lg* (Approximates old English ell of 45_d in.)	3↊.6$_z$=46.5_d in 1.1811_d m
⊡ triqua·lengthel ⊡ *t↑lgℓ* = 10_z^{+3} ⊡ *lgℓ*	⊡ habital·length ⊡ *hb·lg* (Size of a house or "habitation.")	3↊.6$_z$=46.5_d ft 14.1732_d m
⊡ quadqua·lengthel ⊡ *q↑lgℓ* = 10_z^{+4} ⊡ *lgℓ*	⊡ stadial·length ⊡ *st·lg* (Approximates ancient Greek *stadion*.)	3↊6$_z$=558_d ft $0.13\overline{2750}_z$=$0.1056\overline{81}_d$ mi 170.0784_d m
⊡ pentqua·lengthel ⊡ *p↑lgℓ* = 10_z^{+5} ⊡ *lgℓ*	⊡ dromal·length ⊡ *dr·lg* (From Greek *dromos*, "road, racetrack." Good unit for road distances.)	3↊60$_z$=6696_d ft $1.3\overline{2750}_z$=$1.26\overline{81}_d$ mi 2.0409408_d km
⊡ hexqua·lengthel ⊡ *h↑lgℓ* = 10_z^{+6} ⊡ *lgℓ*	⊡ itineral·length ⊡ *itn·lg* (From Latin *iter, itineris* "march." Daily march for Roman legion; recommended limit for a modern daily commute.)	3↊,600$_z$=$80{,}352_d$ ft $13.\overline{2750}_z$=$15.2\overline{18}_d$ mi $24{,}491.2896_d$ m 24.4912896_d km
⊡ septqua·lengthel ⊡ *s↑lgℓ* = 10_z^{+7} ⊡ *lgℓ*	⊡ regional·length ⊡ *rgn·lg* (About the size of a region.)	3↊6,000$_z$=$964{,}224_d$ ft $132.\overline{7502}_z$=$182.6\overline{18}_d$ mi 293.8954752_d km
⊡ octqua·lengthel ⊡ *o↑lgℓ* = 10_z^{+8} ⊡ *lgℓ*	⊡ continental·length ⊡ *cnt·lg* (About the size of a continent.)	3,↊60,000$_z$=$11{,}570{,}688_d$ ft $1{,}327.\overline{5027}_z$=$2191.4\overline{18}_d$ mi $3{,}526.7457024_d$ km
⊡ ennqua·lengthel ⊡ *e↑lgℓ* = 10_z^{+9} ⊡ *lgℓ*	⊡ global·length ⊡ *glb·lg* (A bit more than a global circumference.)	3↊,600,000$_z$=$138{,}848{,}256_d$ ft $13{,}275.\overline{0275}_z$=$26{,}297.0\overline{18}_d$ mi $42{,}320.9484288_d$ km

Primel ⊡ Selected Powers of the ⊡ Areanel			
Quantitel Form *Abbrev*	Colloquial (Rationale)	*Abbrev*	SI and USC Equivalents
⊡areanel ⊡*arℓ*	⊡morsel·area	⊡*mo·ar*	$0.1042751736\overline{1}_d$ in^2 $67.2741710069\overline{4}_d$ mm^2
⊡unqua·areanel ⊡*u↑arℓ* = 10^{+1}_z ⊡*arℓ*	⊡stamp·area (About the size of a postage stamp)	⊡*mo·ar*	$1.25130208\overline{3}_d$ in^2 $8.0729005208\overline{3}_d$ cm^2
⊡biqua·areanel ⊡*b↑arℓ* = 10^{+2}_z ⊡*arℓ*	⊡hand·area	⊡*hd·ar*	15.015625_d in^2 0.9687480625_d dm^2
⊡triqua·areanel ⊡*t↑arℓ* = 10^{+3}_z ⊡*arℓ*	⊡lap·area (About the size of a seated human lap)	⊡*lp·ar*	$1.25130208\overline{3}_d$ ft^2 11.62497675_d dm^2
⊡quadqua·areanel ⊡*q↑arℓ* = 10^{+4}_z ⊡*arℓ*	⊡ell·area	⊡*ℓ·ar*	15.015625_d ft^2 1.39499721_d m^2
⊡pentqua·areanel ⊡*p↑arℓ* = 10^{+5}_z ⊡*arℓ*	⊡tarp·area (Approx size of a painter's dropcloth)	⊡*hb·ar*	180.1875_d ft^2 16.73996652_d m^2
⊡hexqua·areanel ⊡*h↑arℓ* = 10^{+6}_z ⊡*arℓ*	⊡habital·area (Avg floorspace of new home in US)	⊡*hb·ar*	2162.25_d ft^2 2.0087959824_d are
⊡septqua·areanel ⊡*s↑arℓ* = 10^{+7}_z ⊡*arℓ*	⊡jugeral·area (Approximates ancient Roman *jugerum*)	⊡*jg·ar*	$25{,}947_d$ ft^2 0.595661157025_d acre $0.241055517\overline{8}_d$ ha
⊡octqua·areanel ⊡*o↑arℓ* = 10^{+8}_z ⊡*arℓ*	⊡stadial·area	⊡*st·ar*	$311{,}364_d$ ft^2 7.147933884298_d acre 2.892666214656_d ha
⊡decqua·areanel ⊡*d↑arℓ* = $10^{+↊}_z$ ⊡*arℓ*	⊡dromal·area	⊡*dr·ar*	1029.30247933884_d acre 1.60828512396694_d mi^2 4.16543934910464_d km^2
⊡unnilqua·areanel ⊡*un↑arℓ* = 10^{+10}_z ⊡*arℓ*	⊡itineral·area	⊡*itn·ar*	231.59305785124_d mi^2 599.82326627106_d km^2

era. But this consideration has long since become obsolete. Today, it is trivial to specify any length unit using an exact count of caesium transition intervals and the speed of light, both of which are known today with exceeding accuracy.

At this point, you might be questioning whether Pendlebury or I have been "playing fast and loose" with "mundane realities," by picking values for gravity that are convenient for our respective purposes, rather than endeavoring to determine the exact "average" gravity and using that, whatever that may be, convenient or not.[↊]

I would counter that the purist notion that "Earth's gravity" is some kind of "constant of nature" is rather naive. Instead, gravity on Earth's surface is a somewhat "squishy" quantity, in that it *varies* over a certain range, due to a number of factors, the most significant being the counteracting centrifugal force of Earth's rotation, which causes gravitational acceleration to diminish from a maximum at the poles to a minimum at the equator. But so long as a given choice falls somewhere within this natural range, it's fair game to consider it a candidate for "Earth's gravity." If the utility of the metrology is improved in the process, then such a choice is completely legitimate. The important thing is that a metrology pick some *standard* for measuring acceleration. Then local gravity can be measured and quantified against that standard,

[↊] "Puritel" (brand mark: □) is an alternative metrology that is just like Primel, except that all of its "mundane realities" are uncompromisingly "pure," i.e., based on the naturally-occurring values. This is included, for comparison, on the DGW spreadsheet.

Primel ⊡ Selected Powers of the ⊡ Volumel		
Quantitel Form *Abbrev*	Colloquial *Abbrev* (Rationale)	SI and USC Equivalents
⊡ volumel ⊡ *vmℓ*	⊡ morsel·volume ⊡ *mo·vm*	0.111949104136604_d tsp $0.551788356779875578\overline{7037}_d$ ml
⊡ unqua·volumel ⊡ *u↑vmℓ* = 10^{+1}_z ⊡ *vmℓ*	⊡ mascaral·volume ⊡ *msc·vm* (Size of a cosmetic or perfume tube)	0.22389820827321_d fl oz $6.6214602813585069\overline{4}_d$ ml
⊡ biqua·volumel ⊡ *b↑vmℓ* = 10^{+2}_z ⊡ *vmℓ*	⊡ biberonal·volume ⊡ *bb·vm* (Size of a baby bottle, from Fr. *biberon*)	2.6867784992785_d fl oz $79.457523376302083\overline{3}_d$ ml
⊡ triqua·volumel ⊡ *t↑vmℓ* = 10^{+3}_z ⊡ *vmℓ*	⊡ hand·volume ⊡ *hd·vm*	1.00754193722944_d qt 0.953490280515625_d L
⊡ quadqua·volumel ⊡ *q↑vmℓ* = 10^{+4}_z ⊡ *vmℓ*	⊡ bucket·volume ⊡ *bkt·vm* (Typical size of a waste bucket)	3.02262581168831_d gal 11.4418833661875_d L
⊡ pentqua·volumel ⊡ *p↑vmℓ* = 10^{+5}_z ⊡ *vmℓ*	⊡ drum·volume ⊡ *dm·vm* (Typical size of an oil drum)	36.2715097402597_d gal 137.30260039425_d L
⊡ hexqua·volumel ⊡ *h↑vmℓ* = 10^{+6}_z ⊡ *vmℓ*	⊡ ell·volume ⊡ *ℓ·vm*	435.258116883117 gal 1.647631204731_d m^3
⊡ ennqua·volumel ⊡ *e↑vmℓ* = 10^{+9}_z ⊡ *vmℓ*	⊡ habital·volume ⊡ *hb·vm*	3723.875_d yd^3 2847.106721775168_d m^3
⊡ unnilqua·volumel ⊡ *un↑vmℓ* = 10^{+10}_z ⊡ *vmℓ*	⊡ stadial·volume ⊡ *st·vm*	$6{,}434{,}856_d$ yd^2 $4{,}919{,}800.4152275_d$ m^3
⊡ untriqua·volumel ⊡ *ut↑vmℓ* = 10^{+13}_z ⊡ *vmℓ*	⊡ dromal·volume ⊡ *dr·vm*	2.03959795266717_d mi^3 8.5014151175131_d km^3
⊡ unhexqua·volumel ⊡ *uh↑vmℓ* = 10^{+16}_z ⊡ *vmℓ*	⊡ itineral·volume ⊡ *itn·vm*	$3{,}524.425262209_d$ mi^3 $14{,}690.44532306_d$ km^3

and its deviation from that can be factored into physical computations. Gravity is not the only "mundane reality" that is "squishy" in this way, but each such case offers an opportunity to make a metrology more useful.

Further applying the principle of coherence yields a set of Primel base units that are generally smaller than TGM's units. Yet these units clearly bear a familial relationship to TGM units, analogous to the relationship between CGS and the meter-kilogram-second (MKS) system, which eventually became SI. When we scale these coherent units by dozenal powers and simple dozenal factors, many of the resulting auxiliary units show striking resemblances to familiar units in both SI and USC.

Quantitels

A *quantitel* is a generic, formal name for the coherent unit of a given type of physical quantity, within some metrology. A quantitel is formed by appending the suffix **-el**, short for "element," onto the name of the quantity itself. In the same fashion that the word *pixel* designates a "picture-element," likewise a **timel** ("time-element"), a **lengthel** ("length-element"), a **massel** ("mass-element"), etc., would be coherent base units of, respectively, time, length, mass, and so forth.

Each quantitel makes it self-evident what type of quantity it measures. Quantitels entirely bypass the practice of using the names of "dead scientists" as "honor names" for units. There is no attendant need to memorize which obscure historical figure was associated with which science and therefore which type of quantity. How many people

Primel ⊡ Selected Powers of the ⊡ Massel			
Quantitel Form *Abbrev*	Colloquial (Rationale)	*Abbrev*	SI and USC Equivalents
⊡ tricia·massel ⊡ $t{\downarrow}ms\ell = 10_z^{-1}$ ⊡ $ms\ell$	⊡ granular·mass	⊡ *grn·ms*	0.319313024714_d mg
⊡ massel ⊡ $ms\ell$	⊡ morsel·mass	⊡ *mo·ms*	0.019463216516_d oz 0.551772906706_d g
⊡ unqua·massel ⊡ $u{\uparrow}ms\ell = 10_z^{+1}$ ⊡ $ms\ell$	⊡ mascaral·mass (Size of a cosmetic or perfume tube)	⊡ *msc·ms*	0.233558598192_d oz 6.621274880472_d g
⊡ biqua·massel ⊡ $b{\uparrow}ms\ell = 10_z^{+2}$ ⊡ $ms\ell$	⊡ biberonal·mass (Size of a baby bottle, from Fr. *biberon*)	⊡ *bb·ms*	2.802703178304 oz 79.455298565664_d g
⊡ triqua·massel ⊡ $t{\uparrow}ms\ell = 10_z^{+3}$ ⊡ $ms\ell$	⊡ hand·mass	⊡ *hd·ms*	2.10202738372_d lb 0.953463582788_d kg
⊡ quadqua·massel ⊡ $q{\uparrow}ms\ell = 10_z^{+4}$ ⊡ $ms\ell$	⊡ bucket·mass (Typical size of a waste bucket)	⊡ *bkt·ms*	25.224328604736_d lb 11.4415629934556_d kg
⊡ pentqua·massel ⊡ $p{\uparrow}ms\ell = 10_z^{+5}$ ⊡ $ms\ell$	⊡ drum·mass (Typical size of an oil drum)	⊡ *dm·ms*	302.691943256832_d lb 137.298755921467_d kg
⊡ hexqua·massel ⊡ $h{\uparrow}ms\ell = 10_z^{+6}$ ⊡ $ms\ell$	⊡ ell·mass	⊡ *ℓ·ms*	3632.30331907 lb 1647.58507106_d kg

can instantly recognize that *newtons* measure force, whereas *joules* measure energy, while *watts* measure power? But it would go without saying that **forcels** measure force, **energels** measure energy, and **powerels** measure power.

Moreover, quantitels allow us to supply *every* type of quantity with a serviceable unit name, with minimal effort. They're not limited to just a handful of "fundamental" quantities or to a few "important" quantities deemed worthy of honor names. SI's expedient of referring to so many units via often-unwieldy "derived unit expressions" is a ludicrous deficiency, all the more inexcusable for being so unnecessary.

For instance, rather than measure velocity in *lengthels per timel*, you can simply use **velocitels**. Rather than measure volume in *cubic lengthels*, you can simply use **volumels**. Rather than measure momentum in *massel-lengthels per timel* or even *massel-velocitels*, you can just use **momentumels**. And so forth. If you can name the type of quantity you are measuring, you can instantly generate a quantitel for it. If a *new* type of quantity comes along, you can instantly generate a quantitel for *that*. Science has no problem coming up with terminology for the phenomena it studies, so by rights it should be trivial to name the units for measuring said phenomena.

Besides, the choice of which units should be "fundamental" and which should be "derived" is somewhat arbitrary, and can even be a matter of debate. Instead of wasting time and energy on such debates, students of the physical sciences should simply internalize the equations of physical law, and refer to them when they need to do dimensional analysis on their units. If "force equals mass times acceleration" and "acceleration is the second time-derivative of position," then it should be trivial to translate that into "a forcel is a massel times an accelerel" and "an accelerel is a lengthel per timel squared" as needed. But it should not be necessary to declare the dimensional decomposition of a unit every time we make a measurement.

Another point is that we can have synonymous quantitels wherever a quantity can be described with synonymous terms, so long as those terms describe quantities that are truly commensurate. For instance, "width," "height," "breadth," "depth,"

Primel ⊡ Selected Powers of the ⊡ Weightel (⊡ Forcel)		
Quantitel Form *Abbrev*	Colloquial *Abbrev* (Rationale)	SI and USC Equivalents
⊡ tricia·weightel ⊡ $t{\downarrow}wt\ell = 10_z^{-1}$ ⊡ $wt\ell$	⊡ granular·weight ⊡ *grn·wt*	0.318983047680_d mg_f 3.128155104529_d μN
⊡ weightel ⊡ $wt\ell$	⊡ morsel·weight ⊡ *mo·wt*	0.019443103292_d oz_f 0.551202706391_d g_f 5.405452020626_d mN
⊡ unqua·weightel ⊡ $u{\uparrow}wt\ell = 10_z^{+1}$ ⊡ $wt\ell$	⊡ mascaral·weight ⊡ *msc·wt* (Size of a cosmetic or perfume tube)	0.23331723950_d oz_f 6.61443247669_d g_f 64.8654242475_d mN
⊡ biqua·weightel ⊡ $b{\uparrow}wt\ell = 10_z^{+2}$ ⊡ $wt\ell$	⊡ biberonal·weight ⊡ *bb·wt* (Size of a baby bottle, from Fr. *biberon*)	2.79980687401 oz_f 79.3731897203_d g_f 778.385090970_d mN
⊡ triqua·weightel ⊡ $t{\uparrow}wt\ell = 10_z^{+3}$ ⊡ $wt\ell$	⊡ hand·weight ⊡ *hd·wt*	2.09985515551_d lb_f 0.95247827664_d kg_f 9.34062109164_d N
⊡ quadqua·weightel ⊡ $q{\uparrow}wt\ell = 10_z^{+4}$ ⊡ $wt\ell$	⊡ bucket·weight ⊡ *bkt·wt* (Typical size of a waste bucket)	25.1982618661_d lb_f 11.4297393197_d kg_f 112.087453100_d N
⊡ pentqua·weightel ⊡ $p{\uparrow}wt\ell = 10_z^{+5}$ ⊡ $wt\ell$	⊡ drum·weight ⊡ *dm·wt* (Typical size of an oil drum)	302.3791423932_d lb_f 137.1568718366_d kg_f 1.345049437196_d kN
⊡ hexqua·weightel ⊡ $h{\uparrow}wt\ell = 10_z^{+6}$ ⊡ $wt\ell$	⊡ ell·weight ⊡ *ℓ·wt*	3628.549708721 lb_f 1.645882462039_d Mg_f 16.14059324636_d kN

"distance," "displacement," "position," "altitude" are all quantities commensurate with "length," so **widthel**, **heightel**, **breadthel**, **depthel**, **distancel**, **displacementel**, **positionel**, **altitudel** are all just synonyms for **lengthel**. It is a bit more concise to say that a certain box is "50_z ⊡ breadthels by 40_z ⊡ widthels by 30_z ⊡ depthels," than to say it has "a breadth of 50_z ⊡ lengthels, a width of 40_z ⊡ lengthels, and a depth of 30_z ⊡ lengthels." Since "work" and "heat" are just commensurate forms of "energy," **workel** and **heatel** would be synonyms for **energel**. Since "weight" is just an example of "force," **weightel** and **forcel** would be synonyms (but only if the given metrology uses a value for gravity as its **accelerel**).

Sometimes the scientific term for a given type of quantity is already on the longish side, for instance "acceleration" or "momentum." Strict application of the **-el** suffix to these names can yield correspondingly long quantitels, such as **accelerationel** or **momentumel**. It's acceptable to truncate such quantitels, without changing their meaning, as long as this doesn't lead to ambiguity. For instance I have already been referring to the **accelerel**, which can be understood as just a truncated synonym for **accelerationel**. Similarly, **momentumel** might be truncated to **momel**, but perhaps not **momentel**, since this might be confused with the quantitel for "moment."

As a final note, I did not try to make quantitels linguistically "universal." They really are intended as English coinages specifically, and not meant to "work" in all languages. *However*, there is no reason they cannot be *translated* into other languages. Each language would take its own native words for physical quantities and amend them with some common particle appropriate in that language to convey the sense of a piece or portion of the given type of quantity. But I leave that exercise to be worked out by native speakers who are more expert in their own languages.

Branding

My intent for quantitels was that they would be generic terms reusable across many metrologies. An unadorned quantitel could refer to the abstract notion of a coherent unit, allowing us to make general statements such as, "every DGW system begins by choosing a timel;" or "using a value for gravity as an accelerel makes it almost interchangeable to report massels or weightels when 'weighing' something;" or "one energel (as one workel) can raise one massel of water by one heightel (lengthel) against one accelerel, and that same energel (as one heatel) can raise that same massel of water by one temperaturel." Such statements, and similar ones in preceding paragraphs, can apply to any metrology.

On the other hand, if we qualify a quantitel with the "brand name" of a given metrology, it becomes the coherent unit for that specific metrology. For instance, we can talk of Primel's coherent units as the "prime·timel," "prime·lengthel," "prime·massel," etc. When inventing a new metrology, all we need do is come up with a pithy name for the entire metrology. Then we can immediately start discussing and utilizing all its units, and get on with exploring the merits of the metrology itself. This can be a vast time-saver. We need not first engage in some long creative process to find unique names for all of its units, distinct from the units of all other metrologies. (It does not *preclude* such creativity, however. More about that in a moment.)

We can make this even more convenient by choosing a "brand-mark," a single emoji-like character that can serve as an abbreviation for the brand-name. For instance, the brand mark I have chosen for Primel is ⚀, Unicode 'DIE FACE-1' (U+2680$_{\text{x}}$),[ε] which may be pronounced "Primel," or "prime."[10,11]

Scaling Prefixes and Colloquial Names

Beyond the coherent quantitels, Primel defines many auxiliary units for each type of quantity. First, it scales its quantitels to any power of dozen, and sometimes to convenient factors of dozen, using the dozenal scaling prefixes from Systematic Dozenal Nomenclature (SDN) (see page 31$_{\text{z}}$). These are comparable to the decimal scaling prefixes defined for the metric system, but are much more comprehensive, taking full advantage of the high factorability of base twelve.

Primel also introduces many so-called "colloquial" names for its units, as creative alternatives for the formal names derived from quantitels and SDN prefixes. Each colloquial name attempts to provide an intuitive sense of scale by relating the given Primel unit to some comparably-sized physical object known to human experience, or to some customary or ancient unit that it might approximate. In the latter case, I try

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[ε] Originally, I picked the prime character (′) as Primel's brand mark, which may seem the obvious choice. However, compared to brand marks selected for other DGW metrologies, this was rather thin and indistinct. Moreover, it can tend to get lost in other punctuation, making it awkward to discuss Primel units in normal prose. For backward compatibility, the prime character may be considered an alternative, but the die face should be preferred.

[10] Brand marks might even be left silent if the discussion only makes use of one branded metrology. But in any discussion that compares and contrasts branded quantitels from multiple metrologies, or which uses unbranded quantitels in the abstract as well as specifically branded quantitels, it is necessary to pronounce the brand marks to avoid ambiguity.

[11] You can see many more examples of such brand marks, for other notional metrologies in a variety of bases, on the DGW Spreadsheet.

to only reuse existing unit names where the approximation is "close" (within $10\%_z$ or so). The closer the approximation, the more justified the reuse is.

Note that Primel's dozenal divisions of the day (see page 35_z) are identical to those the DSA founders identified for their Do-metric metrology (see page 21_z). However, I have elected to offer a completely new set of colloquial names for these divisions. One thing I strive for is to have colloquial names consist of ordinary English words, as much as possible. Portmanteau neologisms tend to be contrived and awkward, so I try to limit them to a few brand names rather than numerous colloquials. Unfortunately, the DSA founders seemed to favor portmanteaus. Furthermore, their choices for their time units relied too much on references to sexagesimal time and decimal:

- The *duor* is a portmanteau of "double hour," the hour being of course a sexagesimal unit. I suggest the ⊡**dwell** instead, as an allusion to the time the Sun spends each day "dwelling" in each "house" (an astrological term for a 30°_d or 1 uncia·turn sector of the sky relative to the observer).[12] Certainly if you engage in some activity for two hours straight, it's fair to say you are "dwelling" on it.☺[13]
- The *temin* is a portmanteau of "ten minutes," the minute being a sexagesimal unit, and "ten" of course being a decimal number. I suggest calling this the ⊡**breather** instead, as an allusion to "taking a breather" as a hiatus from work. In traditional time, the expression "take ten" also has this meaning, but "taking a breather" avoids the decimal/sexagesimal reference.
- The *minette* is a portmanteau of "minute" and the diminutive suffix "-ette," alluding to this as a shorter analog of a sexagesimal minute. I suggest the ⊡**trice** instead, a slightly archaic but otherwise ordinary word meaning a short period, and a pun on deriving this unit by "thrice" dividing the day by a dozen.
- The *vic* is a portmanteau of "vibration of C," alluding to the period of a musical note. I suggest ⊡**vibe** as a less opaque way to make the same allusion.
- The *grovic* and *dovic* are not even distinct colloquial names, they are just dozenal scalings of the *vic*. I suggest ⊡**lull** for the former, this being enough of a pause to be embarrassing in conversation. For the latter, I suggest ⊡**twinkling**, another slightly archaic word for a brief period, and the time to blink an eye.

The Primel colloquials in each of these cases are ordinary English words from the dictionary without any contrivance or awkward reference to sexagesimal or decimal. We can actually imagine these terms arising organically and completely independently of any knowledge of the terminology for sexagesimal time.

Colloquial Families

In many cases, a colloquial name for a length unit can be the basis for an entire family of colloquial names for related units. For instance, the Primel quantitels themselves (see table on page 34_z) form a "morsel" unit series based on the ⊡lengthel being the ⊡**morsel·length**. Note that Primel colloquial names tend to end in a noun indicating

[12] This oblique allusion to an astrological term is not necessarily an endorsement of the pseudoscience of astrology. It merely takes advantage of astrology as a fertile source of colorful metaphors, which is the name of the game when trying to coin memorable colloquial names.

[13] Primel does accept the traditional hour as an auxiliary unit, the ⊡semi·pentqua·timel, with *hour* as its colloquial name. However, Primel reserves the prerogative to characterize the hour as "half a ⊡dwell," rather than the ⊡dwell as a "double hour."

Primel ⊡ Selected "Hand" Series Units				
Quantity	Quantitel Form *Abbrev*	Colloquial *Abbrev*	Derivation	SI and USC Equivalents
Length	⊡unqua·lengthel ⊡*u*↑*lgℓ* = 10_{z}^{+1} ⊡*lgℓ*	⊡hand·length ⊡*hd·lg*	(resembles customary 4-in hand measure)	0.98425_{d} dm $3.Ɛ6_{\text{z}}$=$3.875\overline{6}_{\text{d}}$ in
Area	⊡biqua·areanel ⊡*b*↑*arℓ* = 10_{z}^{+2} ⊡*arℓ*	⊡hand·area ⊡*hd·ar*	⊡hd·lg^2	0.9687480625_{d} dm^2 15.015625_{d} in^2
Volume	⊡triqua·volumel ⊡*t*↑*vmℓ* = 10_{z}^{+3} ⊡*vmℓ*	⊡hand·volume ⊡*hd·vm*	⊡hd·lg^3	$0.953490280515625_{\text{d}}$ L $1.00754193722944_{\text{d}}$ qt
Mass	⊡triqua·massel ⊡*t*↑*msℓ* = 10_{z}^{+3} ⊡*msℓ*	⊡hand·mass ⊡*hd·ms*	⊡hd·vm × ⊡dsℓ	$0.953463582788_{\text{d}}$ kg 2.10202738372_{d} lb
Force Weight	⊡triqua·forcel ⊡*t*↑*fcℓ* = 10_{z}^{+3} ⊡*fcℓ* ⊡triqua·weightel ⊡*t*↑*wtℓ* = 10_{z}^{+3} ⊡*wtℓ*	⊡hand·force ⊡*hd·fc* ⊡hand·weight ⊡*hd·wt*	⊡hd·ms × ⊡accℓ	$0.952478276643_{\text{d}}$ kg$_f$ 9.34062109164_{d} N 2.09985515551_{d} lb$_f$
Energy Work	⊡quadqua·energel ⊡*q*↑*ngℓ* = 10_{z}^{+4} ⊡*ngℓ* ⊡quadqua·workel ⊡*q*↑*wkℓ* = 10_{z}^{+4} ⊡*wkℓ*	⊡hand·energy ⊡*hd·ng* ⊡hand·work ⊡*hd·wk*	⊡hd·fc × ⊡hd·lg	$0.917728023583454_{\text{d}}$ J
Pressure	⊡unqua·pressure ⊡*u*↑*psℓ* = 10_{z}^{+1} ⊡*psℓ*	⊡hand·pressure ⊡*hd·ps*	⊡hd·fc ÷ ⊡hd·ar	$0.964195073334_{\text{d}}$ kPa

the kind of quantity being measured, often the noun from which the associated quantitel is derived. This makes it easy to have a series of derivative names: ⊡**morsel·length**, ⊡**morsel·area**, ⊡**morsel·volume**, ⊡**morsel·mass**, ⊡**morsel·force**, etc.

Another notable example is the "hand" series starting from the ⊡**hand·length** as a colloquial for the ⊡unqua·lengthel. At $3.Ɛ6_{\text{z}}$ (3.875_{d}) USC inches, this resembles the customary "hand" unit of 4 USC inches. It also bears a remarkable resemblance to an SI decimeter. The derivatives from this (see table on page 42_{z}) turn out to be convenient sizes, mitigating the smallness of the "morsel" series:

Squaring the ⊡hand·length yields the ⊡**hand·area** (⊡biqua·areanel), which resembles a square decimeter. Cubing it yields the ⊡**hand·volume** (⊡triqua·volumel), which resembles a liter or USC quart. Filling the ⊡hand·volume with water at one ⊡densitel yields the ⊡**hand·mass** (⊡triqua·massel), which resembles a kilogram. Multiplying the ⊡hand·mass by one ⊡accelerel yields the ⊡**hand·force** (⊡triqua·forcel) or ⊡**hand·weight** (⊡triqua·weightel), which resembes a kilogram-force (the weight of a kilogram mass in 1 Earth gravity). Applying a ⊡hand·force over one ⊡hand·length yields the ⊡**hand·work** (⊡quadqua·workel) or ⊡**hand·energy** (⊡quadqua·energel) which resembles the joule. Dividing the ⊡hand·force by the ⊡hand·area yields the ⊡**hand·pressure** (⊡unqua·pressurel), which resembles the kilopascal. And so forth.

Similar series of units may be formed from other scalings of the ⊡lengthel. For instance, the colloquial name for the ⊡biqua·lengthel ($3Ɛ.6_{\text{z}}$ or 46.5_{d} USC inches) is the ⊡**ell·length**, because of its resemblance to the old English ell (39_{z} or 45_{d} USC inches).[14] From that, we can derive the ⊡**ell·area** (⊡quadqua·areanel), ⊡**ell·volume**

[14] I stumbled onto the similarity of the ⊡biqua·lengthel to the old English *ell* quite independently, and only later discovered that William S. Crosby, an early member of the DSA, had discovered this same similarity back in 1161_{z} (1945_{d}), as a "harried infantryman" in the US Army at the tail end of World War II. See *Duodecimal Bulletin*, Vol. 52_{z}, No. 1, WN $Ɛ2_{\text{z}}$, page 30_{z}. http://dozenal.org/drupal/sites_bck/default/files/DuodecimalBulletinIssue521.pdf. Crosby also recognized the

Primel ⊡ Selected "Foot" Series Units				
Quantity	Quantitel Form *Abbrev*	Colloquial *Abbrev*	Derivation	SI, USC, TGM Equivs
Length	⊡trina·unqua·lengthel ⊡*t·u↑lgℓ* = 30$_z$ ⊡*lgℓ*	⊡foot·length ⊡*ft·lg*	(resembles customary foot and TGM Grafut)	Ɛ.76$_z$=11.625$_d$ in 0.295275$_d$ m ≈ Grafut
Area	⊡ennea·biqua·areanel ⊡*e·b↑arℓ* = 900$_z$ ⊡*arℓ*	⊡foot·area ⊡*ft·ar*	⊡ft·lg^2	0.9384765625$_d$ ft^2 8.7187325625$_d$ dm^2 ≈ Surf
Volume	⊡bitrina·triqua·volumel ⊡*bt·t↑vmℓ* = 23,000$_z$ ⊡*vmℓ*	⊡foot·volume ⊡*ft·vm*	⊡ft·lg^3	0.909149169921875$_d$ ft^3 25.74423809682744$_d$ L ≈ Volm
Mass	⊡bitrina·triqua·massel ⊡*bt·t↑msℓ* = 23,000$_z$ ⊡*msℓ*	⊡foot·mass ⊡*ft·ms*	⊡ft·vm × ⊡dsℓ	25.7435167353$_d$ kg 56.7547393605$_d$ lb ≈ Maz
Force Weight	⊡bitrina·triqua·forcel ⊡*bt·t↑fcℓ* = 23,000$_z$ ⊡*fcℓ* ⊡bitrina·triqua·weightel ⊡*bt·t↑wtℓ* = 23,000$_z$ ⊡*wtℓ*	⊡foot·force ⊡*ft·fc* ⊡foot·weight ⊡*ft·wt*	⊡ft·ms × ⊡accℓ	25.7169134694$_d$ kg$_f$ 252.196769474$_d$ N 56.6960891987$_d$ lb$_f$ ≈ Mag
Energy Work	⊡hexennea·quadqua·energel ⊡*he·q↑ngℓ* = 690,000$_z$ ⊡*ngℓ* ⊡hexennea·quadqua·workel ⊡*he·q↑wkℓ* = 690,000$_z$ ⊡*wkℓ*	⊡foot·energy ⊡*ft·ng* ⊡foot·work ⊡*ft·wk*	⊡ft·fc × ⊡ft·lg	74.4674011064$_d$ J ≈ Werg
Pressure	⊡trina·unqua·pressurel ⊡*t·u↑psℓ* = 30$_z$ ⊡*psℓ*	⊡foot·pressure ⊡*ft·ps*	⊡ft·fc ÷ ⊡ft·ar	2.892585220827$_d$ kPa ≈ Prem

(⊡hexqua·volumel), ⊡**ell·mass** (⊡hexqua·massel), ⊡**ell·weight** (⊡hexqua·weightel), ⊡**ell·work** (⊡octqua·workel), etc.

Accommodating TGM Units

Primel auxiliary units need not be limited to just pure powers of its quantitels. We can include SDN multiplier prefixes as well, and the results can be granted appropriate colloquial names as well. One particularly interesting example is the "foot" series. (See the table on page 43$_z$.)

The ⊡trina·unqua·lengthel (30$_z$ ⊡lgℓ) approximates the TGM Grafut as well as the USC foot, and therefore gets the colloquial name ⊡**foot·length** (⊡ft·lg). Squaring that gives us the ⊡ennea·biqua·areanel (900$_z$ ⊡arℓ) or ⊡**foot·area** (⊡ft·ar), approximating the TGM Surf. Cubing the ⊡foot·length yields the ⊡bitrina·triqua·volumel (23,000$_z$ ⊡vmℓ), or ⊡**foot·volume** (⊡ft·vm), approximating the TGM Volm. Filling that with water yields the ⊡bitrina·triqua·massel (23,000$_z$ ⊡msℓ), or ⊡**foot·mass** (⊡ft·ms), approximating the TGM Maz. Applying 1 ⊡accelerel to that yields the ⊡bitrina·triqua·weightel (23,000$_z$ ⊡wtℓ), or ⊡**foot·weight** (⊡ft·wt), approximating the TGM Mag. Giving that a 1 ⊡foot·length displacement yields the ⊡hexennea·quadqua·workel (690,000$_z$ ⊡wkℓ), or ⊡**foot·work** (⊡ft·wk), approximating

similarity of the hand·mass to the kilogram and was advocating it as his massel (though not in those terms, of course). In fact, I credit Crosby with being the first to articulate the notion of deriving a metrology from the day, Earth's gravity, and the density of water, some 2 unquennia before Pendlebury. Pendlebury clearly acknowledges Crosby in his *Duodecimal Review* article from 1181$_z$ (1969$_d$). (See page 28$_z$ in this issue.) I've included Crosby's system on the DGW spreadsheet.

the TGM Werg. Dividing the ⊡foot·weight by one ⊡foot·area yields the ⊡trina·unqua·pressurel (30_z ⊡psℓ), or ⊡**foot·pressure** (⊡ft·ps), approximating the TGM Prem. This demonstrates the close family relationship between Primel and TGM. The only reason these correspondences are approximations and not exact, is that Pendlebury and I chose slightly different values for our accelerels.

Note that these colloquials hinge on the ordinary word "foot." As a matter of principle, I will not try to appropriate Pendlebury's unit names as colloquials for Primel analogs. Pendlebury's coinages, after all, are portmanteaus, some of which are rather awkward and oblique. Likewise, I will not appropriate any of SI's "honor names" as colloquials for any Primel units, even where there might be a close analog. Honor names, after all, are completely opaque.

English Binary Series

The resemblance of the ⊡hand·volume to the USC quart is remarkably close (less than a perbiqua off). Scaling this up and down by binary powers yields equally close analogs for all the traditional old English and USC volume units, everything from a ⊡**tun·volume** to a ⊡**gallon·volume** to a ⊡**tablespoon·volume**. Dividing the latter by 3 even yields a ⊡**teaspoon·volume** (consisting of precisely 9 ⊡morsel·volumes) that is equally close to its own analog. (See table on page 45_z.)

I wouldn't say these auxiliary units are "dozenal-metric," per se, but Americans still might find them handy as a form of *mesures usuelles*. Plus, they're an excellent opportunity for students to learn their powers of 2 in dozenal. With two powers of 2 as factors, dozenal is relatively friendly toward binary divisions.

Note that I chose not to use the hypothetical ⊡~~ounce~~·volume (⊡~~oz~~·vm) as the colloquial name for the analog of the fluid ounce. The problem with "ounce" is that it is an English derivative of Latin *uncia*. But this unit isn't a dozenth of anything in Primel. So I've substituted ⊡**swig·volume** instead.

A similar consideration applies to the hypothetical colloquial ⊡~~inch~~·length (⊡~~in~~·lg) for the ⊡trina·lengthel (3 ⊡lengthels). The English word "inch" is another derivative of Latin *uncia*. While it is true that this size is a dozenth of the ⊡foot·length, nevertheless in Primel the latter is not the coherent unit, it is just another auxiliary unit. So I propose the colloquial ⊡**thumb·length** for the former, on the grounds that several languages translate "inch" into whatever word they use for "thumb." (Cf. Latin *pollex*, Italian *pollice*, Spanish *pulgada*, Portuguese *polegada*, French *pouce*, Dutch *duim*, Swedish *tum*, Danish *tomme*, Norwegian *tommer*.) It turns out the ⊡**thumb·volume** (a cubic ⊡thumb·length or ⊡bitrina·volumel) is identical to the ⊡tablespoon·volume.

These volume units would all be associated with corresponding mass units, from ⊡**teaspoon·mass**, ⊡**tablespoon·mass**, ⊡**swig·mass**, etc., to ⊡**gallon·mass**, ultimately to ⊡**ton·mass** (approximating the USC ton and the metric tonne). The ⊡**pint·volume** could be associated with a ⊡**pound·mass** (⊡lb·ms). Likewise, these would be associated with corresponding weight (force) units, from ⊡**teaspoon·weight** to ⊡**gallon·weight** to ⊡**ton·weight**, with ⊡**pint·volume** and ⊡**pound·mass** associated with a ⊡**pound·weight** (⊡lb·wt).

Primel Zoom

I have celebrated the close family relationship between Primel and TGM in a Powerpoint presentation titled "Primel Zoom," which I first presented at the annual meeting of the

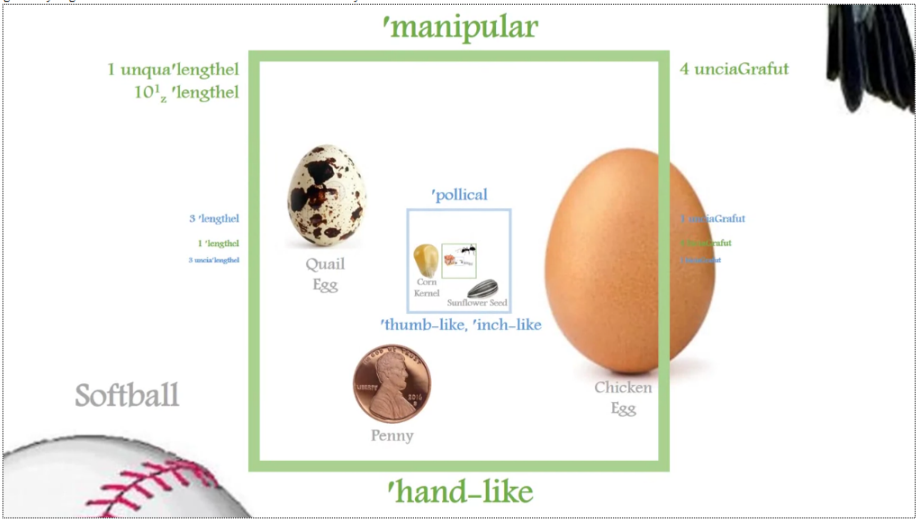
Primel ⊡ Analogs of English Binary Volume Series

Quantitel Form *Abbrev*	Colloquial (Rationale)	*Abbrev*	SI and USC Equivalents
⊡ volumel ⊡ *vmℓ*	⊡ morsel·volume	⊡ *mo·vm*	0.111949104136604_d tsp $0.5517883567798755787\overline{037}_d$ ml
⊡ ennea·volumel ⊡ *e·vmℓ* = 9_z ⊡ *vmℓ*	⊡ teaspoon·volume	⊡ *tsp·vm*	1.00754193722944_d tsp $4.96609521101888020 8\overline{3}_d$ ml
⊡ bitrina·volumel ⊡ *bt·vmℓ* = 23_z ⊡ *vmℓ*	⊡ tablespoon·volume	⊡ *tbsp·vm*	1.00754193722944_d tbsp $14.89828563305664062 5_d$ ml
⊡ quadhexa·volumel ⊡ *qh·vmℓ* = 46_z ⊡ *vmℓ*	⊡ swig·volume (can't use ~~ounce·volume~~)	⊡ *swg·vm*	1.00754193722944_d fl oz 29.79657126611328125_d ml
⊡ ennea·unqua·volumel ⊡ *e·u↑vmℓ* = 90_z ⊡ *vmℓ*	⊡ jack·volume (archaic word for a quarter cup)	⊡ *jck·vm*	2.01508387445887_d fl oz 59.5931425322265625_d ml
⊡ unhexa·unqua·volumel ⊡ *uh·u↑vmℓ* = 160_z ⊡ *vmℓ*	⊡ gill·volume (archaic word for a half cup)	⊡ *gil·vm*	4.03016774891775_d fl oz 119.186285064453125_d ml
⊡ trina·biqua·volumel ⊡ *t·b↑vmℓ* = 300_z ⊡ *vmℓ*	⊡ cup·volume	⊡ *cu·vm*	1.00754193722944_d cup 238.37257012890625_d ml
⊡ hexa·biqua·volumel ⊡ *h·b↑vmℓ* = 600_z ⊡ *vmℓ*	⊡ pint·volume (related mass unit: ⊡ pound·mass)	⊡ *pt·vm*	1.00754193722944_d pt 476.7451402578125_d ml
⊡ triqua·volumel ⊡ *t↑vmℓ* = $1{,}000_z$ ⊡ vmℓ	⊡ hand·volume	⊡ *hd·vm*	1.00754193722944_d qt 0.953490280515625_d L
⊡ bina·triqua·volumel ⊡ *b·t↑vmℓ* = $2{,}000_z$ ⊡ *vmℓ*	⊡ pottle·volume (archaic word for a half gallon)	⊡ *ptt·vm*	0.50477096861472_d gal 1.90698056103125_d L
⊡ quadra·triqua·volumel ⊡ *q·t↑vmℓ* = $4{,}000_z$ ⊡ *vmℓ*	⊡ gallon·volume	⊡ *gal·vm*	1.00754193722944_d gal 3.8139611220625_d L
⊡ octa·triqua·volumel ⊡ *o·t↑vmℓ* = $8{,}000_z$ ⊡ *vmℓ*	⊡ peck·volume	⊡ *pk·vm*	2.01508387445888_d gal 7.627922244125_d L
⊡ unquadra·triqua·volumel ⊡ *uq·t↑vmℓ* = $14{,}000_z$ ⊡ *vmℓ*	⊡ pail·volume	⊡ *pl·vm*	4.03016774891776_d gal 15.25584448825_d L
⊡ biocta·triqua·volumel ⊡ *bo·t↑vmℓ* = $28{,}000_z$ ⊡ *vmℓ*	⊡ bushel·volume	⊡ *bu·vm*	8.06033549783552_d gal 30.5116889765_d L
⊡ pentquadra·triqua·volumel ⊡ *pq·t↑vmℓ* = $54{,}000_z$ ⊡ *vmℓ*	⊡ strike·volume	⊡ *stk·vm*	16.120670995671_d gal 61.023377953_d L
⊡ decocta·triqua·volumel ⊡ *do·t↑vmℓ* = ↊$8{,}000_z$ ⊡ *vmℓ*	⊡ barrel·volume	⊡ *bbl·vm*	32.2413419913421_d gal 122.046755906_d L
⊡ unennquadra·triqua·volumel ⊡ *ueq·t↑vmℓ* = $194{,}000_z$ ⊡ *vmℓ*	⊡ seam·volume	⊡ *sm·vm*	64.4826839826842_d gal 244.093511812_d L
⊡ trihexocta·triqua·volumel ⊡ *tho·t↑vmℓ* = $368{,}000_z$ ⊡ *vmℓ*	⊡ pipe·volume	⊡ *pp·vm*	128.965367965368_d gal 488.187023624_d L
⊡ septunquadra·triqua·volumel ⊡ *suq·t↑vmℓ* = $714{,}000_z$ ⊡ *vmℓ*	⊡ tun·volume (related mass unit: ⊡ ton·mass)	⊡ *tn·vm*	257.930735930737_d gal 976.374047248_d L

Dozenal Media

Video

Here, we have Kodegadulo's *Primel Zoom*, a fantastic romp through the universe from the Planck length to the scale of the whole kit-and-kaboodle. Scientifically interesting, and a great way to get an idea of the scales of the Primel and TGM metric systems.



A frame from the "Primel Zoom" video, about halfway through. The outermost green box represents an ⊡unqua·lengthel (or ⊡hand·length). The blue box within that represents an uncia·Grafut (or ⊡thumb·length). The box within that represents a ⊡lengthel (or ⊡morsel·length). Barely discernible is a bicia·Grafut (or ⊡dermal·length). The next step will expand the view to the Grafut (or ⊡foot·length) level, and the step after that will expand it to the ⊡biqua·lengthel (or ⊡ell·length) level.

Dozenal Society of America in Atlanta in 1200_z (2016_d). This presentation explores all levels of scale in dozenal terms, from the Planck length to the span of the observable universe, interleaving dozenal powers of the Primel ⊡lengthel with dozenal powers of the TGM Grafut.

Like a set of nested Russian dolls, each Primel-measured slide is followed by a TGM-measured slide that expands the view by 3; each TGM-measured slide is followed by a Primel-measured slide that expands the view by 4. Thus every 2 steps constitutes an expansion of the view by a dozenal order of magnitude. I take advantage of Powerpoint's "zoom" transition to give the sense of the view expanding with each step.

Along the way, I populate the view with representative objects that exist at each scale, from quarks and atoms, to everyday objects at the human scale, to galaxies and superclusters. Objects carry through from frame to frame, shrinking in the expanded view, as new objects surrounding them are revealed at the next level of scale.

DSA President Donald Goodman III (member 398_z) was kind enough to convert this presentation into a video, set it to music, and post it at http://dozens.org/drupal/content/media.html. See page 46_z for an illustration.

Reusing Unit Names

Quantitels can be reused across all metrologies, providing formal names for the respective coherent units of each metrology, but their sizes will tend to be very different from metrology to metrology, based on the choices made.[15] Colloquial names can also be reused across many metrologies, but to a certain degree they are more "absolute" than quantitels. They intrinsically allude to particular levels of scale, so if one metrology borrows a colloquial name from another, the proviso is that the new version of the unit should be similar in size to the borrowed version. It need not be identical in size, but the closer of an analog it is, the better.

As an example, another DGW metrology I have experimented with is one I've dubbed "Tertiel" (because it was the third idea that I had for a metrology). (Brand mark ⚂, suggested pronunciation "tersh.") Tertiel starts by selecting the pentcia·day as its ⚂timel, but otherwise it proceeds with the exact same choices as Primel for the other "mundane realities." The ⚂timel is identical in size to the ⚀unqua·timel, so colloquially the ⚂twinkling is identical to the ⚀twinkling. It's just that Tertiel treats that period as its coherent timel, whereas Primel treats it as an auxiliary unit.

This leads to Tertiel's coherent ⚂lengthel being identical in size to Primel's ⚀biqua·lengthel, so, colloquially, the ⚂ell·length is identical to the ⚀ell·length, where "ell" refers absolutely to a size of $3↊.6_z$ (46.5_d) USC inches, resembling the old English ell. In fact, the entire ⚀ell series of derived units is exactly duplicated by corresponding derived ⚂ell series units. The only difference is that Tertiel treats all of those as its quantitels, whereas Primel treats them as dozenal scalings of its quantitels.

Primel units tend to be small; Tertiel units tend to be large. For instance, the ⚂massel or ⚂ell·mass, at over a ton and a half, is $1{,}000{,}000_z$ (a hexqua) times larger than the ⚀massel. The ⚂energel or ⚂ell·energy, at over 19_d kilojoules, is $100{,}000{,}000_z$ (an octqua) times larger than the ⚀energel. However, both metrologies can accept the "hand" unit series as useful auxiliaries that are more convenient in size. So for instance the ⚂hand·mass is identical to the ⚀hand·mass, and both are a fair approximation of a kilogram. But Tertiel treats this as the ⚂tricia·massel (0.001_z of its huge massel), whereas Primel treats it as the ⚀triqua·massel (1000_z times its tiny massel).

My own personal preference is to build up from small units. It is a compelling analogy to take the gram and the ⚀massel and scale both up by three orders of magnitude in their respective bases, to yield the kilogram and ⚀triqua·massel, and have the results approximate each other so closely. However, if you prefer to start with large units and divide them down, you might find Tertiel an interesting alternative, reminiscent of the Meter-Tonne-Second system.[16]

Warming up to Temperature

The temperature scales in common use, Celsius and Fahrenheit, were derived by picking specific anchor temperatures, such as the freezing point and boiling point of water, and dividing the temperature difference by some "convenient" number to define a "degree" unit. But a DGW metrology derives its coherent unit of temperature, or **temperaturel**, by first establishing a coherent relationship between heat and temperature. It takes an intrinsic thermodynamic property of water, its "specific heat

[15] Quantitels can even be used to talk about systems like SI and TGM. The DGW spreadsheet includes SI as the "int'l" metrology with a globe emoji as brand mark. It gives TGM the brand prefix "pendle" and brand mark ⊖ (signifying Pendlebury's choice to cut the day in half). The int'l·timel, int'l·lengthel, and int'l·massel would be the second, meter, and kilogram, respectively. The ⊖timel, ⊖lengthel, and ⊖massel would be the Tim, Grafut, and Maz, respectively.

[16] https://en.wikipedia.org/wiki/Metre-tonne-second_system_of_units.

capacity” — or as I prefer to term it, its “massic heatability”[17] — and identifies that as a “mundane reality.” Some candidate value for this property becomes a coherent unit, the **masselic·heatabilitel**,[18] defined as one heatel per massel per temperaturel. The corresponding temperaturel is thus the temperature change you get when you apply one heatel (one energel in the form of heat) to one massel of water.

For most DGW metrologies, this turns out to be a very tiny temperature difference, because the massic heatability of water is relatively large, and in general heat is a more “concentrated” form of energy than work. In Primel, the ⊡**temperaturel** is equivalent to only about 19_{d} micro·kelvins. So to yield a more convenient temperature unit for everyday use, we need to scale this up with an SDN prefix. The ⊡**quadqua·temperaturel** (abbreviation ⊡q↑tpℓ) turns out to be a fairly useful size.

In the 19th_{d} Century/11st_{z} Biquennium, James Prescott Joule established the mechanical equivalence of work and heat. This means that one ⊡heatel, the amount of energy in the form of heat needed to raise the temperature of one ⊡massel of water by one ⊡temperaturel, is equivalent to one ⊡workel, the amount of energy in the form of work needed to lift one ⊡massel by one ⊡heightel against one ⊡accelerel of gravity. Likewise one ⊡quadqua·heatel, which would raise one ⊡massel of water by one ⊡quadqua·temperaturel, is equal to one ⊡quadqua·workel, which would lift one ⊡massel by one ⊡quadqua·heightel.

Since the ⊡quadqua·lengthel resembles an ancient Greek *stadion* unit, I’ve given it the colloquial name of ⊡**stadial·length**. Similarly, I’ve given the ⊡quadqua·temperaturel the colloquial name ⊡**stadial·temperature** (abbreviated ⊡st·tp), or more concisely, ⊡**stadegree** (abbreviated ⊡ς°).[19]

The massic heatability of water is another example of a “squishy” quantity, because it varies over a certain range depending on conditions of temperature and pressure; this gives us some wiggle room for selecting a quantitel. The strict average value over water’s liquid range ($4190_{\text{d}}\,\frac{\text{J}}{\text{kg}\cdot\text{K}}$) yields a ⊡stadegree very close to $\frac{5}{7}$ of a Fahrenheit degree. In fact, we can get a ⊡stadegree *exactly* equal to $\frac{5}{7}$°F, by judiciously setting the ⊡**masselic·heatabilitel** to a specific value (about $4198_{\text{d}}\,\frac{\text{J}}{\text{kg}\cdot\text{K}}$), that is well within the natural range for water in liquid state, only slightly above the average value, and slightly less than the standard dietary kilocalorie ($4200_{\text{d}}\,\frac{\text{J}}{\text{kg}\cdot\text{K}}$),

This choice has the effect of dividing the liquid range of water, from the freezing point to the boiling point, into exactly 190_{z} (252_{d}) ⊡stadegrees. Compare this with the same range being covered by exactly 180_{d} Fahrenheit degrees, and of course exactly 100_{d} Celsius degrees. So even though the ⊡stadegree is derived from an intrinsic property of water, by sheer coincidence and some careful selection, we get a practical unit that exactly divides the liquid range of water into a fairly round number anyway. Best of both worlds, as it were.

Interestingly, 100_{z} ⊡stadegrees (or 1 ⊡hexqua·temperaturel)[1↊] bears a strong resemblance to 100_{d} Fahrenheit degrees (it is exactly $102\frac{6}{7}{}^{\circ}_{\text{d}}$F).

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[17] ISO 31-0 (see https://en.wikipedia.org/wiki/ISO_31) suggests *massic* as a substitute for “specific,” with the meaning “a quantity divided by its associated mass.” Similar *-ic* endings are used to derive *volumic* to indicate dividing by volume, *areic* for dividing by surface area, and *lineic* for dividing by length. Quantitels for such reciprocal quantities can be formed by appending **-elic**.

[18] masselic = massel^{-1}. heatabilitel = heatel ÷ temperaturel.

[19] I know, I know. I made a big deal about eschewing portmanteaus earlier, and ⊡**stadegree** is undeniably a portmanteau of “stadium” and “degree.” All I can say is, nobody’s perfect.☺ It seems like a catchy name to me, but others might disagree. If you prefer rigorous adherence to principle, then use ⊡**stadial·temperature** as the colloquial.

[1↊] Since the ⊡hexqua·lengthel gets the colloquial ⊡**itineral·length**, the ⊡hexqua·temperaturel could get the colloquial ⊡**itineral·temperature** as part of the same colloquial family.

Primel $\boxdot$ Temperature Scales				
Description	Degrees Celsius	$\boxdot$ Stadegrees Crystallic	$\boxdot$ Stadegrees Familiar	Degrees Fahrenheit
°C = °F	-40°_{d}C	$-84\frac{4}{5}{}_{z}$ $\boxdot\varsigma^{\circ}_{c}$	$-44\frac{4}{5}{}_{z}$ $\boxdot\varsigma^{\circ}_{f}$	-40°_{d}F
	$-38\frac{2}{21}{}^{\circ}_{d}$C	-80_{z} $\boxdot\varsigma^{\circ}_{c}$	-40_{z} $\boxdot\varsigma^{\circ}_{f}$	$-36\frac{4}{7}{}^{\circ}_{d}$F
$-\frac{1}{3}\Delta$Water	$-33\frac{1}{3}{}^{\circ}_{d}$C	-70_{z} $\boxdot\varsigma^{\circ}_{c}$	-30_{z} $\boxdot\varsigma^{\circ}_{f}$	-28°_{d}F
	$-28\frac{4}{21}{}^{\circ}_{d}$C $-23\frac{17}{21}{}^{\circ}_{d}$C	-60_{z} $\boxdot\varsigma^{\circ}_{c}$ -50_{z} $\boxdot\varsigma^{\circ}_{c}$	-20_{z} $\boxdot\varsigma^{\circ}_{f}$ -10_{z} $\boxdot\varsigma^{\circ}_{f}$	$-19\frac{3}{7}{}^{\circ}_{d}$F $-10\frac{6}{7}{}^{\circ}_{d}$F
$\boxdot$ Familiar Zero	$-19\frac{1}{21}{}^{\circ}_{d}$C	-40_{z} $\boxdot\varsigma^{\circ}_{c}$	0_{z} $\boxdot\varsigma^{\circ}_{f}$	$-2\frac{2}{7}{}^{\circ}_{d}$F
Fahrenheit Zero	$-17\frac{7}{9}{}^{\circ}_{d}$C	$-38\frac{4}{5}{}_{z}$ $\boxdot\varsigma^{\circ}_{c}$	$3\frac{1}{5}{}_{z}$ $\boxdot\varsigma^{\circ}_{f}$	0°_{d}F
	$-14\frac{2}{7}{}^{\circ}_{d}$C $-9\frac{11}{21}{}^{\circ}_{d}$C $-4\frac{16}{21}{}^{\circ}_{d}$C	-30_{z} $\boxdot\varsigma^{\circ}_{c}$ -20_{z} $\boxdot\varsigma^{\circ}_{c}$ -10_{z} $\boxdot\varsigma^{\circ}_{c}$	10_{z} $\boxdot\varsigma^{\circ}_{f}$ 20_{z} $\boxdot\varsigma^{\circ}_{f}$ 30_{z} $\boxdot\varsigma^{\circ}_{f}$	$6\frac{2}{7}{}^{\circ}_{d}$F $14\frac{6}{7}{}^{\circ}_{d}$F $23\frac{3}{7}{}^{\circ}_{d}$F
Freezing	0°_{d}C	0_{z} $\boxdot\varsigma^{\circ}_{c}$	40_{z} $\boxdot\varsigma^{\circ}_{f}$	32°_{d}F
	$4\frac{16}{21}{}^{\circ}_{d}$C $9\frac{11}{21}{}^{\circ}_{d}$C $14\frac{2}{7}{}^{\circ}_{d}$C $19\frac{1}{21}{}^{\circ}_{d}$C	10_{z} $\boxdot\varsigma^{\circ}_{c}$ 20_{z} $\boxdot\varsigma^{\circ}_{c}$ 30_{z} $\boxdot\varsigma^{\circ}_{c}$ 40_{z} $\boxdot\varsigma^{\circ}_{c}$	50_{z} $\boxdot\varsigma^{\circ}_{f}$ 60_{z} $\boxdot\varsigma^{\circ}_{f}$ 70_{z} $\boxdot\varsigma^{\circ}_{f}$ 80_{z} $\boxdot\varsigma^{\circ}_{f}$	$40\frac{4}{7}{}^{\circ}_{d}$F $49\frac{1}{7}{}^{\circ}_{d}$F $57\frac{5}{7}{}^{\circ}_{d}$F $66\frac{2}{7}{}^{\circ}_{d}$F
Room Temp	$21\frac{3}{7}{}^{\circ}_{d}$C	46_{z} $\boxdot\varsigma^{\circ}_{c}$	86_{z} $\boxdot\varsigma^{\circ}_{f}$	$70\frac{4}{7}{}^{\circ}_{d}$F
	$23\frac{17}{21}{}^{\circ}_{d}$C $28\frac{4}{7}{}^{\circ}_{d}$C	50_{z} $\boxdot\varsigma^{\circ}_{c}$ 60_{z} $\boxdot\varsigma^{\circ}_{c}$	90_{z} $\boxdot\varsigma^{\circ}_{f}$ ↊0_{z} $\boxdot\varsigma^{\circ}_{f}$	$74\frac{6}{7}{}^{\circ}_{d}$F $83\frac{3}{7}{}^{\circ}_{d}$F
$\frac{1}{3}\Delta$Water	$33\frac{1}{3}{}^{\circ}_{d}$C	70_{z} $\boxdot\varsigma^{\circ}_{c}$	↋0_{z} $\boxdot\varsigma^{\circ}_{f}$	92°_{d}F
Body Temp	37°_{d}C	$79\frac{6}{21}{}_{z}$ $\boxdot\varsigma^{\circ}_{c}$	↋$9\frac{6}{21}{}_{z}$ $\boxdot\varsigma^{\circ}_{f}$	98.6°_{d}F
	$38\frac{2}{21}{}^{\circ}_{d}$C $42\frac{6}{7}{}^{\circ}_{d}$C $47\frac{13}{21}{}^{\circ}_{d}$C $52\frac{8}{21}{}^{\circ}_{d}$C $57\frac{1}{7}{}^{\circ}_{d}$C $61\frac{19}{21}{}^{\circ}_{d}$C	80_{z} $\boxdot\varsigma^{\circ}_{c}$ 90_{z} $\boxdot\varsigma^{\circ}_{c}$ ↊0_{z} $\boxdot\varsigma^{\circ}_{c}$ ↋0_{z} $\boxdot\varsigma^{\circ}_{c}$ 100_{z} $\boxdot\varsigma^{\circ}_{c}$ 110_{z} $\boxdot\varsigma^{\circ}_{c}$	100_{z} $\boxdot\varsigma^{\circ}_{f}$ 110_{z} $\boxdot\varsigma^{\circ}_{f}$ 120_{z} $\boxdot\varsigma^{\circ}_{f}$ 130_{z} $\boxdot\varsigma^{\circ}_{f}$ 140_{z} $\boxdot\varsigma^{\circ}_{f}$ 150_{z} $\boxdot\varsigma^{\circ}_{f}$	$100\frac{4}{7}{}^{\circ}_{d}$F $109\frac{1}{7}{}^{\circ}_{d}$F $117\frac{5}{7}{}^{\circ}_{d}$F $126\frac{2}{7}{}^{\circ}_{d}$F $134\frac{6}{7}{}^{\circ}_{d}$F $143\frac{3}{7}{}^{\circ}_{d}$F
$\frac{2}{3}\Delta$Water	$66\frac{2}{3}{}^{\circ}_{d}$C	120_{z} $\boxdot\varsigma^{\circ}_{c}$	160_{z} $\boxdot\varsigma^{\circ}_{f}$	152°_{d}F
	$71\frac{3}{7}{}^{\circ}_{d}$C $76\frac{4}{21}{}^{\circ}_{d}$C $80\frac{20}{21}{}^{\circ}_{d}$C $85\frac{5}{7}{}^{\circ}_{d}$C $90\frac{10}{21}{}^{\circ}_{d}$C $95\frac{5}{21}{}^{\circ}_{d}$C	130_{z} $\boxdot\varsigma^{\circ}_{c}$ 140_{z} $\boxdot\varsigma^{\circ}_{c}$ 150_{z} $\boxdot\varsigma^{\circ}_{c}$ 160_{z} $\boxdot\varsigma^{\circ}_{c}$ 170_{z} $\boxdot\varsigma^{\circ}_{c}$ 180_{z} $\boxdot\varsigma^{\circ}_{c}$	170_{z} $\boxdot\varsigma^{\circ}_{f}$ 180_{z} $\boxdot\varsigma^{\circ}_{f}$ 190_{z} $\boxdot\varsigma^{\circ}_{f}$ 1↊0_{z} $\boxdot\varsigma^{\circ}_{f}$ 1↋0_{z} $\boxdot\varsigma^{\circ}_{f}$ 200_{z} $\boxdot\varsigma^{\circ}_{f}$	$160\frac{4}{7}{}^{\circ}_{d}$F $169\frac{1}{7}{}^{\circ}_{d}$F $177\frac{5}{7}{}^{\circ}_{d}$F $186\frac{2}{7}{}^{\circ}_{d}$F $194\frac{6}{7}{}^{\circ}_{d}$F $203\frac{3}{7}{}^{\circ}_{d}$F
Boiling	100°_{d}C	190_{z} $\boxdot\varsigma^{\circ}_{c}$	210_{z} $\boxdot\varsigma^{\circ}_{f}$	212°_{d}F

ΔWater = Boiling − Freezing = 100_{d}K = 180_{d}R = 190_{z} $\boxdot\varsigma^{\circ}_{a}$ = 1.9_{z} $\boxdot$h↑tpℓ

Based on these observations, I offer three Primel ⊡**stadigrade temperature scales**[1Ɛ] that all use the ⊡stadegree, but that differ in their choice of zero point:

1. The ⊡**stadigrade·crystallic** scale (abbreviated ⊡ς°_{c}) places zero at the freezing point, like Celsius. In principle, it would be most reminiscent of Celsius, but in practice the numbers for various temperatures bear little resemblance to temperatures in Celsius. (See table on page 49_z for a comparison.)[20]
2. The ⊡**stadigrade·familiar** scale (abbreviated ⊡ς°_{f}) places zero at 40_z ⊡stadegrees below freezing. Four dozen being a third of the way to a gross, this is reminiscent of Fahrenheit's choice to place the freezing point about a third of the way to a hundred. Consequently, many of the dozenal values along the ⊡stadigrade·familiar scale, when interpreted as fractions of a gross, resemble decimal values on the Fahrenheit scale, when interpreted as fractions of a hundred. (See table on page 49_z for a comparison.)
3. The ⊡**stadigrade·absolute** scale (abbreviated ⊡ς°_{a}) places zero at absolute zero, as do the kelvin (K) and rankine (R) scales. It turns out that the freezing point of water falls at 494.41_z ⊡stadegrees·absolute. This bears a greater resemblance to 491.67_d rankine than to 273.15_d kelvin. Similarly, the boiling point of water falls at 664.41_z ⊡stadegrees·absolute, and this is more reminiscent of 671.67_d rankine than of 373.15_d kelvin. Overall the ⊡stadigrade·absolute scale most resembles the rankine scale, all due to the correspondence of the ⊡hexqua·temperaturel to the hecto·rankine.

The Angle on Angles

Technically, units of plane angle are not part of Primel, nor by rights part of any particular DGW metrology. Rather, they are a common adjunct to all metrologies.

For the purposes of most physical sciences and mathematics, the **radian** is clearly the coherent unit of plane angle, and as such, should be used in conjunction with every scientific metrology.

For everyday usages, I also recognize a dozenal-metric set of angle units, based on dozenal divisions of the **turn** or **full angle** (τ radians or 360°_{d}).[21] Of course, these divisions can be named by attaching appropriate SDN power prefixes onto the **turn**. (See table on page $4Ɛ_z$.)

The DSA founders also divided up the turn (or "cycle") in this way, but they used the same names for these angle units as they gave to their time units. The apparent motivation was to equate angular displacement of the sun across the sky with the duration it takes to do so. This seemed a bit off to me, because angles are not commensurate with times.

I take a more nuanced approach: In my view colloquials for angular measures should generally end in the suffix "**·angle**" (or any commensurate synonym, such as "**·latitude**," "**·longitude**," "**·azimuth**," "**·elevation**," "**·direction**," and so on). It's acceptable to reuse time unit colloquials as angle unit colloquials so long as they

[1Ɛ] Again, if you prefer strict principle, use ⊡**stadial·temperature scales** instead.

[20] The "Dozenal Popular Scale" (see page 22_z) from the Do-Metric System makes a better analog for Celsius than ⊡stadigrade·crystallic. But of course that scale is not based on any coherent relationship between heat and temperature. It merely takes the same range between the same two anchor temperatures and divides it by a "convenient" gross instead of a "convenient" hundred.

[21] $\tau = 2\pi$.

Uncial Divisions of the Turn (⊙) and Associated Circumferal Units					
Formal Name *Abbrev*	Colloquials and Abbrevs Temporal	 Geographic	Sexagesimal Equivs	Circumferal Unit *Abbrev*	USC, SI Equivs
turn ⊙	day·angle *dy*∡	global·angle *glb*∡	360°_{d}	©global·length ©*glb·lg*	$24{,}883.2_{\text{d}}$ mi 40045.6286208_{d} km
uncia·turn $u{\downarrow}⊙ = 10_{\text{z}}^{-1}⊙$	dwell·angle *dw*∡	continental·angle *cnt*∡	30°_{d}	©continental·length ©*cnt·lg*	$2{,}073.6_{\text{d}}$ mi 3337.1357184_{d} km
bicia·turn $b{\downarrow}⊙ = 10_{\text{z}}^{-2}⊙$	breather·angle *br*∡	regional·angle *rgn*∡	$2^{\circ}30'_{\text{d}}$	©regional·length ©*rgn·lg*	172.8_{d} mi 278.0946432_{d} km
tricia·turn $t{\downarrow}⊙ = 10_{\text{z}}^{-3}⊙$	trice·angle *tr*∡	itineral·angle *itn*∡	$12'30''_{\text{d}}$	©itineral·length ©*itn·lg*	14.4_{d} mi 23.1745536_{d} km
quadcia·turn $q{\downarrow}⊙ = 10_{\text{z}}^{-4}⊙$	lull·angle *lu*∡	dromal·angle *dr*∡	$1'02.5''_{\text{d}}$	©dromal·length ©*dr·lg*	1.2_{d} mi 6336_{d} ft 1931.2128_{d} m
pentcia·turn $p{\downarrow}⊙ = 10_{\text{z}}^{-5}⊙$	twinkling·angle *tw*∡	stadial·angle *st*∡	$5.208\overline{3}''_{\text{d}}$	©stadial·length ©*st·lg*	0.1_{d} mi 528_{d} ft 160.9344_{d} m
hexcia·turn $h{\downarrow}⊙ = 10_{\text{z}}^{-6}⊙$	vibe·angle *vb*∡	habital·angle *hb*∡	$0.43402\overline{7}''_{\text{d}}$	©habital·length ©*hb·lg*	44_{d} ft 13.4112_{d} m
septcia·turn $s{\downarrow}⊙ = 10_{\text{z}}^{-7}⊙$		ell·angle ℓ∡	$36.1689\overline{814}_{\text{d}}$ milli·arc·sec	©ell·length ©*ℓ·lg*	44_{d} in 1117.6_{d} mm
octcia·turn $o{\downarrow}⊙ = 10_{\text{z}}^{-8}⊙$		hand·angle *hd*∡	3.01408179_{d} milli·arc·sec	©hand·length ©*hd·lg*	$3.\overline{6}_{\text{d}}$ in $93.1\overline{3}_{\text{d}}$ mm
enncia·turn $e{\downarrow}⊙ = 10_{\text{z}}^{-9}⊙$		morsel·angle *mo*∡	0.25117348_{d} milli·arc·sec	©morsel·length ©*mo·lg*	$0.30\overline{5}_{\text{d}}$ in $7.76\overline{1}_{\text{d}}$ mm

get one of these suffixes, indicating angles *associated with* times. (See the Temporal Colloquials column in the table on page 4Ɛ$_{\text{z}}$.)

However, that is not the only correspondence suggestive of colloquial names for these angle units. We can give angles a "Geographic" interpretation by correlating fractions of a turn with fractions of Earth's circumference, and in the process derive a set of **Circumferal** length units useful for navigation purposes. (See relevant columns in the table on page 4Ɛ$_{\text{z}}$.) This is analogous to the correlation of minutes of arc to nautical miles, but I support this notion at all levels of scale, not just miles.

Circumferal units are not strictly part of Primel, but adjunct to it. They use © (pronounced "circum") as their brand mark, and recapitulate all the same colloquial names for Primel's length units. "Circum" suggests both that these lengths are derived from Earth's circumference and also that they are "around" as large as the corresponding Primel analogs. (This reuse of colloquial names is justified because the sizes are fairly close.)

For convenience, I've elected to set the ©stadial·length to exactly a tenth of a statute mile, making each Circumferal unit exactly $\frac{74}{79}_{\text{z}}$ or $\frac{88}{93}_{\text{d}}$ of the corresponding Primel unit. This makes the ©hand·length exactly 3.8_{z} or $3.\overline{6}_{\text{d}}$ inches and identical to H. C. Churchman's "metron" from his Metronic system.[22] This also makes the ©dromal·length exactly 1.2_{d} statute miles, or one "naire" from Churchman's system. The ©global·length is exactly $24{,}883.2_{\text{d}}$ statute miles, a reasonable compromise between meridional and equatorial circumference. This is identical to Churchman's "dominaire" unit, but with a name that is a little less contrived and a little more transparent.

[22] See *Duodecimal Bulletin*, Vol. 16_{z}, No. 1, WN 30, October 1176_{z} (1962_{d}), p. 17_{z}, http://dozenal.org/drupal/sites_bck/default/files/DuodecimalBulletinIssue161-web_0.pdf.

More to Come

This article by no means exhausts the subject of Primel and DGW systems. In future articles, I hope to cover some more advanced topics, including but not limited to:

- **Units for Angular Mechanics**: Although the radian is not metrology-specific, angular mechanics works with quantities that combine angles with mechanical units. My approach differs from SI's in that I treat plane angle as a distinct dimension rather than as a dimensionless quantity, with interesting results.
- **Units for Electromagnetism**: In order to make sense of all the quantities called out in Maxwell's Equations, and provide reasonable quantitels for them, I found it necessary to diverge even more radically from both SI and TGM, to the extent of overhauling the terminology used for electromagnetic phenomena, and even the interpretation of words such as "magnetism," "force field," and "flux." This is quite a large topic just by itself.
- **Units for Chemistry**: It took some finessing to provide quantitels for "amount of substance" and the various forms of solution concentrations in use in chemistry.
- **Units for Radiometry and Photometry**: There's a variety of quantities surrounding radiant energy and radiant power; and then a similar variety surrounding luminous energy and luminous power, with the luminous efficacy of the human eye as another "mundane reality" of human life.
- **Other DGWs**: Members of the DozensOnline forum have applied the techniques and tools I've outlined here to develop their own systems organized around other bases, including octal, hexadecimal, senary — even tetradecimal, and more! This meant generalizing from Systematic Dozenal Nomenclature to Systematic *Numeric* Nomenclature, to accommodate any base. The DGW Spreadsheet has proven to be an indispensable tool for this, and deserves a thorough introduction of its own.

Primel Online

Most of the development of Primel occurred (and continues to occur) in the following thread on the DozensOnline Forum: `https://www.tapatalk.com/groups/dozensonline/the-primel-metrology-t666.html`. I've maintained the original post of that thread as an overall summary of the metrology, and try to keep it up to date with my latest ideas. In fact, that thread is part of an entire subforum dedicated to Primel now. If you have questions or suggestions about Primel, DGW's, and so forth, by all means post them there.

I am also developing a Confluence wiki about Primel, at `https://primelmetrology.atlassian.net/wiki/spaces/PM/overview`. There is quite a bit of material there already, but it is still a work in progress, so it is by no means complete. Eventually, however, it will be the definitive resource.

In Conclusion

Thank you for taking a look at the Primel metrology. I hope this introduction to Primel *measures* has sparked some *measure* of interest in what life would be like using a coherent dozenal-metric system!

Dozenal Timekeeping

Paul Rapoport

I have been interested in dozenal time for more than four unquennia (dozen-year periods). That makes me a relative newcomer to the subject! Clock time is specifically mentioned in the DSA's second bulletin in 1161$_{\text{z}}$ (1945$_{\text{d}}$), and F. Emerson Andrews included it, as well as calendar time, in "An Excursion in Numbers," his pioneering article in the *Atlantic Monthly* in 1152$_{\text{z}}$ (1934$_{\text{d}}$).

Before and since then, many have discussed dozenal time favorably. It's not a hard concept to grasp, because the traditional Western divisions of the day and the year already have twelve as the first or second subdividing factor.

In 1174$_{\text{z}}$ (1960$_{\text{d}}$), the DSA explained how to divide the day, dozenally, in its *Manual of the Dozen System*. In 1183$_{\text{z}}$ (1971$_{\text{d}}$), in England, Tom Pendlebury first published his coherent dozenal metrology, called TGM (since revised a few times). It bases units of measure on dozenal divisions of the half-day, along with a constant for gravity.

Since the arrival of microchips and the Internet, it's been easier to create or access dozenal timekeeping, resulting in keen interest in it among more people. That interest generates discussion of theory and design mostly. If practice is discussed, it's necessarily without much practice itself. Dozenalists have, however, created notable software to make practice possible.

For some years I've been using daily both a clock and a calendar in dozenal. Since late 11ƐƐ$_{\text{z}}$ (2015$_{\text{d}}$), I've cheerfully been using them a maximal amount while considering whether and how to translate dozenal time systems into and out of traditional decimal ones in everyday life. I'll briefly describe the clock and the calendar I use, then turn to issues of actually using them.

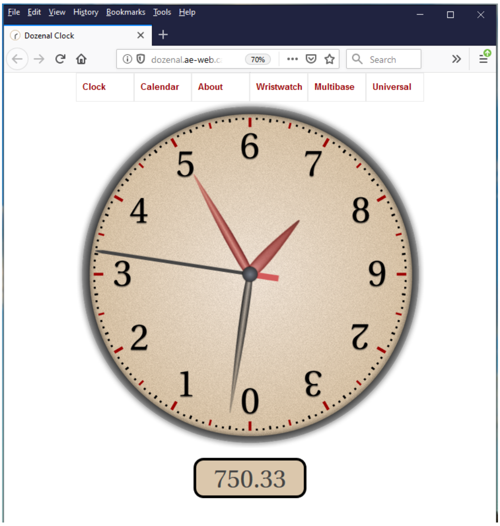
The Diurnal and Semidiurnal Clock

The two main systems of dozenal clock time divide the day into successive powers of a dozen only (diurnal), or first in half and then by dozens (semidiurnal, with equivalents to AM and PM). The latter, a foundation in Pendlebury's TGM, is the one I don't use. I wanted to design something from the basics, with as little regard as possible to traditional decimal practice or deviation from dozens. I find it simpler and more efficient in this case to divide by dozens only.

I designed and commissioned two mechanical dozenal diurnal clocks, one in 1183$_{\text{z}}$ (1971$_{\text{d}}$) and another in 1197$_{\text{z}}$ (1987$_{\text{d}}$). In December 11Ɛ8$_{\text{z}}$ (2012$_{\text{d}}$), I decided to go for electronic. The result, created by Rodrigo Flores and Tom Cassidy, is on a website (`http://dozenal.ae-web.ca`), showing three versions of a dozenal clock: one semidiurnal and two diurnal. The only difference between the latter two is where the analog clock puts midnight: top or bottom. There are also two digital time readouts on the site, for each kind of division of the day, as well as clocks in many formats using Coordinated Universal Time (UTC).

The next move was to get those clocks onto a smartphone. That was completed by Jasper Chan for Apple's iPhone in October 11ƐƐ$_{\text{z}}$ (2015$_{\text{d}}$). Then I could conveniently carry it around and also use the clock to set a dozenal timer or alarm, which I do at least daily.

Not yet satisfied, I also wanted a dozenal wristwatch. That desire goes back at



Paul Rapoport's online clock, diurnal version with midnight 0 at bottom

least to 1178_{z} (1964_{d}). Attempts to alter traditional watches, including digital, went nowhere. The programmable Pebble watch I acquired in December 11ƐƐ$_{z}$ (2015_{d}) is what enabled me to use dozenal time much more than previously. I modified an existing C program to produce a digital readout for time, and Andrew Cenko wrote the code for the calendar I wanted.

Tom Cassidy then expanded the time and calendar functions and added current local temperature, relative humidity, and wind speed, all in dozenal units, of course. The whole, completed in December 1200_{z} (2016_{d}), is based on and uses units from both TGM and Primel metrology, the latter a cousin of the former, developed over the last few years by the Bulletin's Editor, John Volan (see page 32_{z}), but previously discovered, quite independently, by William S. Crosby in 1161_{z} (1945_{d}).

The Holocene Calendar

The calendar I use does not follow the principle of least change, which governs the usual dozenal calendar. Dozenalists usually number the Christian year in dozenal and the days of the month likewise, but leave everything else as is. For convenience, the dates mentioned in this article are in that calendar, which is available on the Pebble watch installation as well.

But again I wanted to design something from the basics. My solar calendar starts not 1203_{z} years ago but near the beginning of the Holocene era, with an astronomical event. Every year begins on the December solstice. (There are arguments for starting in other months, as there are for starting the day at other times, and for moving the International Date Line, affecting determination of the seasons.) The distribution of the 5 or 6 days beyond the 260_{z} (10_{z} months of 26_{z} days each) differs notably from traditional Western practice, because it maximizes seasonal accuracy.

Unfortunately Pebble watches stopped being made in late 1200_{z} (2016_{d}). I expect to transfer my weather data to another watch. Meanwhile, in 1201_{z} (2017_{d}) I produced

Sample of Paul Rapoport's online Holocene appointment calendar, for Holocene date 6852-0↋-20$_z$, or Gregorian date 1203-0↋-13$_z$ (2019-11-15$_d$), showing daily appointments.

a web-based interactive calendar on the above principles, including a six-day week. Users may schedule appointments and events, including recurring ones, in either the Gregorian calendar or this dozenal calendar. Gregorian dates may be shown along with the dozenal ones. There are also search and time zone adjustment functions. The calendar is at http://calendar.wmdev.ca.

The Experience

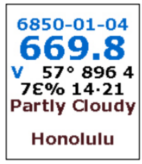
I use the calendar in a few simple ways. My exercise routine has a 2-day cycle, so I use the odd-numbered days to begin it, making a simple adjustment when a month has 27_z days. Those who have to do something strict (e.g. take medication every 2nd day without exception) would adjust slightly differently. That's no harder than dealing with the traditional calendar and its irregular sequence of 26_z and 27_z days. (In mine, the 27_z-day months are consecutive.)

I have a few other things to do every 3rd day. That's easy; I just do them when my watch indicates a date divisible by 3 (3, 6, 9, 10_z, etc.), also adjusting for a month of 27_z days. I water one plant once a week. It doesn't mind that I choose a 6-day week.

Keeping track of events every 2nd and every 3rd day is easy in a 6-day week, although I admit to not noticing weeks much (except for the plant watering), because the conflict between 6-day and 7-day weeks is difficult. Yes, to me many events in the traditional world recur every 1.2_z or 2.4_z weeks.

Some notes I make are dated according to my calendar. One set keeps track of distances our car goes, and some of its charging schedule. My web-based calendar automatically converts Gregorian dates between 1033_z (1767_d) and 1271_z (2101_d), into Holocene dates between 6682_z and 6900_z. Using dozenal time for both clock and

Watch face in the Primel metrology



This calendar format is not part of Primel. It is Holocene (Ordinal) with the day of the week indicated, V. The time is diurnal.

Temperature
1 stadigree = 0.7143° Fahrenheit
1° Fahrenheit = 1.4 stadigrees
80.0°F = $_{\text{z}}$57 stadigrees crystallic

Not shown:
80.0°F = $_{\text{z}}$97 stadigrees familiar

Barometric pressure
1 pressurel = 0.8035 millibar
1 millibar = 1.2446 pressurels
1017 millibars = $_{\text{z}}$896 pressurels

Not shown:
1 lengthel = 0.3229 inch
1 inch = 3.0968 lengthels
1 inch Hg = 33.8639 millibars
1 millibar = 0.0914 lengthel Hg
1 millibar = 1.0974 uncialengthels Hg
1017 millibars = $_{\text{z}}$790 uncialengthels Hg

UV index
1 intensitel = 22.7762 W/m^2
1 W/m^2 = .0439 intensitel
1 mW/m^2 = 1 in the UV index = 0.9104 quadciaintensitel
4.7 in the UV index = $_{\text{z}}$4 quadciaintensitels

Relative humidity
66% = $_{\text{z}}$7↋%

Wind speed
1 velocitel = 1.0205 km/h
1 km/h = 0.9799 velocitel
16.3 km/h = $_{\text{z}}$14 velocitels

Paul Rapoport's Wrist Watch – Example Documentation Page

calendar is easy and fun. They're much better than the traditional Western versions, which combine a variety of historical idiosyncrasies into a mishmash in an awkward number base.

There are challenges, however. Often I don't know the traditional date. A watch displaying 6852-0↋-20$_{\text{z}}$ isn't much help for that. Even though I know the year is 2019$_{\text{d}}$ (1203$_{\text{z}}$) — because that doesn't change for a while — I have to exert some effort to remember the month and day number. I can't convert what I see on my watch. (You know that mental acuity question that asks what today's date is? Dangerous!)

To know the traditional time is easier, because converting to or from dozenal is quick. The problem is doing just that: converting. I have to fight the tendency to look at the time 7↊6$_{\text{z}}$ and think 15:45$_{\text{d}}$, because that keeps me in the traditional system, using my watch only as a code for that.

Far better to think of an appointment at 15:45$_{\text{d}}$ to be at 7↊6$_{\text{z}}$. Then if the current time is 610$_{\text{z}}$, I know I have 196$_{\text{z}}$ trices remaining. I don't want to think of that period as 3 hours and 35$_{\text{d}}$ minutes. (Despite thinking as dozenally as possible, I may still be able to satisfy the mental acuity question requiring a clock face to be drawn with a specific traditional time on it. I just have to not put the 6 at the top!)

It's rare to find anyone else wanting to use the clock or calendar as much as I do. The calendar is idiosyncratic, doing more than dozenalizing what we know, and subscribes to a calendar reform different from just about any proposed in the past.

The chances for the clock's use are better, especially the digital readout.

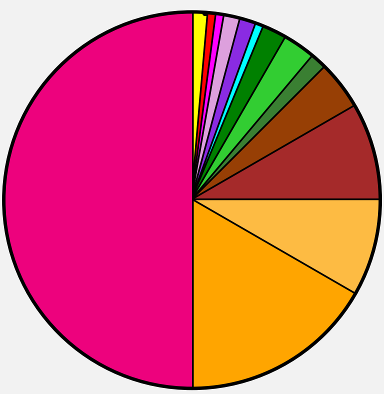
Someone once said that clock time was a poor choice for promoting dozenal because twelve is such an obvious part of it already. That's a reason it's a *good* choice to show dozenal in practice: there's not much change from the usual. Despite that, in early 11Ɛ9$_z$, a well-known radio interviewer said my dozenal clock would "melt your brain." That would be the analog version, which I find necessary in explanations before the digital, unless someone understands the concept of dozenal metric immediately. Then a time like 7↊6.4$_z$ makes almost immediate sense.

The hardest display on the watch to use may be the local outdoor temperature in one of the dozenal scales available. What Primel metrology calls "stadigrade crystallic" gives a scale that doesn't immediately relate to anything commonly used. In order to achieve 1:1 coherence of units in other parts of Primel's physics system, stadigrade temperatures are 2.52$_d$ times Celsius, dozenalized; the individual degrees are 5/7 the size of those in Fahrenheit. I use the crystallic scale (zero at the freezing point) because that appeals best to my own experience with Celsius.

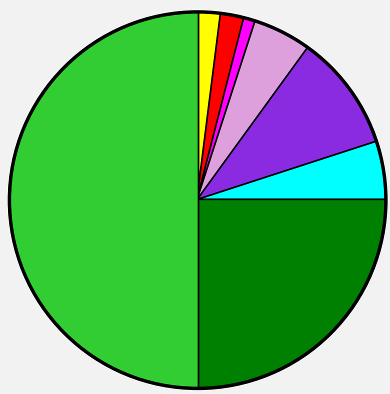
Even if interested in time or weather, most people have no use for different ways to measure it. But all they need are to be open to a certain kind of creative arithmetic and to be willing to challenge longstanding traditions at least a little.

Use Perbiquas!

It's plain to see that a biqua (or gross) has many, many more even divisions than a hundred has—*six* more, to be precise! You'll have a much easier time getting a clean fraction with dozenal perbiquas than with decimal percentages.



Perbiquas



Percents

DOZENS IN THE MEDIA

Along with all the goodies we have at www.dozenal.org, *check out this round-up of the latest dozenal delicacies gleaned from the greater infosphere.*

Book Review: The *Orthogonal* Trilogy

Book One:	*The Clockwork Rocket*	• 2011$_\mathrm{d}$ (11↋7$_\mathrm{z}$)
Book Two:	*The Eternal Flame*	• 2012$_\mathrm{d}$ (11↋8$_\mathrm{z}$)
Book Three:	*The Arrows of Time*	• 2014$_\mathrm{d}$ (11↋↊$_\mathrm{z}$)
Author:	Greg Egan	
Website:	www.gregegan.net	
Publisher:	Nightshade Books	

Are you a fan of hard science-fiction, and dozenal numbers? Then you're in for a treat. Greg Egan, an Australian SF author, has completed a trilogy of novels that turn conventional assumptions, about both physics and numbers, upside-down. (Perhaps not surprising, coming from "down under.")

Orthogonal Relativity? *Orthogonal* is set in a universe with a difference. Or rather, it *lacks* a certain difference that exists in *our* universe. To put it more plainly, our universe has a four-dimensional spacetime, and so does *Orthogonal*. But our time dimension involves a crucial minus sign that makes it distinct from our three spatial dimensions. This leads to something called Lorentzian geometry and Einsteinian relativity. But in the *Orthogonal* universe, the time dimension does not bear this minus sign, so it acts entirely like another spatial dimension, The resulting geometry is actually *Euclidean* (well, technically speaking, *Riemannian*): Acceleration doesn't subject your reference frame to the Lorentz transformation, it simply *rotates* it. Velocity is a *slope*.

There is no "speed of light," nor any speed limit in this universe; anything, even light, can travel at any speed. In fact the *color*, or wavelength, of light is a direct function of its speed, from *stationary* light of the deepest "infrared," to *infinite* velocity light of the extreme "ultraviolet." Stars in the night sky aren't twinkling points, but streaks of rainbow, depending on their proper motion. Red light takes longer to reach the eye, so needs to start earlier along a star's trajectory. But things get truly bizarre at relativistic speeds. Egan's website has a couple videos to demonstrate the effects, as well as extensive write-ups about his physics.

Somehow, this makes light a negative sort of energy. Plants don't *absorb* light to feed themselves, they must *emit* it to capture chemical energy. Vegetation shines. Flowers glow. It's *animals* that need to absorb light to stay healthy. Indeed, most matter is inherently unstable. Given the right provocation, it can start emitting light, gain heat, and go incandescent. Stars are simply planets that were too massive to resist. Our universe is doomed to end in ice; the *Orthogonal* universe will end in fire.

Over the course of the novels, the inhabitants of this universe discover "rotational physics" and all its implications, including its own version of the "twin paradox": There, the twin who goes off on a relativistic journey experiences *more* proper time than the one who stays home. This is exactly the opposite of what happens in our universe—and it's critical to the plot.

Orthogonal Dis-aster? The inhabitants discover that their planet (it's never named, it's just "the world") is threatened with an impending cosmic catastrophe. They don't have the technology to avert calamity, nor the time to develop any. The only way to buy time is to send a spacecraft off into the void at relativistic speeds, on a voyage that will take generations, perhaps even biquennia, for its crew—while only a few years pass back home. During that borrowed time, the hope is that the crew and their descendants might be able to develop the science to save their world.

It's quite a saga. Starting with little more than Victorian-era technology, they manage to launch a *mountain* into space, Mount Peerless. It happens to sit upon a massive vein of an element they call *sunstone*, which is particularly prone to ignition. The mountain becomes the *Peerless*, a star ship carrying the population of a small city, underground farms of glowing crops, a small forest, everything they'll need to survive—hopefully.

There's enough fuel to accelerate, at one of their gravities, up to *infinite* velocity, rotating their arrow of time until it points in the direction of their motion, *orthogonal* to the time axis of their home world. (Hence, the title of the series.) Coasting at infinite velocity, each year of proper time spent aboard the *Peerless* will move it a light-year through space. But to the home world, that journey, no matter how long, will appear *instantaneous*. There's enough fuel to decelerate to a stop again, but not enough to return. Somehow, somewhere along the way, they must find the resources—or invent the technology—that will bring them back home.

Orthogonal People? The species of the inhabitants is never named, they're just "men" and "women." But they're definitely not human. They have about human intelligence, with roughly human psychology and personalities. They have the usual factional rivalries and internecine ideological conflicts, so there is drama to be had. Physically, they stand on two legs, have a torso, with a head on top. But there the resemblance ends.

They have four eyes, two in front and two in back, so they have a "front gaze" and a "back gaze." They (usually) have four arms, but they're shapeshifters, able to extrude and resorb extra limbs at will, even completely absorb all their usual limbs if necessary. Endoskeleton configurable at will. They can even, chameleon-like, induce marks to appear on their skin in real time—writing, diagrams, even animation!

They don't breathe. Air is the most inert substance they know; it plays no metabolic role. But it does help keep their bodies *cool*. Decompression and asphyxiation in the vacuum of space aren't issues—the chief risk is *hyperthermia*. A single organ—a "tympanum," located somewhere in the neck below the mouth—lets them both hear and speak. So they only use their mouths for eating.

They have neither blood nor blood stream. There's nothing even analogous to *water* in that universe. The closest thing to a fluid most encounter is "resin." In fact, any substance achieving actual "liquid" phase is an exceedingly rare—and dangerous—occurrence. How their nervous system can possibly function under these conditions—well, that's something they (and you) will discover in the course of the journey.

The "women" are larger, and the "men" have the nurturing instinct. This is because the females give birth by *fission*. Mothers never see their children—they *become* their children. Their bodies literally coalesce into a blastocyst and then divide overnight into (usually) four squalling babies, two males and two females. This necessarily ends the mother's existence, although her flesh is effectively immortal. Only the males can die of old age and rot in a grave. The mating act triggers fission, and imprints a deep bond between the father and the resulting offspring. But of course, it can only happen once. Couples usually hold off reproduction until they're provisioned for the father to raise the kids, or the female is ready to make the choice. But even for an unmated female, fission is ultimately inevitable. After too many years, she'll spontaneously divide—a tragedy without someone on hand to tend to the young. Modern contraceptives are able to prolong female lives, to a point. Like in our world, the males seem to be in charge, although there is an ongoing struggle for gender equality.

What's even stranger about this, is that, of the four offspring, each male-female pair is a *mating* pair. The other pair is their brother and sister, but their complementary sibling—their "co"—is (usually) their lifetime mate. It's not incest. Mating doesn't seem to have anything to do with exchanging genetic information. (That happens by a completely different mechanism, which the crew only discovers during the trip.) So reproduction is essentially parthenogenesis. We must presume the males are actually infertile drones evolved to provide care for relatively helpless offspring during their development.

A charming conceit Egan has come up with is that everyone has recognizably Italianate names, with "co's" always given complementary names: Carlo and Carla, Angelo and Angela, Eugenio and Eugenia, and so on. Egan skillfully weaves in homely details like this to lull readers into perceiving his protagonists as human—only to jar us with some aspect of their intrinsic alienness.

Orthogonal Numbers? Along with everything else, these creatures are hexadactyls. So naturally, they use base twelve. The words "hundred," "thousand," "million," and so forth, never appear in these novels. Instead, they seem to do just fine using "a dozen" and "a gross," and multiples and halves of these. Egan finesses the third power of twelve as simply "a dozen gross." His protagonists prove by demonstration that even just this much perfectly ordinary dozenal English is sufficient for day-to-day purposes, and even for the work of scientists. He evidently intends this to contribute to the atmosphere of familiar-yet-strange.

Egan does include an appendix that lists a number of scientific prefixes these people use. Here they are with their SDN[1] equivalents:

12_d^{+3}	10_z^{+3}	**ampio-**	triqua-	12_d^{-3}	10_z^{-3}	**scarso-**	tricia-
12_z^{+6}	10_z^{+6}	**lauto-**	hexqua-	12_z^{-6}	10_z^{-6}	**piccolo-**	hexcia-
12_z^{+9}	10_z^{+9}	**vasto-**	ennqua-	12_z^{-9}	10_z^{-9}	**piccino-**	enncia-
12_z^{+12}	10_z^{+10}	**generoso-**	unnilqua-	12_z^{-12}	10_z^{-10}	**minuto-**	unnilcia-
12_z^{+15}	10_z^{+13}	**gravido-**	untriqua-	12_z^{-15}	10_z^{-13}	**minuscolo-**	untricia-

As you can see, Egan is continuing with the "Italianate" theme here. However, these actually appear only very sparingly within the narrative itself.

Orthogonal Distance? Instead, Egan gets a lot of mileage out of the units of measurement he has endowed these people with. He provides an appendix listing a rich set of units for distance, time, angle, and mass. He gives most of these units names that are straightforward English words, mostly self-explanatory. They are all built systematically upon powers of twelve, yet it's quite plausible that each of these developed organically. The protagonists of his novels make liberal use of these units within the narrative in a variety of contexts, and they all seem to flow quite naturally.

Here is Egan's table of length/distance units, compared to some analogous units from metrologies developed by human dozenalists.[2,3]

Orthogonal Units			Analogous Units		
Distance		In strides		Primel	Do-Metric
1 **scant**		$1/144_d$	(bicia)	⊡ morsel	quan
1 **span**	$= 10_z$ scants	$1/12_d$	(uncia)	⊡ hand	palm
1 **stride**	$= 10_z$ spans	1_d		⊡ ell	yard
1 **stretch**	$= 10_z$ strides	12_d	(unqua)	⊡ habital	doyard
1 **saunter**	$= 10_z$ stretches	144_d	(biqua)	⊡ stadial	groyard
1 **stroll**	$= 10_z$ saunters	$1{,}728_d$	(triqua)	⊡ dromal	mile
1 **slog**	$= 10_z$ strolls	$20{,}736_d$	(quadqua)	⊡ itineral	domile
1 **separation**	$= 10_z$ slogs	$248{,}832_d$	(pentqua)	⊡ regional	gromile
1 **severance**	$= 10_z$ separations	$2{,}985{,}984_d$	(hexqua)	⊡ continental	momile

Given that we're talking about another universe with a different set of physical laws, it's hard to know exactly what size-scale Egan's protagonists exist at, and how it compares to our own. But if we assume that they are approximately human-sized and -shaped, then we could compare a "stride" to a human yard or perhaps an ell, a "span" to a human palm or hand measure, and a "scant" to a quarter or third of an inch. This would make a "stroll" something like a mile. This is plausible, because the height of Mount Peerless is described as 5 strolls and 5 saunters, which would make it comparable to our Mount Everest's 5.5_d (5.6_z) mile height.

For planetary and astronomical distances, Egan's protagonists make use of the "severance," which would be something on the order of 2000_d (1200_z) miles. The equatorial circumference of the home planet is described as 7.42_d (7.5_z) severances, which would be something like $15{,}000_d$ (8800_z) miles. That would make the home planet only about $60\%_d$ ($72\%_z$) the size of the Earth. The distance to their sun is described as $16{,}323_d$ (9543_z) severances; this would be something like $33{,}000{,}000_d$ (Ɛ,000,000$_z$) miles, only about $35\%_d$ ($43\%_z$) that of Earth's orbit.

On the other hand, one of the protagonists, contemplating the equivalence of space and time, and trying to work out the conversion factor between them, muses over the serendipity of customary units, and notes that the "scant" was the arbitrary width of some ancient ruler's thumb. A quarter to a third of an inch makes a rather skeletal thumb on a human, but perhaps Egan's species has very spindly fingers. But following human proportions, the "scant" would need to be about 3 times larger, perhaps 3/4 inch. This would make the "span" closer to 9 inches, which is the same as a customary human "span" measure. But then the "stride" would be 9 feet! Even if we take a "stride" as equivalent to a 2-step pace, this would give Egan's species a gait, and likely a height, more than one and a half times that of humans. The "stroll" would then be more like a league. Mount Peerless would tower 3 times as high as

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[1] See page 31_z.

[2] https://primelmetrology.atlassian.net/wiki/spaces/PM/overview

[3] http://www.dozenal.org/drupal/sites/default/files/DuodecimalBulletinIssue012-web_0.pdf

Everest, and the planet would be some $80\%_d$ ($97\%_z$) larger than Earth. However, the planet's distance to its sun would be more analogous to Earth's orbit.

So these guesses should be taken with a grain of salt.

Orthogonal Time? Egan has his species divide up their planet's day in pure powers of twelve, exactly as many human dozenalists have suggested we divide Earth's day. Here is Egan's table of day-based time units, compared to analogous Earth-based dozenal units:

Orthogonal Units				Analogous Units		
Time		In pauses		Primel	Do-Metric	
1 **flicker**		$1/12_d$	(uncia)	⊡twinkling	dovic	0.42_z sec
1 **pause**	$= 10_z$ flickers	1_d		⊡lull	grovic	4.2_z sec
1 **lapse**	$= 10_z$ pauses	12_d	(unqua)	⊡trice	minette	50_d sec
1 **chime**	$= 10_z$ lapses	144_d	(biqua)	⊡breather	temin	10_d min
1 **bell**	$= 10_z$ chimes	$1{,}728_d$	(triqua)	⊡dwell	duor	2_d hrs
1 **day**	$= 10_z$ bells	$20{,}736_d$	(quadqua)	⊡day	day	
1 **stint**	$= 10_z$ days	$248{,}832_d$	(pentqua)	⊡unquaday	doday	

We cannot know how long the home planet's day is, compared to Earth's. However, it's fair to say that Egan's species, having evolved to be adapted to their day length, would likely *perceive* their day similar to the way we perceive ours. So they would likely perceive the subunits of their day similarly to the way we perceive our own subunits. Apparently, their clocks ring a bell the equivalent of every two hours, and sound a chime the equivalent of every ten minutes. Egan's species seem to mark the "lapse" of time in the equivalent of 50_d second periods comparable to minutes. The "pause," analogous a little over 4 seconds, seems to be what they use where we would use SI seconds, for things like frequencies and velocities.

The "stint" of a dozen days is their equivalent of a week, with eleven days of work and one day off—quite a work-ethic! Their year is described as 43.1_d (37.1_z) stints, which would be 517_d (371_z) of their days, rather longer than an Earth year if we assume their days are equivalent to ours. The dozenal powers of the year get a series of unique names:

Orthogonal Units				Equivalents?
Time		In years		Earth years?
1 **year**	$= 371_z$ (517_d) days	1_d		1.4_d
1 **generation**	$= 10_z$ years	12_d	(unqua)	17.0_d
1 **era**	$= 10_z$ generations	144_d	(biqua)	203.9_d
1 **age**	$= 10_z$ eras	$1{,}728_d$	(triqua)	$2{,}446.9_d$
1 **epoch**	$= 10_z$ ages	$20{,}736_d$	(quadqua)	$29{,}363.1_d$
1 **eon**	$= 10_z$ epochs	$248{,}832_d$	(pentqua)	$352{,}357.7_d$

A "generation" of only a dozen years seems short, but a couple factors mitigate this. First, Egan's species seems to mature rather more quickly than humans do. Indeed, before modern contraception, the average lifespan of a female before she typically fissioned was around a dozen of their years. Second, the longer year means that, in equivalent days, a generation is more like 17_d (15_z) human years, which is rather within the normal range for humans reaching adulthood. The powers above the generation seem like a reasonable series of terms for grander and grander units of time.

Orthogonal Angles? Egan has his species divide up the circle into pure powers of twelve to provide a set of angle units, and even bases the names for these on the names for the divisions of their day. This is exactly equivalent to how many dozenalists have suggested dividing up the circle following Earth's day.

Orthogonal Units			Equivalent Units		
Angles	In revolutions		Primel	Do-Metric	
1 **arc-flicker**	$1/248{,}832_d$	(pentcia)	⊡twinkling·angle	arc-dovic	$5.208\overline{3}''_d$
1 **arc-pause**	$1/20{,}736_d$	(quadcia)	⊡lull·angle	arc-grovic	$1'02.5''_d$
1 **arc-lapse**	$1/1{,}728_d$	(tricia)	⊡trice·angle	arc-minette	$12'30''_d$
1 **arc-chime**	$1/144_d$	(bicia)	⊡breather·angle	arc-temin	$2°30'_d$
1 **arc-bell**	$1/12_d$	(uncia)	⊡dwell·angle	arc-duor	$30°_d$
1 **revolution**	1_d		⊡turn	cycle	$360°_d$

This makes sense, because the times indicated are exactly how long it takes for their planet, or Earth, to rotate over the corresponding angular distance.

Orthogonal Mass? Here is Egan's table of mass units, compared to a similar breakdown in a human dozenal metrology:

Orthogonal Units					Analogous
Mass			In Hefts		Do-Metric
1 **scrag**			$1/144_d$	(bicia)	DM-gram
1 **scrood**	$= 10_z$	scrags	$1/12_d$	(uncia)	DM-ounce
1 **heft**	$= 10_z$	scroods	1_d		pound
1 **haul**	$= 10_z$	hefts	12_d	(unqua)	DM-stone
1 **burden**	$= 10_z$	hauls	144_d	(biqua)	DM-burden

There's very little to indicate how heavy any of these actually are. However, it's plausible that a "heft" is something like a pound, a weight easily "hefted" in one hand. A "haul" would then be analogous to a British stone; the term seems apt for an amount that can readily be hauled or carried by a person. A "burden" would be analogous to the Do-Metric unit of the same name, and would be reasonable as a mass requiring a vehicle to transport. A "scrag," being something on the order of a few grams, makes a reasonable unit for dosages.

Orthogonal Conclusion? Not to reveal any spoilers, but suffice to say, the crew of the *Peerless* manage to make a number of amazing discoveries about their universe, as well as about their own biology, sufficient to change *everything*—even their culture. This is a classic example of truly high-concept, hard SF, in that many of its protagonists are scientists or inventors, struggling to unlock the mysteries of their universe, and struggling with the impact their discoveries have upon themselves as individuals, as well as their society at large. The characters are well-drawn and memorable. The occasional glimpse into the "Uncanny Valley" of their alien origin will not dissuade you from looking at them as *people*. The fact that they are also natural-born dozenalists can only add to the charm of these works, for human readers with a predisposition to regard base twelve favorably.

This is well worth the read!

2016 – The start of a new (dozenal) century

Featuring: Dr. James Grime • Member 482_z (674_d)
URL: https://www.youtube.com/watch?v=EsLgiffa9Cc

Dr. Grime was kind enough to put out a quick video just before the rollover from $11ƐƐ_z$ to 1200_z, to commemorate the turning of the biquennium. He actually featured the words "unquennium" and "biquennium," the Pitman digits, and even counting to twelve on the phalanges of one hand. He gives it a cute finish about partying like it's "one-dozen-one gross eleven-dozen-eleven." The discussion in the comments is actually interesting and surprisingly civilized for a Youtube comment section, even with the inevitable debate about whether the biquennium actually turned at the start or end of 1200_z.☺

BBC Ideas: Is there a better way to count. . . ? 12s anyone?

Featuring:
Stephen Wood, Physics Teacher, Base 12_d Enthusiast
Dr. Vicky Neale, Mathematics Lecturer, University of Oxford
Dr. Philip Beeley, Historian of Mathematics, University of Oxford
Dr. Chris Hollings, Lecturer in Mathematics and History, University of Oxford
URL: https://www.bbc.com/ideas/videos/is-there-a-better-way-to-count-12s-anyone/p06mdfkn

This is one in a series of short videos titled "Is there a better way. . . ?" that the BBC put out in 1202_z (2018_d). The production quality is hip and fresh, very appealing to the Millennial and Post-Millennial demographic. Stephen Wood from the DSGB features prominently, but they also got no less than three Oxford professors to comment on the advantages of dozenal, interspersed with quick clips of a young "mathsy person" and a young "non-mathsy person" in front of a shared whiteboard casually discussing some feature of dozenal counting or the dozenal multiplication table.

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