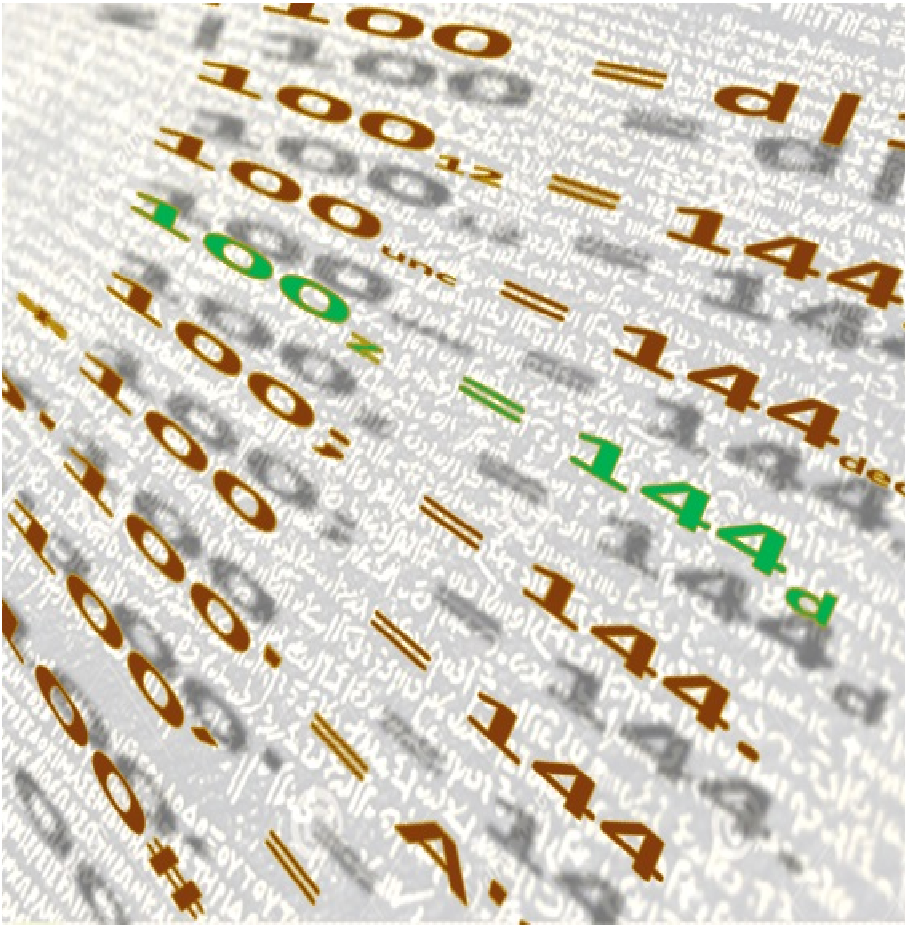
The Duodecimal Bulletin • Volume 52_z (62_d), Number 1 • Whole Number $↊2_z$ (122_d) • July $11↋↋_z$ (2015_d)

The Duodecimal Bulletin

Vol. 52_z (62_d), No. 1 • July $11↋↋_z$ (2015_d)



Base Annotation Schemes

Whole Number

$↊2_z$

(122_d)

ISSN $0046\text{-}0826_d$

The Dozenal Society of America

is a voluntary, nonprofit corporation, organized to conduct research and educate the public in the use of base twelve in calculations, mathematics, weights and measures, and other branches of pure and applied science.

Basic membership is free, but as a nonprofit we depend upon the generosity of our members to continue operating. An annual subscription of US\$$18.00_d$ (US\$$16.00_z$) entitles the member to delivery of paper copies of *The Duodecimal Bulletin.*

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The DSA does not endorse particular symbols for digits in alternate bases. Authors are free to propose their own symbols. However, unless otherwise specified, this publication defaults to the Pitman numerals for the digits ten (↊) and eleven (↋).

The DSA does not endorse any particular scheme for disambiguating the base of a number. Authors are free to propose their own notations. However, this publication will annotate the base of any number containing more than one digit, and as a default will do so using a single-letter subscript: **d** *for* ***D****ecimal,* **z** *for do****Z****enal,* **x** *for he****X****adecimal,* **o** *for* ***O****ctal,* **b** *for* ***B****inary, etc. By default, numbers are annotated individually, although whole numeric expressions in parentheses may be given a single annotation applying to all of their components. A tabular display may omit annotations, as long as the base used by the entire display is indicated.*

Volume Five Dozen Two (52_z) • Number 1 • Whole Number Ten Dozen Two ($↊2_z$)

The Duodecimal Bulletin

Volume Sixty Two (62_d) • Number 1 • Whole No. One Hundred Twenty Two (122_d)

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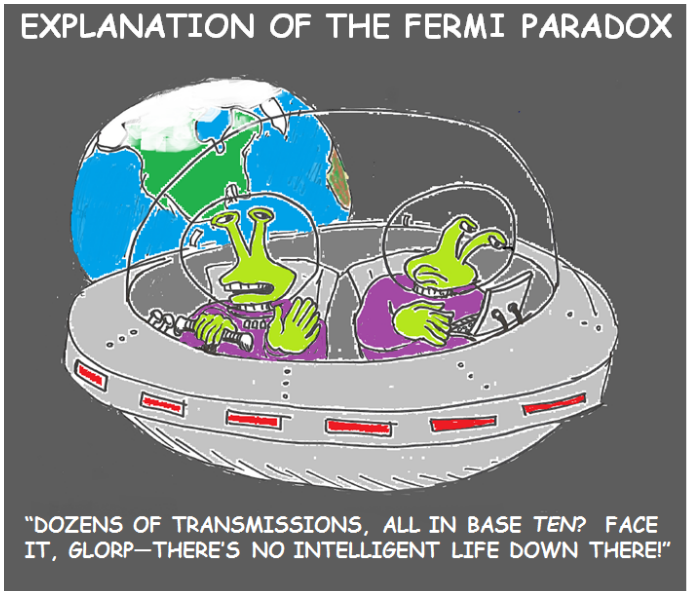

President's Message

Greetings to our membership; we hope this long-awaited issue of our *Bulletin* finds you well. A great deal has come and gone since our last issue, and we are excited to have this issue in our hands, ready to reveal to us new aspects of the dozenal base. This issue brings us change, and with that change growth; the DSA is continuing its march into the future at the dawning of the 1200$_z$'s.

First, we must regretfully bid farewell to the past editor of the *Bulletin*, Michael De Vlieger (Member 37↋$_z$). Since his first issue, in 11↋4$_z$ (2008$_d$), Mike has continually pushed the DSA forward, and it is with great sorrow that the Board accepted his resignation earlier this year. Unfortunately, the press of business was simply too great for him to continue his duties, and we wish him the best of luck in the future.

Mike's first issue was a lot of firsts: the first digitally-produced *Bulletin*; the first full-color *Bulletin*; the first *Bulletin* in many years to return to our original Dwiggins numerals. But fittingly, it also was headlined by a history of the DSA, uniting both our long and distinguished past with our journey into the future.

We continue that journey with this issue. Our new editor, John Volan (Member 418$_z$) has endeavored to keep the same look-and-feel of Mike's original and beautiful design; he has succeeded brilliantly, despite using different software. John has contributed to the *Bulletin* in the past, notably an extensive article on Systematic Dozenal Nomenclature; he submits an article in this issue on "Base Annotation Schemes" which will surely provoke an equal amount of thought and discussion. Like Mike's initial issue, this article hearkens back to the very beginnings of our Society, with a "new/old" proposed solution to a problem inherent in proposing a change of base.

Readers will be delighted to see Prof. Jay Schiffman (Member 2↋8$_z$) returning to our pages once again, providing us great food for thought on integer sequences and why dozens are so powerful a tool in that field.

We also explore some recent contributions to mass media which highlight our favorite number. Online videos expositing not only the utility of base twelve, but also its beauty, are just some of the joys that we will look at in these pages.

Our membership has been exploding; our Internet presence has been doing an incredible job spreading knowledge of and interest in the Society. In the past two years alone, we have gained over 90$_z$ new members; this is significantly more than the several *years* prior to our expanded website and the ability to join online. This membership surge is just another sign of encouragement as we move forward into the future.

Our *Bulletin*, thanks to Mike De Vlieger, has already entered the Twenty-First Century; join us as we usher it into the One Dozen Third *Biquennium*[1]. The future is full of possibilities, and the DSA intends to be fully a part of it.

Donald P. Goodman III, current President of the DSA, also publishes a monthly email newsletter, The DSA Newscast. *It's free to all DSA members—so be sure to join!*

[1] See page 31$_z$.

from the Editor's Desk

Radically Conventional

If you're a long-time member of the Dozenal Society of America, some changes you see in this issue of the *Duodecimal Bulletin* may take you aback. First, you'll have noticed that the Dwiggins ten digit (𝒳) has been replaced with the Pitman ten (↊). Second, something seems to have happened to all the usual Humphrey points (semicolons acting as "duodecimal" points, in contrast to periods acting as "decimal" points). In their stead there seem to be a lot of "z" and "d" subscripts. Gentle Readers, I beg your indulgence, as I address both of these developments in turn.

Pitman versus Dwiggins

As long anticipated, the glyphs for the Pitman characters have achieved official recognition by a mainstream, international standards body: the Unicode Consortium. As of June 15th$_z$, 11↋↋$_z$ (June 17th$_d$, 2015$_d$), version 8.0.0 of the Unicode standard[1] has been released, including the following two code-points of dozenal interest:[2]

U+218A$_x$	↊	TURNED DIGIT TWO • digit for 10$_d$ in some duodecimal systems
U+218B$_x$	↋	TURNED DIGIT THREE • digit for 11$_d$ in some duodecimal systems

Kudos to our Israeli friend "Treisaran" for alerting the DozensOnline Forum about this.[3] Also, someone has already updated the "Duodecimal" article on Wikipedia.[4]. Of course, it will be some time before operating system fonts catch up with the new standard, so that we can actually see these glyphs rendering on our web browsers. But the significance of this milestone for the DSA, as well as for its sister organization, the Dozenal Society of Great Britain, cannot be overstated. The time has come for this publication to begin using the Pitman characters as its default convention.

This is feasible, thanks to the efforts of our current president, Don Goodman (Member 398$_z$), who some time ago developed a package for typesetting these characters in LaTeX[5]. Not to mention the brilliant work of our previous editor, and past president, Mike De Vlieger (Member 37↋$_z$), using Adobe Illustrator to render these, and numerous other alternate characters.

The official position of the DSA has long been, and continues to be, not to endorse particular characters, but rather uphold the freedom of individual dozenalists to experiment with characters they prefer. This publication has always been a friendly

[1] http://www.unicode.org/versions/Unicode8.0.0/
[2] http://www.unicode.org/charts/PDF/Unicode-8.0/U80-2150.pdf
[3] http://z13.invisionfree.com/DozensOnline/index.php?showtopic=1324
[4] https://en.wikipedia.org/wiki/Duodecimal
[5] http://www.ctan.org/tex-archive/fonts/dozenal

place for members to field their symbology suggestions, and that will not change.[6]

Nevertheless, the DSA has also long recognized the importance of settling upon some *default* which we can all count upon as a convention, and which the publications of the Society endeavor to adhere to in the interest of fostering understanding. As the new editor of the *Duodecimal Bulletin*, I take it as my responsibility to see to it that this publication continues to fulfill that obligation.

The Dwiggins characters were certainly serviceable as a default convention. For my own part, I admit I've grown a bit fond of them, and regret their eclipse. The obvious provenance of the Dwiggins ten from the ancient Roman numeral ten strikes a definite "Least-Change" chord.[7] For some time to come, typing X and E for ten and eleven will continue to be a necessary expedient in disadvantaged media, such as email.

However, we must concede that the Pitman ten predates the Dwiggins, having been introduced by Sir Isaac Pitman in 1860_{d} ($10Ɛ0_{\text{z}}$). It has a more number-like appearance, without the Dwiggins ten's unfortunate similarity to both the algebraic unknown (x) and the multiplication sign ($\times$). The Unicode Consortium evidently observed signs of usage of the Pitman numerals on both sides of the Atlantic, whereas the Dwiggins appears to have been an exclusively American peculiarity. Further, it turns out that the ↊ character has been independently suggested more than once, in more than one country, in more than English: For instance, Don Vicente Pujals de la Bastida came up with exactly the same shape for a dozenal ten in 1844_{d} (1098_{z}), in his work *Filosófia de la Numeración, ó Discubrimiento de un Nuevo Mundo Científico*.[8] This underscores the *international* appeal of the Pitman transdecimals.

Humphrey-Free Zone

As to the second matter, I have written an article in this issue entitled "Base Annotation Schemes," exploring the history of how the members of the DSA (and DSGB) have undertaken to annotate (or, as the case may be, *not* annotate) the bases of numbers. To summarize, I make the case that we really need an annotation method that is

- *equitable*—one that treats all bases alike, neither favoring any particular base, nor disadvantaging any base;
- *explicit*—one that presents some kind of *positive* statement of a number's base, rather than relying on some implicit assumption;
- *comprehensive*—one that can tackle *any* base, and scale to *all* bases;
- *modular*—one that implements *only* the function of base-annotation, while neither participating in, nor interfering with, the function of any other textual feature, whether in the syntax of numbers or of prose; by implication, one that can be *omitted*, when appropriate, without disturbing any other function of text; and, as much as possible, one that is "lightweight" rather than "cumbersome";

[6] Personally, I have taken a fancy of late to using a mirror-reversed six (∂) as a stylized "d" evocative of "dek". It is quite number-like, has an obvious seven-segment representation, is easy to hand-write with a single stroke, and, as you can see, is readily typeset in LaTeX.

[7] Ralph H. Beard, "The Opposed Principals", *Bulletin* Vol. 1, No. 3, WN 2, Oct 1161_{z} (1945_{d}).

[8] http://www.dozenal.org/drupal/sites/default/files/pujals_de_la_bastida_filosofia_de_la_numeracion_0.pdf

- *familiar*—one that requires as little deviation as possible from what everyday folks are used to ("Principle of Least Change").

I argue that the base annotation techniques that have been most popular among dozenalists, in particular the Humphrey point, fail to meet these criteria.

On the other hand, the conventions of mainstream mathematics include a technique for base annotation that satisfies nearly all of these goals. In one important respect, however, it falls short on the first goal. It's a technique which most of you likely learned in secondary school. As near as I can tell, it has been around even longer than the DSA and the DSGB. If the founders of these societies learned this technique as students, they evidently ignored it.

Interestingly, certain dozenalists, in particular another past president, our esteemed emeritus member Gene Zirkel (Member 67_{z}), have at one time or another touched upon ideas which could have been grafted into this mainstream technique to let it satisfy even my first bullet point. All that it would take is a simple synthesis—which you see demonstrated here.

Why am I doing this? I am a relative newcomer to dozenalism. By trade, I am a software engineer, and therefore very detail-oriented and mathematically inclined, and something of an amateur linguist. The architectures I deal in are entirely in the abstract (versus the architectures Mike deals with, which are often in concrete). As a kid growing up in the Chicago area, I fondly remember enjoying the "Little Twelvetoes" cartoon from Schoolhouse Rock,[9] with its "dek-el-do". But I had no idea that "dozenal societies" existed, until I happened to stumble across the DozensOnline Forum in $11Ɛ7_{\text{z}}$ (2011_{d}). I am surprised now at how much the subject has captivated me since then.

So, to me, something like the Humphrey point is not an old, familiar, well-worn tradition, hewn out of the living rock by titans of old and lovingly polished over the ages, but more of a rank, avant-garde, hot-off-the-presses-and-rough-around-the-edges neologism, the work of ardent, but evidently naive, amateurs. The fact that they happened to have been located some six dozen years down-time doesn't change that.

"It is often easier to ask for forgiveness than to ask for permission."
Grace Hopper

The nuns and priests at my Catholic high school, who drilled into me the fundamentals of English prose style and the principles of mathematics, were quite academically rigorous, and insisted on high standards. The things that dozenalists have done with punctuation and numerals ... well, they just aren't *done*. Worse, they shouldn't *need* to be done. Worst of all, in failing to be *equitable*, these techniques single-out one base in particular to place at greatest *dis*advantage ... and that is base *twelve*.

Now I find myself asked to edit this publication, contemplating whether I should support something some of you may cherish as a sacred tribal practice, or an emblem of dozenalist solidarity, but which I see as just a weight holding dozenal back, and I find that ... I can't. I just *can't*. I *have* to try to persuade you that there is a better

[9] https://www.youtube.com/watch?v=_uJsoZheTR4&index=12&list=PLnx6r9S_SJ7I_Msib-Nj-zgR0aicmyqA8

way, and prove it by demonstration. Money-where-my-mouth-is. If this means I'll be voted off the island, so be it.

You may feel that I'm being a radical iconoclast. But from my perspective, I feel I'm standing up for a more conservative, conventional, indeed "Least-Change" approach to annotating bases. In doing so, I'm trying to stand up for base twelve, pull it out of a mathematical ghetto that we have inadvertently created, assert its legitimacy to go mainstream, and help make sure that it fits comfortably there.

What you will find in these pages is that, for the most part, every number, whether dozenal or decimal or some other base, has its base explicitly annotated somehow. All without violating generally-accepted rules of English prose style and mathematical symbology, that readers of any serious publication have a right to expect. As editor, I consider it my obligation to satisfy that expectation.

You may see a number annotated individually. Or it might be part of a parenthesized expression that has been annotated as an aggregate. Or it might be part of a table or row or column, or some other structure, which carries a blanket annotation. If there is *no* annotation at all, it's either because it's a single-digit number, and therefore unambiguous; or there's a deliberate reason to not identify any base at all, in which case the *lack* of annotation should stand out like a sore thumb. There is actually a specific case in Jay Schiffman's paper in this issue, where he needs to be indefinite about the base in order to make a particular point.

If it isn't already, I think it ought to be the policy of the DSA *not* to promote any particular scheme for disambiguating the base of a number. This publication should be a friendly place for anyone wishing to propose a solution of their own, and it will be. Of course, I think it's fair to subject any such proposals to analysis against the criteria I've outlined above. Meanwhile, we still need some *default* convention that we can all rely upon, for the sake of communicating clearly with each other. Hence I'm offering the one you see here, which I've laid out in detail in my article.

Along with a lot of carefully-annotated dozenal numerals, another thing you may notice on these pages is a lot of carefully-annotated *decimal* numerals, often side-by-side. Almost as if I meant for this publication to act as some kind of "Berlitz Guide" supplying translations between the language of a dozenal world and the language of this predominantly decimal one. In point of fact, I *do* think that this would be an important role for this publication to fulfill.

If, as I do, you hope for the DSA to attract many new members, as we head into a new biquennium,[↊] it will be important for such folks to get used to translating back and forth between their "native" decimal and this "second language" of dozenal. Having a "Rosetta stone" of sorts to practice with could come in handy. Perhaps some of you could benefit from that sort of immersive exercise yourself. I know I do.

Make a Dozenal Difference! *The DSA no longer charges dues; membership is free. Our officers volunteer their time as a labor of love. If you're a lover of base twelve too, please consider making a modest donation to help us produce* The Duodecimal Bulletin *and* The Dozenal Newscast, *maintain the website, as well as advocate and educate the world about the usefulness of the dozen.* **Thanks!**

↊See page 31$_z$.

New Members

We've had a bumper crop of new members since the last issue! Many joined in the wake of the publicity around 12/12/12$_d$. Members highlighted in red have paid subscriptions to receive hard-copies of the *Bulletin.* (The electronic version is free to all members.)

415$_z$ (593$_d$) Thomas W. Carter-Thompson
416$_z$ (594$_d$) Robert Sherwood
417$_z$ (595$_d$) Joseph B. Vadella
418$_z$ (596$_d$) John Volan
419$_z$ (597$_d$) Brian Carroll
41↊$_z$ (598$_d$) Alex J. Harkleroad
41↋$_z$ (599$_d$) Hudson J. Therriault
420$_z$ (600$_d$) Marton F. Szocs
421$_z$ (601$_d$) Adam A. Straub
422$_z$ (602$_d$) Neil A. Batt
423$_z$ (603$_d$) Rock Brown
424$_z$ (604$_d$) Benjamin J. Cullen
425$_z$ (605$_d$) James B. Robinson
426$_z$ (606$_d$) Brian Hetrick
427$_z$ (607$_d$) Halley T. Haruta
428$_z$ (608$_d$) Devanney T. Haruta
429$_z$ (609$_d$) Sean Hartung
42↊$_z$ (610$_d$) Mateus M.F. Mendonca
42↋$_z$ (611$_d$) James J. Zamerski
430$_z$ (612$_d$) Fernando Flores
431$_z$ (613$_d$) Jake L. Duzyk
432$_z$ (614$_d$) Jim Carruthers
433$_z$ (615$_d$) Wayne D. McManus
434$_z$ (616$_d$) Orlando A. Giovanni
435$_z$ (617$_d$) Jeffrey A. Kreindler
436$_z$ (618$_d$) Kim Scarborough
437$_z$ (619$_d$) Matteo Biasin
438$_z$ (620$_d$) Adam Partridge
439$_z$ (621$_d$) Stephen Van Sickle
43↊$_z$ (622$_d$) Bryce Wedig
43↋$_z$ (623$_d$) Philip D. Preston
440$_z$ (624$_d$) James McComb
441$_z$ (625$_d$) Christopher D. Elce
442$_z$ (626$_d$) Patrick Louis
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444$_z$ (628$_d$) Nathaniel A. Turtel
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446$_z$ (630$_d$) Paul Sobczak
447$_z$ (631$_d$) Walter de Brouwer
448$_z$ (632$_d$) Gabe O. Pridemore
449$_z$ (633$_d$) Ajay R. Karpur
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44↋$_z$ (635$_d$) Jehsuamo A. Casas
450$_z$ (636$_d$) Caroline B. Bonnett
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452$_z$ (638$_d$) Steven M. Williams
453$_z$ (639$_d$) Dean Clark
454$_z$ (640$_d$) Shanmugam
455$_z$ (641$_d$) Ben Huxham
456$_z$ (642$_d$) Rudy H. Terriquez
457$_z$ (643$_d$) Miranda Elliott Rader
458$_z$ (644$_d$) Nicholas A. Knibbs
459$_z$ (645$_d$) Steve Cohen
45↊$_z$ (646$_d$) James C. Peterson
45↋$_z$ (647$_d$) Max B.E. Rudig
460$_z$ (648$_d$) Daniel T. Crocker
461$_z$ (649$_d$) Virgil J. Lomocso
462$_z$ (650$_d$) Christian Modery
463$_z$ (651$_d$) Ronnie E. Johnson
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465$_z$ (653$_d$) Jason Pruski
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486$_z$ (678$_d$) Jack E. Tisdell
487$_z$ (679$_d$) Simeon Moses
488$_z$ (680$_d$) Brandon Ward

Accuracy of Common Roundings

Donald P. Goodman III

DSA President • Member 398$_z$ (548$_d$)

Introduction

The use of place notation is a huge boon for mathematics, making calculation and expression much easier in many cases than that of vulgar fractions, which had previously been the only way of referencing values less than a whole unit. However, while vulgar fractions can *always* exactly express their real value (such as $\frac{1}{7}$), place-value fractions (which we will henceforth call "inline" fractions) sometimes cannot (such as the inline expression of $\frac{1}{7}$, $0.\overline{186↊35}_z$).

Still, the convenience of inline fractions is such that we often wish to use them despite their inherent inaccuracy in such cases. Therefore, we *round* them; that is, we select a point in the inline expansion of the fraction which we will deem an acceptable level of inaccuracy. One common such level, used in the trigonometric and logarithmic tables common before digital calculators, was four digits; but any number of digits can be selected. The acceptable degree of inaccuracy is typically gauged in precisely this way: number of digits. The amount by which that number of digits is actually varying from the true value is rarely considered.

But not all roundings to the same number of digits reflects the same variation from the true value. Let's take a relatively simple, terminating fraction as an example: 0.0↊5$_z$. We decide that three digits is too long, and we want to round it to two, giving us 0.0↊$_z$. (Remember, this is dozenal; we round up at 6, the half, not at 5 as in decimal.) We have another fraction, 0.0↊2$_z$, which we also want to round to two digits; this gives us 0.0↊$_z$, as well. Clearly, the second rounding is more accurate; it varies from the true value by only 0.002$_z$, while the first varies from the true value by 0.005$_z$. So simply being rounded to the same number of digits doesn't indicate how close the rounded value is to the true value, except within certain fairly broad limits.

With non-terminating fractions, of course, the accuracy calculations are more difficult; we must round our inaccuracy values themselves to make them manageable. However, the basic concept is the same.

Methodology

The nature of non-terminating fractions means that, no matter how far we take our expansions, we never really have the true value; we merely have increasingly accurate approximations of it. So to compare the accuracy of approximations of such numbers, we must choose a certain level of expansion as normative; that is, we must take a certain approximation of the numbers and simply declare that value to be "true." Then we have a value that we can compare our approximations to in order to determine their accuracy.

In this article, we take one dozen digits as normative. So, for example, for the purposes of this article, $\pi =$ 3.184809493↋91$_z$ in dozenal, and $\pi = 3.14159265358 9_d$ in decimal. These "normative" values we obtain by simple truncation, not by rounding. We will compare dozenal approximations to the normative dozenal values, and decimal approximations to the normative decimal values, to ensure that we're not comparing apples to oranges.

For dozenal, the errors are calculated by taking the absolute value of the difference of the normative value and the estimation; e.g., |3.184809493↋91 − 3.18|$_z$. For decimal, the errors are calcualted the same way, but the resulting error quantity is converted to dozenal.

	Dozenal			Decimal		
	Normative	Approx.	Error	Normative	Approx.	Error
π	3.184809493↋91$_z$	3.18$_z$	$4.8\times10_z^{-3}$	3.141592653589$_d$	3.14$_d$	$2.9\times10_z^{-3}$
		3.185$_z$	3.↋$\times10_z^{-3}$		3.142$_d$	$8.5\times10_z^{-4}$
		3.1848$_z$	$9.5\times10_z^{-6}$		3.1416$_d$	1.↊$\times10_z^{-5}$
e	2.875236069821$_z$	2.87$_z$	$5.2\times10_z^{-3}$	2.718281828459$_d$	2.72$_d$	$3.0\times10_z^{-3}$
		2.875$_z$	$2.4\times10_z^{-4}$		2.718$_d$	5.↊$\times10_z^{-4}$
		2.8752$_z$	$3.6\times10_z^{-5}$		2.7183$_d$	$4.6\times10_z^{-5}$
φ	1.74↋↋6772802↊$_z$	1.75$_z$	$5.4\times10_z^{-5}$	1.618033988749$_d$	1.62$_d$	$3.5\times10_z^{-3}$
		1.750$_z$	$5.4\times10_z^{-5}$		1.618$_d$	$8.5\times10_z^{-5}$
		1.7500$_z$	$5.4\times10_z^{-5}$		1.6180$_d$	$8.5\times10_z^{-5}$
		1.75↋↋7$_z$	$4.5\times10_z^{-6}$		1.61803$_d$	↋.↋$\times10_z^{-6}$
$\sqrt{2}$	1.4↋79170↊07↋8$_z$	1.50$_z$	$4.3\times10_z^{-3}$	1.414213562373$_d$	1.41$_d$	$7.3\times10_z^{-3}$
		1.4↋8$_z$	2.↊$\times10_z^{-4}$		1.414$_d$	$4.5\times10_z^{-3}$
		1.4↋79$_z$	$1.7\times10_z^{-5}$		1.4142$_d$	$3.4\times10_z^{-5}$

Irrational Numbers

So let's examine some primary irrational numbers for the relative accuracies of their roundings in both dozenal and decimal, and see what we arrive at. π, of course, we all know well as the ratio of the circumference of a circle to its diameter. e, Euler's number, is less well known as the base of natural logarithms. φ is the "mean and extreme ratio," that ratio of a line segment such that the ratio of the larger part to the smaller is equal to that of the whole to the larger part. And the last is the square root of two, $\sqrt{2}$. Errors are themselves rounded, in these cases to two significant digits.

A few things to note about the numbers we've explored here:

- Decimal is *more* inaccurate than dozenal for the same number of digits almost every single time. The only exceptions are the two-digit roundings for π, $\sqrt{2}$, and e. But in the case of $\sqrt{2}$, the difference is miniscule, and even in these cases the dozenal three- and four-digit roundings are significantly more accurate than the decimal. Indeed, the four-digit rounding of $\sqrt{2}$ in dozenal is more than twice as accurate as in decimal.
- In the case of φ, the dozenal two-digit rounding is more accurate than the

	Dozenal			Decimal		
	Normative	Approx.	Error	Normative	Approx.	Error
$\frac{1}{3}$	0.4_z	0.4_z		$0.333333333333_\mathrm{d}$	0.33_d	$5.9\times10_\mathrm{z}^{-3}$
					0.333_d	$6.\text{Ɛ}\times10_\mathrm{z}^{-4}$
					0.3333_d	$8.3\times10_\mathrm{z}^{-5}$
$\frac{1}{5}$	$0.249724972497_\mathrm{z}$	0.25_z	$2.5\times10_\mathrm{z}^{-3}$	0.2_d	0.2_d	
		0.24↊_z	$4.\text{↊}\times10_\mathrm{z}^{-4}$			
		0.2497_z	$2.5\times10_\mathrm{z}^{-5}$			
$\frac{1}{7}$	$0.186\text{↊}35186\text{↊}35_\mathrm{z}$	0.19_z	$5.2\times10_\mathrm{z}^{-3}$	$0.142857142857_\mathrm{d}$	0.14_d	$4.\text{Ɛ}\times10_\mathrm{z}^{-3}$
		0.187_z	$2.4\times10_\mathrm{z}^{-4}$		0.143_d	$3.0\times10_\mathrm{z}^{-4}$
		0.186↊_z	$3.6\times10_\mathrm{z}^{-5}$		0.1429_d	$\text{↊}.8\times10_\mathrm{z}^{-5}$

decimal *four*-digit rounding. Decimal does not beat out dozenal's two-digit accuracy for φ until it reaches five digits; and the dozenal five-digit rounding is still more accurate than that.

- Often, the dozenal rounding is not only more accurate, but *much* more accurate.
  - The three-digit rounding of π, for example, is more than twice as accurate in dozenal than in decimal.
  - The four-digit rounding of π is an entire order of magnitude more accurate in dozenal than in decimal.
  - The three-digit rounding of e is nearly a third more accurate in dozenal than in decimal.
  - The roundings of φ have already been reviewed above.

All in all, dozenal clearly comes out on top in these calculations.

Repeating Rational Fractions

Let's now examine some other difficult fractions; not irrational numbers this time, but just difficult rational ones. These are not the same in all bases; so, for example, $\frac{1}{3}$ in decimal will be compared with $\frac{1}{5}$ in dozenal, since $\frac{1}{3}$ is trivially simple in dozenal (0.4_z) while $\frac{1}{5}$ is trivially simple in decimal (0.2_d). On the other hand, $\frac{1}{7}$ is a six-digit-period repeating fraction in both bases, so it will be compared to itself.

Surprisingly, for $\frac{1}{7}$, decimal is slightly more accurate for a two-digit rounding; two thousandths as opposed to nearly seven tricia[1] explains that discrepancy. But otherwise, dozenal holds its own quite well even in sevenths; the three-digit rounding is slightly more accurate in dozenal than in decimal, while the four-digit rounding is more than twice as accurate.

Decimalists will always point to the fifth as the Achilles heel of dozenal; but this table gives that claim the lie. Not only is the third arguably a more important fraction anyway, but *dozenal handles fifths better than decimal handles thirds.* Observe our roundings here. The two-digit rounding of the dozenal fifth is more than twice as accurate as the two-digit rounding of the decimal third; the three- and four-digit roundings continue to blow decimal's out of the water. At four digits, dozenal fifths are more than three times as accurate as decimal thirds.

Even in the most difficult numbers; even in the numbers that are some of dozenal's few weaknesses; even in these cases, dozenal is the best base.

[1] See page 31.

EDITOR'S NOTE: Don Goodman's observations on the generally better accuracy per digit of dozenal over decimal can largely be explained by the greater information capacity per digit in dozenal. Consider that every dozenal digit can take on twelve possible values while each decimal digit can only take on ten possible values. That means that each digit of these bases is, on average, equivalent to a different number of binary digits, or "bits," of information:

$$\begin{aligned}\log_2 10_\mathrm{z} &= 3.584962501_\mathrm{d} = 3.702994803_\mathrm{z}\\ \log_2 10_\mathrm{d} &= 3.321928095_\mathrm{d} = 3.3\text{↊}4360183_\mathrm{z}\\ \text{difference} &= 0.263034407_\mathrm{d} = 0.31\text{Ɛ}634631_\mathrm{z}\end{aligned}$$

This means that every dozenal digit can carry more than a quarter bit more information than a decimal digit. Every four digits grants a dozenal numeral greater than 1 bit of information more than the decimal numeral, so errors in dozenal roundings will be cut in half compared to decimal. (This makes sense, because $10{,}000_\mathrm{z} = 20{,}736_\mathrm{d}$). After one dozen one digits, a dozenal numeral gains the equivalent of a whole decimal digit, so errors in dozenal roundings will be more than a decimal order of magnitude better than decimal roundings. After one dozen two digits, the errors will be better by more than a full dozenal order of magnitude.

Of course, hexadecimal trumps both:

$$\log_2 10_\mathrm{x} = 4.000000000_\mathrm{d} = 4.000000000_\mathrm{z}$$

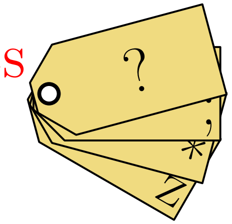
In other words, a hexadecimal digit is exactly equivalent to four full bits of information, making it that much more accurate than both decimal and dozenal.

This just goes to show that we should not rely on only one consideration to evaluate the merits of different bases. If dozenal is the "best" base, it is not solely due to its better accuracy compared to decimal, since hexadecimal would beat it there. I think divisibility may be as important a factor, if not more so. Rounding ceases to be an issue if dividing by the factors you are most interested in yields exact results. When those factors include 3, dozenal beats decimal and hexadecimal, hands down!

A Limerick

A base boasting reason and rhyme,
Sported factors full four at a time.
"Why the fourth and the third?"
"'Cause just three is absurd;
And only two, sir, is a crime."

Base Annotation Schemes



John Volan

Bulletin Editor • Member 418_z (596_d)

The Problem

The overwhelming majority of the population presumes all numbers are expressed in base ten, often with no awareness that there is even an alternative. To them, "100" means simply "a hundred"—ten times ten—and that's that.

Many advocates of base twelve fondly envision a world where dozenal is the presumed base. In such a world, ordinary folk would naturally read "100" as "a gross."

Of course, the reality is that supplanting decimal as the "civilizational" base has proven a stubbornly distant goal. This circumstance has persisted since the founding of the Dozenal Society of America nearly six dozen years ago—half a biquennium![1] No doubt this state of affairs will continue for the foreseeable future.

Consequently, the more sober advocates of dozenalism have long been reconciled to the need to be "bilingual" (perhaps a better term would be "binumeral") in our mathematical discourse. We recognize the need to be able to switch back and forth, as needed, between base ten and base twelve—and even other bases—preferably, in as neutral and equitable a manner as possible, favoring no base over any other. This is particularly important when introducing the subject to newcomers, a perennial task. ("Each One, Teach One" has been a motto of the DSA since its inception.[2])

Of course, admitting more than one base into the discussion renders any number longer than one digit ambiguous—unless care is taken to stipulate the base in use at any given moment. Various schemes to achieve this have been devised over the years.

An Early Expedient: Stylistic Marking

Largely at the behest of F. Emerson Andrews, co-founder of the DSA and author of the book *New Numbers*,[3] this publication in its earliest days established a convention of distinguishing dozenal numbers from decimal, by typesetting the former in italic style. A number typeset in normal style would simply default to a decimal interpretation.

$$\begin{aligned} \mathit{100} &= 144 \\ \mathit{16.9} &= 18.75 \end{aligned}$$

This convention persisted for over three unquennia.[1]

An advantage of this approach is that it is fairly non-intrusive, at least as far as conventions for mathematical notation are concerned. This means that readers

[1] See page 31_z.
[2] Ralph H. Beard, "Propagation", *Duodecimal Bulletin*, Vol. 1, No. 2, WN 1, Jun 1161_z (1945_d).
[3] F. Emerson Andrews, *New Numbers: How Acceptance of a Duodecimal Base Would Simplify Mathematics*, 1944_d (1160_z).

can take all their prior experience with how numbers work in decimal, and all their expectations about the "look and feel" of numbers, and simply transfer that to dozenal numbers, with minimal adjustment. Except for new symbols for digits ten and eleven, there is no other notation to learn. All other mathematical symbols and operators that people have been comfortable with for generations will largely look the same, and continue to behave in the same way.

However, a disadvantage is that this makes for a rather subtle distinction. Seeing italicized or non-italicized numerals in isolation affords the reader with no *positive* prompting about which base is being applied. If the italicization is not particularly strong, the intent may not be clear.

A stronger objection to this scheme is the fact that it interferes with other common usages of italic style. For instance, italics are generally used to show emphasis, or to set off foreign or quoted text. If there were ever an occasion to emphasize a decimal number, or to *not* emphasize a *dozenal* number, there would be no way to do that.

An even stronger objection is that this scheme is neither neutral nor equitable. It requires dozenal numbers to be marked in a particular—and peculiar—way, while requiring no marking or change at all for decimal numbers. This implies a favored status for decimal and relegates dozenal to an "also-ran" position.

This is problematic enough in typeset print. But consider what this requires of people writing by hand. Either they must go out of their way to artificially distinguish the degree of slant in their cursive, or they must represent italics via underscoring, which means branding every dozenal numeral as somehow out-of-place—a sore thumb, as it were. In an era before word-processing, when the typical mechanical typewriter provided one and only one font, this meant laboriously backspacing over a dozenal numeral and superimposing it with underscores. It is not surprising that aficionados of base twelve would seek out a more streamlined scheme for base annotation.

Humphrey's Radical Radix-Point

Very early in its history, one of the pioneers of the DSA, Herbert K. Humphrey, hit upon an idea: If a period is known as a "decimal" point, separating whole digits from fractional digits in decimal, then perhaps dozenal numbers need a radix point of their own too—a "dozenal" point, as it were. He began using a semicolon to that end:

$$16;9 = 18.75$$

The obvious advantage of this is that it allows us to mark a number as dozenal with no need for any change of font or style, nor any laborious backtracking and retyping. All it takes is the use of another key already available on the typewriter.

(Interestingly, this was not an entirely new idea. More than an unquennium prior the DSA's founding, Grover Cleveland Perry made a similar proposal, in his pamphlet "Mathamerica."[4] He suggested the colon, rather than the semicolon, for this purpose.)

Humphrey proposed this use of the semicolon in an early letter to the *Bulletin*,[5]

[4] Grover Cleveland Perry, "Mathamerica, or The American Dozen System of Mathematics", 1149_z (1929_d). Reprinted in *Bulletin*, Vol. 6, No. 3, WN 14_z (16_d), Dec 1166_z (1950_d).
[5] Herbert K. Humphrey, letter in "Mail Bag", *Duodecimal Bulletin*, Vol. 1, No. 3, WN 2, Oct 1161_z (1945_d).

and others were slowly influenced to adopt this practice. However, it was not until more than an unquennium later that it really took off, under the intensive and enthusiastic lobbying of Henry Clarence Churchman.[6] Churchman at that time had become editor of the *Bulletin* (and a prolific contributor), and even, for a time, DSA president.

The so-called “Humphrey point” did not, at first, supplant italics, despite its clear potential to do just that. This was not simply a case of inertia or incipient traditionalism.

Under the prevailing syntax rules for numbers, a “decimal” point can only appear as part of a number, if it is actually followed by fractional digits—i.e., “decimals” (meaning, “minuscule quantities in base ten”). In a pure integer, of course, no “decimal” point appears.

This rule ensures that a period only admits to an interpretation as a radix point, if it is embedded between digits (or at least, followed by one or more digits), without intervening whitespace. In any other context, it is interpreted as a terminator of a sentence. Another way of saying this is that a period is only interpreted as a radix point if it appears in “medial” position or “initial” position (in the middle or at the start of a numeral); in “terminal” position (at the end of a word or number), it is always interpreted as prose punctuation.

In conventional prose, a semicolon can only appear in terminal position, where it is only interpreted as punctuation (a separator between clauses in a sentence). However, it’s certainly reasonable to consider using it in medial or initial position for some purpose, such as an alternate radix point, or for Internet jargon such as “tl;dr”.

At first dozenalists limited themselves to using the semicolon as a “duodecimal” point in medial and initial positions only, where it would actually be followed by “duodecimals” (“miniscule quantities in base twelve”). They initially refrained from using it in terminal position, in deference to its role as prose punctuation.

So the italics were still needed to mark dozenal integers. In fact, for quite a few years, italics continued to be used for all dozenals, even while the non-integer dozenals began sporting Humphrey points:

$$\begin{aligned} \mathit{100} &= 144 \\ \mathit{16};\mathit{9} &= 18.75 \end{aligned}$$

However, in later issues Churchman and his followers became even more creative:

$$\begin{aligned} \mathit{100};\mathit{0} &= 144 \\ \mathit{16};\mathit{9} &= 18.75 \end{aligned}$$

In other words, they got into the habit of taking what otherwise would have been a pure integer, and appending a spurious 0 fractional digit, simply for the sake of embedding a “dozenal” marker—logic they never thought to apply to decimal integers.

In fact, it appears that Humphrey himself had always been sanguine about using a terminal semicolon to mark an integer as dozenal, without following it with any fractional digits at all. It took many years for other dozenalists to wear down their

[6] Henry Clarence Churchman, “A Dozenal Point Worth Making”, *Duodecimal Bulletin*, Vol. 11_z (13_d), No. 1, WN 22_z (26_d), May 1171_z (1957_d).

inhibitions and accept this practice, but eventually it caught on. This was enough to abolish the former italic scheme:

$$\begin{aligned} 100; &= 144 \\ 16;9 &= 18.75 \end{aligned}$$

For an outsider looking in, this is a rather curious practice, with clear drawbacks. First, it falls short on the goal of being neutral and equitable. It requires a radical change to the syntax of numbers, but only for dozenal numbers, so they can be marked as such. Meanwhile, it imposes no change at all to the syntax of decimal numbers, leaving them essentially unmarked. This confers a privileged default status to decimal base, and relegates dozenal to secondary status, as surely as italicization did.

In an attempt to reclaim some neutrality, in the last few years we even see the period being appended onto decimal integers, so that it acts as a “decimal” base marker, even in terminal position:

$$\begin{aligned} 100; &= 144. \\ 16;9 &= 18.75 \end{aligned}$$

But this merely compounds the problem. Now the scheme is quite intrusive, interfering with the normal interpretation of key punctuation marks fundamental to commonly accepted prose style. For we can easily imagine a sentence such a this:

> A gross, in decimal, is 144; whereas in dozenal, it’s 100.

Here, the semicolon ends a clause while following a *decimal* integer, and the period ends the sentence while following a *dozenal* integer. Yet, under the regime of “dozenal” and “decimal” points, that sentence would be impossible. Instead we’d need to write:

> A gross, in decimal, is 144.; whereas in dozenal, it’s 100;.

If we simply wish to reverse the sentence, the result is even more unfortunate:

> A gross, in dozenal, is 100;; whereas in decimal, it’s 144..

The circumstances where such statements would occur are quite ordinary. As awkward as these forms are, the circumlocutions necessary to avoid them are just as awkward.

Modularity in Design … and Its Lack

What this comes down to is that the Humphrey point is a classic example of a design which achieves very poor *modularity*. Modularity is the principle of good design that stipulates that, ideally, there should be a one-to-one correspondence between the functions that a system implements, and the specific features that implement them.

Features should implement their functions as independently of each other as possible. Proliferating new features that duplicate functions already implemented elsewhere—“reinventing the wheel”—should be avoided. Piggybacking multiple functions into a single feature can be tempting to naive designers, because it feels like “killing two birds with one stone.” But when a single feature is overloaded trying to satisfy too many functions, it makes it difficult to adjust how one function is being handled, without interfering with other functions.

The Humphrey point is just such an example of a naive design. It attempts to implement more than one function at once: It tries to act both as a *base-indicator*, marking a number as dozenal, and at the same time as a *fraction-point*, marking the boundary between whole digits and fractional digits. As a fraction point, it unnecessarily duplicates the function already being adequately served by the medial-period. It interferes with the normal role of the terminal-semicolon, usurping its established function as a clause-separator, in order to overload it with a new function as a base-indicator. This leads logically and inevitably to interfering with the normal role of the terminal-period, co-opting its established function as a sentence-terminator, in order to make it into a decimal base-indicator to contrast with the Humphrey point.

The fact of the matter is that when Simon Stevin coined this usage of a medial-period, inventing the so-called "decimal point," the role he intended for that was simply to act as an indicator that subsequent digits are "miniscules," fractional powers of the base. It acquired the name "decimal point," simply because it was primarily applied to base ten, and in base ten, the miniscules are known as "decimals" (meaning, "divisions of ten").

By a different etymological route, the word "decimal" has also become a term for base ten itself, in contrast with other bases. But it was never Stevin's intention that the medial-period be *limited* to base ten. He meant for it to be a fraction-point, applicable to any base. Indeed, he is actually reputed to have considered applying it to dozenal. He most certainly never intended it as a base-*indicator*. It only acquired that connotation because of the unfortunate overloading of the term "decimal."

Mainstream mathematicians have studied *many* non-decimal bases, for biquennia now (at least as far back as Gottfried Leibniz's studies of binary base back in the Dozenth Biquennium). They have done so, and continue to do so, with apparently no idea that any particular base requires its own special punctuation to mark its fractional digits. Such a requirement is simply not scalable to all the bases we might like to employ. How many different punctuation marks can we co-opt?

Nearly four unquennia ago, Churchman himself discovered, much to his chagrin, just how untenable this program of punctuation-reassignment could become. In response to the burgeoning interest in hexadecimal base due to the rise of computing machinery, he wrote an article in the *Bulletin* entitled "Welcome, Hexadecimalists!"[7] In it, he proposed using the exclamation mark as the "hexadecimal identification point" (or "HIP" for short). This would then let us say:

$$\begin{aligned} 90! &= 100; = 144 \\ 12!\mathrm{C} &= 16;9 = 18.75 \end{aligned}$$

The very next issue saw letters to the editor, from correspondents in both England and the U.S., objecting to how this proposal would usurp the role of the exclamation mark as the symbol for the factorial operator![8] The HIP was never heard from again.

[7] Henry Clarence Churchman, "Welcome, Hexadecimalists!" *Duodecimal Bulletin*, Vol. 1Ɛ_z (23_d), No. 1, WN 36_z (42_d), Sep 1180_z (1968_d).

[8] Letters from Shaun Ferguson, Stan Bumpus, *Duodecimal Bulletin*, Vol. 1Ɛ_z (23_d), No. 2, WN 37_z (43_d), Dec 1180_z (1968_d).

Apparently, this misadventure had been inspired the year before by Tom Pendlebury, a member of the Dozenal Society of Great Britain, and the creator of the Tim-Grafut-Maz measurement system.[9] In a short editorial note, Churchman enthusiastically relates Pendlebury's suggestion to call the Humphrey point the "Dozenal Identification Tag," or "DIT" for short.[Ƨ] Bestowing such a convenient handle upon it, with a concise pronunciation counterpointing the "dot" for the period, seems to have helped cement the Humphrey point's dubious appeal.

If the "DIT" had truly been nothing more than a "tag" indicating a base, there would be nothing to object to. But its role as a fraction point, overloaded onto its established role as punctuation, make it problematic.

Honourable (?) Mentions

Meanwhile, as the "DIT" was insinuating itself into the consciousness of most dozenalists, some members of the DSGB (including Pendlebury) had gotten into the habit of marking some dozenal numbers with an asterisk prefix. On face value, this potentially could have been a somewhat more modular solution than the Humphrey point, if it had been applied both to integers and to fractionals:

$$\begin{aligned} {*}100 &= 144 \\ {*}16.9 &= 18.75 \end{aligned}$$

This would neatly avoid any interference with the normal syntax of integers as well as the normal radix point of fractionals.

On the other hand, it does risk clashing with the use of the asterisk as a multiplication operator, and it rather gets in the way of prefixing a minus sign to make a negative number. But the chief disadvantage of this is that it would have been no more equitable or neutral than the Humphrey point. Once again, dozenal would be given the sole burden of carrying the special marking, while decimal would retain the privileged position of being able to remain unmarked.

However, what asterisk proponents actually suggested was the following:

$$\begin{aligned} {*}100 &= 144 \\ 16;9 &= 18.75 \end{aligned}$$

In other words, they made use of two completely different base-indicators for dozenal: the asterisk prefix for dozenal integers, and the Humphrey point for dozenal fractionals. This makes for worse modularity, because this proliferates multiple features implementing the same function of marking dozenal numbers—while still providing no feature to mark decimal numbers.

For a counterpoint to the preceding, let us go back more than a century (eight unquennia) prior to this. Sir Isaac Pitman, the Englishman who invented shorthand, was promoting both spelling reform (a phonetic alphabet for English) and "reckoning

[9] T. Pendlebury/D. Goodman, *TGM: A Coherent Dozenal Metrology*, 11Ɛ8_z (2012_d)

[Ƨ] Henry Clarence Churchman, editorial note relating "DIT" suggestion from Tom Pendlebury, bottom of p. 4, *Duodecimal Bulletin*, Vol. 1Ƨ_z (22_d), No. 0, WN 35_z (41_d), Sep 117Ɛ_z (1967_d).

reform" (adoption of base twelve).[Ɛ] He advocated a system of base annotation where *decimal* numbers would be marked, but dozenal numbers would be left unmarked:

$$
\begin{aligned}
100 &= (\!|144|\!) \\
16.9 &= (\!|18.75|\!)
\end{aligned}
$$

The express purpose of these awkward-looking parentheses was to mark "obsolescent" numbers. Pitman's clear intent was to declare dozenal the superior base, and to stipulate that decimal was henceforth deemed obsolete. While this approach was certainly modular, it was also clearly inequitable—although in this instance, on the opposite extreme from the cases we have considered so far. It appears this approach did not persuade many of Pitman's Victorian-era countrymen to abandon decimal.

Bottom line, we shall see how all of these infelicities could have been avoided in the first place—once we examine how folks in the mainstream annotate their bases today.

The Mainstream Solution

Mainstream mathematicians and textbooks on mathematics actually have a fairly straightforward approach for annotating the base of a number, an approach that has been in existence for unquennia (perhaps biquennia): They simply suffix the number with a subscript expressing the base. Usually this is itself a numeral:

$$
\begin{aligned}
90_{16} &= 100_{12} = 144_{10} = 220_{8} = 400_{6} = 1001{,}0000_{2} \\
12.\mathrm{C}_{16} &= 16.9_{12} = 18.75_{10} = 22.6_{8} = 30.43_{6} = 1{,}0010.11_{2}
\end{aligned}
$$

One advantage of this scheme is that it is *comprehensive*: This syntax lets us express a number in any base we please. This assumes, of course, that we have sufficient digit characters to support that base. In fact, the convention is to use the letters of the standard Latin 1 alphabet (the English letters A through Z) as transdecimal digits ten through two dozen eleven, thereby supporting up to base three dozen. This convention is promoted both by the educational community and, to varying degrees, by several modern computer programming languages. The letters A through F are well-known as the transdecimal digits for hexadecimal.

Another advantage of this scheme is that it is highly modular. It augments the syntax of numbers with an additional feature, which serves *only* to identify the base of the number. It does this, while neither participating in, nor interfering with, any function of any other feature. Whether the number is an integer, or has a radix point and a fractional part; whether it is a positive number, or a negative one; whether it is expressed in scientific notation, or otherwise; and so forth—none of these have any bearing upon, nor are they perturbed by, this additional subscript annotation. A long and complex expression can be couched in parentheses, and such a subscript can be applied to the whole. Readers can take all their prior experience with how decimal numbers work, and transfer that to numbers in *any* other base. There is no need to reinterpret existing punctuation, nor to learn any new operators or symbols, other than any additional digits the new base requires; all other mathematical symbols and operators that people are familiar with continue to behave the same way.

In terms of how based number values are formatted, this scheme is entirely equitable. All bases are treated the same; none is favored over any other. If we decide, within a given context, to designate one particular base as the assumed default, then we can simply make a blanket statement about that, and then omit the subscripts from numbers of that base, without changing any other aspect of their syntax. We can do this equivalently, no matter which base we choose to favor. Once the annotation feature has been removed from those selected numbers, no lingering trace remains that it was ever there.

The main disadvantage of this convention is that it begs the question: What base is the annotation itself expressed in? The conventional answer, of course, is simply to assume decimal. But this grants decimal favored status, at least within the subscripts. If dozenal were ever to become the preferred base, would these subscripts be recast?

$$
\begin{aligned}
90_{14} &= 100_{10} = 144_{\text{Ɛ}} \\
12.\mathrm{C}_{14} &= 16.9_{10} = 18.75_{\text{Ɛ}}
\end{aligned}
$$

Ultimately, this does not eliminate the ambiguity, it merely pushes it into the subscripts. We need some way to express the annotations themselves that is neutral to any base.

One way to mitigate this is to spell out the subscripts as words:

$$
\begin{aligned}
90_{\text{sixteen}} &= 100_{\text{twelve}} = 144_{\text{ten}} = 220_{\text{eight}} = 400_{\text{six}} = 1001{,}0000_{\text{two}} \\
12.\mathrm{C}_{\text{sixteen}} &= 16.9_{\text{twelve}} = 18.75_{\text{ten}} = 22.6_{\text{eight}} = 30.43_{\text{six}} = 1{,}0010.11_{\text{two}}
\end{aligned}
$$

School textbooks teaching alternate bases will often use this style. (Indeed, even as Churchman was promoting the semicolon, Shaun Ferguson of the DSGB ably demonstrated this spelled-out technique in correspondence to the *Bulletin*.[10,11])

An obvious disadvantage of using spelled-out base names, is that they make rather unwieldy subscripts. They are fine enough for isolated demonstrations of fundamental principles in a textbook setting. As tools for everyday handling of numbers, where switching between competing bases may become a frequent occurrence, such long words become tedious to write, as well as read.

Interestingly, in a letter to the *Bulletin*, published in its very second issue,[12] William S. Crosby, then a U.S. Army private in World War II, suggested the following:

$$
\begin{aligned}
100_{\text{unc}} &= 144_{\text{dec}} \\
16.9_{\text{unc}} &= 18.75_{\text{dec}}
\end{aligned}
$$

where "dec" is short for "decimal," and "unc" is short for "uncial" (Crosby's preferred term for base twelve). Here we have the germ of an idea: To annotate a based number, use an *abbreviation* for the name of its base. How far might we abbreviate these annotations? We will revisit this question shortly.

[Ɛ] Sir Isaac Pitman, "A New and Improved System of Numeration", *The Phonetics Journal*, London, 9 Feb. $10\text{↊}8_{z}$ (1856_{d}), http://www.dozenal.org/drupal/sites/default/files/DSA_pitman_collected.pdf.

[10] Shaun Ferguson, "Number Base Oddments," *Duodecimal Bulletin*, Vol. 1Ɛ_{z} (23_{d}), No. 2, WN 37_{z} (43_{d}), Dec 1180_{z} (1968_{d}).

[11] Shaun Ferguson, letter, *Bulletin*, Vol. 20_{z} (24_{d}), No. 0, WN 38_{z} (44_{d}), Apr 1181_{z} (1969_{d}).

[12] William S. Crosby, "Uncial Jottings of a Harried Infantryman," *Duodecimal Bulletin*, Vol. 1, No. 2, WN 1, Jun 1161_{z} (1945_{d}). Entire letter reprinted in full on page 29_{z}.

Approaches from Programming Languages

Even as the Humphrey point was rising to prominence within the dozenalist societies, the rise of computing machines led to a different sort of prominence for the semicolon: In numerous programming languages, the semicolon became the marker for the end of an "executable statement" of code. This makes it perhaps *the* premiere character of punctuation in most software.

If we deemed the Humphrey point to be an indispensible feature of dozenal numbers, we would run the risk of branding them incompatible with the design of most programming languages. Yet this is demonstrably unnecessary. While the dozenalist societies have been focused for generations on the rather narrow problem of how to distinguish numbers of just two bases, decimal versus dozenal, programming languages tend to support several bases besides decimal. Usually there is at least support for octal and hexadecimal, and often binary as well, and in some cases, many other bases, including dozenal.

For example the Ada programming language has built-in support for all bases between binary and hexadecimal:

```
16#90#   = 12#100#  = 10#144#   = 8#220#  = 6#400#   = 2#1001_0000#
16#12.C# = 12#16.9# = 10#18.75# = 8#22.6# = 6#30.43# = 2#1_0010.11#
```

In this syntax, a based number (integer or real) is flanked by number-sign characters and prefixed with the base. The base itself must be expressed as a decimal number between 2 and 16, so Ada's syntax exhibits the same decimal bias as the mainstream subscript solution. It also is rather verbose and heavy-weight.

Other programming languages favor a more terse, streamlined style of annotation. For instance, languages such as C, C++, and Java, allow the following:

```
0x90 = 144 = 0220 = 0b10010000
```

In other words, a numeric literal always starts with a digit, but if the initial digit is 0, it is a signal that the base is non-decimal. If the zero is followed only by digits, then the literal is interpreted as octal base. If, however, the initial zero is followed by an "x", then the literal is hexadecimal. If it is followed by a "b", then the literal is binary.

Thus, these C-style languages have managed to reduce base annotations down to one or two alphanumeric characters, without resorting to any radical redefinition of punctuation. The downside is they provide only a limited repertoire of alternate bases, and once again, they single out decimal for special status as the unmarked base.

Gene Zirkel's "Unambiguous Notation"

The better part of three unquennia ago, our very own Gene Zirkel (Member 67_{z} (79_{d}), a past *Bulletin* editor and president of the DSA, today a member of its board) observed the base annotation syntaxes demonstrated in these and other programming languages. He was inspired to write an article for the *Bulletin* titled "Unambiguous Notation for Number Bases."[13] In it, he raised the issue of the ambiguity of the mainstream subscript notation. He proposed an alternative: assign each base a unique single-letter abbreviation, and use that as an annotation. In Zirkel's formulation, the annotation would be a prefix, with the value set off by bracketing apostrophes:

[13] Gene Zirkel, "Unambiguous Notation for Number Bases," *Duodecimal Bulletin*, Vol. 28_{z} (32_{d}), No. 3, WN 48_{z} (56_{d}), Fall 1193_{z} (1983_{d}).


```
x'90'   = z'100'  = d'144'   = o'220'  = h'400'   = b'1001,0000'
x'12.C' = z'16.9' = d'18.75' = o'22.6' = h'30.43' = b'1,0010.11'
```

Such a scheme is comprehensive, because it can accommodate a good number of bases. It is equitable, because all bases are treated the same, with none singled out for special consideration. It is relatively lightweight, because the annotation makes use of characters readily available on the keyboard, and does not require any additional fancy typesetting—although on the downside, couching every number in apostrophes does add a bit of weight. It is also a very modular solution, because the annotations only focus on specifying the base; within the bracketing apostrophes, the existing syntax for numbers can reside, unaffected by the annotation. Finally, this notation is unambiguous, because each annotation is a single letter uniquely associated with a particular base, without itself requiring any interpretation as a numeral in some base.

(The choice of base abbreviations shown above will be explained in a moment. They are slightly different than those which Zirkel selected in his original article. Nevertheless, they demonstrate the principles that Zirkel was promoting.)

A New/Old Solution

Let's revisit the mainstream subscript annotation solution. But instead of using decimal numerals in the subscripts, suppose we substitute single-letter abbreviations similar to those from Zirkel's notation:

$$\begin{aligned} 90_{\mathrm{x}} &= 100_{\mathrm{z}} = 144_{\mathrm{d}} = 220_{\mathrm{o}} = 400_{\mathrm{h}} = 1001{,}0000_{\mathrm{b}} \\ 12.\mathrm{C}_{\mathrm{x}} &= 16.9_{\mathrm{z}} = 18.75_{\mathrm{d}} = 22.6_{\mathrm{o}} = 30.43_{\mathrm{h}} = 1{,}0010.11_{\mathrm{b}} \end{aligned}$$

This seems to make for an ideal solution. It shares with Zirkel's notation the traits of being comprehensive, neutral, equitable, and unambiguous. It is light-weight and modular: Subscripts such as these are fairly unobtrusive, interfering little with any other aspect of numeric syntax, nor with any surrounding punctuation. We can demonstrate this with our previous example sentences:

> A gross, in decimal, is 144_{d}; whereas in dozenal, it's 100_{z}.
> A gross, in dozenal, is 100_{z}; whereas in decimal, it's 144_{d}.

The subscript suffix position also avoids clashing with important unary functions, such as negation (additive inverse, or the "minus" sign), which by convention are prefixes:

$$100_{\mathrm{z}} - 100_{\mathrm{x}} = -112_{\mathrm{d}} = -94_{\mathrm{z}} = -70_{\mathrm{x}}$$

Subscripts do require a bit of formatting effort. However, modern word processors, typesetting software such as LaTeX, as well as software supporting on-line blogs, wikis, and forums, all readily provide the capability to do subscripts and superscripts.[14]

[14] This option is even available to people posting on the DozensOnline Forum. This author has been using this convention there for months.

Annotations		Base Names			
Nominal	Digital	SDN	Classical	English	Dozenal English
b	2	binal	binary	two	two
t	3	trinal	ternary	three	three
q	4	quadral	quaternary	four	four
p	5	pental	quinary	five	five
h	6	hexal	senary	six	six
s	7	septal	septenary	seven	seven
o	8	octal	octal	eight	eight
e	9	enneal	nonary	nine	nine
d	A	decial	decimal	ten	ten
ℓ	B	levial	undecimal	eleven	eleven
z	C	unqual	duodecimal	twelve	one dozen, dozenal
	D	ununial	tridecimal	thirteen	one dozen one
	E	unbinal	tetradecimal	fourteen	one dozen two
	F	untrinal	pentadecimal	fifteen	one dozen three
x	G	unquadral	hexadecimal	sixteen	one dozen four
	H	unpental	heptadecimal	seventeen	one dozen five
	I	unhexal	octadecimal	eighteen	one dozen six
	J	unseptal	nonadecimal	nineteen	one dozen seven
v	K	unoctal	vigesimal	twenty	one dozen eight
	L	unenneal	unvigesimal	twenty-one	one dozen nine
	M	undecial	duovigesimal	twenty-two	one dozen ten
	N	unlevial	trivigesimal	twenty-three	one dozen eleven
	O	binilial	tetravigesimal	twenty-four	two dozen
	P	biunial	pentavigesimal	twenty-five	two dozen one
	Q	bibinal	hexavigesimal	twenty-six	two dozen two
	R	bitrinal	septavigesimal	twenty-seven	two dozen three
	S	biquadral	octavigesimal	twenty-eight	two dozen four
	T	bipental	nonavigesimal	twenty-nine	two dozen five
	U	bihexal	trigesimal	thirty	two dozen six
	V	biseptal	untrigesimal	thirty-one	two dozen seven
	W	bioctal	duotrigesimal	thirty-two	two dozen eight
	X	bienneal	tritrigesimal	thirty-three	two dozen nine
	Y	bidecial	tetratrigesimal	thirty-four	two dozen ten
	Z	bilevial	pentatrigesimal	thirty-five	two dozen eleven
	Ω	trinilial	hexatrigesimal	thirty-six	three dozen

Table 1: “Nominal” and “Digital” Base Annotations

The best aspect of this scheme, however, may be its *familiarity*. It is a relatively minor twist on a notation that mainstream mathematicians, along with many reasonably educated people, are already quite familiar with. People not necessarily invested in dozenalism might find it easier to accept and adopt this syntax.

“Nominal” and “Digital” Annotations

All that is needed is to settle on a suitable convention for single-letter abbreviations for the bases. The first column in Table 1 specifies one possible standard, supporting the previous examples. These are termed “nominal” base annotations, because these single-letter abbreviations derive from names used for the bases.

For bases under one dozen, the abbreviations from Systematic Dozenal Nomenclature[15] are apropos, since the SDN digit roots were expressly designed to start with unique letters that would be amenable to single-letter abbreviations. These include “d” for decimal. The “z” for dozenal can be rationalized based on the fact that “zen,” as a contraction for “dozen,” was historically favored both by F. Emerson Andrews and by Tom Pendlebury. It can also be seen as a reference to the astrological Zodiac, the dozen constellations along the ecliptic. The “x” for hexadecimal reflects the existing convention in programming languages. The “v” for vigesimal is straightforward.

[15] John Volan, “Systematic Dozenal Nomenclature,” *Duodecimal Bulletin*, Vol. 51_z (61_d), No. 1, WN $↊1_z$ (121_d), $11↋9_z$ (2013_d). See also SDN Summary in this issue on page 31_z.

The second column specifies another possible standard, supporting the following:

$$\begin{aligned} 90_G &= 100_C = 144_A = 220_8 = 400_6 = 1001{,}0000_2 \\ 12.C_G &= 16.9_C = 18.75_A = 22.6_8 = 30.43_6 = 1{,}0010.11_2 \end{aligned}$$

These are termed “digital” base annotations, because they systematically exploit the character assignments for transdecimal digits typically used in modern programming languages for (digital) computers. For any given base, the numbers and/or letters up to but not including the base letter can act as the digits of that base. The base letter is always one greater than its largest digit. For bases two through nine, the actual digit characters suffice as base annotations, since they are not ambiguous in isolation. Bases ten through two dozen eleven are represented by the letters A through Z of the Latin 1 (English) alphabet. The Greek letter omega is included to represent base three dozen, rounding out the set. That base must utilize all ten decimal numerals and all two dozen two Latin 1 letters, in order to represent its digits.

As specified, the “nominal” annotations all use lowercase letters, while the “digital” annotations all use uppercase. This contrast allows both types of annotation to coexist without conflict. Users may employ whichever standard best suits their needs. The lowercase nominal forms are a bit more pleasant on the eye, and more suggestive of the names of the bases, so they might be good for frequent everyday usage. Whereas the digital annotations, being more exhaustively comprehensive, might be better suited to technical analyses about multiple number bases.

To Subscript or Not to Subscript

Subscripting might be problematic in certain disadvantaged environments, such as when writing by hand, or in email or other impoverished forms of text communication. In that case, a suitable inline syntax, utilizing the same annotation abbreviations, might be able to substitute for subscript notation.

One possibility would look at how mainstream mathematicians have inlined subscripts in other contexts. For instance, when a variable represents an array or set of quantities, or a vector quantity, mathematicians often use a subscript as an index referring to a specific element of the array, set, or vector. When subscripting is not available, the substitute is often to suffix the variable with the index in brackets:

$$a_0 = \texttt{a[0]}, \quad a_1 = \texttt{a[1]}, \quad a_2 = \texttt{a[2]}, \quad \text{etc...}$$

This syntax might also work as an inline substitute for base annotation subscripts:

```
  90[x] =  100[z] =   144[d] =  220[o] =   400[h] = 1001,0000[b]
12.C[x] = 16.9[z] = 18.75[d] = 22.6[o] = 30.43[h] =   1,0010.11[b]
```

One way to rationalize this is to view these inlined annotations as parenthetical remarks about the preceding values. Indeed, we could read these numerals off as follows: “nine-zero (hexadecimal) equals one-zero-zero (dozenal) equals one-four-four (decimal) ...” In fact, such a reading might be just as applicable to the fully typeset subscript annotations. The suffix-subscript position is pretty much the “oh-by-the-way” position in mathematical notation.

The First Few Squares											
N			N^2			N			N^2		
[d]	[z]	[x]	[d]	[z]	[x]	[d]	[z]	[x]	[d]	[z]	[x]
1	1	1	1	1	1	13	11	D	169	121	A9
2	2	2	4	4	4	14	12	E	196	144	C4
3	3	3	9	9	9	15	13	F	225	169	E1
4	4	4	16	14	10	16	14	10	256	194	100
5	5	5	25	21	19	17	15	11	289	201	121
6	6	6	36	30	24	18	16	12	324	230	144
7	7	7	49	41	31	19	17	13	361	261	169
8	8	8	64	54	40	20	18	14	400	294	190
9	9	9	81	69	51	21	19	15	441	309	1B9
10	↊	A	100	84	64	22	1↊	16	484	344	1E4
11	↋	B	121	↊1	79	23	1↋	17	529	381	211
12	10	C	144	100	90	24	20	18	576	400	240

Table 2: Example table with blanket column-wise base annotations

Inline bracketed suffixes manage to remain about as unobtrusive as suffixed subscripts. For instance, they avoid interfering with unary operators in prefix position:

$$\mathtt{100}[\mathrm{z}] - \mathtt{100}[\mathrm{x}] = -\mathtt{112}[\mathrm{d}] = -\mathtt{94}[\mathrm{z}] = -\mathtt{70}[\mathrm{x}]$$

Moreover, bracketing the base abbreviations in this way might also make for convenient stand-alone tags useful as blanket annotations for whole regions of text. For instance, we could use them in table headers to annotate the bases for entire rows or columns of a table. This would allow us to avoid having to annotate each cell individually, making the table less cluttered overall, yet not shirking the obligation to explicitly specify the base in use at every point. Table 2 provides an example demonstrating this.

International Neutrality

Thus far, we have been presuming the Anglo-American convention for punctuating numbers, in which the period is used as the fraction point, and the comma is used as a grouping separator in long numbers:

$$\begin{aligned}\left(2^{36}+2^{-12}\right)_{\mathrm{d}} &= 68{,}719{,}476{,}736.000\ 244\ 140\ 625_{\mathrm{d}}\\ \left(2^{30}+2^{-10}\right)_{\mathrm{z}} &= 11{,}39\text{↊}{,}01\text{↋}{,}854.000\ 509_{\mathrm{z}}\end{aligned}$$

On the continent of Europe, and elsewhere, the convention is the exact opposite:

$$\begin{aligned}\left(2^{36}+2^{-12}\right)_{\mathrm{d}} &= 68.719.476.736{,}000\ 244\ 140\ 625_{\mathrm{d}}\\ \left(2^{30}+2^{-10}\right)_{\mathrm{z}} &= 11.39\text{↊}.01\text{↋}.854{,}000\ 509_{\mathrm{z}}\end{aligned}$$

Since the subscripted annotations proposed here provide a modular solution, they are completely independent of these considerations. So Continentals could readily adopt the same base annotations, while retaining their preferred punctuation.

A solution such as this, compatible with the local standards of other nations regarding number format, is much more likely to gain international acceptance than one that usurps their preferences. Even though the Humphrey point disrupts American/British standards as much as it does Continental standards, nevertheless there can be a *perception* that it constitutes a veiled attempt to impose Anglophile cultural hegemony. Base annotation should simply be a question of what is most practical. We should prefer a solution that avoids seeming political.

"Why Change?"[16]

Dozenalists are people who wish to bring the use of base twelve into the mainstream, because it is demonstrably a better base than decimal. As such, it would behoove us to do as much as possible to demonstrate how *normal* base twelve can be, how *little* people really need to change in order to make use of it.

It is therefore a great irony to see the earliest proponents of dozenalism in this country actually accepting—indeed, vigorously embracing—practices better geared to emphasize decimal as the "normal" or "default" base, and dozenal as a base set apart as "marked" and "different" and "peculiar"—and by implication, "second-rate".

In this author's opinion, the Humphrey point was a chief culprit. Yet today, it has become something of a cherished tradition within the dozenal societies, with roots spanning more than a human lifespan. Perhaps the foregoing has persuaded the reader to reconsider whether this was really a good thing. The Humphrey point should not persist merely for the sake of nostalgia.

The alternative set forth in these pages also has roots that go back at least as far, if not further. Its elements have been present since the founding of the DSA, and aspects of it have been touched at by contributors to this publication, at various times throughout its history.

The DSA has made major changes in the past, notably the adoption of the "Bell" characters as transdecimal digits, and later the abandoning of these to return to the Dwiggins characters. So it is not impossible to decide to change something seemingly fundamental, upon better judgment.

Going into a new biquennium, we should opt for a solution for base annotation that is more neutral, equitable, modular, and versatile, than the Humphrey point. We need a technique that marks all bases equally, without clashing with mainstream standards of mathematical notation and prose style—indeed, one that derives from, and extends upon, mainstream practices. A convention assigning single-character alphanumeric abbreviations to bases, with handy, and generally-familiar, places to position these, can satisfy these goals.

[16] Title of an editorial essay by Ralph H. Beard, first editor of the *Bulletin*. First published in *Duodecimal Bulletin*, Vol. 4, No. 1, WN ↋_{z} (11_{d}), Dec 1164_{z} (1948_{d}). Remastered in $11\text{↋}7_{\mathrm{z}}$ (2011_{d}) by Michael T. De Vlieger as `http://www.dozenal.org/drupal/sites/default/files/db043r2_0.pdf`. Quote: "Then, shouldn't we change? No! No change should be made and we urge no change. All the world uses decimals. But people of understanding should learn to use duodecimals to facilitate their thinking, and to ease the valuative processes of their minds. Duodecimals should be man's second mathematical language. They should be taught in all the schools. In any operation, that base should be used which is most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity. But no change should be made. Perhaps by the year 2000, or maybe by *1200*, which is *14* years later, duodecimals may be the more popular base. But then no change need be made, because people will already be using the better base." (Original italic marking of dozenal numbers retained.)

Patterns and Palatable Morsels in Duodecimals

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Abstract: Base twelve furnishes one with a plethora of palatable patterns and morsels that are often absent in the awkward decimal base. This paper will serve to furnish illuminating results which render dozenals their appeal. Seven excursions are included. Among these are patterns emerging from recursive sequences where dozens play a prominent role. These ideas are certainly in the spirit of engaging mathematics which can succinctly be categorized as the science of patterns.

Rationale

A number of years ago, several members of the Dozenal Society of America (DSA) furnished lists of popular integer sequences in our favorite number base. For example, the first three dozen powers of two, the first three dozen rows in Pascal's triangle, as well as the initial three dozen factorials, graced the pages of our *Bulletin*. They served as morsels to whet one's appetite, as well as to view any possible patterns.

More recently, the excellent On-Line Encyclopedia of Integer Sequences (OEIS), managed by Dr. Neil A.J. Sloane, has transformed the manner in which number sequences are viewed. It serves as the basis for both stimulating mathematical research, as well as engaging recreations on the lighter side of mathematics.

Alas, the over quarter million sequences in the OEIS database are in decimal. As base twelve enthusiasts, I feel it is advantageous to view integer sequences in dozenal. The purpose of this article is to revisit the idea of dozenal integer sequences, an idea that was initiated in the pages of our *Bulletins* more than two dozen years ago.

Computer algebra system technology, such as Mathematica, lends itself well to partaking of such explorations. Mathematica has the capability to convert very large integers between decimal and any other number base, including dozenal.

Let us now embark on our dozenal journey.

Morsel #1: Twin Primes

We initiate our tour by considering one of the classical problems in number theory: namely, twin primes. Twin primes are odd primes that differ by two, for instance 3 and 5, or ↋ and 11_{z}. The following constitutes the set of all twin prime pairs less than one great gross:

$$\left\{\begin{array}{rrrrrrrrrrrr}
3, & 5; & 5, & 7; & ↋, & 11; & 15, & 17; & 25, & 27; & 35, & 37; \\
4↋, & 51; & 5↋, & 61; & 85, & 87; & 8↋, & 91; & ↋5, & ↋7; & 105, & 107; \\
12↋, & 131; & 13↋, & 141; & 145, & 147; & 16↋, & 171; & 17↋, & 181; & 1↊5, & 1↊7; \\
1↋5, & 1↋7; & 21↋, & 221; & 24↋, & 251; & 2↊↋, & 2↋1; & 2↋↋, & 301; & 325, & 327; \\
375, & 377; & 3↋5, & 3↋7; & 41↋, & 421; & 435, & 437; & 455, & 457; & 46↋, & 471; \\
575, & 577; & 585, & 587; & 58↋, & 591; & 5↋5, & 5↋7; & 615, & 617; & 70↋, & 711; \\
71↋, & 721; & 745, & 747; & 76↋, & 771; & 795, & 797; & 7↋↋, & 801; & 865, & 867; \\
8↊5, & 8↊7; & 8↋5, & 8↋7; & 905, & 907; & 91↋, & 921; & 9↊↋, & 9↋1; & ↊↋5, & ↊37; \\
↊3↋, & ↊41; & ↋1↋, & ↋21; & ↋2↋, & ↋31; & ↋6↋, & ↋71; & ↋95, & ↋97; & ↋↋5, & ↋↋7
\end{array}\right\}_{\mathrm{z}}$$

One finds that there are 46_{z} pairs of twin primes less than one great gross.

With the exception of 3 and 5, the sum of any twin prime pair is a multiple of twelve. To prove this, we argue as follows: Let a, b, and c be three consecutive integers larger than 3, with a and c both prime. The sequence must therefore be odd-even-odd. Since b is even (and larger than 2), it is not prime.

Among any three consecutive integers, exactly one is divisible by three. But unless that number is 3 itself, it is is not a prime. Since b is the only number of our sequence that is not prime, it must be the one divisible by three. Since it is also divisible by two, it is therefore divisible by six. In other words:

$$b = 6n \qquad n \in \mathbb{N}$$

But note that b is the average of a and c:

$$a + c = 2b$$

Substituting for b from the first equation above yields:

$$a + c = 2(6n) = 10_{\mathrm{z}}n \qquad n \in \mathbb{N}$$

Hence the sum of two twin primes is a multiple of twelve.

For example, consider the twin prime pair $\{15, 17\}_{\mathrm{z}}$. Observe that they lie on either side of the integer 16_{z}. Since this ends with a 6 in dozenal, it is self-evidently a multiple of 6. The sum of the pair is 30_{z}, which is twice 16_{z}, and divisible by twelve.

On the other hand, consider the twin prime pair $\{4↋, 51\}_{\mathrm{z}}$. Observe that they flank the integer 50_{z}. Since this ends with a 0 in dozenal, it is self-evidently a multiple of 10_{z}, as well as 6. The sum of the pair is $↊0_{\mathrm{z}}$, which is twice 50_{z}, and divisible by twelve.

Examining the above data set, a related question arises: Notice that the numerals

$$\{37, 107, 131, 181, 251, 421, 457, 577, 587, 617, 797, 907\}$$

are primes in both decimal and dozenal. Hence one might have a numeral that is a prime in different number bases. On the other hand, is it possible to have the same numerals comprise a twin prime pair in both decimal and dozenal? Unfortunately, the answer is in the negative. Twin prime pairs in decimal must necessarily terminate in the digits 1 and 3, 7 and 9, or 9 and 1. In dozenal, an integer terminating in the digits 3 or 9 is divisible by three, and thus not a prime. On the other hand, in dozenal, twin prime pairs must necessarily terminate in the digits 5 and 7, or ↋ and 1. The second case is not applicable to decimal numerals, and the first case is not possible; a decimal integer ending in the digit 5 is divisible by five, and consequently not a prime.

Morsel #2: Pythagorean Triples

Our next stop in our dozenal journey takes us to the world of Pythagorean triples. A Pythagorean triple is a set of three integers that constitute the sides of a right triangle, such as $\{3, 4, 5\}$ and $\{5, 10, 11\}_z$. Moreover, a Pythagorean triple is classified as primitive if the three components comprising the triple have no factors in common other than one. It can be shown that one can generate primitive Pythagorean triples (PPT's) by virtue of the following criteria:

> $\{x, y, z\}$ is a PPT for $m,n \in \mathbb{N}$ such that m and n are of opposite parity, $(m, n) = 1$, and $x = m^2 - n^2$, $y = 2mn$, and $z = m^2 + n^2$.

The following is a short table of PPT's:

m [z]	n [z]	{	$x = m^2 - n^2$ [z]	,	$y = 2mn$ [z]	,	$z = m^2 + n^2$ [z]	}
2	1	{	3	,	4	,	5	}
3	2	{	5	,	10	,	11	}
4	1	{	13	,	8	,	15	}
4	3	{	7	,	20	,	21	}
5	2	{	19	,	18	,	25	}
5	4	{	9	,	34	,	35	}
6	1	{	2↋	,	10	,	31	}
6	5	{	↋	,	50	,	51	}
7	2	{	39	,	24	,	45	}
7	4	{	29	,	48	,	71	}
7	6	{	11	,	70	,	71	}
8	1	{	53	,	14	,	55	}
8	3	{	47	,	40	,	61	}
8	5	{	33	,	68	,	75	}
8	7	{	13	,	94	,	95	}
9	2	{	65	,	30	,	71	}
9	4	{	55	,	60	,	81	}
9	8	{	15	,	100	,	101	}
↊	1	{	83	,	18	,	85	}
↊	3	{	77	,	50	,	91	}
↊	7	{	43	,	↋8	,	105	}
↊	9	{	17	,	130	,	131	}
↋	2	{	99	,	38	,	↊5	}
↋	4	{	89	,	74	,	↋5	}
↋	6	{	71	,	↋0	,	111	}
↋	8	{	49	,	128	,	135	}
↋	↊	{	19	,	164	,	165	}
10	1	{	↋↋	,	20	,	101	}
10	5	{	9↋	,	↊0	,	121	}
10	7	{	7↋	,	120	,	141	}
10	↋	{	1↋	,	1↊0	,	1↊1	}

Examining this table can demonstrate the following redeeming features: Since m and n are of opposite parity, the even component in the triple is divisible by four. Notice that the product of the two legs of the right triangle is a multiple of one dozen, and the product of all three sides is a multiple of five dozen. Those triples where m and n differ by one have the length of the longer leg being one less than the length of the hypotenuse, by virtue of the equations generating Pythagorean triples. In some cases, both odd components in the triple are prime numbers, for instance in $\{3, 4, 5\}$, $\{5, 10, 11\}_z$, and $\{17, 130, 131\}_z$. For this to occur, m and n differing by one is a necessary, though certainly not sufficient, condition.

There is an interesting relationship between Fibonacci numbers and Pythagorean triples which we will explore shortly.

Morsel #3: Pisano Periods

Consider the Fibonacci sequence, which is recursively defined as follows:

$$F_1 = F_2 = 1$$
$$F_n = F_{n-1} + F_{n-2} \qquad n \geq 3.$$

The first dozen Fibonacci numbers are

$$\{1, 1, 2, 3, 5, 8, 11, 19, 2↊, 47, 75, 100\}_z .$$

In the awkward base ten, the period of the units digit is five dozen before it repeats. One can employ arithmetic modulo ten to see this. On the other hand, when appealing to arithmetic modulo twelve, the period of the units digit is a much tidier two dozen!

Similarly, the period of the last two digits modulo one hundred is three hundred. On the other hand, when using arithmetic modulo one gross, it is only two dozen!

In the same manner, the period of the last three digits, in arithmetic modulo one thousand, is one thousand five hundred. Whereas in arithmetic modulo twelve, the period modulo one great gross is a far more palatable two gross! The repeating block in the Fibonacci sequence is really cheaper by the dozen!

It is easy to secure the period of the units digit modulo twelve, which is of length two dozen. The sequence of remainders is

$$\{1, 1, 2, 3, 5, 8, 1, 9, ↊, 7, 5, 0, 5, 5, ↊, 3, 1, 4, 5, 9, 2, ↋, 1, 0, ...\}_z$$

which repeats from that point forward. Similarly the period of the last two digits modulo one gross is likewise two dozen, as their remainder are

$$\{1, 1, 2, 3, 5, 8, 11, 19, 2↊, 47, 75, 0, 75, 75, 2↊, ↊3, 11, ↋4, 5, ↋9, 2, ↋↋, 1, 0, ...\}_z$$

which thereafter repeat.

Morsel #4: Lucas Sequences

A companion recursive sequence is the Fibonacci-like sequence known as the Lucas sequence. The initial two terms are 1 and 3 respectively and each term thereafter is the sum of its two immediate predecessors. The first twelve terms of the Lucas sequence are

$$\{1, 3, 4, 7, ↋, 16, 25, 3↋, 64, ↊3, 147, 22↊\}_z$$

Appealing to arithmetic modulo ten, the sequence of remainders for the units digit is

$$\{1, 3, 4, 7, 1, 8, 9, 7, 6, 3, 9, 2, ...\}_d$$

and the repeats are seen from that point on. The length is one dozen. The period of the units digit in arithmetic modulo twelve is

$$\{1, 3, 4, 7, ↋, 6, 5, ↋, 4, 3, 7, ↊, 5, 3, 8, ↋, 7, 6, 1, 7, 8, 3, ↋, 2, ...\}_z$$

and then repeats, a period of length two dozen. Similarly the period of the last two digits modulo one gross is likewise two dozen, as the remainders are

$$\{1, 3, 4, 7, ↋, 16, 25, 3↋, 64, ↊3, 47, 2↊, 75, ↊3, 58, 3↋, 97, 16, ↋1, 07, ↋8, 03, ↋↋, 02, ...\}_z$$

and then repeats. In base ten, the period of the last two digits modulo one hundred is sixty, as can be verified by the interested reader.

In decimal, the period of the last three digits modulo one thousand is three hundred. In dozenal, the period of the last three digits modulo one great gross is two gross.

Morsel #5: Fibonacci-Pythagorean Connection

One method of generating Pythagorean triples is to consider any four consecutive terms in the Fibonacci sequence (or any Fibonacci-like sequence):

$$..., F_n, F_{n+1}, F_{n+2}, F_{n+3}, ...$$

First we form the product of the first and fourth terms.

$$x = F_n \cdot F_{n+3}$$

Next take twice the product of the second and third terms.

$$y = 2 \cdot F_{n+1} \cdot F_{n+2}$$

Finally take the sum of the squares of the second and third terms.

$$z = F_{n+1}^2 + F_{n+2}^2$$

Presto: A Pythagorean triplet $\{x, y, z\}$ is formed, which will be primitive if the first (and hence the fourth) term is odd.

To cite an example, consider the sequence of consecutive Fibonacci numbers $\{3,5,8,11\}_z$:

$$\begin{aligned} x &= 3 \cdot 11_z = 33_z \\ y &= 2 \cdot 5 \cdot 8 = 68_z \\ z &= 5^2 + 8^2 = 21_z + 54_z = 75_z. \end{aligned}$$

The Primitive Pythagorean triplet $\{33, 68, 75\}_z$ is formed, and corresponds in our table to $m = 8$ and $n = 5$.

Morsel #6: Square Numbers

We next examine patterns in square numbers. In the decimal base, the repeating block for the units digit is of length ten, and forms the palindrome

$$\{1, 4, 9, 6, 5, 6, 9, 4, 1, 0, ...\}_d$$

which is of length ten before repeating. For base twelve, the repeating block is

$$\{1, 4, 9, 4, 1, 0, ...\}_z$$

and is of length 6 before repeating. Again observe the palindrome that is formed with regards to the repeating block. In decimal, if one examines the period of the last two digits, think of arithmetic modulo one hundred and observe the emerging pattern:

$$\left\{ \begin{array}{l} 01, 04, 09, 16, 25, 36, 49, 64, 81, 00, \\ 21, 44, 69, 96, 25, 56, 89, 24, 61, 00, \\ 41, 84, 29, 76, 25, 76, 29, 84, 41, 00, \\ 61, 24, 89, 56, 25, 96, 69, 44, 21, 00, \\ 81, 64, 49, 36, 25, 16, 09, 04, 01, 00, ... \end{array} \right\}_d$$

before repeating. The period is of length fifty.

Based on the above analysis, there are exactly twenty-two combinations for the last two digits for a decimal integer to possibly be a perfect square. The period of the last three digits using arithmetic modulo one thousand is of length five hundred.

In our favorite number base, the period of the last two digits is obtained by examining the arithmetic modulo one gross.

$$\left\{ \begin{array}{l} 01, 04, 09, 14, 21, 30, 41, 54, 69, 84, Ɩ1, 00, \\ 21, 44, 69, 94, 01, 30, 61, 94, 09, 44, 81, 00, \\ 41, 84, 09, 54, Ɩ1, 30, 81, 14, 69, 04, 61, 00, \\ 61, 04, 69, 14, 81, 30, Ɩ1, 54, 09, 84, 41, 00, \\ 81, 44, 09, 94, 61, 30, 01, 94, 69, 44, 21, 00, \\ Ɩ1, 84, 69, 54, 41, 30, 21, 14, 09, 04, 01, 00, ... \end{array} \right\}_z$$

The length of the repeating block is six dozen. Similarly, the period of the last three digits using arithmetic modulo one great gross is of length six gross.

Morsel #7: Triangular Numbers

In our final activity, we examine triangular numbers, which are numbers of the form

$$\frac{n\,(n+1)}{2} \qquad n \in \mathbb{N}$$

In dozenal, the triangular numbers are

$$\{1, 3, 6, Ɩ, 13, 19, 24, 30, 39, 47, 56, 66, ...\}_z$$

Observe that the numeral 66 represents a triangular number in both decimal and dozenal. Can one find any others, apart from 1, 3, and 6, of course? In base ten, the repeating block for the units digit is

$$\{1, 3, 6, 0, 5, 1, 8, 6, 5, 5, 6, 8, 1, 5, 0, 6, 3, 1, 0, 0, ...\}_d$$

The length is one score (twenty). In base twelve, we have the repeating block

$$\{1, 3, 6, Ɩ, 3, 9, 4, 0, 9, 7, 6, 6, 7, 9, 0, 4, 9, 3, Ɩ, 6, 3, 1, ...\}_z$$

which has length two dozen.

One can check that the period of the last two digits is two hundred for base ten and two gross for base twelve, by considering arithmetic modulo one hundred and modulo one gross, respectively.

Finally one can check that the period of the last three digits is two thousand in base ten and two great gross in base twelve, by considering arithmetic modulo one thousand and modulo one great gross, respectively.

Conclusion

This article served to whet one's appetite with several useful integer sequences in base twelve, and secured a number of curiously interesting patterns, especially with the recursive sequences and their period lengths in dozenal.

It is my sincere hope that over the course of the next year, members and contributors augment these lists. The seemingly modest goal is for our data base to consist of at least one gross of dynamic dozenal integer sequences, by 1200_z (2016_d). At that time, the DSA commemorates six dozen (half a gross) years as a recreational mathematics society, whose goal since its inception in 1160_z (1944_d) has always been to educate the general public on the advantages of the base twelve numeration system, in mathematics, weights and measures, as well as in pure and applied science.

FROM THE ARCHIVES:

Duodecimal Bulletin, Vol. 1, No. 2, WN 1, June 1161_z (1945_d).

"UNCIAL JOTTINGS OF A HARRIED INFANTRYMAN" by PVT. WILLIAM S. CROSBY[1]

On Propaganda: I favor great restraint. Advocates of the Metric System (and opponents of it), of the World Calendar, of various schemes of Nu Spelling and the like, in their printed outbursts seem to me to overstate their case, to sink their important arguments in a sea of minor points, to seek favor with too many separate interests at once, and consequently to sacrifice their dignity. Such passion as they display may better, I think, be saved for issues of larger importance.

The merits of counting by dozens don't need much arguing; the facts are pretty eloquent, given opportunity and time to do their work. The person to whom I've had the least trouble in explaining the system was a lad with a grade school education with whom I worked in Alaska, cutting and forming concrete-steel reinforcing. Having wrestled with feet and inches, and with dividing lengths into halves and thirds for so long, with him, the idea clicked with no exhortation or argument on my part.

Especially, argument for the twelve-system should not be even slightly chauvinistic. As I recall them, Grover Cleveland Perry's pamphlets laid objectionable stress on the Anglo-Saxon-ness of twelve.[2]

On Nomenclature, Notation, and Numeration: Maybe I am a factionalist, but here are some of the prejudices I stick by:

Duodecimal, (two more than ten) is a derived concept as well as a clumsy word. What is needed is a word expressing "counting by the scale of twelve", but as far as possible not depending on any other concept. "Uncial" is a suitable word to replace "decimal" in naming point-form fractions, and I myself use the word for the whole field of counting by dozens. Its chief

[1] *EDITOR'S NOTE:* I very recently discovered this remarkable letter (here, reproduced in full), hiding in plain sight in the "Mail Bag" column of an old issue of the *Bulletin*, in fact its very second issue. This astute young soldier, possibly going through basic training before joining an engineering unit, right at the very tail end of World War II, managed to touch on a number of interesting—even prescient—ideas. I find him echoing many of my own opinions, which is both gratifying and frustrating. One wishes to have heard more from such a promising source, in later years.

[2] Here here! We should strive to make dozenal as international as possible, positioning it as something useful across all cultures. But as Crosby urges, we shouldn't overstate the case. It's something that can make everyday arithmetic incrementally easier, and lead to more natural and coherent systems of measure—it won't cure cancer or bring world peace.☺ —*Ed.*

drawback is that only a specialized meaning (in the field of paleography) is given in most dictionaries. "Dozenal" I consider beneath contempt.[3]

𝒳 and Ɛ. I heartily approve of the flat-bottomed Ɛ. It is distinctive, elegant, and "looks like a numeral". In 𝒳, we are not, I think, so fortunate; the most that can be said for it is that it is not likely to be confused with any other numeral and that it has an internationally acceptable origin. But it sticks out on a page of print like a black "WHEREAS"; and even Bill Dwiggin's artistry on the Society's seal has not much tamed its outlandishness; moreover it is liable to confusion with other X symbols commonly used in mathematics. For these reasons I have been using the Irish ↊ for the past four years. I can recall only one instance of my confusing it with any other symbol, and I now transcribe numbers from the Terry Tables, substituting ↊ for 𝒳 without conscious thought. Like Ɛ, ↊ is a handsome character, looks plausibly "like a numeral", and can be improvised, thought not handsomely, on the typewriter by having a repairman mount an inverted 2 on one of the type bars.

Numeration. I consider the names dek, el, do, gro, bizarre and unnecessary, and instead read uncial numbers with the names at present corresponding to the digit-groups of identical appearance, except for 10, 11, 12, which I render as "twelve, oneteen, twenteen"; as for numbers involving ↊ and Ɛ, I simply make appropriate use of "ten" (or "tendy-") and "eleven" or ("eleventy"), as 1↊, ↊4, Ɛ4 - "tenteen, tendy-three, and eleventy-four", and so forth.

Italics. We unnecessarily cripple our typographical resources, in my opinion, if we continue to rely on italics to differentiate uncial from decimal numbers. Ambiguity can almost always be prevented by the context, and where this is not possible, by phrasing spelt-out numbers with the use of the words "dozen", "gross", etc., and by writing algorismic numbers with the subscripts, "dec" or "unc". Incidentally, I have found it convenient to use the symbol R as an operator indicating "transradication" from one base to another; as R $128_{\text{dec}} =$ ↊8_{unc}.

On a System of Weights and Measures.[4] The convenience of the common man should be the main consideration; that of specialists like chemists, navigators, astronomers, and physicists, should be subordinate. For this reason the system will be earth-bound, and will not give special prominence to such "unusual constants" of physics as the speed of radiation, or Planck's Constant, nor even to such rather nearer quantities as the dimensions of the earth.

It seems to me desirable to define the units of the system arithmetically in terms of existing international (Metric) standards, especially for the time being, so that the arithmetic of conversion from one system to the other can be unambiguous – in particular, the ratios of the "workshop" units of length should be simple, to facilitate conversion of machine-tools from one to the other. (e.g., The U.S. legal ratio of $100{,}000{:}3{,}937_{\text{dec}}$ for the inch to the millimeter has been rejected by industry both in this country and Great Britain in favor of the new inch whose ratio to the millimeter is $127{:}5_{\text{dec}}$, or $1{:}25.4_{\text{dec}}$.)

A system based on these considerations that I have been doing some playing with, works out as follows:

Angle. Uncial subdivisions of the circle, as universally advocated hitherto. However, why not denote the whole circle, one cycle, abbreviated c, and use the millicycle (0.001_{unc} c, or 1 mc) and the microcycle ($0.000\,001_{\text{unc}}$ c, or 1 Mc) as derived units when convenient?

Time. Uncial subdivisions of the day, as advocated hitherto. The unit ${10^{-5}}_{\text{unc}}$ day (approximately 1/3 sec.)[5] is a convenient one to use in defining the basic units of the more complicated kinds of physical quantities, such as acceleration, force, action, power, and energy. Especially:

Acceleration, whose unit it is desirable to set at something approximating the average acceleration of the earth's gravity, for otherwise any measure system will split into two – a physical system and a gravitational system – as both the English and the Metric systems have done. Making the "gee" the basic unit of acceleration will enable units of mass to be spoken of and used also as units of force, with scarcely a lifted eyebrow from the physicist. An acceleration of 118.2_{dec} cm$({10^{-5}}_{\text{unc}}$ day$)^{-2}$ is the unit required, and accordingly the factor 118.2_{dec} cm, or some convenient approximation to it, should become the fundamental unit of

Length. Now considering the needs of the workshop - the simple ratio

[3] I've also argued that "duodecimal", "dozen", and even "twelve" carry decimal etymological baggage, so we need more purely dozenal terms. I wonder what Crosby would have thought of SDN, which actually incorporates the Latin *uncia* as a metric-style prefix? (See SDN Summary on page 31_z.) —*Ed.*

[4] Here Crosby has captured the idea of basing a metrology on "fundamental realities" of human existence, rather than on global or universal quantities abstracted away from everyday life. He's articulated this principle at least one—perhaps two—unquennia before Tom Pendlebury embarked on developing his own TGM metrology starting from the very same principle. Moreover, Crosby's about to identify the most important of these fundamental realities as (1) the mean solar day, (2) the acceleration due to Earth's gravity, and (3) the density of water—the same as Pendlebury would do years later. Apparently Pendlebury was aware of this letter. Perhaps it inspired him? —*Ed.*

[5] This is the *pentciaday* (p↓Dy), equivalent to 0.42_z ($0.347\overline{2}_d$) SI seconds. Colloquially, I like to call this the *twinkling*, because it's about the time it takes to blink an eye. As a base unit of time for a metrology for everyday human use, it's quite serviceable. Time units shorter than this start to get too painfully fleeting for human comfort.

The mean solar day is arguably the one unavoidable time unit with the greatest impact on "every*day* life". So dividing the day by pure dozenal powers seems to be a natural assumption for many newcomers to dozenalism. (It happens to be my own preference.) It was certainly the prevailing practice in the early DSA.

Years later, Pendlebury defied this instinct, opting instead to start with a *binary* division of the day into two *semidays*, before dividing those dozenally. He selected the *pentcia* of the semiday, dubbed the *Tim*, to be the base time unit for his TGM metrology. This makes the traditional hour a *quadquaTim* (q↑Tm). Unfortunately, the day itself, so fundamental to human life, is not a dozenal power of the Tim.

Therefore it's nice to see that the *first* person to work out a dozenal metrology based on the day, gravity, and water actually divided the day dozenally. —*Ed.*

between the millimeter and the $10^{-3}{}_{\text{unc}}$ part of the "ell" of 118_{dec}-odd cm[6] that we desire to establish - it appears that to define a milliell as $13/19_{\text{dec}}$ mm (or 1 mm = $17/11_{\text{unc}}$ milliell) gives a very satisfactory result: an acceleration of 1 ell($10^{-5}{}_{\text{unc}}$ day)$^{-2}$ = 980.660_{dec} cm/sec^2, a very decent approximation to the present standard value of 980.665_{dec} cm/sec^2.

Mass. Here we can be guided fairly well by the metric example of giving the density of water a value of some power of the numerical base in units of the system. At the same time I have allowed myself some leeway here, since the density of water changes with temperature and purity, and have used this leeway to try to insure that the new unit of pressure will work out so that the present "Standard Atmosphere" can be fairly accurately expressed in some round number. The result of these efforts to date is a compromise: let the unit of mass be defined such that one metric gram = $0.001\,986_{\text{unc}}$ of it, precisely; then the density of water will be around 1000_{unc} unit mass/ell^3 [7] at ordinary temperature, and the present "Standard Atmosphere", now expressed as 760_{dec} mm of mercury, will be almost exactly 8900_{unc} of the new unit of pressure. The preservation of the "Standard Atmosphere" is of some importance because of the tremendous volume of data that has been recorded in terms of it - including even the foundation of the

Temperature Scale. It happens here that the degree Fahrenheit works out quite well as uncials. Measured from a fictitious zero (a fraction, negligible for most purposes, below the Absolute Zero) equivalent to -$460^{\text{o}}{}_{\text{dec}}$ F., the icepoint is $350^{\text{o}}{}_{\text{unc}}$ and the steampoint $480^{\text{o}}{}_{\text{unc}}$.

Electrical Units offer a big field for controversy, and I have come to no definite conclusions so far. Defining the permeability of space as 10 or 10 of unit permeability, and writing Ampere's law in Heaviside's "rational" form would define units of fairly convenient size.

Units of Musical Interval offer a very pleasant exercise for the uncial enthusiast; the octave has already been divided into twelve equal semitones. For calculations in theoretical harmony a table of logarithms to the base two in uncial notation is helpful.

—William S. Crosby[8]

[6] Indeed, Earth's gravity does come out to about 118_{d} cm, or 46.5_{d} ($3↊.6_{\text{z}}$) inches, per twinkling per twinkling. This length is fairly close to an obsolete English unit of measure known as the "ell", which was 45_{d} (39_{z}) inches. I noticed this myself some time ago, and have been calling this gravity-based length a kind of "ell" ever since. I was pleasantly surprised to find that Crosby had discovered this same association way back when!—*Ed.*

[7] The cubic ell of water, or *ellmass*, makes a rather unwieldy mass unit. But notice that the uncia-ell is actually a pretty close approximation of both the decimeter and the customary 4-inch "hand", so we could style it a gravity-based "hand." That means a cubic hand, or *handvolume*, is a pretty close fit to both the liter and the quart. And a handvolume full of water, or *handmass* (equivalent to a tricia-ellmass), is a pretty close fit to a kilogram. This is what Crosby has picked for his base mass unit. It's too bad he didn't highlight these close correspondences with SI units; SI users might find these units attractive. —*Ed.*

[8] All in all, an intriguingly perspicacious set of "jottings," from a lifetime ago. Would that there had been more! —*Ed.*

[z] • Systematic Dozenal Nomenclature Summary • [z]

N	Root	Abbr	Multiplier Prefix $N\times$	Reciprocal Prefix $\frac{1}{N}\times$	Power Prefixes: Positive $10^{+N}\times$	Power Prefixes: Negative $10^{-N}\times$
0	nil	n	nili	nilinfra	nilqua	nilcia
1	un	u	uni	uninfra	unqua	uncia
2	bi	b	bina	bininfra	biqua	bicia
3	tri	t	trina	trininfra	triqua	tricia
4	quad	q	quadra	quadinfra	quadqua	quadcia
5	pent	p	penta	pentinfra	pentqua	pentcia
6	hex	h	hexa	hexinfra	hexqua	hexcia
7	sept	s	septa	septinfra	septqua	septcia
8	oct	o	octa	octinfra	octqua	octcia
9	enn	e	ennea	enninfra	ennqua	enncia
↊	dec	d	deca	decinfra	decqua	deccia
↋	lev	ʟ	leva	levinfra	levqua	levcia
10	unnil	un	unnili	unnilinfra	unnilqua	unnilcia
11	unun	uu	ununi	ununinfra	ununqua	ununcia
12	unbi	ub	unbina	unbininfra	unbiqua	unbicia
13	untri	ut	untrina	untrininfra	untriqua	untricia
14	unquad	uq	unquadra	unquadinfra	unquadqua	unquadcia
15	unpent	up	unpenta	unpentinfra	unpentqua	unpentcia
16	unhex	uh	unhexa	unhexinfra	unhexqua	unhexcia
17	unsept	us	unsepta	unseptinfra	unseptqua	unseptcia
18	unoct	uo	unocta	unoctinfra	unoctqua	unoctcia
19	unenn	ue	unennea	unenninfra	unennqua	unenncia
1↊	undec	ud	undeca	undecinfra	undecqua	undeccia
1↋	unlev	uʟ	unleva	unlevinfra	unlevqua	unlevcia
20	binil	bn	binili	binilinfra	binilqua	binilcia

etc...

uncia was Latin for *one twelfth* • retains same meaning • *inch* and *ounce* are English derivatives
Concatenating roots = positional place-value • Suggested pronunciation: -cia = /ʃə/ ("-sha")
Concatenating prefixes = multiplication • mix & match freely • Commutative Law applies
Prefer Unicode abbreviations where supported • ASCII abbreviations for email, text, etc.

SDN Form	Example Value [z]	Example SDN	Abbreviation: Unicode	Abbreviation: ASCII
Root Form	46	quadhex	qh	qh
Multiplier Prefix	$46\times$	quadhexa	qh•	qh*
With Fractional Part	$4.6\times$	quaddothexa	q.h•	q.h*
Ordinal	46^{th}	quadhexal	qh′	qh'
Reciprocal Prefix	$\frac{1}{46}\times$	quadhexinfra	qh\	qh\
Positive Power Prefix	$10^{+46}\times$	quadhexqua	qh↑	qh@
Negative Power Prefix	$10^{-46}\times$	quadhexcia	qh↓	qh#
Rational Number	$4\times\frac{1}{5}\times$	quadrapentinfra	q•p\	q*p\
Rational Number	$\frac{1}{5}\times4\times$	pentinfraquadra	p\q•	p\q*
Scientific Notation	$4\times10^{+6}\times$	quadrahexqua	q•h↑	q*h@
With Fractional Part	$4.5\times10^{+6}\times$	quaddotpentahexqua	q.p•h↑	q.p*h@
Scientific Notation	$10^{+6}\times4\times$	hexquaquadra	h↑q•	h@q*
With Fractional Part	$10^{+6}\times4.5\times$	hexquaquaddotpenta	h↑q.p•	h@q.p*
Euphonic "n" Between Vowels	$4\times10^{+1}\times$	quadranunqua	q•u↑	q*u@
one dozen years	$10^{+1}\times$ year	unquayear, unquennium	u↑Yr	u@Yr
one gross years	$10^{+2}\times$ year	biquayear, biquennium	b↑Yr	b@Yr
one galore years	$10^{+3}\times$ year	triquayear, triquennium	t↑Yr	t@Yr
two hours (a "dwell")	$10^{-1}\times$ day	unciaday	u↓Dy	u#Dy
ten minutes (a "breather")	$10^{-2}\times$ day	biciaday	b↓Dy	b#Dy
fifty seconds (a "trice")	$10^{-3}\times$ day	triciaday	t↓Dy	t#Dy

For more info see:
Original article: http://www.dozenal.org/drupal/sites/default/files/DSA_kodegadulo_sdn.pdf
Wiki page: http://primel-metrology.wikia.com/wiki/Primel_Metrology_Wiki#Systematic_Dozenal_Nomenclature
Forum: http://z13.invisionfree.com/DozensOnline/index.php?showforum=29
Original thread: http://z13.invisionfree.com/DozensOnline/index.php?showtopic=463

DOZENS IN THE MEDIA

Along with all the goodies we have at www.dozenal.org*, check out this round-up of the latest dozenal delicacies gleaned from the greater infosphere.*

The Ancient Melodies

Jim Zamerski • Member 42Ɛ$_\text{z}$ (611$_\text{d}$)
Website: www.theancientmelodies.com

Jim Zamerski is a talented musician and composer with a mathematical bent, and a fan of base twelve. He's created some remarkable videos that set transcendental numbers, expressed in base twelve, to music. The dozenal digits, mapped to the twelve semitones of the chromatic scale, dictate the melody, but Zamerski picks the rhythm and phrasing, and the accompanying chord structure. You would think that transcendental numbers would just produce random noise, but Zamerski's results are amazingly lyrical, even orchestral, and full of feeling. Perhaps there is some deep structure to these numbers that we don't understand, but which are revealed as music.

In "The Melody of Pi" (https://youtu.be/A0aR4NS7ObI), Zamerski takes the first 16Ƨ$_\text{z}$ (226$_\text{d}$) digits of π (in base twelve), and sets them to an upbeat waltz rhythm, creating something that sounds like the opening movement of a piano concerto. He has a more somber version clouded with minor-key arpeggios, which he calls "The Requiem of Pi" (https://youtu.be/WxYcAM-oU2Y), that he would like played at his funeral. (Not for a long time, we hope!)

He has also taken the first Ɛ9$_\text{z}$ (141$_\text{d}$) digits (in base twelve) of Euler's constant, e, the base of the natural logarithms, and turned them into a jazzy dance tune which he dubs "The Euler Tango" (https://youtu.be/0hxutro259o).

Other numbers—pardon the pun—are in the works. Zamerski's site has mp3 versions and even sheet music for sale for very nominal prices; and, much in keeping with public-performer tradition, there is a virtual tip-jar—so feel free give him your support. You'll be glad you did.

Numberphile: Base 12

Producer: Brady Haran
Featuring: Dr. James Grime • Member 482$_\text{z}$ (674$_\text{d}$)
URL: http://numberphile.com/videos/base_12.html

In the hype over the (decimal) date 12/12/12$_\text{d}$, the subject of dozenalism attracted attention from many quarters, including the folks responsible for the popular "Numberphile" video channel. This video was the result—if you didn't see it when it came out, it's a must! In it, Dr. James Grime, one of talents from Brady Haran's stable of bright young British "maths" professors, gets out the

brown paper and gives an enthusiastically spot-on introduction to dozenalism. He shows how to count using the Dwiggins numerals and the old DSA nomenclature of "dek-el-do-gro-mo"; explains why twelve's factorability makes it a more practical base for division and fractions, and an easier one for teaching children their times tables; recounts the history of dozens in ancient measures; points out the madness of the French Revolution's attempt to decimalize the clock, the calendar, and the compass; ponders what if the French Metricists had decided to dozenalize our counting rather than decimalizing all our measurement units; and when Haran challenges him with the inevitable question, "What about finger-counting?" Grime demonstrates how to count to "do" on the segments of his fingers.

The latest news about this is that Grime has graciously accepted an honorary membership to the DSA! We're all delighted to count him within our ranks. Grime's own website is www.singingbanana.com (we have no idea why). It's chock full of fun "maths" stuff, so do check it out.

In a related tidbit, you may want to peruse the last couple dozen seconds (last half triciaday) or so of Numberphile's "Tau vs. Pi Smackdown" video (http://numberphile.com/videos/tau_vs_pi.html), wherein Steve Mould advocates for τ, while Matt Parker defends π, as the circle constant. There's a little treat for us at around time-mark :10:37$_\text{d}$ (twinkling 1090$_\text{z}$).

Multipication Tables In Various Bases

Author: Michael Thomas De Vlieger • Member 37Ɛ$_\text{z}$ (527$_\text{d}$)
URL: http://www.dozenal.org/drupal/sites/default/files/MultiplicationSynopsis.pdf

Our preceding editor, Michael De Vlieger, has enlisted the aid of the Wolfram Mathematica tool to update his excellent set of multiplication tables for numerous bases. Automation has allowed him to expand this document so that it now flawlessly covers all bases from binary (binal) to sexagesimal (pentanunqual), completely free from human error.

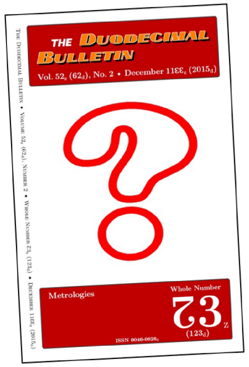
De Vlieger is able to assign a unique glyph to every digit in each of these bases because of the unquennia of work he has put into his fascinating "argam" numeral set, now in excess of four hundred (three gross) distinct characters. He has included a short summary explaining the origin and evolution of his numerals, as well as the names of the first ninty-nine (eight dozen three).

For each successive prime, De Vlieger has devised a unique monosyllabic name, along with a unique character shape. Composite numerals combine the shapes of their factors in much the same way that Chinese ideograms combine "radicals" from component ideograms. Likewise, the names that De Vlieger has assigned to composite numbers combine syllables of their factors.

For instance, his name for ninety-nine (eight dozen three), "novell", reflects its factorization into nine ("nove") times eleven ("ell"). Its glyph builds upon the backward-seven shape of De Vlieger's ell, but takes the top leg of the ell and curls its tip into a miniature, horizontal, nine.

Definitely worth a look-see!

Preview of the Next Issue



The theme of the *Duodecimal Bulletin's* ten dozen third issue, planned for this December, will be "Metrologies." Look for articles covering everything to do with systems of weights and measures, not only showcasing specific metrologies of interest, but also criteria for analyzing them.

Your Editor will set the stage with a piece on the classification and assessment of metrologies; as well as introducing "Quantitels", generic terms for units of measure, that can be used both to talk about units in the abstract, but also as default "starter names" for units in any metrology.

Look for articles that revisit metrologies out of the past, such as Do-Metric and Do-re-mic, giving them a new polish using Systematic Dozenal Nomenclature. DSA president Don Goodman will be writing about TGM, Tom Pendlebury's Tim-Grafut-Maz metrology. Your Editor will also be presenting an introduction to the Primel metrology, a new/old system of measure that synthesizes ideas from many quarters.

We hope to showcase several other interesting metrologies, which is why we need you to get involved! We are actively soliciting articles from our membership to include in this issue. If you have a favorite metrology you'd like to talk about, or you'd just like to share an idea or voice an opinion regarding the mathematics or science of dozens, be sure to contact `editor@dozenal.org`!

The Annual Meeting of the DSA for 11ƐƐ$_z$ (2015$_d$) will be held in Cincinnati, at the Embassy Suites Cincinnati-RiverCenter, 10 East RiverCenter Boulevard, Covington, KY 41011, on October 17$_d$ (15$_z$), from 9:00$_d$AM to 5:00$_d$PM (from triciaday[1] 460$_z$ to 860$_z$). All members are encouraged to join us there! This will be the last Annual Meeting of the 1100$_z$'s ("One Dozen One Grosses"), so there's sure to be big plans for the dawning of the 1200$_z$'s ("One Dozen Two Grosses")! (In SDN terms, this is the last year of the "Ununibiquas"; next year kicks off the "Unbinabiquas".)[2] We're currently experimenting with virtual meeting software such as Google Hangouts, so if you can't join us in person, consider attending online! For updates on the details, watch for our monthly *DSA Newscast* email (free to members, so be sure to join!), as well as our posts on the DozensOnline Forum.[3] For up-to-the-triciaday dozenal updates, follow our Twitter feed @dozenal.

[1] See page 31$_z$.
[2] Ordinal purists, never fear! ☺ We'll also be celebrating at the end of *next* year, to mark the end of the 12$^{nd}_z$ ("One Dozen Second") Biquennium and the eve of the 13$^{rd}_z$ ("One Dozen Third"). (In SDN terms, that's the end of the UB′ ("Unbinal") Biquennium and the eve of the UT′ ("Untrinal").)
[3] `http://z13.invisionfree.com/DozensOnline`.

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A subscription membership entitles you to a paper copy of *The Duodecimal Bulletin* when it is published, for a period of one year from your donation; regular members receive only a digital copy. Donations of any amount, large or small, help keep the DSA going and are greatly appreciated.

To facilitate communication, do you grant the DSA permission to furnish your name and contact information to other DSA members? ☐ Yes ☐ No

We'd be delighted to see you at our meetings, and are always interested in your thoughts, ideas, and participation. Please tell us about your particular interests here:

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