THE Duodecimal Bulletin

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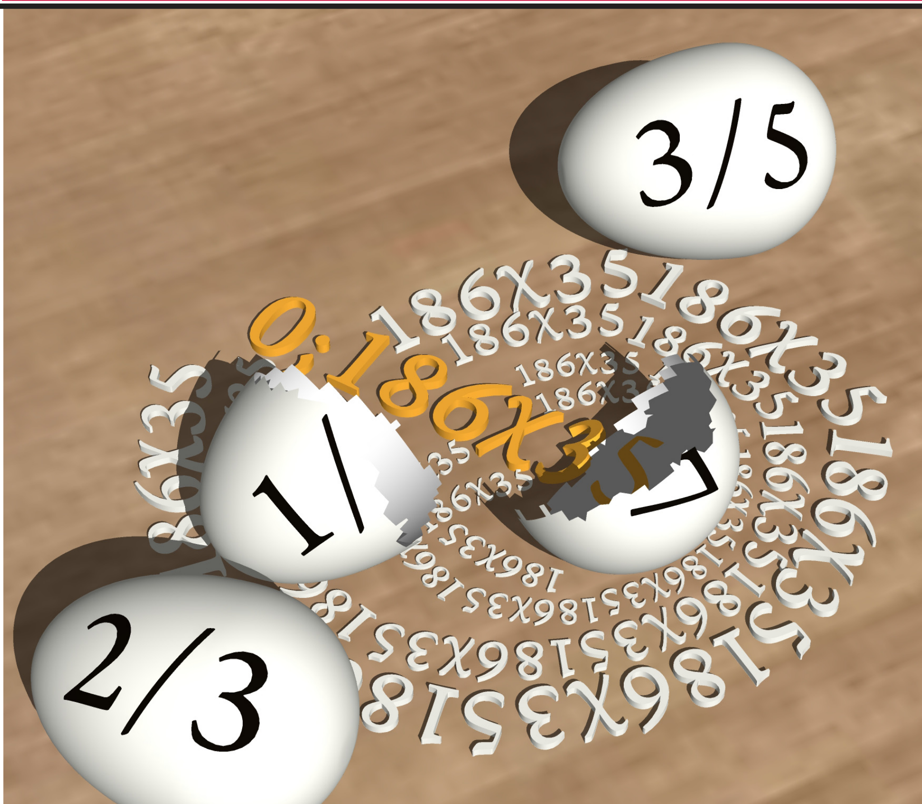

Digital Fractional Expansions page 9

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The Dozenal Society of America

is a voluntary nonprofit educational corporation, organized for the conduct of research and education of the public in the use of base twelve in calculations, mathematics, weights and measures, and other branches of pure and applied science

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The Duodecimal Bulletin

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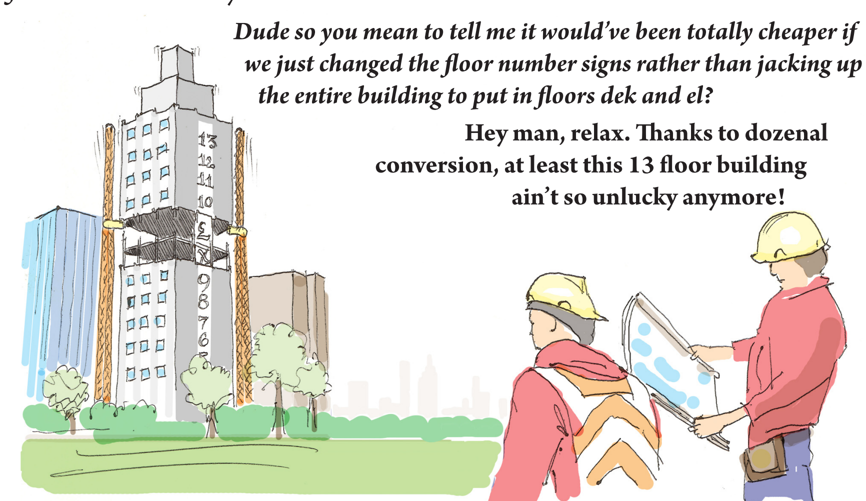


Dozenal conversion is good for business in more ways than you can imagine!

president's message

In this issue we continue our review of numeral symbols from Vol. 4↊; №. 2. The next issue, Vol. 4↋; №. 2 is about 60 p/g complete. We expect to publish it before the end of 11↋6; (2010.) Look for the minutes of the June 11↋6; (2010.) Annual Meeting in that issue.

In July and August of this year, the Duodecimal Society of America has posted an archive of back issues at http://www.Dozenal.org/archive/archive.html. Read about it on page 5. Most of the back issues of our Society's publication can now be accessed through the Internet or searched in Google.

In June, Jay Schiffman was elected Treasurer; his address appears below. Please send any dues to his attention.

Prof. Jay Schffman, Treasurer
604-36 S. Washington Square
Apt. 815
Philadelphia, PA 19106-4115

Michael Thomas De Vlieger,
President and Editor

Seen in a Missouri preschool: a train of numbers that runs up to … one dozen!

Electronic Publication of the Bulletin

Starting with the last issue (WN 99;, Vol. 4↊; № 2), the Dozenal Society of America produced its first electronic *Duodecimal Bulletin*. It was delivered electronically in March 11↋6; (2010.) using a third-party service called **YouSendIt.com**. This service sends Members in good standing messages notifying them when the *Duodecimal Bulletin* is available, along with instructions on how to download the file. Generally, the *Bulletin* will be available for download for two dozen six days. You may download the *Bulletin* at your leisure; this delivery method avoids stuffing your mailbox with a large file. Though one might consider using a mass emailing, many IT departments, security software systems, and internet service providers prevent such transmissions, as mass email is a principle method of spamming.

Members seem to like the electronic *Bulletin*. The electronic version emerges as soon as the digital file is completed. It transmits a week or so before the press finishes publication and we mail out the physical copies. The digital version is always in full color. Some have suggested that the electronic version be formatted rather differently than what is conve-

Continued in the middle of page 4

Robert Roy McPherson
DSA Member № 4↋; ~ rest in peace

Longtime Member, Fellow, and member of the Board of Directors of this Society, Mr. McPherson passed away 21 November 2009 at age seven dozen two.

The Society celebrated Mr. McPherson's avid devotion to dozenal with the Beard Award in 11↋0; (2004).

Rob Roy was born in 1923 in Jackson, TN, moving to Gainesville, FL at a young age. His mother was a schoolteacher. Mr. McPherson was a Navy man, a veteran of World War II, and an engineer, strongly interested in physics and quantum theory. Dr. Eric Barber, the developer of the solar panel, was a friend who lived closeby and walked with Rob Roy on occasion. Jerry Combs described his uncle as a strong-minded individual and a fervent Methodist. Mr. McPherson "made himself a little different from everyone else". Mr. Combs related his uncle's passion for dozens: Rob Roy's goal in life was to "spread the word of base twelve". Many neighbors knew and liked him as he was fond of walking his Gainesville neighborhood. On his walks, Rob was known to collect items he'd find along the way. In his final month, Rob Roy refused to eat. He leaves three nephews, two nieces, and a sister in Atlanta, GA.

Electronic Publication, continued from page 3

nient for print. Because publication of the *Bulletin* is a volunteer effort, I regret that we'll need to live with some of the effects associated with print as long as a physical edition is printed. The DSA is not considering elimination of the printed *Duodecimal Bulletin*.

The dissemination of the electronic version has reduced the number of printed copies of a given issue by about 76; p/g (pergross, or 62.5%). Because of this, the DSA is less encumbered by a standard document length, like the former two dozen eight page average. We can avoid abridging or dividing articles among issues which are half a year apart. Electronic publishing allows us to focus on content rather than regulate the number of printed pages, giving you the Member greater value. The organization also saves money by printing fewer issues.

Those who are Life Members or Supporting Members receive both the electronic and printed copies of the *Bulletin*. Standard Memberships still enjoy the *Bulletin* in its new, electronic form. If you are a Standard Member ($18 per year) and are interested in receiving physical issues of the *Duodecimal Bulletin*, please consider the Supporting Membership level at $36 per year. The DSA thanks all of its Members for their continued support.

Sincerely, Michael Thomas De Vlieger, Editor

Symbology & Nomenclature

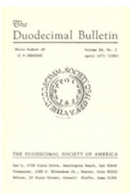
The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use Dwiggins dek (↊) for ten and his el (↋) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "dough" in the duodecimal system.

When it is not clear from the context whether a numeral is decimal or dozenal, we use a period as a unit point for base ten and a semicolon, or Humphrey point, as a unit point for base twelve. Thus ½ = 0;6 = 0.5, 2⅔ = 2;8 = 2.66666…, 6⅜ = 6;46 = 6.375

The
Duodecimal Bulletin
ONLINE ARCHIVE

The Dozenal Society of America welcomes you to visit its online archive of back issues of the *Duodecimal Bulletin*. A system of online indexes is available to help you navigate.

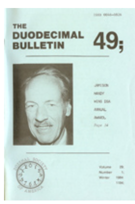
Archive Index. The archive index, at http://www.dozenal.org/archive/archive.html, is a simple listing of each individual issue. Issues are listed chronologically, in four columns according to cover appearance and editorship, with the decimal year of publication given in parentheses after the first issue of that year. Each issue is noted in a five character Volume-Number-Whole Number format for concision. (Differences in the numeral design over the years has been ignored for simplicity on the website: dek or digit-ten is represented online as "X", el or digit-eleven by "E".) Simply click on the designation of any given issue to download a 3 to 5 megabyte PDF of the entire issue. The four "eras" of the *Duodecimal Bulletin* appear below:



011-00 to 251-41, 1945—1974

Vol. 1 №. 1 wn 0 through Vol. 25; №. 1 wn 41;

"Classic Issues" with the black and white DSA emblem on the cover, produced by Editors Ralph Beard, George S. Terry, Jamison Handy, and Henry Churchman.



261-42 to 352-6Ɛ, 1981—1992

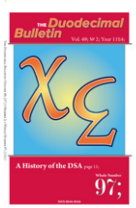
Vol. 26; №. 1 wn 42; thru Vol. 35; №. 2 wn 6#;

"Second Generation" issues, printed on colored paper according to year of publication, edited by Dr. Patricia Zirkel. From 291-49 (1984), a picture is often seen on the cover, whereas before, only the DSA logo.

361-70 to 491-96, 1993—2008

Vol. 36; №. 1 wn 70; through Vol. 49; №. 1 wn 96;

"Second Generation" issues, with a firm cover featuring a light blue background and the DSA logo in a white square, edited by Prof. Jay Schiffman, later with Prof. Gene Zirkel as Associate Editor.



492-97 to present, 2008—

Vol. 49; №. 2 wn 97; through current issues

"Third Generation" digital and print issues, with a red masthead and color printing, edited by Mike De Vlieger, with Gene Zirkel as Associate Editor.

Continued on the next page

See page 2Χ; for a list of the Editors of the *Duodecimal Bulletin* throughout its history.

Pictorial Synoptic Indexes have been prepared, wherein one can browse the covers of any past issue, along with a brief synopsis of the main content of the issue. The main index is retrievable at http://www.dozenal.org/archive/dbpict.html. On this page, one can select from one of the ten "duodecades" (sets of one dozen Whole Numbers) of *Bulletin* issues. Click on the duodecade of interest, and you'll visit a second index showing the dozen individual issues of that duodecade. The duodecade is used in the pictorial indexes for consistency and efficiency of presentation. Since each year of publication yielded an inconsistent number of issues, the duodecades present a set of issues which are not quite intuitive and equal in the lengths of their periods of coverage. Here are the coverages for each duodecade:

011-00—043-0Ɛ	Vol. 1 №. 1 wn 0 (1945)	— Vol. 4 №. 3 wn Ɛ; (1948)
051-10—0Ɛ2-1Ɛ	Vol. 5 №. 1 wn 10; (1949)	— Vol. Ɛ; №. 2 wn 1Ɛ; (1955)
101-20—152-2Ɛ	Vol. 10; №. 1 wn 20; (1956)	— Vol. 15; №. 2 wn 2Ɛ; (1961)
161-30—230-3Ɛ	Vol. 16; №. 1 wn 30; (1962)	— Vol. 23; №. 0 wn 3Ɛ; (1972)
240-40—293-4Ɛ	Vol. 24; №. 0 wn 40; (1973)	— Vol. 29; No. 3 № 4#; (1984)
2Χ1-50—313-5Ɛ	Vol. 2Χ; №. 1 wn 50; (1985)	— Vol. 31; №. 3 wn 5#; (1988)
314-60—352-6Ɛ	Vol. 31; №. 4 wn 60; (1988)	— Vol. 35; №. 2 wn 6#; (1992)
361-70—3Ɛ2-7Ɛ	Vol. 36; №. 1 wn 70; (1993)	— Vol. 3#; №. 2 wn 7#; (1998)
401-80—452-8Ɛ	Vol. 40; №. 1 wn 80; (1999)	— Vol. 45; №. 2 wn 8#; (2004)
461-90—4Ɛ2-9Ɛ	Vol. 46; №. 1 wn 90; (2005)	— Vol. 4Ɛ; №. 2 wn 9Ɛ; (2010)
501-Χ0—	Vol. 50; №. 1 wn Χ0; (2011)	—

At the bottom of the pictorial indexes, we provide a brief narrative describing the apparent *zeitgeist* of the Society as seen through its *Bulletin*. The latest digitally-produced issues are available in electronically-produced vs. scanned format. The very latest issues are not publicly available, but are sent to each Subscriber/Member. The latest and some of the earliest issues have their tables of contents (TOC) available online. These TOCs will be prepared over the next couple years until all are available.

Over the past three years, nearly all back issues of the *Bulletin* have been scanned and digitally bound as PDFs. These have been available online since June 2009 in a rudimentary way. (Visit the legacy page at http://www.dozenal.org/archive/archiveOld.html.) In July, the new set of reference indexes described above were developed. In the future, the archive will be linked to the home pages, as the latter will be overhauled to resemble the layout of the former.

A similar effort is underway for the publications of the DSGB. Currently, only the *Duodecimal Newscast* Volumes 1 through 8 (Continuity Numbers 1 through one dozen nine, 1959-1966) are available online. Visit the provisional DSGB index at http://www.dozenal.org/archive/archiveb.html. This index will be transferred over to the DSGB upon its completion, but will be linked to from the DSA website.

As a consequence of posting each scanned *Bulletin* on the Internet, you can search the archive using Google. Because dozenal publishing is unique in that it often involves printing novel characters (dozenal numerals), some of the Google optical character recognition algorithms read dek or el as the letters they happen to resemble.

I hope you enjoy navigating the back issues of this *Bulletin*. Much of the discussion on the Internet chat rooms among today's dozenalists has been covered by dozenal enthusiasts of earlier generations. Perhaps we can learn from these thinkers, maybe their thoughts crystallized in the back issues of our *Bulletin*, can stoke new ideas today.

A Multiplication Oddity

by Jean Kelly

Many people are aware of the multiplication procedure of halving one factor and doubling the other. Thus:

$$\begin{aligned} & 14; \times\ 9;6 \\ = \ & 8; \times 17; \\ = \ & 4; \times 32; \\ = \ & 2; \times 64; \\ = \ & 108; \end{aligned}$$

But what if the number being divided in half is not a power of 2? If we repeatedly halve 18; we obtain:

18; → X; → 5; → 2;6 → 1;3 → 0;76, etc.

Repeated halving produces fractions and does not terminate in 1.

However, it is fairly well known that the following procedure does work: Divide one factor in half and double the other discarding any fractions. (They are a pain in the neck.) Now (to make the work less tiresome) delete the rows in the column produced by halving which resulted in even numbers. Then add the remaining numbers in the doubling column. Voila!, the product appears. Thus for example to multiply 19; × 1Ɛ; we obtain:

19;	1Ɛ;
~~X;~~	~~3X;~~
5	78;
~~2~~	~~134;~~
1	268;
	343;

How It Works

a) Halving 19;:

$$\begin{aligned} \underline{19} \times 1Ɛ &= ? \\ (\underline{2\times X} + 1) \times 1Ɛ &= ? \\ (2 \times (\underline{2\times 5} + 0) + 1) \times 1Ɛ &= ? \\ (2 \times (2 \times (\underline{2\times 2} + 1) + 0) + 1) \times 1Ɛ &= ? \\ (2 \times (2 \times (2 \times (2\times 1 + 0) + 1) + 0) + 1) \times 1Ɛ &= ? \end{aligned}$$

b) Halving 19;:

$$\begin{aligned} (2 \times (2 \times (\underline{2 \times (2\times 1 + 0)} + 1) + 0) + 1) \times 1Ɛ &= ? \\ (2 \times (\underline{2 \times (4\times 1 + 2\times 0)} + 1) + 0) + 1) \times 1Ɛ &= ? \\ (\underline{2 \times (8\times 1 + 4\times 0 + 2\times 1)} + 0) + 1) \times 1Ɛ &= ? \\ (14\times 1 + 8\times 0 + 4\times 1 + 2\times 0 + 1) \times 1Ɛ &= ? \ ❦ \end{aligned}$$

where the left side, 14; + 4 + 1, is the 19; we started with.

Multiplying, we obtain:

$$\begin{aligned} 14\times 1Ɛ + 0 + 4\times 1Ɛ + 0 + 1\times 1Ɛ &= ? \\ 268 + 0 + 78 + 0 + 1Ɛ &= ? \\ 343 &= ? \end{aligned}$$

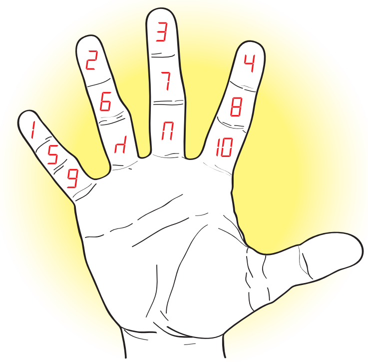
Note that these are the same addends we summed above. But what does this have to do with number bases? What we are doing is basically finding the binary representation of the factor we are dividing in half.

Note that (14×1 + 8×0 + 4×1 + 2×0 +1) in the line marked with (❦) above is $1\times 2^4 + 0\times 2^3 + 1\times 2^2 + 0\times 2^1 + 1\times 2^0$ or 10101 in binary, and thus 19 × 1Ɛ is simply being treated as $10101_2 \times 1Ɛ$.

~ Quite Easily Done

Dozenal Counting on Your Fingers

by Timothy F. Travis, DSA Member № 342;



We have a number system based on ten because we have ten fingers and started counting on them. But by using our thumb for making contact and the twelve pads on the other four fingers, we can also have a good way of counting in base twelve. Please refer to the drawing and try it using your hand.

The extra dozenal symbols are called "dek" and "brad". These symbols look like a chair for "dek", and like a staple or brad for "brad". Note that the symbols are consistant with seven-stroke digital representation. The "dozens" of dek and brad are called "deka" and "brada".

It would be fun to have gloves with these numbers on them. Children would love them. They could also be electronic and when touched by the thumb, omit the count sound.

If we had been using a base twelve system and counting this way on our hands for as long as we have been counting in base ten, we would undoubtedly have come up with games based on this method. If you can come up with any, please contact me, raenbo@verizon.net, or submit them to *The Duodecimal Bulletin.*

~ *Editor's Note: see some of Mr. Travis' original website at the following archived address:* http://web.archive.org/web/20031212123420/http://www.raenbo.com/

problem from last issue:

problem corner

Skip Scifres proposed the following problem to readers back in 1199; (1989.) We reprint it here, noting that we are yet to receive a solution.

Find a procedure which will generate a set of integers each of which cannot be partitioned into four nonzero square numbers. Example:

The year 1199; (1989.) can be so partitioned:

$$1199; = 36;^2 + X^2 + X^2 + 5^2$$

The number 2540; (4224.) cannot.

Can anyone solve Skip's teaser?

~ Reprinted from WN 61; Vol. 32; № 1, page 24;.

~ Solution on page 1Ɛ;

On Maximal Repeating Sequence of Decimal Expansions in Base-Twelve

by T. J. Gaffney, DSA Member № 399; • 8 June 11Ɛ5;

Introduction

When learning the decimal expansions of fractions in the base-ten system, the first one that seems to be a huge pain is $1/7$. This decimal expansion is a worst case scenario, meaning that there could not possibly be fewer digits involved; we say that it has maximal repeating sequence. However, we'll learn that fractions of this type are actual somewhat friendly, in that any $m/7$, where m is an integer such that $0 < m < 7$, also has maximal repeating sequence with the digits being cyclical permutations of the $1/7$ case. In dozenal, $1/5$ and $1/7$ have this same maximal repeating sequence property and, for the same reason, have this same nice property.

Worst Case Scenario

I will demonstrate how $1/7$ is a worst case scenario, or "maximal repeating sequence", because it will come in useful later on.

When dividing one by any number you basically face an infinite number of zeros after the decimal. When doing long division you look at the first number, and after dividing the first zero by the divisor you take the remainder and "carry" it or move it front of the proceeding zero (which in decimal means to multiply by ten and add. For example:

```
     0.1
7 ) 1.000...
    - 7
      30
```

Where, in this case, on the bottom row 3 is the remainder and 0 is the carried down digit. This paper assumes that the reader knows how to do long division; I only provide the detail in case that the reader hadn't previously considered this 3 the remainder.

Now, there are very few values possible values for the remainder. Mathematicians will know by the Fundamental Theorem of Arithmetic and the layman will know by intuition, that this remainder cannot be less than zero and that it cannot be greater than or equal to the divisor (7 in the example). If the remainder is not less than the divisor, then you increment the solution you've listed (the 1 in the example) until it is.

If you ever get a zero as the remainder, then it is clear to see (when dividing along infinite zeros) that the remaining solutions will be zeros, which we don't write because they're after the decimal. This is called a terminating fraction. (Try the calculation of $1/8$ for an example.) The other option is that the remainder has already been encountered in the long division, in which case (since we're working along infinite zeros, that all act the same) it is clear that the solutions would simply repeat themselves. This is a repeating sequence. We see this in the $1/7$ case (here I write the remainder as super-scripts to the zeros to save space; the way you would see them in subtraction):

$$7\,\overline{)\,1.{}^{1}0\;{}^{3}0\;{}^{2}0\;{}^{6}0\;{}^{4}0\;{}^{5}0\;{}^{1}0\;\ldots}\quad\text{quotient: } 0.1\;4\;2\;8\;5\;7\;1\;\ldots$$

Obviously, the pattern will continue forever.

Since, there is a finite option for remainders, then the solution (for fractions with one as the dividend) must repeat or terminate. (In fact, it is a well known mathematical fact that the decimal expansions of all fractions must repeat or terminate. The proof is a generalization of the preceding arguments.) Further an expansion of this type (if not terminating) must repeat within m digits after the decimal place, where m is the divisor. That is to say that at most m-1 digits will be repeated, since we don't allow zero to be a remainder (for that would be a termination), then there are only m-1 unique remainders. This produces the maximal repeating sequence (our "worst case scenario").

A Nice Property

It is a fun bit of trivia that every mathematician should know that multiples of $1/7$ use permutations of the same repeating sequence. That is to say that the repeated digits are the same, but in different order. We show the first six digits of the sevenths in decimal in Figure 1 at right.

$1/7 = 0.142857\ldots$
$2/7 = 0.285714\ldots$
$3/7 = 0.428571\ldots$
$4/7 = 0.571428\ldots$
$5/7 = 0.714285\ldots$
$6/7 = 0.857142\ldots$

Fig. 1: Decimal Sevenths

The reader may immediately see why this is. A hint for mathematicians is to notice that the permutations are of the particular, cyclic type.

The proof of this actually quite simple. In the first example, the calculation of $1/7$, I write the 1 in front of the first zero in the dividend, as though it is the remainder when dividing only the digit 1 by 7. (Although, it is usually taught to just start the division by looking at the 1 and 0 together for 10, what you're really doing here is just exactly the same "carrying" of the remainder that you're doing everywhere else.) But, if instead of a 1 for the first digit there was a 2 or 3 or some other number (as it would be in these other fractions), then the first remainder would also be this other number.

From there, and because you're moving along infinite zeros, the division would proceed exactly as it did in the $1/7$ case, with the same repeating sequence as well, but merely starting at a different place.

If it is not immediately clear why these expansions use the same 6 digits, it may be a useful exercise for the reader to calculate a few until he or she can see the pattern on his or her own.

Dozenal

1/2 = 0;6
1/3 = 0;4
1/4 = 0;3
1/6 = 0;2
1/8 = 0;16
1/9 = 0;14

Fig. 2: Some Dozenal Terminating Fractions

A driving argument for the usage of dozenal is the ease of expanding fractions. The fractions shown in Figure 2 at left have much cleaner and easier expansions in dozenal.

But this argument seems to be weakened by two fractions, 1/5 and 1/7. One seventh is just as "bad" in base-twelve as it is in base-ten, and one fifth seems much worse. However, by simply calculating these two dozenally, you can see that these both have the property of having a maximal repeating sequence, and thus inherit this nice property of just cyclically repeating the same digits.

The expansions of the fifths and sevenths in dozenal appear in Figures 3 and 4 below.

What's So Nice?

1/5 = 0;2497…
2/5 = 0;4972…
3/5 = 0;7249…
4/5 = 0;9724…

Fig. 3: Dozenal Fifths

1/7 = 0;186X35…
2/7 = 0;35186X…
3/7 = 0;5186X3…
4/7 = 0;6X3518…
5/7 = 0;86X351…
6/7 = 0;X35186…

Fig. 4: Dozenal Sevenths

1/8 = 0.125
2/8 = 0.25
3/8 = 0.375
4/8 = 0.5
5/8 = 0.625
6/8 = 0.75
7/8 = 0.875

Fig. 5: Decimal Eighths

"So, what's so great about the maximal repeating sequence property? I still say that 1/7 is the hardest to memorize." Try to memorize your eighths in decimal (1/8 to 7/8, shown in Figure 5 below) and you'll find that you'll be memorizing more numbers than when you memorize the sevenths.

When memorizing the sevenths in decimal (this will work the same with the fifths and sevenths in dozenal). You need to first memorize 1/7 = 0.142857. It's not pretty, but it is only six digits. Now say you want to recall $m/7$ on cue. You only need to remember the first digit of this expansion, because, as we know, the rest cyclically follows the order in the 1/7 case. However, this is easy; since the leading digits of the numbers 1/7 to 6/7 must be in increasing numerical order. (Because, otherwise, it would suggest that 2/7 is less than 1/7, for example.)

Say you want to impress your friends by "calculating" 6/7. You start by looking at the digits in 1/7: 142857 in numerical order: $1 < 2 < 4 < 5 < 7 < 8$; since 8 is the 6th smallest here, you take that to be the leading digit. Then you recall the rest of 1/7, in order: 0.857142 (Where, upon reaching the "end", or the 7 digit, you just continue from the beginning.)

For example, you might use this method to calculate the fifths in dozenal, where 1/5: 0;2497. Which gives you $2 < 4 < 7 < 9$. Thus we obtain the leading digits of the fractions as seen at left below. Adding the remaining digits cyclically, we obtain the situation seen at right below:

1/5 = 0;2…	1/5 = 0;2497…
2/5 = 0;4…	2/5 = 0;4972…
3/5 = 0;7…	3/5 = 0;7249…
4/5 = 0;9…	4/5 = 0;9724…

Thus it is easy to recall these seemingly unfriendly fractions. For this reason, along with many others, dozenal is a superior counting system.

featured figures

1/7

Building on Mr. Gaffney's examination of dozenal fractions, we thought about other such fractions. Here we explore these for you, first examining the first one dozen ten reciprocals and their unique multiples. In the tables below, all reciprocal multiples ignore zero in the unit place or "integer part" of the figure. Each reciprocal multiple appears in one of three states. The first state terminates, like 2/3 = ;8. The second state repeats after the unit point, like 2/5 = ;4972.. , the repetition indicated by ellipsis (..). The last is a repeating series after an initial quantity of digits, like 5/12 = 0;4⁝35186X.., the repeating series appearing here between a vertical series of dots (⁝) and the ellipsis. The reciprocals of Ɛ and 11; are fun. Do you recognize their patterns? One dozen five displays the behavior Mr. Gaffney described for fifths and one sevenths. Can you explain some of the pattern changes that happen, say, in the multiples of the reciprocal of 17; (19.)?

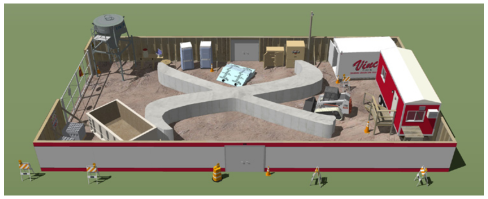
2	**Halves**	
	1/2	;6
3	**Thirds**	
	1/3	;4
	2/3	;8
4	**Quarters**	
	1/4	;3
	3/4	;9
5	**Fifths**	
	1/5	;2497..
	2/5	;4972..
	3/5	;7924..
	4/5	;9247..
6	**Sixths**	
	1/6	;2
	5/6	;X
7	**Sevenths**	
	1/7	;186X35..
	2/7	;35186X..
	3/7	;5186X3..
	4/7	;6X3518..
	5/7	;86X351..
	6/7	;X35186..
8	**Eighths**	
	1/8	;16
	3/8	;46
	5/8	;76
	7/8	;X6
9	**Ninths**	
	1/9	;14
	2/9	;28
	4/9	;54
	5/9	;68
	7/9	;94
	8/9	;X8
X	**Tenths**	
	1/X	;1⁝2497..
	3/X	;3⁝7249..
	7/X	;8⁝4972..
	9/X	;X⁝9724..

Ɛ	**Elevenths**			
	1/Ɛ	;1..	6/Ɛ	;6..
	2/Ɛ	;2..	7/Ɛ	;7..
	3/Ɛ	;3..	8/Ɛ	;8..
	4/Ɛ	;4..	9/Ɛ	;9..
	5/Ɛ	;5..	X/Ɛ	;X..

10;	1	;1	7	;7		
	5	;5	Ɛ	;Ɛ		
11;	1	;0Ɛ..	5	;47..	9	;83..
	2	;1X..	6	;56..	X	;92..
	3	;29..	7	;65..	Ɛ	;X1..
	4	;38..	8	;74..	10	;Ɛ0..
12;	1	;0⁝X35186..				
	3	;2⁝6X3518..				
	5	;4⁝35186X..				
	9	;7⁝86X351..				
	Ɛ	;9⁝5186X3..				
	11	;Ɛ⁝186X35..				
13;	1	;0⁝9724..	8	;6⁝4972..		
	2	;1⁝7249..	Ɛ	;8⁝9724..		
	4	;3⁝2497..	11	;X⁝4972..		
	7	;5⁝7249..	12	;Ɛ⁝2497..		
14;	1	;09	9	;69		
	3	;23	Ɛ	;83		
	5	;39	11	;99		
	7	;53	13	;Ɛ3		
15;	1	;085792 14Ɛ364 29X7..				
	2	;14Ɛ364 29X708 5792..				
	3	;214Ɛ36 429X70 8579..				
	4	;29X708 579214 Ɛ364..				
	5	;36429X 708579 214Ɛ..				
	6	;429X70 857921 4Ɛ36..				
	7	;4Ɛ3642 9X7085 7921..				
	8	;579214 Ɛ36429 X708..				
	9	;6429X7 085792 14Ɛ3..				
	X	;708579 214Ɛ36 429X..				
	Ɛ	;79214Ɛ 36429X 7085..				
	10	;857921 4Ɛ3642 9X70..				
	11	;9214Ɛ3 6429X7 0857..				
	12	;9X7085 79214Ɛ 3642..				
	13	;X70857 9214Ɛ3 6429..				
	14	;Ɛ36429 X70857 9214..				
16;	1	;08	Ɛ	;74		
	5	;34	11	;88		
	7	;48	15	;Ɛ4		
17;	1	;076Ɛ45..	X	;639582..		
	2	;131X8X..	Ɛ	;6Ɛ4507..		
	3	;1X8X13..	10	;76Ɛ450..		
	4	;263958..	11	;826395..		
	5	;31X8X1..	12	;8X131X..		
	6	;395826..	13	;958263..		
	7	;45076Ɛ..	14	;X131X8..		
	8	;5076Ɛ4..	15	;X8X131..		
	9	;582639..	16	;Ɛ45076..		
18;	1	;0⁝7249..	Ɛ	;6⁝7249..		
	3	;1⁝9724..	11	;7⁝9724..		
	7	;4⁝2497..	15	;X⁝2427..		
	9	;5⁝4972..	17	;Ɛ⁝4972..		
19;	1	;0⁝6X3518..	Ɛ	;6⁝35186X..		
	2	;1⁝186X35..	11	;7⁝5186X3..		
	4	;2⁝35186X..	14	;9⁝186X35..		
	5	;2⁝X35186..	15	;9⁝86X351..		
	8	;4⁝6X3518..	17	;X⁝X35186..		
	X	;5⁝86X351..	18	;Ɛ⁝5186X3..		
1X;	1	;0⁝6..	11	;7⁝1..		
	3	;1⁝7..	13	;8⁝2..		
	5	;2⁝8..	15	;9⁝3..		
	7	;3⁝9..	17	;X⁝4..		
	9	;4⁝X..	19	;Ɛ⁝5..		

A Numeral Toolbox

WRITTEN BY
Mike De Vlieger



This article explores various strategies and tools which can be drawn upon to develop new numeral systems. Like the tools in a master carpenter's toolbox, these methods can help produce work; only the master carpenter's experience and planning can guide these tools to produce good work. One will need good criteria for the effective design of new numerals. This article is strictly concerned with tools.

Simple Graphic Tools

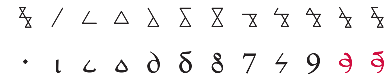
Figure 1: *Examples of four simple tools to derive new numerals from existing symbols.*

Presented below are simple graphic tools for deriving a new symbol from an antecedent. Rotation and reflection are usually pure wholesale transformations of a given existing symbol, while smoothing and justification are governed by intuition and the author's creativity. These tools apply to symbols derived from antecedent symbols in the public lexicon, or to invented symbols implied by a pure strategy.

Rotation and Reflection are simple transformations of everyday symbols to derive new symbols. These simple tools can be observed at work within the Hindu Arabic numeral set itself in the 6 and 9, and in the alphabet in the letters p, q, b, and d.

Elision and Appendage simply involve eliminating or introducing an element to a symbol to produce a new symbol. A. Chilton arrives at a "hatless" five (ƽ). Diacritical marks, such as those used by J. Halcro Johnson to build a "negative four" (4̄) out of a four (4), are common. F. Ruston uses an appended curl at the lower right of his numerals to signify that six has been added to their value (Π = 3, Π̥ = 9). In this article, we'll distinguish "free-floating" additions to a symbol as diacritical marks, whereas those additions which are attached to the numeral are appendages.

Smoothing simply involves adapting a symbol to everyday handwriting. The evolution of today's letters and numbers is an empirical process that takes place over generations; redundancies and hard angles are lost; differences between similar symbols tend to accentuate, further distinguishing them. Smoothing is an author's attempt to anticipate the evolution of figures. Gwenda Turner produced a set of symbols which derive from arrangements of matchsticks. The matchstick arrangements, Turner argues in *Dozenal Journal* 4 page 4, can be used to teach children the relationship of quantity to number. The numerals thus serve as an analog of the quantity they represent. Turner adds "If a dozen were the standard base it would be necessary to write the numerals at speed. For this, a cursive form is more efficient, and △ [her numeral three] would become ▵." Turner adds that the cursive, rounded triangle, perhaps taken as a circle, would force the zero be conveyed by "another symbol, such as the Arabic · [a raised dot]". In the written form of her analog numerals, the complex of "matchsticks" that convey numeral six [⧖] can be bundled and represented as a leftward stroke at the bottom of the numerals larger than six:



Cultural Resonance. The producer of a new set of numerals may be inclined to force a symbol produced by a pure strategy to conform to a familiar configuration. This is seen mostly in "least change" strategies where the existing "Hindu-Arabic" numerals are retained, as most people are already familiar with their value.

Justification. An existing symbol, like the numeral "4", can be forced to comply with a technical constraint. This may include compliance with a grid as in a dot-matrix screen or a tiling layout. The 7 or 13 segment LCD/LED digital readout is a common constraint.

Don Hammond [1], honorary DSA Member active in the DSGB, and Niles Whitten [2] modified Sir Issac Pitman's 1857 transdecimals to make these more compliant with the 7 segment LCD/LED digital readouts. (See Vol. 4a; № 2 page 17;):

Pitman (1857):	0	1	2	3	4	5	6	7	8	9	↊	↋
7-segment:	0	1	2	3	4	5	6	7	8	9	↊	↋
Hammond:	0	1	2	3	4	5	6	7	8	9	↊	↋
Whitten-1:	0	1	2	3	4	5	6	7	8	9	↊	↋
Whitten-2:	0	1	2	3	4	5	6	7	8	9	↊	↋

Strategy. The following techniques relate more to a governing methodology rather than sheer graphic manipulation. We'll back up the concepts with examples. Strategies can be "pure", meaning they are allowed to function as ideas without intervention by the author. The author can also interrupt a strategy to serve other strategies: multiple strategies may be at work. Some ideas are permitted to run over a sequence of numerals, then are interrupted: this sequence we'll call a *run sequence*. Some authors smooth or justify the "pure output" of a given strategy. It might be clearly observed by the reader that some "pure" strategies yield a set of symbols which are difficult to distinguish from one another when seen in mass. This tends to serve as impetus to distinguish some numerals from others, sacrificing purity for legibility. No judgments will be made regarding an author's application of strategy; we'll simply observe which strategy appears to be at work.

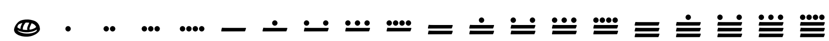


Figure 2: *Mayan numerals zero through one dozen seven.*

The Analog Strategies. The simplest means of developing graphic symbols intended to convey numerals is by the use of an *analog*, or a representation of a real-world concept through the use of a symbol which more or less directly resembles that concept itself. A clear example of this occurs among the Mayan numerals. One dot represents the number one, two dots represent "two", etc. The Mayan numerals one through four constitute the basic run sequence for Mayan numerals. This type of basic run sequence, wherein each element can be counted and

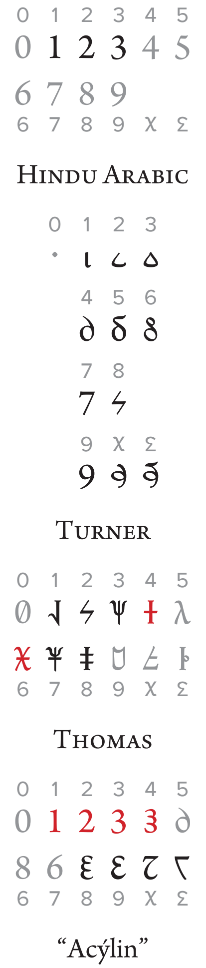


Figure 3: *Studies of the apparent incomplete or interrupted additive analog run sequences of several separate identity symbologies. The small gray numerals above or below the symbols are the "standard" duodecimal digits which the symbols convey.*

yield the digit the numeral represents we will call an *additive analog*. At number five, the Mayans introduce an element intended to convey the idea of a grouping we'll call a *bundle*. In this case, the bundle is a "bar" that signifies a group of five. As we count to nine, the basic run sequence set above the bundle generates the numerals six through nine. At ten, the Mayans stack their bundles, one atop the other, "two fives". The bundles stack again at one dozen three. Thus the numerals five, ten, and one dozen three represent a bundle sequence. This bundle sequence can be called a *multiplicative analog*. The "standard" Mayan numeral set is a relatively pure system strategically. The numeral zero symbolizes emptiness; its numeral is symbolic. Zero seems to be symbolic in every system; it's difficult to be literal about the numeral zero without puzzling spaces cropping up to confuse the reader.

We've already examined the run sequences and bundles apparent in Gwenda Turner's "matchsticks" and smoothed numerals. It's interesting to observe the handwritten numerals to see that a different set of run sequences become apparent. (See Figure 3).

Roman numerals are also analogous in the first three numerals: I is one stroke and means "one", II is two strokes and means "two", etc. Beyond III, in most cases (but not in the case of clock faces), the numeral is generated using a different, more complex concept. The first three Roman numerals constitute the basic run sequence for Roman numerals. (The Roman numeral system is not a positional-notation system and really can't be compared meaningfully with dozenal symbologies recently proposed, except for Brother Louis Francis' "Roman Dumerals" described in Vol. 10; № 1 page 7;, which are beyond the scope of this article).

The Hindu Arabic numerals are somewhat analogous in the first three nonzero numerals; the system having undergone evolution from earlier forms which were perhaps even more clearly analogs. Let's look at analogs among dozenal symbologies.

Additive Analog Strategy. Rafael Marino, professor at Nassau Community College in New York, described a system in Vol. 38; № 2 page 10; wherein a set of numerals are devised and function similar to the Mayan system. The numerals in the basic run sequence, numerals {1, 2, 3, 4, 5}, are defined simply by producing a number of strokes, one above the other, in analogy to the number the numeral represents. At {6}, a vertical bar bundles 6; thereafter, appending the bundle element to the basic run sequence supplies {7, 8, 9, χ, Ɛ}. Zero is symbolized by a box.

F. Ruston's numerals, conveyed in 1961 by the DSGB's *Dozenal Newscast*, Year 3, № 2, page 3, establishes its basic run sequence {1, 2, 3, 4, 5} using segments in the numerals. The numeral corresponding to 5 (X) has five segments but can be efficiently produced in three strokes. At {6}, Ruston introduces a "curlicue" bundle, which thereafter can be appended to the lower right end of each of the basic run sequence numerals to obtain {7, 8, 9, χ, Ɛ}.

P. D. Thomas, based in South Australia, produced a couple pamphlets revised in 1987 entitled "Modular Counting" and "The Modular System". The proposed symbology features number forms which are purely creative, except for the fact that for the digits {1, 2, 3, 4, 6}, the number of "appendages" equals the integer it represents. In this interrupted range, Thomas' numerals function as analogs. Some symbols, such as "dow" (ϟ), representing two, derive "from the mirror image of the [Hindu-Arabic] figure 2". Others, such as "okt" (ǂ), representing 8, "being twice kaw [Thomas' symbol for four], it has the vertical line with two horizontal arms," the four having four points, the symbol for eight "doubling" the symbol for four ideogrammatically, but ending up with six points. Some elements have to be ignored in order to construct the numeral Thomas intends. In order to construct the notion of "one" from Thomas' digit-one, ignore the "flag" at its lower left. The appendages to Thomas' two and three sprout from a horizontal bar and a vertical pole, respectively. Thomas writes that some of his numerals, for example, "vin" (λ), are "entirely arbitrary". Thus Mr. Thomas' symbology is a less-purely analogous system, with an interrupted and impure basic additive run sequence:

R. J. Hinton's proposal, published in the *Dozenal Newscast*, Year 3, № 2, page 7, were developed having "visually consider[ed] each [numeral] with the amount it represents". The basic run sequence {1, 2, 3} enumerates loops. A second run sequence {4, 5, 6} presumes 3 is added, and loops are again enumerated, the new loops being arranged in a distinct manner from the first sequence. The sequence {7, 8, 9} continues the same idea presuming 6 is added to the enumeration. Hinton's article conceptually employs a system of dots similar to those seen on dominos, around which he builds new numerals. The numerals for digit-ten and digit-eleven appear to be mildly related to the pattern of dots Hinton used to build his set [3]. Hinton's symbology basically runs an

Figure 4. *Studies of the additive analog run sequences of several separate identity symbologies.*

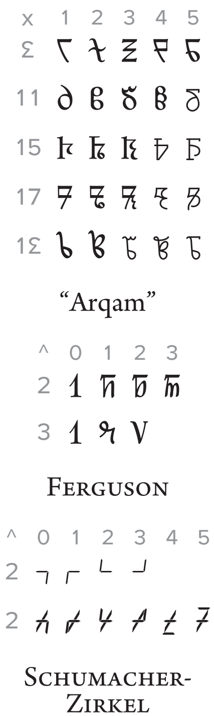
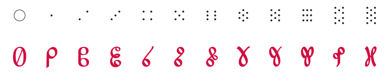


Figure 5. *Studies of the multiplicative strategy exhibited by the large "arqam" least change symbology, and exponential strategy for Shaun Ferguson's and the Schumacher and Zirkel-modified binary-coded analog separate identity sequences.*

analog strategy in three groups of three, then abandons the strategy at digit ten:



Fred Newhall, past DSA President, introduced his system of "efficient number symbols" in Vol. 2ᘔ; № 3 on page 16;. The system functions similar to shorthand and is unique in that it is truly cursive. The system is not analog but is arranged in a series of brief run sequences. Mr. Newhall generally arranged odd and even numerals in pairs, with the odd number reversed or repeated to obtain the next numeral. The digit-eleven perhaps supplies the symbol for zero through rotation:

Positional / Rotational Additive Analogs. A system of numerals can be produced by rotating one symbol to indicate the numeral's value, much like on a clock face. The purest manifestation of this idea follows. In the nineties, Prof. Bill Lauritzen devised a set of "gravity-generated numerals" in his paper "Nature's Numbers," retrieved at http://www.earth360.com/math-naturesnumbers.html. The numerals function as miniature six-position analog clocks with one hand that indicates the integer the figure represents. Numbers smaller than 6 have hands on the interior of a circle, those six or greater have hands outside the circle. Lauritzen's symbology is perhaps among the more purely geometrical and abstract:

Other Analog Strategies. Symbologies can be constructed using operations other than addition to produce numerals. Numerals can be related visually with their multiples or exponents (see Figure 4). The "arqam" system, illustrated in a later part of this article, produces a more or less arbitrary symbol for a transdecimal prime number (transdecimal meaning a numeral representing a quantity larger than the decimal single-digit numerals), modifying these in a way such that all of the numeral's multiples bear some distinguishing element from the basis prime numeral. Shaun Ferguson's system, illustrated in the *Dozenal Newscast*, Year 2, № 1, page 10; at first blush appears to be a system of purely creative symbols. There is a relationship among the numerals {2, 4, 8}, the exponents of 2, and somehow digit-ten. However, no similar relationship appears to be shared between the multiples or exponents of three [4]:

Indexed Value Strategy. The 7 segment LCD/LED readout plays an important role in Bill Schumacher's hexadecimal symbology, described in Vol. 33 № 3 page ᘔ. The numerals are read clockwise, representing bits, starting from the bottom right element. This element represents the unit bit. The bottom left element represents the presence of two when activated; the top left element four, and the top right element eight. One simply totals up the bits to read the numeral. The Schumacher numerals were expanded by Prof. Gene Zirkel to encode five dozen four digits. The Schumacher symbology and its Zirkel extension (illustrated later in this article) employ an indexing strategy wherein elements are assigned values, then these values are totalled to construct the overall value of the digit:

George P. Jelliss posted an entry in the DozensOnline internet forum describing a set of numerals which function much like the Schumacher / Zirkel symbologies. The Jelliss separate identity symbology takes advantage of the 7 segment LCD/LED readout to supply a matrix of elements which can be assigned indexed values. The two leftmost segments serve as a placeholder in all digits, effectively representing a value of zero. The middle horizontal segment has a value of 1, the top and bottom horizontals 2. When the two rightmost segments are activated, a value of 6 is added to the digit[5]:

Modular Symmetry Strategy. Rebuilding numerals for each dozenal digit enables one to take advantage of the high divisibility of the dozen and express the many modular relationships among twelve's digits. Decimal numerals might take advantage only of the relationship between 2 and 5, but dozenal numerals may be crafted to express the sequences {0, 2, 4, 6, 8, ᘔ}, {0, 3, 6, 9}, {0, 4, 8}, and {0, 6} in a manner that accentuates the fact, for example, when you see a 3, it's ¼ of a dozen. J. Halcro Johnson's reverse notation numerals possess a symmetry which expresses relationships between the thirds {0, 4, ⱡ}: any numeral involving the form "4", whether erect or reversed, signifies an exact third of a dozen. Odd quarters are likewise represented in the sequence {0, 3, 6, Ɛ} by the "3" form, and the half is represented by "6".

My own 2003 "acýlin" or cyclical dozenal symbology repurposes the Hindu-Arabic numbers to fit the cycle of dozenal digits. In this system, the sequence {0 3 6 9} appears as {0 3 8 Ɛ}, so all quarters or multiples of three appear as some manifestation of the "three" base form. The sequence {0 4 8} appears as {0 3 Ɛ}, so all thirds or multiples of 4 use the "3"

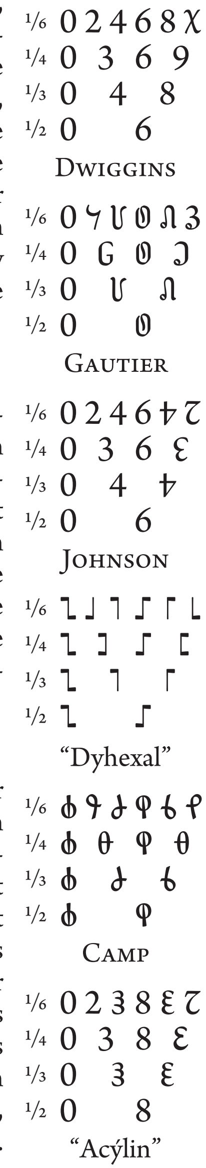


Figure 6. *Studies of various modular symmetries of some separate identity symbologies. The rows of each study lay out digits divisible by 2, 3, 4, or 6.*

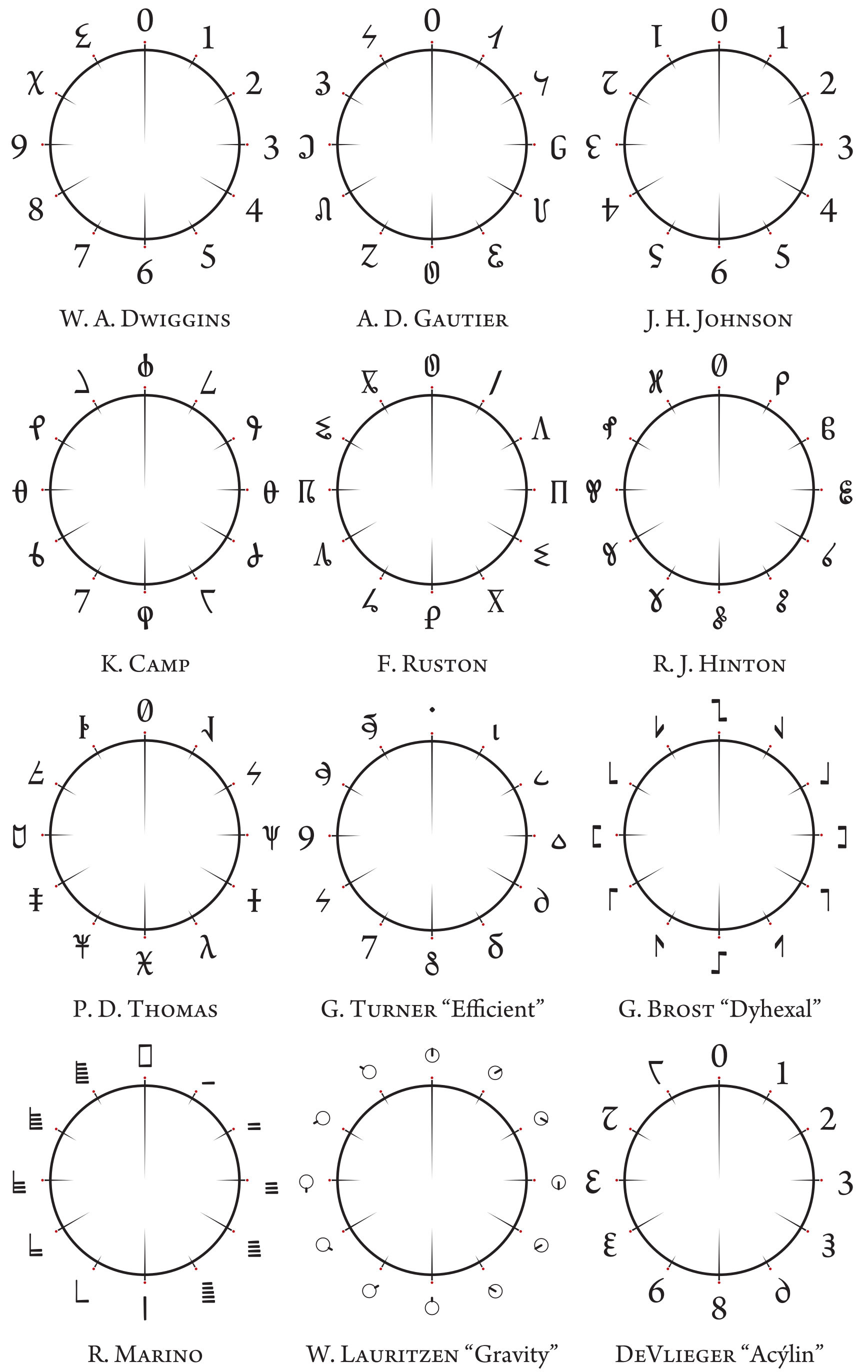


Figure 7. *Cyclical studies of several separate identity symbologies.*

base form. The symbol "8" is repositioned, because of its symmetry and likeness to a "twisted zero", to signify six. The concern here is appearance, although the set could be used to represent reverse notation as well.

0 1 2 3 Ɛ ∂ 8 6 Ɛ Ɛ Շ ⌜

A. D. Gautier's symbology of 1858, published in the *Dozenal Newscast*, Year 2, № 1, page 10; accentuates dozenal halves, quarters, and thirds. Gautier seems to have derived his numeral 6 (0) by splitting the zero in half. The odd quarters {3, 9}, represented by {G, Ͻ} are either half of the zero, with minor distinguishing marks added, perhaps, to reduce their confusion. The thirds {4, 8} are represented by {Ʊ, Ω}, more or less a "v" or "u" shape pointing downward for 4 and upward for 8.

0 1 ϥ G Ʊ Ɛ 0 Z Ω Ͻ 3 ϟ

Gerard Robert Brost's "dyhexal" symbology, which appears in Vol. 35; № 2 page χ;, uses numerals which function as analogs which might be read like hands on a clock to literally indicate which digit the symbol represents. He states, "Identification is aided because when the numbers are arranged in order they form a symmetric pattern". Later he adds "All numbers that are evenly divisible by two but not three (i.e. two, four, eight, and dec) have one horizontal feature in one of four positions (┘ ┐ ┌ └)." Thus Brost's symmetrical symbology is devised to aid the user in adapting to and interpreting the new symbols. Due to Brost's conscious inclination to produce a symmetrical set of symbols, his system automatically accentuates the modular relationships among the dozenal digits. The halves {0, 6} are represented by {ℸ, ʃ}, odd quarters by {ℶ, ℂ}, thirds by {┐, ┌}, and the sequence of sixths, {0, 2, 4, 6, 8, χ} by {ℸ, ┘, ┐, ʃ, ┌, └}. Additionally, relatively prime digits {1, 5, 7, Ɛ} are characterized by numerals with slanting "flag" strokes {ꜜ, ꜛ, ꜛ, ꜜ}:

ℸ ꜜ ┘ ℶ ┐ ꜛ ʃ ꜛ ┌ ℂ └ ꜜ

Kingsland Camp described more than one "ideal" set of numerals in his article "Number Symbols" in Vol. 2 № 1 page 16;. These he contrasted with the Least Change or "transitional" symbologies, including the DSA's classic Dwiggins numbers. The symmetrical geometries of his numerals were intended to more purely convey the digit they represent, by their rotational orientations. Halves are represented by {ȸ, φ}, circles with vertical strokes through their centers, odd quarters by {θ, θ}, circles with strokes left or right. The numerals representing the range of quarters {0, 3, 6, 9} can be read like clock faces: {ȸ, θ, φ, θ}. Camp's notation expresses thirds by {ɗ, ɓ}, and the sequence of sixths by {ȸ, ɖ, ɗ, φ, ɓ, ƍ}. Thus any numeral representing a sixth of a dozen not already covered by the quarters appears as a "fish" shaped symbol. Camp's notation also treats the relatively prime digits {1, 5, 7, Ɛ} in a way that accentuates them, a sharp angular numeral opening up to the numeral's position on a standard clock face: {∠, ⌜, 7, ⅃}:

ȸ ∠ ɖ θ ɗ ⌜ φ 7 ɓ θ ƍ ⅃

Application to Subdecimal Bases. We've considered dozenal and some hexadecimal proposals in this article and in Vol. 4χ; № 2. Symbologies are not necessarily limited to these bases. Arguably the solution for "subdecimal" bases (those

Figure 8. *The first six dozen arqam.*

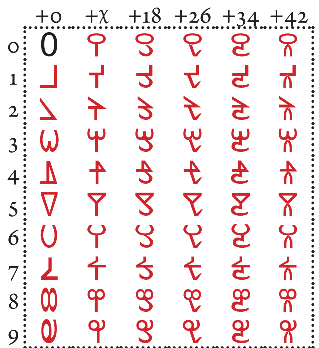


Fig 9. *Smith's sexagesimal numerals*

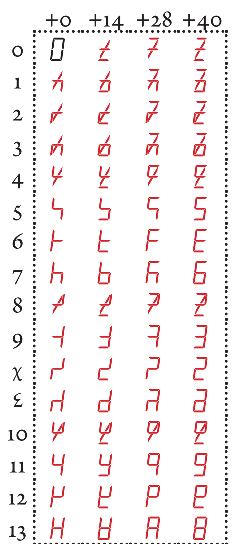


Fig Χ. *Zirkel's expansion of Schumacher numerals*

smaller than ten) is simply to use the requisite portion of the Hindu-Arabic numeral set, ignoring the rest. This is what is encountered in practice, concerning octal and binary. In theory, one could devise other solutions for subdecimal symbologies. The binary used on NASA's Voyager Golden Record (see http://voyager.jpl.nasa.gov/spacecraft/images/VgrCover.jpg) uses a horizontal line to signify zero, a vertical line to signify one. This is a simple and pure rotational strategy:

An octal proposal which functions much like Schumacher's hexadecimal proposal can be seen at http://www.octomatics.org/. This symbology uses a pure exponential indexed value strategy, similar to the Schumacher and Zirkel hexadecimal and base five dozen four symbologies:

Exceedingly Transdecimal Bases. When we are working with "human scale" number bases, between about 6 or 7 and around one dozen or one dozen four, it's not too difficult to devise sufficient numeral symbols. The challenge increases in proportion to the size of the number base. When the base under consideration is several multiples of ten, the opposed principles begin to resemble one another. This is because the retention of the ten Hindu-Arabic symbols represents an increasingly smaller subset of the total number of necessary numerals.

Least Change. I produced and use a series of transdecimal digits called "arqam", Arabic for "numbers", which at first covered hexadecimal, then was extended to sexagesimal and beyond. Currently there are over two and a half gross total contiguous symbols. The digits zero through four dozen eleven appear below. This is a "creative" least change proposal writ large. The system uses a multiplicative strategy to generate digits (see Figure 5). The multiples of one dozen five begin {ꝁ ꝁ ꝁ}, of one dozen seven {ꟻ ꟻ ꟻ}, and extend themselves ever higher using basic graphic elements called "radicals". A small tight circle, normally at the foot of a base form, tends to double the value of that base form, as seen in two dozen ten (ꝁ) compared to one dozen five (ꝁ). A three-pointed wavelike radical at the top of a glyph normally means the base form is tripled: as seen in one dozen nine (ꟻ), compared to seven (7). There are radicals for relatively large numbers like one dozen four (seven times 14; = 7 × ρ = ꟻ, ten times 14; = Ϛ × ρ = ꟻ) and even three dozen nine (three times 39; = 3 × ꝶ = ꝶ, six times 39; = 6 × ꝶ = ꝶ), as ever larger symbols are created. The symbols at first resemble plausible Hindu-Arabic-like numerals, then devanāgarī letters divorced from their top line, then Mandarin characters. The largest symbols in the set, which represent powers of prime numbers, use an exponential strategy similar to Ferguson in Figure 4 (powers of three: 1, 3, 9, ꝫ, ꝫ, ꝫ). See Figure 8 for the first six dozen of the "arqam" numerals.

Separate Identity. A rationalized, derivative Separate Identity sexagesimal symbology invented by Jean-Michel Smith appears at http://autonomyseries.com/Canon/Sexagesimal/. This symbology uses the "six on ten" or decimal "sub-base" arrangement also used by Babylonian cuneiform numerals, but applies this to Hindu-Arabic numerals. The numerals zero through nine are reinterpreted, rotated on their sides, and serve as a basic run sequence. Smith appends diacriticals to the bottom of the basic run sequence to shift the value of these ten basic glyphs up by multiples of ten. An example of this diacritical is the vertical line extending down from the "body" of the glyph, to add ten to the integer the body represents. This results in a relatively transparent "transitionary" system (to borrow the term from Kingsland Camp). Smith's numeral set is perhaps a direct conceptual descendant of Babylonian cuneiform numerals. (These symbols were created and produced by Jean-Michel Smith and appear in Figure 9 via the GNU license which is posted on his site.)

Prof. Gene Zirkel expanded Bill Schumacher's notation in Vol. 49; № 1 page 15;, extending the digits to represent the base of the sixth power of two. The kernel notion of this symbology was expressed in Vol. 33; № 3 page 11;. The bottommost horizontal element, when activated, represents the presence of one dozen four; while the topmost horizontal element represents the presence of two dozen eight. Zirkel's figures add a "slash" in the digits which preserves the integrity of the digits. See Figure Χ.

We have reached the end of our tour through dozenal symbologies and beyond, and some of the tools and strategies one can use to design them. This does not rule out further possibilities, nor does it mean that the classification or strategies presented here are the only valid ones. Again, this framework and the strategies mentioned herein are intended merely to serve as tools. We look forward to your own creations, or comments on the numeral sets shown here.

Notes:

1 Retrievable at time of publishing at www.dozenalsociety.org.uk/basicstuff/hammond.htm, part of the official website of the Dozenal Society of Great Britain.

2 Retrieved in early 2009, originally at www.geocities.com/nigellus_albus/whittenswords/measure/dozchar.htm, currently at www.angelfire.com/whittenwords/measure/dozchar.htm. Mr. Whitten recently devised a third generation of his numerals.

3 The dot patterns displayed above Hinton's numerals do not precisely correspond to his original article.

4 This set of Ferguson numerals appears as Mr. Ferguson conveyed through personal correspondence in early December 2009.

5 Retrievable from the DozensOnline internet forum. George P. Jelliss (2005). Symbols for TEN and ELEVEN? (Internet forum thread in the "On Topic" Forum, "Number Bases" topic) http://z13.invisionfree.com/DozensOnline/index.php?showtopic=11, entry by user "GPJ" at 10:54 am 14 August 2005.

problem corner

solution from page 8; *by Jean Kelly*

Note: Skip asked for a *procedure* to construct *a* set of integers which have the property that they can not be expressed as the sum of 4 non-zero square numbers.

We will find not only *a set*, but *the set of all* such numbers less that any desired limit. Our procedure is as follows:

First we choose an integer j and construct a set of some numbers N which do *not* have this property, that is a set of numbers which *can* be expressed as the sum of four squares. This set may not be complete — it may have skipped some such numbers. (Note, this is a solution to Skip's original problem.)

Next we determine a limit L such that the subset of all the numbers < L which are not members of N are a solution to Skip's teaser.

We use the following subroutine (in pseudocode below at left) to generate the set N:

```
SUBROUTINE Construct N
  READ(j)
  Loop FOR a = 1 to j DO
    Loop FOR b = a to j DO
      Loop FOR c = b to j DO
        Loop FOR d = c to j DO
          SET p = a² + b² + c² + d²
          WRITE LINE(a, b, c, d, p) {1}
        END FOR d
      END FOR c
    END FOR b
  END FOR a
END Subroutine Construct N
```

Four Numbers	Sum of their Squares
1 1 1 1	4
1 1 1 2	7
1 1 2 2	χ
⋮	⋮
1 2 2 2	11;
2 2 2 2	16;

For example with j = 2 this produces the output above at right.

These are all the possible numbers p, $3 < p < 4j^2$ which can be expressed as the sum of four squares *using only the numbers from 1 to j*. The smallest number that was omitted from this list is $L = 3 + (j+1)^2 = 3 + (3)^2 = 10;$ and hence all numbers < L which are not in the above list satisfy Skip's criteria, namely {1, 2, 3, 5, 6, 8, 9, χ}.

Finally, to find all the numbers less than a desired limit L — for example, say < than 2 dozen — we look for j such that $20; \le L = 3 + (j+1)^2$ [See work below at right.]

$$20; \le L = 3 + (j+1)^2$$
$$19; \le (j+1)^2$$
$$\sqrt{19;} \le j+1$$
$$4;7 \le j+1$$
$$3;7 \le j$$

The smallest integer j is 4, and $L = 3 + (4+1)^2 = 24;$

Now replace line {1} in the subroutine above to simply read:

```
WRITE(p).
```

With j = 4 the output of our subroutine will be the set N:

4, 7, χ, 10, 11, 13, 14, 16, 17, 18, 19, 1χ, 1χ, 21 ... along with other numbers larger than 2 dozen. Therefore the numbers less than 2 dozen satisfying Skip's problem are: 1, 2, 3, 5, 6, 8, 9, Ɛ, 12, and 15.

QED

Postscript

For a complete program to find all the integers less than a given number G, we solve:

$$G < L = 3 + (j+1)^2$$

for j which yields: j = the smallest positive integer > $\sqrt{(G-3)}$ -1

In addition, instead of printing the numbers not satisfying our criteria, we can store them in an array and then output the numbers which we do want.

DozensOnline Forum: Symbols Debate

A. Introduction

Objectives of the Digest. This digest aims to bring one discussion thread at the DozensOnline internet forum regarding symbology to our readers' attention. From time to time we will visit the Forum to sample what others, some within our Societies and many outside of them, are discussing. A digest can't truly be a complete record of the proceedings of the Forum; it's only a sample, and samples can be party to bias. Posts cited here are in no way the only views on the Forum. You are welcome to visit the Forum and read the posts for yourself, and come to your own conclusions. Better still, you can add your own views to this or any other discussion thread.

About DozensOnline. Two individuals from the Dozenal Society of Great Britain maintain the DozensOnline Forum (retrievable in 2010 at http://z13.invisionfree.com/DozensOnline). The board is moderated by Mr. Parry and DSA Honorary Member Shaun Ferguson. The Forum was initiated 2 August 2005, by Mr. Bryan Parry. One may visit the board and read posts as a guest, however to start new discussion threads or to reply to posts, one needs to register. Anyone may join the Forum simply by registering a username and password; registration is free of charge. There are simple rules of etiquette posted within every subforum. At the time of writing, there were about one and a half gross members, only 15; of which have posted more than three dozen messages, and four have posted around three gross or more messages, out of a total of nearly two dozen six gross posts.

The discussion thread of interest in this digest is titled "Symbols for ten and eleven?", directly accessible by entering http://z13.invisionfree.com/DozensOnline/index.php?showtopic=11. We'll nickname this discussion thread "Topic 11" for simplicity. If you're familiar with the Forum, you can also navigate to this thread via the board index, clicking "On Topic", then "Number Bases", examining the menu until you see the discussion thread title, and then finally clicking on it. There are 101; posts in this thread.

Forums (fora) work a bit differently than some other literature formats. Firstly, only one's username appears to identify who's posted an item. We've indicated usernames by placing «double angle brackets» around the username. One can click on the username and get some information on the user, however, most users on the Forum have elected not to disclose their true identity. The true name of the contributor is used when known to the Editor, while some information about those contributors whose true identities are not explicitly revealed are included after their username. Secondly, all posts have a time stamp, and are sorted chronologically. Theoretically, one can find any post by knowing under which topic the post occurs, and the time stamp. For brevity, the time stamp is listed for each post citation, in the year-month-day-24 hour time format. Over time, content in the Forum, especially linked material, tends to be lost; this article cites the Forum as it appeared in June 2010. Finally, internet forum discussions tend to be informal; some of the text has been corrected so that it is understandable.

Some of this work derives from a document written by Mr. Ray Greaves, which can be found at http://base12.plus.com/DozensOnline/Symbols.doc, or at Topic 11 at time stamp 2006 0712 1058. This digest includes a broader scope of comments on the Forum. A more in-depth examination of the discussion of seven segment display numerals follows.

B. Selected Forum Statements Regarding Dozenal Numerals.

Bryan Parry *(«The Mighty Dozen», from Middlesex, England)*

2005 0802 2056: I personally favour the DSGB's symbols over the DSA's. However, whatever symbols we need should exist right now in fonts that are easily available.

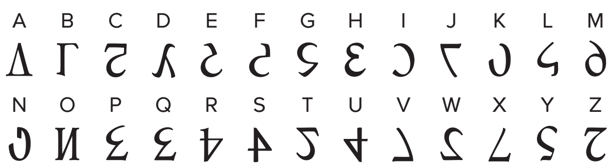
2005 0803 1239: Cyrillic is useful because we don't use them in this country, of course. And the symbols 'slot' quite well into the Roman alphabet, mostly. Plus the symbols are easily available. And that point is key — new symbols is well and good (and is the ideal solution), but sadly, if we don't have those symbols in word or unicode, then how can we ever use them but by handwritten correspondences? It's not ideal, but *if* we can find good symbols that already exist, then we ar[e] on to a winner."

«Rosie» *(Born 1979, from Roehampton, England)*

2005 0803 1232: "My thinking is that the new symbols should be easy to write and not overlap with any other commonly used symbols (so I'd avoid Greek letters or those from the regular alphabet)."

Daniel White *(«Twinbee», from Bedford, England)*

2005 0806 1620: "I agree that it would be nice for them to not clash with any mathematical symbols. Below are some of my favourites. One or two of them are from the DSGB range. Others are borrowed from here and there (including this thread), and flipped, chopped and twisted. And a few of them I made myself. Some of them may look a bit weird initially, but I think you'll find that they look more like the numbers 0 – 9 than most symbols do. Aesthetically, I think that's quite important.

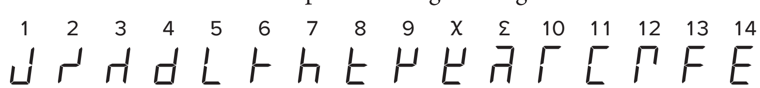


«finlay» *(Born 1987, British time zone)*

2005 0806 1008: "whatever we end up with, it should be writable with a single stroke of a pen or two short ones." «finlay» likes 'E' for eleven provided it is executable in "one stroke", as 'Ɛ'.

«Dan» *(Born 1982, from Houston, TX)*

2005 0814 1716: "First of all, [the numeral-candidates] should "look like" digits. Like the current decimal digits, they should be connected, and require the full height of the display. But, to prevent confusion, they should not be identical to the current digits (including the variant forms of 1, 6, 7, and 9). Symbols that differ from the current digits by only one segment should also be avoided, to avoid misreading broken displays. These rules reduce the number of possible 7-segment digits to sixteen:"



«DoubleG» *(Guillaume G., US Eastern time zone)*

2005 1208 1929: "[Some] worry about selecting characters among those found on keyboards (without too much acrobatics) and typewriters. First off, typewriters are a thing of the past. Secondly, this is way too restraining to do and pointlessly so. Same goes with calculator-style displays."

"Choosing among Greek characters would have frustrating consequences for math people, as has been noted. And broader interest just might be easier to get if people of various 'alphabetical-heritage' don't see conflict with the characters they already work with everyday. … Handwriting has to be easy and quick."

«Shaun» *(Shaun Ferguson, DSGB)*

2005 1211 1243: "1: The trouble with a handwritten x with a bar at the bottom [X] is that sloppy writing makes it look very much like an 8. I've known people who wrote their 4 starting at the bottom - making it look more like a 9. Whatever we choose will have to be something sloppy handwriting cannot spoil.

2: 'T' and 'E' — as someone else pointed out — are fine for the English-speaking world (Romance languages might prefer 'D', as in dix and diez). But they would do as a temporary standard if we can achieve agreement — which leads me on to

3: Apart from [Ɛ], which stood some chance of being acceptable to both DSGB and DSA until the DSA started using [Henry]Churchman's proposal of the 'hash' (also called 'octothorpe') [#] for eleven, there never has been agreement on the symbol for 'ten'. At one stage there was the possibility that a form of 'X' might do, as [Issac Pitman's ↊] was not acceptable to many, but the DSA abandoned [Dwiggin's X] for the 'star' [*]. The hash and star might well be on the phone keyboard but that doesn't make them counting numbers."

[Point] 3 is the sticking point. We have been discussing the symbols for some sixty years and still haven't come up with two that are acceptable to the (small) group of people who make up the two societies. Maybe we need some sort of internet questionnaire aimed at everybody, along the lines of 'If you were asked to invent a single symbol to represent ten, what would you suggest?' noting in passing that 'A' is unacceptable."

«växan» *(Stockholm, Sweden)*

2006 0209 2008: "for new numerals to really work (in any number base) they must :

1. match the other (existing) numerals
2. work with 7-segment displays
3. be easy to write by hand with as few strokes as possible"

"so i would first suggest re-arranging 7-segment elements into shapes that are not mistaken for other numerals, and also that don't look like a burned out display element then take these shapes and work them into hand written numerals which match the other 9 indian numerals (yes they are indian, not arabic)"

"one obvious clue that a numeral is not working is that it stands out like a borg in a nudist camp [= like a sore thumb.]"

Andrew Patterson *(«Endi», European time zone)*

2006 0223 1022: "I do appreciate the worry over 7 segment displays but cannot over-emphasise that new display technologies are already out that are as cheap or nearly as cheap as 7-segment displays but which are able to display a far greater variety of symbols.

"… I would say the only criteria should be that they:

1. are no more complicated than letters,
2. are easily distinguishable from letters,
3. are easy to write, and

Subjectively, they:

4. are aethetically pleasing to the eye, and
5. just look like numbers"

C. Criteria for Numerals as Implied on the Forum

Table 1 lists many of the numeral-candidates proposed in Topic 11. Each symbol is listed according to the contributor, the time stamp of the post, and a seven-segment display rendition of the symbol. Mr. Greaves' document "Symbols.doc" contains a set of criteria which, coupled with the postings quoted above, serves as the basis for the following criteria for selection of dozenal numerals for digit-ten and digit-eleven.

1. **Distinction**

   It should be easy to read the proposed numeral as a unique symbol, minimizing potential confusion with other symbols in the public lexicon. Candidate numerals that resemble Latin, Greek, or other characters should be avoided. By some, this avoidance extends to such characters in any of their major treatments (handwritten, printed, computer-generated, segmented display).

   - Easily distinguishable from English-use letters (Parry, Patterson).
   - Avoid Greek letters or mathematical symbols (White, «DoubleG», «Rosie»).
   - Multicultural consideration; avoid appearance of nonwestern characters (Ferguson, «DoubleG»).

2. **Clarity**

   Proposed numerals should be robust, to resist confusion by hurried or sloppy handwriting. Preliminary evaluation of possible malformed proposed numerals and test-execution of the proposed numerals can help control their clarity.

   - Numerals should be resistant to confusion by sloppy handwriting (Ferguson).
   - Seven-segment display numerals should avoid the appearance of regular numerals with "burned out" elements («växan», «Dan»).

3. **Ease of Writing**

   The ideal result seems to be numerals that require at most 2 strokes and don't involve lifting the pen.

   - Writable in a minimal number of strokes («finlay», «växan»).
   - Quick and easy handwriting (Patterson, «DoubleG», «växan»).
   - Are no more complicated than letters (Patterson).

4. **Visual Unity**

   Proposed numerals should work well with the existing decimal numerals [implying Ralph Beard's "Least Change" philosophy, cf. Vol. 1 № 3 pp. Ɛ–11;.].

   - New numerals should resemble the Hindu Arabic numerals or "look like" numbers (White, Patterson, «växan»).
   - Symbols like the Churchman-proposed { *, # } may be fine in their original application, but don't make acceptable numerals (Ferguson).

5. **Minimization of Impact**

   Some support creation of new numerals which fit existing representational technology, in the spirit of reducing or eliminating resistence to the introduction of a dozenal system.

   - Work with seven-segment displays («växan»).
   - Don Hammond, Niles Whitten, Ray Greaves, George Jelliss, William Schumuacher, and others have produced numeral sets which mesh well with the seven segment LCD/LED displays.

Table 1. DozensOnline Suggested Numerals

	Source «Username»	Topic Number=11 Timestamp	Digit-ten Symb.	Digit-ten Disp.	Digit-eleven Symb.	Digit-eleven Disp.
1	DSGB [Issac Pitman, DB 03·2·01]		↊	5	Ɛ	E
2	DSGB [Don Hammond, DB 4χ·2·13]		ƻ	ᒧ	Ɛ	Ŀ
3	DSA [Kramer-Bell, NR 02·1·11, DB 25·1·02]		*	—	#	—
4	DSA [Dwiggins, DB 01·1·02]		χ	H	Ɛ	E
5	Bryan Parry [citing the common "T & E"]		T	—	E	E
6	Bryan Parry [citing the use of Dwiggins]		X	—	E	E
7	Bryan Parry (in resonance with "A & B")		Γ	Γ	Э	Ǝ
8	Bryan Parry	2005 0802 2056	β	B	Σ	E
9	Bryan Parry ("Δ Λ resembles 'A'")		Δ Λ	Π	Σ	E
χ					Π	Π
Ɛ	Bryan Parry (analogs to X=10, II=11)		Ж	—	Њ	—
10					И	—
11	«GPJ» [George P. Jelliss, DB 36·2·14]	2005 0803 1516	δ	d	Ɛ	E
12	«genito» [Gene Zirkel, "Bell" numerals]	2005 0803 1603	Ӿ	H	♮	E
13		2005 0805 1842	Ø	—	Î	—
14	«adolfzero»		†	—	‡	—
15			Ç	—	€	—
16	Daniel White	2005 0806 1620	*See Section B*			
17	Jean Essig [DB 10·2·48, posted by Parry]	2005 0807 1817	↊	—	∠	L
18	«EAP»	2006 0123 0134*	⍟	—		
19	«växan»	2006 0210 0226*	ɑ	Π	ɑ	Ɦ
1χ			Ↄ	⅃		
1Ɛ	«ruthe» [Ray Greaves, DB 4χ·2·13]	2006 0712 1058	⊣	ᖷ	ᒥ	Γ
20	«Dan»	2005 0814 1716	*See Section B*			

Table 2. Other DozensOnline Symbology Discussions

Topic Title	№
"Symbols for TEN and ELEVEN"	11
":A & :B <design compo! fun!>"	125
"More symbols, from the Cherokee font"	161
"Digits, Ideas for new digits"	208
"Number representation, != 0-9 in base12"	262
"New Symbols"	328
"Hexadecimal Digits"	331
"Ligatures for hexadecimal and duodecimal glyphs"	333

All topics are accessible from the "On Topic: Number Bases" board. Visit the topics in question by entering the number in the right column after "http://z13.invisionfree.com/DozensOnline/index.php?showtopic=". *This list is not necessarily complete.*

Figure 1. *Daniel White «Twinbee» 2005 0806 1620 posts a graphic containing two dozen two numeral candidates which "look more like the numbers 0 – 9 than most symbols do."*

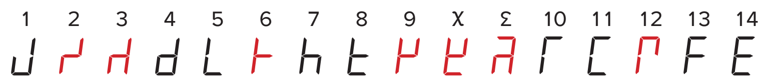


Figure 2. *«Dan» 2005 0814 1716 posts seven-segment configurations which are not identical to the current digits and their variants, but are contiguous and stand the full height of the display. Additionally, the candidates must not differ from the current digits and their variants by one segment. Digits in red are among the two dozen later selected by «Twinbee» (see Figure 3). None of these candidates are considered by «Dan» in his later post as acceptable (see Figure 4).*

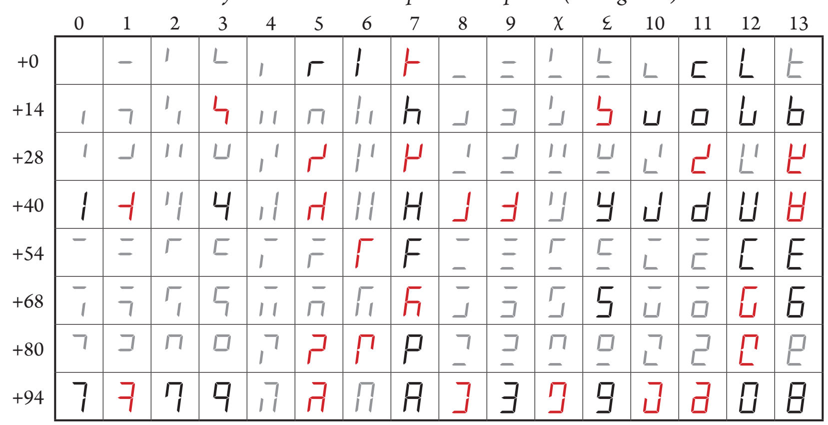


Figure 3. 2^7 *possible ways to compose seven-segment display-style numeral candidates, as posted by «Dan» 2006 0814 2314. User «Twinbee» 2006 0816 1831 used Mr. Greave's criteria similar to that shown in section C to identify 20; numeral candidates shown in red above, and in Figure 4.*

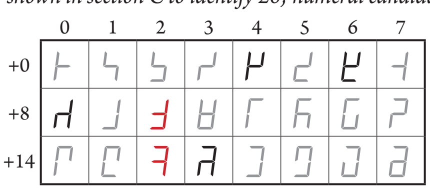


Figure 4. *«Dan» 2006 0817 0513 examines the Twinbee selection, deeming only two shown in red as acceptable, and four as "awkward to write".*

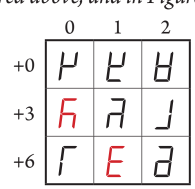


Figure 5. *«Twinbee» 2006 0830 0042 posts his favorite nine candidates; the two red candidates are deemed by Mr. Ferguson at 2006 0830 1358 as his selection out of this group. Note that 'E' is selected in violation of the section C criteria.*

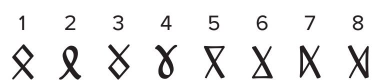


Figure 6. *A selection of other symbols discussed in the thread. The "Elder Futhark éðel/ethel" rune, and its rounded and inverted variants, and various ways of writing "X" in one stroke.*

D. Further Seven Segment Evaluations.

A contingent of supporters of a strong criterion 5 test and some simply interested in discussing the problem examined the permutations of a seven-segment display character to determine which of these configurations were acceptable. User «Dan» initially posted one dozen four configurations (see Figure 2). Seven of these were among two dozen configurations Mr. White selected in response to Dan's posting all χ8; possible configurations (see Figure 3). Mr. White, as «Twinbee», added "The missing two characters for dozenal are in there somewhere!" after having identified the two dozen symbols in Figure 4. Mr. Greaves requested comments, directing someone to screen the Twinbee candidates against his criteria, similar to section C. User «Dan» responded by evaluating each of Mr. White's choices given Greaves' criteria, narrowing the Twinbee candidates to the two shown in red in Figure 4 (See Figure 7, inadvertently skipping number 15 { }). The discussion of seven segment candidates moved away from criteria and back to preferences soon afterward. Mr. White posted his favorite configurations, from which Mr. Ferguson identified two as favorite. Discussion began to consider pixilated displays and moved away from exclusively considering seven-segment configurations.

Figure 7. *Evaluation of symbols in Figure 4 by user «Dan» at 2006 0817 0513:*

0) Looks like '+'.
1) Looks like '4'.
2) Looks like a broken '5'.
3) Could possibly be confused with '2' or 'Z'.
4) Awkward to write.
5) Looks like a broken '2' or a Spanish '¿'.
6) Awkward to write.
7) Looks like '+'.
8) Awkward to write.
9) Looks like 'J'.
χ) Requires lifting the pen but is otherwise fine.
Ɛ) Requires lifting the pen and could be confused with for-all symbol.
10) Looks like Gamma.
11) Looks like 'h'.
12) Looks like 'C'.
13) Looks like '?' or a broken '2'.
14) Looks like Gamma.
16) Requires lifting the pen but is otherwise fine.
17) Awkward to write.
18) Looks like '7' or ')'.
19) Looks like a broken '9' or '0'.
1χ) Looks like ')'.
1Ɛ) Looks like 'a' or '∂'.

problem solution in next issue

problem corner

Find the base, *b*, used in each of the following.
Hints: Each equation is written in its base, *b*. For example $47 = 4b + 7$ and $b > 7$. The base of a logarithm is an integer > 1.

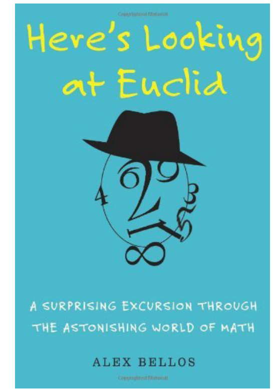
1.) $\text{Log}_b\, 24 - \text{Log}_b\, 3 = \text{Log}_b\, 8$
2.) $2\,\text{Log}_b\, 5 = \text{Log}_b\, 31$
3.) $\text{Log}_b\, 4 + \text{Log}_b\, 30 = \text{Log}_b\, 100$
4.) $\text{Log}_b\, 100{,}000 = 101$
5.) $-\text{Log}_b\, 100 = -2$
6.) $\text{Log}_b\, 5 = -2$

Book Review

Here's Looking at Euclid:
A Surprising Excursion Through the Astonishing World of Math

Author: Alex Bellos ~ Free Press, NY 2010

Reviewed by Gene Zirkel



The above subtitle of this wonderful book uses the words "Surprising" and "Astonishing". These are not merely pr superlatives. I have seen many such books which covered a variety of mathematical topics but never one that started out relating the story of an Amazon tribe who have no words or symbols—not even a concept—for more than five!

A man with half a dozen children, if asked how many children he has, will respond, "I don't know." For him a number greater than five is impossible to express; the idea is simply ludicrous. In fact he might easily become suspicious of the questioner thinking, "Why is he asking such a question?".

It is difficult for me to understand, to grasp, a culture in which counting is not an everyday activity which is easily understood by everyone. And this is merely Chapter Zero!

I said that this is a wonderful book. And it is. And that is not only because Alex Bellos does a wonderful job explaining dozenal counting in his second chapter, namely Chapter One.

> [Bellos] has a knack of conveying his topics clearly with a minimum of technical language

In addition, when he wrote about Amazonia, Bellos went to Paris to interview a scientist who had been there and studied the people. To write about dozenals, he interviewed our President, Mike DeVlieger. Before he wrote about probability, he went to Reno and observed the gamblers in action. In his book, he takes us on a world tour of the places he visited in order to research this book, including Germany, India, France, etc.

After he interviewed Mike—and I cannot think of anyone better that he could have picked to get the real skinny on dozenals—Bellos didn't give just a few sentences to the topic. No, as with each topic he devoted several pages to one of our favorite subjects.

One of the advantages that Bellos brings to his book is that altho he studied math along with philosophy at University, he never used it. He is not a mathematician. He has a knack of conveying his topics clearly with a minimum of technical language and of using some very good examples. I seldom had to resort to my dictionary to look up a word (altho *farinaceous* was one exception).

Many books include the famous (infamous?) dissection of a square of 64. square inches into a rectangle of 65. square inches(!), but Bellos neatly ties this old favorite to the Fibonacci numbers.

Another favored topic is Georg Cantor's transfinite numbers. Here the author's gift for using simple examples shines thru. Thus instead of the (more or less) standard example of the diagonal method of counting the rational fractions with the slight complication that 2/4 and 3/6 are two different fractions which represent the same rational number, he presents the proof as seats in busses where 2/4 represents the 4th seat in bus number 2 while 3/6 stands for the distinct seat number 6 in bus number 3, and so the extra complication disappears.

Similarly, when looking for an example of a hyperbolic surface (that is, one with negative curvature) he eschews the usual infinitely long, trumpet shaped psuedosphere and instead opts for the ubiquitous Pringle potato chip.

Cantor is only one of many old friends we meet again in Euclid. There are also many other standbys such as Nicolai Lobachevsky and Martin Gardner. What was an unexpected pleasure was coming upon not only my friend Mike, but also the most memorable teacher I ever had, Raymond Smullyan, who taught me advanced mathematical logic and Gödel's Incompleteness Theorem in graduate school along with many other things.

One gift I enjoyed was Haga's theorem about a single folding of a square piece of paper which produces not only three similar right triangles, but three Egyptian 3-4-5 right Triangles. Not being a text book, no proof was given, and I enjoyed proving it for myself.

Another gift Bellos gave me (and you, when you read his book) was the title to two movies I want to view and the titles of a book and a magazine article I want to read.

Bellos refers to Gardner as an "enthusiastic amateur". He could well be describing himself. If you are also in that category, get this book! Better yet, get your library to buy a copy so that others may enjoy it too.

Lest you think that I am looking at Euclid wearing rose colored glasses, let me warn you that the book is not perfect. One of the pluses that I mentioned above is that the author is not a mathematician. Unfortunately, this is one of the minuses also.

There are some errors, some typos and some careless wording scattered here and there in an otherwise very enjoyable book, As an author of several books I feel that some of these are the fault of the editors and the proofreaders. It is their job to find them. No one can write more than two gross pages of mathematics and not make a slip. Furthermore, from my own experience, once you make a mistake, it is nigh on impossible to find it yourself, for you keep reading what you thought you wrote, not what is on your manuscript. Here are some of the ones I found:

- The worst of these was the statement that a diameter wraps around the circumference of a circle "just over three times" instead of "just under three times".

- When speaking of Bolyai, Bellos says that he "dared to consider that the postulate might be false." Dared? Postulates are neither true nor false. They are assumptions which, as their name indicates, are postulated or hypothesized. This is merely the first step in a proof by contradiction, a method of proof the author himself uses in one of the appendices.
- In Chapter One, in a table illustrating some exact numbers versus some unending fractional numbers the three dot ellipsis has been left off giving the impression that 1/7 is exactly 0.14285 instead of 0.14285…
- At the end of Chapter Three, the adjective "decimal" wrongly implies that our place value system is elegant and efficient because it is based on ten, when the readers of this *Bulletin* know well that one based on a dozen is even more elegant and more efficient.
- In the same vein, dozenalists object to saying that in base twelve "a third becomes 40 percent" when it clearly is 40; pergross.
- Several times in the text, the author mentions that you will find further material in his appendices at the website for this book. But nowhere does he tell us the name of the website. Since I don't read at my computer, I simply made a mental note to check the appendices later. However, when I finished the book not only was I without the website, I was also without the references. They were not in the index. They were not in footnotes but were tucked in the text, and not easy to find.

  A search for the title led to more than 50,000 hits, many of them from booksellers. When I finally found the appendices[1], it took me quite a while to find the pages in the book to which they referred. I was quite disappointed when I did. Five of the six appendices were only one page in length and could have been easily printed in the text. Only one was worth the web search.

There are several other minor quibbles, but all-in-all I really enjoyed this book, and on a scale of 1 to 10; I give it a Ɛ;6!

[1] At alexbellos.com, simply highlight The Book and then click on Appendices.

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2005—2008	461-90—491-96	Prof. Jay Schiffman with Prof. Gene Zirkel
2008—	492-97—	Michael De Vlieger

the mailbag

Mr. John Earnest, DSA Vice President, № 250; suggests the following:

When writing a dozenal number, for example five dozen seven, write

$$57z \text{ in place of } 57;$$

which is reminiscent of the subscript notation

$$57_{12} = 67_{10}$$

What do you think? ::::

⁝ ⁝ ⁝ ⁝ ⁝ ⁝

Mr. Erich Kothe, DSA Life Member № 210; writes the following in reference to the article "Why is 1 One?" by Prof. Gene Zirkel, which appears in WN 99; Vol. 4X; № 2 p. Ɛ:

»Dear Gene,

Although I found the attempt to explain the shapes of our presently used numerals very interesting, it does not match with the references I have. As I can make it out, the presently most common numeration system which can be described as a "positional numeration system with the base ten" originated in India about 500. AD. The Indians (or Hindus) used it for recording a "result" after doing some manipulation on their abacus (counting board). In the beginning a dot marked an empty column, but later the "zero" appeared (possibly derived from the Greek letter "omega" [Ω] ?). There are several different sets of Indian or Hindu numerals. And Indian scholars started already way back to investigate "numbers", in particular the "zero".

The Arabs recognized the inherited advantages of these types of numeration systems. Al-Khwarizmi around 780. AD, developed multiplication tables and invented algebra. Several sets of Arabic numerals have been developed. The Western world got introduced to the western Arabic numerals by Fibonacci as they were in use around 1200. AD in Spain and north-western Africa.

I came up with symbols for "dek angles" and for "el angles". The "dek" would look like an eight but have an additional line from either the upper right corner to the lower right corner or from the upper left corner to the lower left corner. The "el" numeral would look like three triangles stacked upon each other where either the right or the left side of each triangle would form a continuous vertical line.

I would prefer the following symbols which are readily available on the computer: Þ [thorn, unicode 00DE] or Đ [eth, unicode 00D0] for dek, and Є [unicode 0404, Cyrillic space] or # for el. What are the other suggestions beside X or ↊ for dek? – My major reservation would be only against using the asterisk for dek because it is used already as a multiplication symbol on the computer.

Sincerely, Erich ::::

»Thanks,

Do you [the reader] have any suggestions for symbols with dek or el angles?

Gene Zirkel ⁝⁝

Mr. Jeff Wells sent in an email 26. December 2009.:

»Hello,

I've been interested in the dozenal system for most of my life, and have a proposal for reforming our clocks, calendars, and circular measure in general:

Imagine a clock face divided into a dozen sections as it is now, but with the zero hour (midnight) at the bottom, where the sun would be looking south from anywhere in the northern hemisphere (or looking north from the southern hemisphere).

Each section would represent two hours instead of one, and as the big hand moves clockwise it matches the path of the sun, rising in the east at 3 (on the left side of the northern hemisphere clock face), getting to 6 at midday, 9 at sunset, and back to zero at midnight.

Each of these 1/10; (1/12.)-day divisions is X0; (120.) of our current minutes long, and divided by a dozen again you get ten-minute segments, 100; (144.) of them per day (one more division by a dozen gives us our new minutes, each exactly 42; (50.) old seconds long). Using just a two-digit dozenal number then, we can specify any time of day to ten-minute accuracy, sufficient for most practical appointments and a great simplification over what we have now.

This same clock can also serve as a calendar, with the twelve divisions representing months instead of fractions of a day—and if the year were adjusted by merely ten days to start at the northern winter solstice, the seasonal cycle would nicely match the daily cycle - just add a "month hand" to the clock face and you have your calendar.

Circles can be much more coveniently divided into 100; (144.) degrees, and if these are further divided into 100; (144.) segments (let's call them anything but minutes to avoid confusion with clocks), each of these 10,000; (20,736.) segments of a circle would by an interesting coincidence be almost exactly 1;25 (1.2) English miles long on the surface of the Earth. The Earth's circumference could then be defined as 10,000; (20,736.) new miles (or "nyles") - and the difference between statute and nautical would disappear in the bargain.

It could happen!

Jeff Wells
San Diego, CA ::::

»Dear Jeff,

Thanks for contacting us. Your proposal is similar to (but not identical to) others' ideas. For example, Dr. Paul Rapoport has actually constructed a duodecimal clock with divisions into what some call "duors" and others "dours" which are then divided into ten minute periods. I am forwarding this email to him and to our editor for consideration in a future *Bulletin*. Unfortunately, [Vol. 4X; №. 2 WN 99;] is in production and it is too late for this issue whose theme is nomenclature and symbols. I think it will be one of our best.

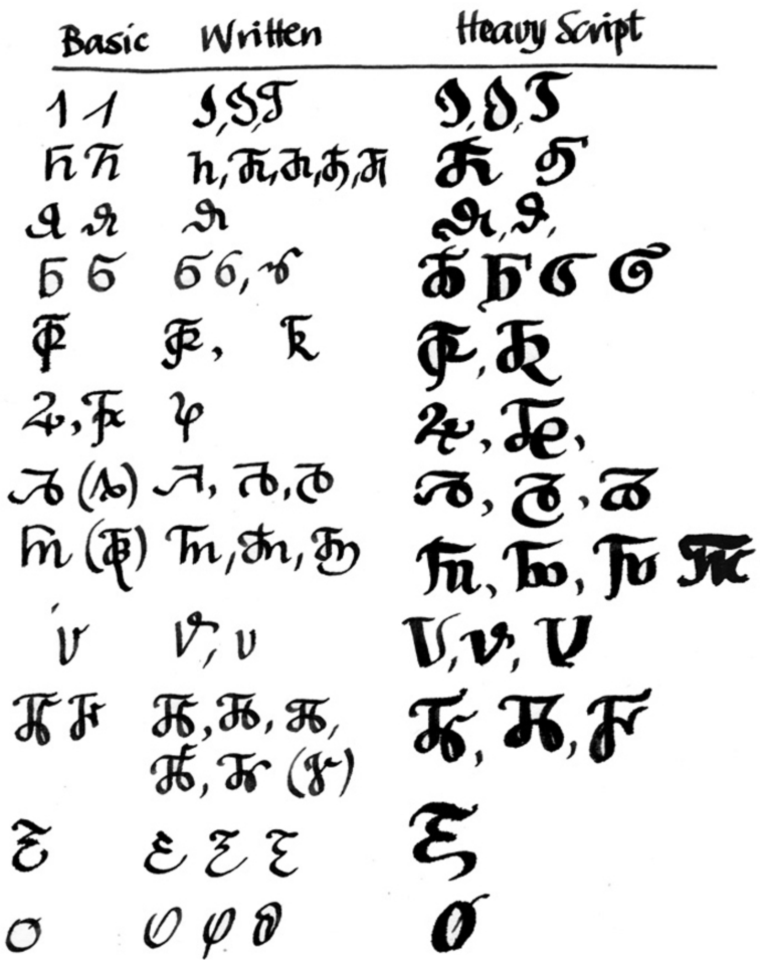
How many new symbols do you use: two or twelve? What are they? Have you ever considered joining us? Annual dues are only $18 and entitle you to an electronic *Bulletin*. Student dues are a mere $3 and they too receive an electronic copy. Supporting members pay $36 and can receive a paper copy.

Dozens of good wishes and a gross of good luck,
Gene Zirkel ⁝⁝

Mr. Shaun Ferguson, member of the DSGB, sent us this beautiful example of his calligraphic and creative symbology studies:

»I attach some ideas I wrote down on number base scripts (over 40; years ago …) which might interest you.

Best Wishes,
Shaun ⁘⁘



Mr. Ferguson's dozenal symbols, above. Shaun also sent in symbols for bases seven and eight. See our Bulletin, "Featured Figures: Symbology Overview", Vol. 4Χ; № 2 pp.13;-14; WN 99; *for some of Shaun's duodecimal numerals, and those of many others.*

»Shaun,

You certainly are a gifted calligraphist. The numerals simply look gorgeous! I really love the evolution or interplay between the numerals.

Cheers,
Mike ⁞⁞

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