THE Duodecimal Bulletin

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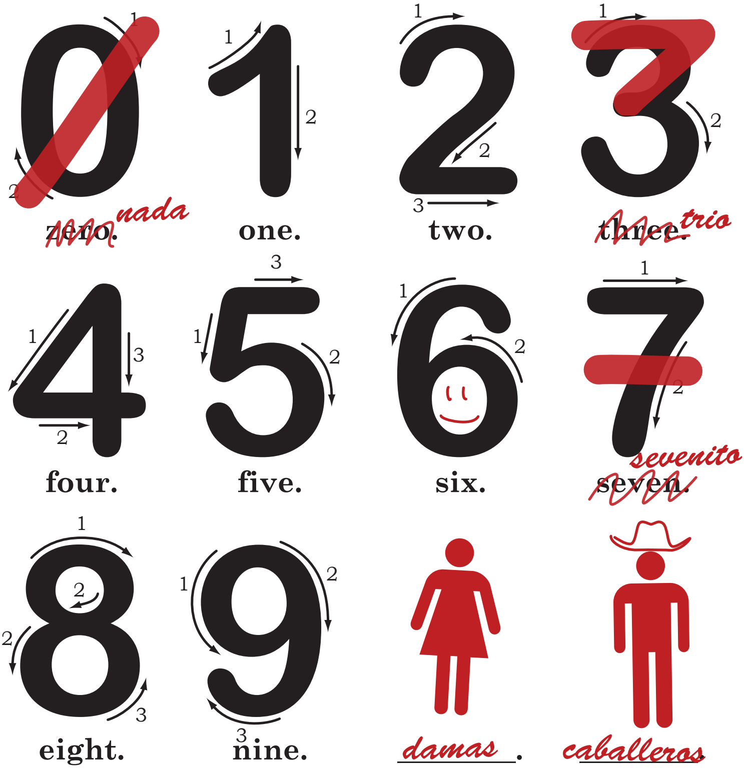
Exercise 1. Fill in the missing numerals.
You may change the others on a separate sheet of paper.



All About Our New Numbers

Whole Number

99;

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The Dozenal Society of America

is a voluntary nonprofit educational corporation, organized for the conduct of research and education of the public in the use of base twelve in calculations, mathematics, weights and measures, and other branches of pure and applied science

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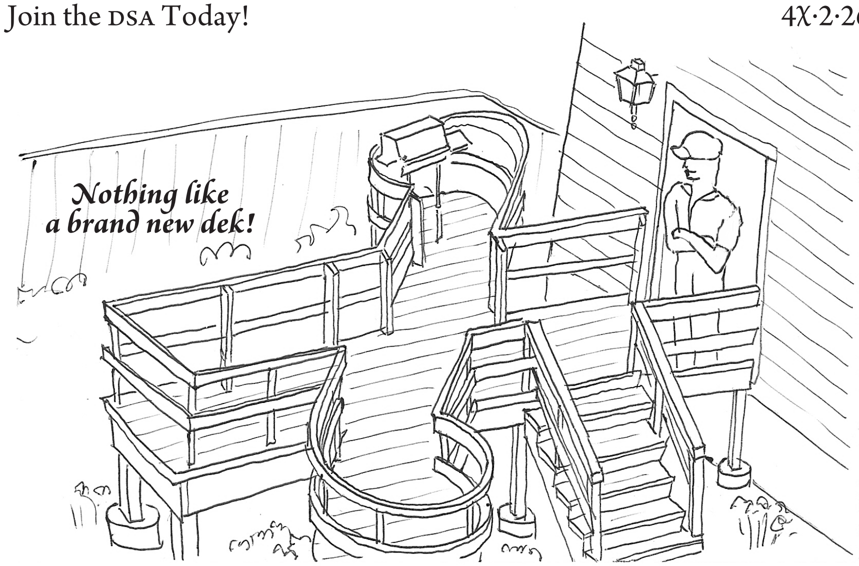
The Duodecimal Bulletin

Contents

The page numbers appear in the format Volume·Number·Page



president's message

Every dozenalist eventually encounters the question of new numerals—*symbology*. Unlike the comedic cover of this magazine, it's a serious system of questions. What are the appropriate numerals for such a flexible and optimum number base? Should we keep our decimal numerals, or dump them? Should we borrow symbols from elsewhere, or should we come up with completely new ones? If we're inventing new numerals, what are the criteria by which we should design them? What tools can we use to devise new numbers? What about scrawling them on a jobsite or at the supermarket? What about gas stations, how will those digital readouts display new numerals? Finally, once we do all this, can we type them on our laptops?

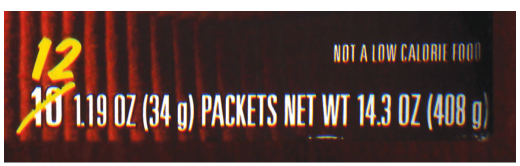
This issue will deeply explore the issue of symbology, as it is one of the foremost considerations on the minds of dozenalists. We invite you to take part in the debate; send in your thoughts and participate in our teleforum at our upcoming June meeting. Got your own numerals? Send them in. This issue provides some tools to get you started. We've taken many of the symbologies presented in the last six dozen years, rebuilt them, got them together for a group photo, and we studied them. Maybe you will be the designer of the "killer application", the symbology that proves to be the most appropriate dozenal set of numerals anyone might devise. Look for more information in the next issue. Remember, we're considering only the written shape of a number, not it's name. After a breather of a couple regular issues, we will tackle another topic.

If you like what you see, please, get involved. The material presented in this issue brings together in one place the passions of so many who've thought about dozens long before we did. Join us at our meeting, call us on the phone, write an email. We can also use your help producing articles like the ones in this magazine. Any donation would be appreciated, all our work is volunteered, but things like producing the magazine require some money. Look for information on our upcoming Membership Meeting in the next issue.

The DSA would like to honor the passing of two great dozenalists. Eugene "Skip" Scifres, a Life Member, № 11;, is featured on page 8. Mr. Rob Roy McPherson passed away in late November; his notice will be featured in the next issue.

I hope you enjoy this issue celebrating our New Numbers!

Michael Thomas De Vlieger,
President and Editor



Changing 10 to 12: our sentiments, exactly!

An Error in Arithmetic

by Jean Kelly

Lieutenant Kirk, having saved the peace-loving people of Zendo from being overrun and captured by the evil Malcides, was confused. Somehow he had managed to offend Queen Wevelta as she was thanking him and his crew. Looking back at what had happened he was baffled.

Wevelta declared that she had authorized her prime minister Sorg to give them 10 large gold pieces telling Kirk that 4 were for him that the other 8 were to be given to his 8 companions.

When he very politely attempted to point out that her 8 + 4 did not add up to 10 she at first merely laughed at him. However, when he persisted she flew into a rage and ordered him imprisoned for insulting her. "Women", he mused, "they are so emotional, so defensive especially when they make a mistake."

With that, Sorg entered the cell. He surprised Kirk, by trying to explain that it was Kirk who was wrong. When it became apparent to the prime minister that Kirk didn't follow his explanation, Sorg began to count on his six fingers…

Kirk immediately realized what had happened and apologized to Sorg. He explained his mistake and humbly begged her majesty's forgiveness. Realizing that the earthlings were biologically deficient, Wevelta not only felt sorry for them but she graciously forgave them.

As she explained to Kirk the multitude of advantages of dozenal counting, he clearly understood the points she made. Years later, as he climbed the ranks, he stressed the advantages of the duodecimal system of counting, weights, and measures. When his home planet finally did adopt the obviously better system Kirk was honored for his role in the improvement. They named the unit of currency which contained one gross of sub units as the Goldkirk.

Symbology & Nomenclature

The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use Dwiggins dek (χ) for ten and his el (Ɛ) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "dough" in the duodecimal system.

When it is not clear from the context whether a numeral is decimal or dozenal, we use a period as a unit point for base ten and a semicolon, or Humphrey point, as a unit point for base twelve. Thus ½ = 0;6 = 0.5, 2⅔ = 2;8 = 2.66666..., 6⅜ = 6;46 = 6.375

THE OPPOSED PRINCIPLES

Ralph Beard, October 1161; (1945.)

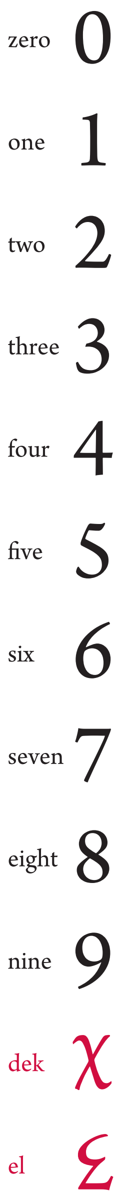


Figure 1: A least change symbol set, this by William Dwiggins

Members of the Duodecimal Society are somewhat accustomed to the mention of the Principle of Least Change, and the Principle of Separate Identity. But comment from the members seems to show that a clearer exposition of these principles would be appreciated.

Since it is important that these principles, and the divergence which they represent, be thoroughly understood so that they may be advantageously applied, an attempt will be made to bring these principles into better definition.

Duodecimal proposals divide themselves, readily, into two groups. These groups are named for the principle which typifies each.

Classification under the Principle of Least Change

Most duodecimal proposals are conceived with the fundamental purpose of making that specific proposal most acceptable to the mind of the general publis. They are quite easily characterized as embodying the Principle of Least Change.

Least change

They usually contemplate no change in the names and the symbols for the first nine numbers, and sometimes propose to retain the customary names for ten and eleven when used duodecimally.

They exhibit a similar approach to the duodecimal weights and measures. The sizes and names of the accepted Anglo-American standards are retained as faithfully as possible, and these are adjusted by minor changes into a duodecimally unified metric system.

Classification under the Principle of Separate Identity

The outstanding characteristic of duodecimal proposals that fall within this group, is that they are designed to prevent any possible confusion with decimal quantities of measures. They generally propose entirely new symbols for all numbers and new names for these numbers.

Since they already embrace the necessity for complete change, they afford the opportunity for the suggestion of every novel practice and method that may seem to improve our current procedures. New practices in grouping, denominating, and punctuating numbers are typical.

There is a corresponding revisionary attitude as to the weights and measures. These are generally to be based upon some specific method of determining a new unit of length, and around this unit is erected a conformal duodecimal metric system.

Traits typical of this group, then, are the general disregard of customary methods and practices, and the proposal of radical and novel procedures in numeration, notation, nomenclature and metrology.

Separate identity

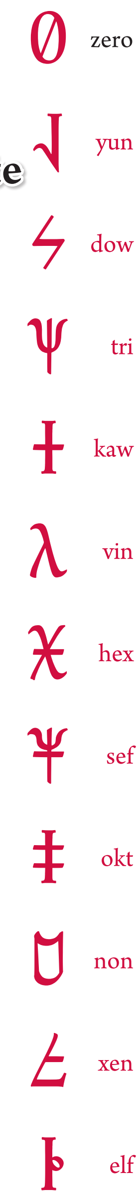


Figure 2: A separate identity symbol set by P. D. Thomas

Rationale

These classifications seem simple and clear. But confusion will continue unless it is comprehended that this separation means more than at first appears. There is a fundamental difference in ideologies involved.

As one becomes more familiar with duodecimals, and duodecimal proposals, one begins to perceive that there are supporting factors for both groups. One begins to see that in some applications there would be greater advantage in the one type of system than in the other. And that under other conditions, the reverse would be true, and that what had been considered essential had become secondary.

As an analogy, the general public makes little use of the Kelvin Temperature Scale which is based on Absolute Zero, but prefers a scale emphasizing the freezing and boiling points of water. For some scientists, however, there are advantages in the use of the Kelvin Scale which makes it indispensable.

It must be realized that it is from the proposals under the Principle of Separate Identity that the innovations and inventions are developed which constitute progress. And these new ideas are valuable. But to the general public, the idea of changing all the names and symbols for numbers would be simply repulsive, and entirely unthinkable, and proposals involving as little change as possible are required.

So both systems are necessary. It should be the responsibility of the Society to develop both. And when a practical degree of unanimity is expressed in a proposal under either of the two principles, that proposal should be endorsed by the Society. It must be clear that the endorsement does not mean acceptance of the one principle and the suppression of the other. Nor does it imply the necessity of blending both principles into a single solution. Both are necessary. Both are important. But they are opposed, and relatively unblendable.

Different necessities, different viewpoints, different logics, are inherent in each. We will only create confusion and useless dissension if we apply to some proposal under the Principle of Least Change the arguments and critiques that are entirely proper to the Principle of Separate Identity. And the reverse. This just won't work. Since opposed lines of thought are involved, there must be a corresponding change of attitude as we consider the one or the other.

Since each of these groups has its own definite factors of preference, it would be well to avail ourselves of these advantages intelligently, – to analyze each new proposal from the viewpoint specifically proper to it, – and to aid in the development of a consensus as to each, by making our judgments known.

"there will be unbiased presentation of all such proposals"

Recently there has been a considerable amount of discussion of the duodecimal terms and symbols used by the Society. Perhaps it would be well to set forth the Society's attitude in the matter.

When formal organization of the society was undertaken, it was decided that we would continue to use the Dek (Χ), El (Ɛ), and Do, which, over a period of some eight or nine years, had become accepted as the usage of the informal society.

All of the Society's duodecimal material, currently in the hands of the public, employs this usage. Moreover, there is a solid basis for its preference. The symbol "X" for ten, was used exclusively throughout western civilization from early Roman days until the last years of the thirteenth Century. And all European names for ten are derived from the Latin "decem," pronounced "dekem."

A review of all duodecimal proposals has shown that there is a preponderance of preference for these terms, Dek and El, over any other names and symbols. No other terms which have been considered can marshall an equal weight of argument. Since confusion of the public mind is to be avoided if duodecimals, and the Society, are to make solid progress, this usage is not to be lightly changed.

When the weight of preference shifts to some other usage, and we can be confident of unanimity and finality in that choice, then the change should be made through official action of the Society. This possibility is not to be neglected. For this reason, there will be unbiased presentation of all such proposals, and the adoption of an accepted usage by the Society does not in the least preclude consideration of any and every proposal under the Principle of Least Change.

For our personal use, of course, we shall employ those terms and symbols preferable to each of us. When any of our papers are selected for publication it will be easy to substitute the accepted usage.

See our *Bulletin*, Vol. 1 № 3 pp. Ɛ;–11;, for Mr. Beard's original article.

Got a friend into numbers who would appreciate a sample copy of our *Bulletin*? Send in his or her name and address—we'll send one their way.

Eugene Maxwell "Skip" Scifres

DSA Member № 11; ~ rest in peace

Lt. Eugene M. Scifres, a Reconnaissance Officer in the U.S. Army Air Corps, was born on the thirteenth day of February in 1916. The Duodecimal Society of America was founded in 1944, and in 1945 "Skip" became our thirteenth member (№. 11;) of our fledgling Society. A baker's dozen—twelve plus one—was certainly a lucky number for the DSA.

His nickname, Skip, was bestowed upon him when his schoolmates and teachers had trouble with his surname, "Scifres".

In 1963 he was elected to our Board of Directors joining the illustrious Class of 1966, whose other members were the dozenal stalwarts F. Emerson Andrews, a founder and first President who later had served as Board Chair; Henry C. Churchman, Vice President who later edited our *Bulletin* for many years; and Jamison "Jux" Handy, who served as assistant editor and editor at various times. Skip served on the Board for 1½ dozen years. In 1965 he was elected to the position of Treasurer, an office he held for a dozen years.

Skip was an engineer, a pioneer computer programmer and systems analyst, and a prize winning photographer.

For many years Skip belonged to the Two by Two Fellowship at Washington Park Community Church (now Washington Park United Methodist), serving as their president in 1957. Later he attended Calvary Temple, where he belonged to a Bible study group, served as an usher and volunteer (doing computer projects, of course).

He was devoted to his family and is survived by his daughter Beverly, his granddaughters, Meghan and Robin, his great granddaughter, Lola, his wife Georgette and her children, Melanie and Dean, as well as four nephews, Bill, Dennis, Wally and Rick Scifres.

Skip was a wonderful man—always kind, loving, helpful, positive, and interesting. Having lived almost eight dozen years, he outlived a great many of his friends, but those that remain will mourn his passing including all of us in the DSA who knew him.

See the Problem Corner, page 23; of this issue
for a problem that Skip submitted to us 18; (20.) years ago.

presenting … symbology

an editorial

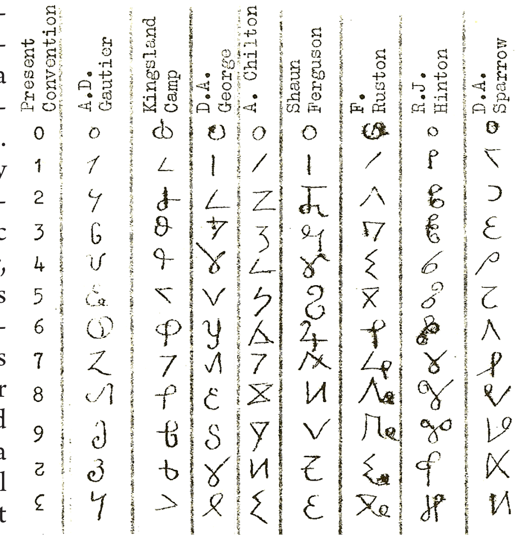
The next two issues of this Bulletin delve into the foremost subject on many a dozenalist's mind: dozenal *symbology*, the set of symbols one uses to convey dozenal numerals or digits. In order to write a number in base twelve, generally we need twelve symbols. Decimal, through the "Hindu-Arabic" numerals, gives us only ten numerals. Many dozenalists have devised their own set of numerals, or they adopt the symbols prevalent in their dozenal society. Some fiercely guard their numerals under their own banners of reason; the subject is hotly debated in the DozensOnline forum. So we're focusing on *numerals* here. Thus, the DSA now offers you a deep study of symbology, and perhaps ultimately, "what are the appropriate symbols for duodecimal numerals?"

First, this study is not intended to definitively answer the question all by itself: it merely aims to equip you, the dozenalist, with as much information as possible so that the question can be debated more efficiently, with a thorough and broad common frame of reference. We anticipate your correspondence and participation in this effort. Here are a few notes about what we've done.

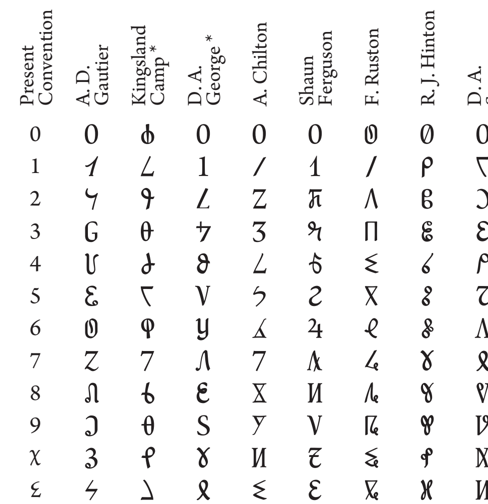
For this current symbology synopsis, the Dozenal Society of America has examined nearly every system of dozenal numeral symbols ever printed in its own journal, or those of our sister society, the Dozenal Society of Great Britain. We have also contacted dozenalists on internet forums and social media over a period of up to one year to elicit symbologies which have not been published, provided we are able to name the author to properly assign credit. These have been "remastered" (to borrow a term from digital audio) and appear throughout this and related works to serve as tools by which you and other dozenalists may use to get better acquainted with the various symbologies. Perhaps in the future these tools may help our societies and dozenalists everywhere agree on a standard set of duodecimal numerals. What you have before you is the fruit of a deep and massive effort.

We attempted to eliminate most of the differences in presentation style of symbologies, so you can focus attention as much as possible on their form and intent. Each symbology has been digitally reproduced and made into a typeface which blends as cleanly as possible with the typeface used in this *Bulletin*. By doing this, each symbology arguably appears as though it already serves as a standard in print, and all symbologies appear equally well treated. Now that each set has at least one font file, the DSA can reproduce tables and manipulate data in any of these symbologies. Compare this to reproducing the symbol sets by hand, or cutting and pasting facsimiles of these symbols here and there throughout this work. Refer to the tables at right to see how symbologies presented in original work have been remastered for use today.

There are some consequences to remastering the symbologies. You may notice that there are many ways to print a symbol, like a letter, which are not often seen in handwriting. The letters "a" and "g" in this article are probably not the way you see them handwritten. We are also familiar with differences in font treatment (regular vs. *italic*), between a given symbol's appearance in different typefaces (serif vs. sans serif). So there is some "natural" variability even among the symbols we use everyday. A lot of this is due to "artistic license" or "style" a type designer, calligrapher, or even each one of us exercises whenever we write a symbol. This "license" or "style" extends to this work. In order to remaster the symbologies, the editor had to interpret them, then produce a symbol that fit into the new digital constraints. To be honest, this must have also happened in the legacy articles when editors of years past synopsized symbologies; they reproduced a symbol they may have heard described or saw in a book, and some unintentional alteration may have occured then, too. Certain sets were checked by their authors; most of them have not, simply because the authors are no longer available. This said, the symbologies presented here appear to be acceptable renditions of the symbols proposed by dozenalists throughout the history of dozenal publication.



Excerpt from a table which originally appeared in "New Duodecimal Notations and Names", Duodecimal Newscast, *Year 3, № 2, p. 3, 1165; (1961.)*



New interpretation of the original symbologies for consideration today

Although this symbology study began as an independent effort to summarize what we know about dozenal symbols, much is owed the DSGB's *Duodecimal Newscast*. In summer 2009, Mr. Shaun Ferguson sent copies of their publications. The summaries at Year 2 № 1 page 10; and Year 3 № 2 page 3 illustrated the diligence with which the early Members of the DSGB approached the problem. It is hoped that this current effort approaches theirs.

I hope you enjoy our excursion into the world of new numerals, and that you correspond with us in the coming months about what you've seen here.

** See note 1 page 18; for notes regarding differences between the tables.*

problem corner

problem from last issue:

In our last issue we presented the following decimal example of mathematical symmetry, asking, "Can you find a similar pattern in base twelve?" Prof. Jay Schiffman suggests we can extend the problem as indicated in red below.

$9 \times 9 + 7 = 88$
$98 \times 9 + 6 = 888$
$987 \times 9 + 5 = 8{,}888$
$9876 \times 9 + 4 = 88{,}888$
$98765 \times 9 + 3 = 888{,}888$
$987654 \times 9 + 2 = 8{,}888{,}888$
$9876543 \times 9 + 1 = 88{,}888{,}888$
$98765432 \times 9 + 0 = 888{,}888{,}888$
$987654321 \times 9 - 1 = 8{,}888{,}888{,}888$
$9876543210 \times 9 - 2 = 88{,}888{,}888{,}888$

Continued on page 23;

Why is 1 One? by Professor Gene Zirkel

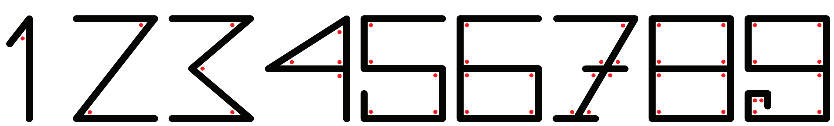
I received the following in an email. It is presented here with the original spelling and punctuation:

The Numbers

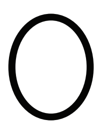
The numbers we write are made up of algorithms, (1, 2, 3, 4, etc) called arabic algorithms, (*sic*) to distinguish them from the roman algorithms (I; II; III; IV; etc.).

The arabs popularise these algorithms, but their origin goes back to the penecian merchants that used them to count and do their commercial contability.

Have you ever asked the question why 1 is "one", 2 is "two", 3 is "three" … ? What is the logic that exist in the arabic algorithms ? Easy, very easy … ! There are angles! Look at these algorithms written in their primitive form and check it up!...



And the most interesting and intelligent of all … 0 zero angle !



Moral of the story: It is never late to learn!

I cannot vouch for the accuracy of this account. I have never heard of it before. If any of our readers know anything about it please let us know.

However the idea is intriguing. Many of us play with creating new symbols for dek and el. Can any one come up with an idea for dek that had dek angles and/or and idea for el that has el angles? We eagerly await your submissions.

Book Review

A Good History of the Different Notations for Numbers

By Charles Ashbacher, DSA Life Member, № 2Ɛ8;

A review of **Man and Number**,
a Dover Science paperback by Donald Smeltzer

Counting is the point of origin of all mathematics, and it can be done in many ways. There are the trivial differences in the names of the numbers, and the more significant ones concerning the base and the notation used to abbreviate the names of the numbers. For example, the words "twenty one" are but one name for 21, which means two tens and one one. In base twenty, this would be one twenty and one one. We tend to think of the base ten as a mathematical law, but that is largely an anatomical accident. Humans have ten fingers and that is the origin of the base ten.

Depending on the circumstances, other bases are more natural than ten. For example, base two is more natural for computers, base twelve is more natural for some areas of commerce due to the larger number of factors, and base 360 works very well for astronomical computations. Smeltzer takes us through the historical and cultural records of the different bases used by humans and when bases other than ten make more sense. It is an excellent demonstration of how humans have tightened up the notation used for numbers and the abstractions that have developed over centuries.

We use the representation of numbers and the symbols for arithmetic operations routinely, and rarely realize that they developed over centuries. I believe all students of mathematics, especially those who are on track to teach elementary and middle school mathematics, should know the history of the development of these notations. This readable book is an excellent way to learn the fundamentals of number representation and how those notations developed over time.

* *This book review is very appropriate for this issue of our Bulletin. In addition to several positive references to dozenal counting (e.g., pp 21, 55, 111), the author covers numerical symbols from various ancient civilizations up to 1958, when it was published. Erich Kothe, Life Member № 210;, alerted us to this book, and a web search uncovered this review by Mr. Ashbacher, also a Life Member.*

Adapted from a review which first appeared at Amazon.com, reproduced here with permission of the author and Amazon.

We Depend on You

Annual dues are due as of 1 January 2010. If you forgot, please forward your check for only one dozen six dollars ($18.) to Treasurer Ellen Tufano, 95 Holst Drive West, Huntington NY 11743-3939, USA. Student dues are $3.

Or take it up a notch, to three dozen dollars and receive a one-year paper-copy subscription of the *Duodecimal Bulletin* as a Supporting Member. As you know, our continued work depends very much upon the tax deductible dues and gifts from our Members.

Symbology Overview

by michael de vlieger

A good first step in the consideration of systems of numerals for dozenal and other number bases is reviewing and classifying all systems. "The Opposed Principles" which Ralph "Whiskers" Beard presented in our *Bulletin* in 1945 seemed to frame the debate about "symbology", the practice of crafting new numerals for use in the representation of dozenal numbers, as well as "nomenclature", the names which pertain to the numerals, early on in the history of our society.

Nearly six dozen years have elapsed between our first years and this issue. Because of this we benefit from having a plethora of "symbologies", here the word applies to sets of symbols which serve as numerals, which we may observe and compare. We can put into practice what Mr. Beard was extolling, the "unbiased presentation" of our proposals, laid out side by side before all to see.

In the production of this issue, the DSA has surveyed and developed typefaces that might convey the disparate proposals across the dozens of years. In doing this, we have found a need to classify the proposals with finer resolution than Whiskers' readily discernable dichotomy of "Least Change" and "Separate Identity". *(See page 5 of this* Bulletin *for a full reprint of Mr. Beard's article).*

In the spirit of Mr. Beard's editorial, we do not intend to judge, or worse, disparage any symbology, but offer the refined classifications as an aid for your discernment of their value in your own estimation.

This article is purely concerned with so-called Western numerals, especially those of the "Anglo-American" cultural sphere, speakers of English. This article doesn't cover alterations to Eastern Arabic, Hindi, or Chinese numerals; perceivably those cultures and speakers will devise their very own symbology or nomenclature. Presented below is what this article is calling "Hindu-Arabic numeral set":

0 1 2 3 4 5 6 7 8 9

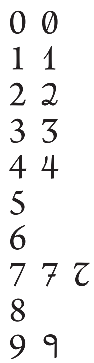


Figure 1: *Variants*

One might observe that for many of the numerals presented above, there exist variant number forms which most people would recognize. The table in Figure 1 offers a few of these. When these variants are used in society they seem to convey a regional "accent", in the case of the "Continental one" (1), to reduce confusion, as in the stroked zero (0) or seven (7). Others are merely different styles of a numeral. If a symbology designer specifies a variant as a numeral in his or her set, we will interpret that here as a departure from "Hindu Arabic", presuming there is an underlying reason for the specification.

For the sake of unity and clarity, the symbols which appear in this article have been crafted to fit the Bulletin's standard typeface, Adobe Arno Regular, as if they have perhaps already been accepted in print as numerals in use by the general public. Certain more abstract proposals are left to appear more like geometric shapes. Digits which are altered or invented by the author of a symbology are illustrated in red. Let's embark on our journey, shall we?

Figure 2: *The retention scale and index.*

Mr. Beard's dichotomy of "least change" and "separate identity" may be regarded as a continuous spectrum of "retention" of the existing Hindu-Arabic numerals. The above graphic illustrates the retention spectrum, with strict "separate identity" on the left wherein all existing decimal digits are discarded, and strict "least change" on the right, wherein all existing decimal digits are preserved. If we want to quantify the retention of a given symbology, we might define a "retention index" or RI, with ten (X) representing strict "least change" and zero representing strict "separate identity". The symbologies which appear below illustrate a more or less continuous spectrum of retention:

Figure 3: *A spectrum of retention of Hindu Arabic numeral forms among some dozenal symbologies.*

Between the extremes of strict "least change" and strict "separate identity", there lies a spectrum of symbologies involving partially-retained Hindu Arabic numerals, dutifully fulfilling their former roles. The most popular numeral retained among many so-called "separate identity" symbologies appears to be the zero, closely followed by the numeral one. If we count the variants of zero as "retained", the figure then seems to be retained in all but a dozen of the six dozen eleven cases studied in the "Featured Figures" spread *(See pages 13;-14;).*

Because the number of Hindu Arabic numerals retained can be quantified discretely, we can assign a rating to each symbology to measure their position on the spectrum. This rating might be divided by the total number of numerals in the set to measure the proportion of Hindu-Arabic numerals retained in a symbology. A higher number in both cases indicates a more conservative symbology. Issac Pitman retains all Hindu Arabic numerals, thus his retention index is X, while Marino's is zero. All dozenal symbologies have a maximum retention proportion of X0 pergross; hexadecimal symbologies max out at 76 P/G; sexagesimal at 20 . Even a strict "least change" sexagesimal numeral system is, in effect, as "creative" as Zirkel's or Turner's in Figure 3 above.

Continued on page 15;

Symbology Overview

from the *Duodecimal Bulletin* and Beyond

featured figures

See page 19; for notes.

Style Symbology	0	1	2	3	4	5	6	7	8	9	χ	Ɛ	*Reference**
Least Change: Repurposing: Sequential													
"IBM" (applied to dozenal)	0	1	2	3	4	5	6	7	8	9	A	B	DB 27·2·10
Alphanumeric lowercase	0	1	2	3	4	5	6	7	8	9	a	b	
Least Change: Repurposing: Rationalized													
D'Alambert & Buffon	0	1	2	3	4	5	6	7	8	9	X	Z	NR 02·1·11
"Hall"	0	1	2	3	4	5	6	7	8	9	t	e	DB 2♯·1·1♯
G. Chrystal (1150;)	0	1	2	3	4	5	6	7	8	9	τ	ε	DB 03·1·11
Henry Parkhurst (1115;)	0	1	2	3	4	5	6	7	8	9	X	Λ	DB 10·2·33
"Delta-Epsilon"	0	1	2	3	4	5	6	7	8	9	δ	Ɛ	Note 2
H. K. Humphrey (Strict)	0	1	2	3	4	5	6	7	8	9	d	k	DB 01·3·23
"Alice"	0	1	2	3	4	5	6	7	8	9	Λ	Ɛ	Note A
"Decker"	0	1	2	3	4	5	6	7	8	9	$\overset{1}{0}$	$\overset{1}{1}$	Note E
Least Change: Repurposing: Creative													
Edna Kramer	0	1	2	3	4	5	6	7	8	9	*	#	NR 02·1·11
Lancelot Hoghen	0	1	2	3	4	5	6	7	8	9	♀	♂	NR 02·1·12
T. Pendlebury	0	1	2	3	4	5	6	7	8	9	?	&	NR 08·2·03
F. S. Whellams	0	1	2	3	4	5	6	7	8	9	♯	♭	NR 09·2·09
Least Change: Derived: Aesthetically Rationalized													
Juan C. Lobkowitz (Ɛ50;)	0	1	2	3	4	5	6	7	8	9	ρ	∩	NR 02·1·11
H. G. G. Robertson	0	1	2	3	4	5	6	7	8	9	θ	ɤ	DB 03·4·07
Tom Johnson	0	1	2	3	4	5	6	7	8	9	θ	♯	Note A
Peter Barlow (106χ;)	0	1	2	3	4	5	6	7	8	9	φ	γ	NR 02·1·11
Peter Barlow (106Ɛ;)	0	1	2	3	4	5	6	7	8	9	φ	π	NR 02·1·11
Vicente Pujals de la Bastida	0	1	2	3	4	5	6	7	8	9	↊	ŧ	NR 02·1·11
Sir Issac Pitman (10χ9;)	0	1	2	3	4	5	6	7	8	9	Ɛ	V	NR 02·1·11
Sir Issac Pitman (10χ9;)	0	1	2	3	4	5	6	7	8	9	↊	Ɛ	DB 03·2·01
William S. Crosby	0	1	2	3	4	5	6	7	8	9	↊	Ɛ	DB 02·2·14
William Dwiggins	0	1	2	3	4	5	6	7	8	9	χ	Ɛ	DB 01·1·02
G. Elbrow	0	1	2	3	4	5	6	7	8	9	χ	Ɛ	DB 04·1·11
T. Wood	0	1	2	3	4	5	6	7	8	9	χ	ℓ	NR 02·1·12
DSA-"Bell" via Churchman	0	1	2	3	4	5	6	7	8	9	✗	♯	DB 25·1·02
William Schumacher	0	1	2	3	4	5	6	7	8	9	∂	℘	DB 37·2·19
H. K. Humphrey	0	1	2	3	4	5	6	7	8	9	∂	ƙ	DB 01·3·23
Paul Van Buskirk	0	1	2	3	4	5	6	7	8	9	ɗ	ɇ	DB 03·4·18
Ray Greaves / David James	0	1	2	3	4	5	6	7	8	9	ɬ	↋	Note C
Ray Greaves	0	1	2	3	4	5	6	7	8	9	⅃	Γ	Note C
De Vlieger "Arqam" (1193;)	0	1	2	3	4	5	6	7	8	9	↊	↋	DB 45·2·1♯
Least Change: Derived: Technically Rationalized													
Don Hammond	0	1	2	3	4	5	6	7	8	9	⅃	Ŀ	Note B
Don Hammond	0	1	2	3	4	5	6	7	8	9	↊	Ɛ	Note B
Niles Whitten 1	0	1	2	3	4	5	6	7	8	9	↊	Ɛ	Note S
Niles Whitten 2	0	1	2	3	4	5	6	7	8	9	↊	∠	Note S
Paul Rapoport	0	1	2	3	4	5	6	7	8	9	5	E	DB 31·3·04
"Bell" numerals via Zirkel	0	1	2	3	4	5	6	7	8	9	H	E	DB 32·1·12
"Bell" via Radio Shack	0	1	2	3	4	5	6	7	8	9	⊦	H	DB 32·1·12
Least Change: Improvisation: Rationalized													
Charles Bagley 2	0	1	2	3	4	5	6	7	8	9	ρ	ʃ	DB 1Ɛ·2·37
Tom Linton	0	1	2	3	4	5	6	7	8	9	Ɔ	ʃ	DB 1Ɛ·2·37
Least Change: Improvisation: Creative													
Handy/Norland	0	1	2	3	4	5	6	7	8	9	Ɛ	∠	DB 01·2·22
Jean Essig	0	1	2	3	4	5	6	7	8	9	↊	∠	DB 10·2·48
Charles Bagley 1	0	1	2	3	4	5	6	7	8	9	Ɉ	ℓ	DB 11·2·48

Style Symbology	0	1	2	3	4	5	6	7	8	9	χ	Ɛ	*Reference**
"Compromise"	0	1	2	3	4	5	6	7	8	9	χ	∠	DB 1Ɛ·1·15
Frank Plevin	0	1	2	3	4	5	6	7	8	9	ꝺ	ℓ	NR 06·2·07
Dr. Paul Rapoport	0	1	2	3	4	5	6	7	8	9	ɤ	ℓ	DB 2χ·2·24
Shaun Ferguson 2	0	1	2	3	4	5	6	7	8	9	ħ	Ƒ	Note J
Separate Identity: Repurposing: Sequential													
"Ernest Stryver"	a	b	c	d	f	g	h	i	l	m	n	o	DB 01·3·22
J. Halcro Johnson-1	0	1	2	3	4	5	6	5	4	3	2	1	DB 06·2·25
Louis Loynes 1	Z	I	A	B	C	D	E	F	G	H	J	K	NR 03·2·03
Louis Loynes 2	M	A	B	C	D	E	F	G	H	J	K	L	NR 03·2·03
Separate Identity: Repurposing / Derived: Rationalized: Modular Symmetry													
De Vlieger "Acylin" (11χƐ;)	0	1	2	3	Ɛ	∂	8	6	Ɛ	Ɛ	↊	↋	Note E
J. Halcro Johnson-2	0	1	2	3	4	5	6	ϛ	ŧ	Ɛ	↊	I	DB 02·1·18
Separate Identity: Improvised: Rationalized: Additive Analog													
Rafael Marino	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	DB 38·2·10
F. Ruston	0	/	Λ	Π	≤	χ	ρ	∠	ʎ	Π	≤	χ	NR 03·2·03
Gwenda Turner	·	ι	∠	△	∂	δ	8	7	ϟ	9	∂	9	DJ 04·04
R. J. Hinton	0	ρ	ß	Ɛ	6	8	8	ɣ	ɣ	♆	♆	ϗ	NR 03·2·07
P. D. Thomas "Modular"	0	ɺ	ϟ	Ψ	ŧ	λ	✗	¥	ǂ	Ʊ	∠	⊦	Note D
D. A. Sparrow	0	ↄ	Ɔ	Ɛ	ρ	↊	Λ	ℓ	V	∀	χ	И	NR 03·2·0Ɛ
Fred Newhall "Efficient"	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	DB 2χ·3·16
Separate Identity: Improvised: Rationalized: Multiplicative / Exponential													
Shaun Ferguson 1	0	ɺ	ħ	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	ħ	Ʊ	Ƒ	Ɛ	Note C
Separate Identity: Improvised: Rationalized: Rotational Run Sequence													
Lauritzen "Gravity" (11χ0;)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	DB 3♯·1·04, L
Kingsland Camp[1]	ȸ	∠	ϥ	θ	∂	ↄ	φ	7	ɓ	θ	ρ	ɭ	DB 02·1·16
Separate Identity: Improvised: Rationalized: Indexed Value													
Smooth Binary Coded	0	1	ʃ	Ո	L	ƽ	⊦	h	ɉ	ɫ	ẓ	d	
George P. Jelliss	/	⊦	L	F	Ɛ	E	ll	H	U	A	0	8	Note F
Separate Identity: Improvised: Modular Symmetry													
A. D. Gautier (10χχ;)	0	ɩ	ɣ	G	Ʊ	Ɛ	0	Z	ɲ	Ɉ	3	ϟ	NR 03·2·03
George Brost "Dyhexal"	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	DB 35·2·0χ
James Conlon	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Note A
Separate Identity: Improvised: Creative													
Dudley George (±1146;)	0	1	∠	ɀ	ϑ	V	ʮ	Λ	Ɛ	S	ɣ	ℓ	DB 02·2·17
A. Chilton	0	/	Z	Ʒ	∠	ƽ	∡	7	χ	ɣ	И	≷	NR 02·1·10
Shaun Ferguson	0	ɩ	ħ	[illegible]	[illegible]	Ƨ	[illegible]	Λ	И	V	Ƹ	Ɛ	NR 02·1·10
Louis Loynes 3	▽	1	2	3	ℓ	И	6	7	8	9	↊	Ɛ	Note A
Mohan Kala	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	DJ 31·35
Raymond Mason	θ	1	ʅ	Ƨ	L	V	Π	˥	ɣ	9	↊	Ɛ	Note A

Hexadecimal Digit

Symbology	0	1	2	3	4	5	6	7	8	9	χ	Ɛ	10	11	12	13	*Reference**
Least Change: Repurposing: Sequential																	
"IBM"	0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F	DB 27·2·10
Least Change: Derived: Rationalized																	
Whillock Com.	0	1	2	3	4	5	6	7	8	9	χ	∠	↊	Ɛ	ŧ	ϛ	DB 27·3·07
Least Change: Repurposing: Creative																	
"WordPerfect"	0	1	2	3	4	5	6	7	8	9	/	\	:	*	?	+	DB 33·2·1χ
Least Change: Improvised: Creative																	
"Arqam" (1193;)	0	1	2	3	4	5	6	7	8	9	↊	↋	ɣ	∂	Ɛ	℘	DB 45·2·1♯
Separate Identity: Improvised: Rationalized: Indexed Value																	
Bruce Martin	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	Note M
Schumacher	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	H	DB 33·3·0χ
Binary Coded	0	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	H	DB 49·1·15
T. Hauptman	8	φ	9	ℓ	ȸ	ǀ	ƽ	Ƨ	∂	?	3	2	6	ς	S	Ɛ	Note T
Separate Identity: Repurposing: Rationalized: Indexed Value																	
Whillock Com.	0	1	2	3	4	5	6	7	8	9	H	L	d	E	h	5	DB 27·3·07

Continued from page 12;

Figure 4: *The inspiration scale, related to the source of inspiration for new numerals.*

Retention represents a key consideration in the development of a new set of numerals. Another consideration regards the source of inspiration for any new numerals. One may draw from other sets of symbols in the public lexicon, such as from alphabets or musical notation, and append the set in sequence where seen fit. This "sequential repurposing" represents a more conservative extreme. New symbols are added, but they are somehow familiar and in sequence. One might also devise new symbols entirely randomly and ascribe a meaning to them, "random improvisation", representing the other extreme. This scale is more complex than the retention scale; let's examine examples.

Repurposing. The first set of strategies involves using existing symbol sets, such as the Latin, Greek, or other alphabets, the symbols of the planets or zodiac, etc., to extend the Hindu-Arabic numerals. Repurposing is typically the province of least-change or more retentive symbologies.

Sequential repurposing involves simply appending a more or less contiguous series of symbols to the existing numerals to attain the requisite number of digits to convey the base. The following is a common hexadecimal example:

0 1 2 3 4 5 6 7 8 9 A B C D E F

J. Halcro Johnson's reverse notation, appearing in Vol. 6 № 2 page 25;, extracts the numeral sequence 0 through 5 of the Hindu-Arabic numeral set, echoing it backward. For structural reasons, i.e., in order to represent "balanced" dozenal notation, the reverse notation can be seen as redefining the latter portion of the dozenal numeral set, here denoted as "Johnson-1" as follows:

0 1 2 3 4 5 6 $\bar{5}$ $\bar{4}$ $\bar{3}$ $\bar{2}$ $\bar{1}$

This can be seen as reincorporating the symbols 1 through 5 and mildly modifying them with a diacritical bar to distinguish these negative numbers from their positive cousins, as they march in reverse from 0. Alternatively, the set can be regarded as including seven Hindu-Arabic symbols 0 through 6, and a diacritical bar that is placed above a digit to represent negativity.

Selective repurposing involves choosing some symbols from an existing foreign set and reprogramming them in a significantly interrupted sequence or in a sequence which has no relation to the original foreign sequence. "Ernest Stryver" submitted a tongue-in-cheek letter in Vol. 1 № 3 page 22; which repurposes the letters of the alphabet (with some exceptions) to serve as dozenal digits. This would be a less-pure manifestation of a sequentially repurposed Separate Identity symbology: perhaps it is a selectively repurposed set.

a b c d f g h i l m n o

Symbols can be chosen which are related to the names of the digits, whether these names be the English decimal names, or other names. Symbols from existing sets may be selected for their aesthetics; perhaps these convey some aspect of the integer they represent. DSA Past President Harry Robert proposed the following extension in Vol. 2 № 1 page 1↊;. Perceivably his scheme uses the Greek initials of the words "dek" or "δεκα", standing for digit ten and "el" or "ενδεκα" for eleven[2].

0 1 2 3 4 5 6 7 8 9 δ ε

H. K. Humphrey set out the following symbology, suitable for the typewriter, in Vol. 1 № 3 page 23;. The lowercase "k" as el or "kel", filled the entire line height, unlike "e", eliminated the need to shift, and is positioned next to the lowercase "l", which in the day was used as a numeral "1".

0 1 2 3 4 5 6 7 8 9 d k

Creative/aesthetic repurposing, finally, encompasses the use of symbols from existing sets which may be chosen for no particular or identifiable reason. Edna Kramer borrowed two "punctuation" marks available on typewriters to serve as digit-ten and digit-eleven in the 1951 book *The Main Stream of Mathematics*.[3] These same characters became the so-called "Bell" numerals, which were introduced in meeting minutes from 1973 in in Vol. 25 № 1 page 1, initially began as the now-familiar "star" and "pound" we use rather universally today on telephone keypads.

0 1 2 3 4 5 6 7 8 9 * #

Derivation. Symbols selected from existing symbol sets can be altered to adjust for any combination of perceived constraints. The author of new numerals can use an existing "antecedent" symbol as a starting point, making minor alterations to suit the intent.

Visual harmony and appearance is a chief constraint for some authors, who try to produce transdecimal digits which "blend in" or resemble the existing Hindu-Arabic numeral set. Sir Issac Pitman proposed a classic "aesthetically derived Least Change" set of transdecimal symbols in 1857, mentioned at length in Vol. 3 № 2 page 1, where he writes that he adds a "'T' modified to 'Ꞇ' for ten, and 'E' altered to 'Ɛ' for eleven".

0 1 2 3 4 5 6 7 8 9 Ꞇ Ɛ

Our own numerals, devised by William Addison Dwiggins, are likewise aesthetically rationalized derivations of letters. The ↊ is described by many former issues of the *Bulletin* as inspired by the Roman numeral X, standing for decimal ten. The ↋ is described in Vol. 1↋; № 2 page 44; as a "fancy form of the italic *E* known to printers as 'swash E'".:

0 1 2 3 4 5 6 7 8 9 ↊ ↋

The "Bell" numeral forms which now appear in the *Duodecimal Bulletin* are slightly altered to resemble Roman X with a single cross bar through it to represent "*not*-ten", and the number 11 with two cross bars through it to represent "*not*-eleven". This set can be considered a derivation of the "strict" Kramer/Bell numerals which appeared under creative repurposing.

0 1 2 3 4 5 6 7 8 9 Ӿ ♮

H. K. Humphrey, author of an above-mentioned symbology, seemed inclined to make the "d" more graceful. His compatriot in symbology, John Jarndyce (a pen name used by H. C. Churchman), writes in Vol. 24; page 15; that the Humphrey symbology may persist letter-like initially, "until someone attempts to pretty them up", thus:

0 1 2 3 4 5 6 7 8 9 ∂ ꝁ

A common constraint since the advent of "digital" readouts in the mid twentieth century is legible expression of a digit using 7 or 13 segment LCD/LED readouts. Don Hammond presented a set of numerals which altered Sir Issac Pitman's 1857 transdecimals in the interest of making these more amenable to 7 or 13 segment LCD/LED readouts[4]. Hammond attempted to make the handwritten version of his numerals more amenable to these readouts. Niles Whitten further enhances Hammond's pull towards the readout constraints, in two waves[5]:

Pitman (1857):	0 1 2 3 4 5 6 7 8 9 ↊ ↋
7-segment:	0 1 2 3 4 5 6 7 8 9 ↊ ↋
Hammond:	0 1 2 3 4 5 6 7 8 9 ↊ ↋
Whitten-1:	0 1 2 3 4 5 6 7 8 9 ↊ ↋
Whitten-2:	0 1 2 3 4 5 6 7 8 9 ↊ ↋

Improvisation. New numerals can be invented in complex, subjective, or random ways. The rationally improvised low-retention symbologies exhibit a wide array of organization which will be the subject of an article in the next issue which explores the tools by which one can produce one's own symbology. We'll examine some of the simpler symbologies here.

Rationalized improvisation. An author of a symbology may have specific reasons for inventing wholly new symbols for transdecimal digits. The reasons can include attempts to convey some aspect of the integer, its English decimal name, or handwriting efficiency.

The example below, presented in the article "New Symbols", Vol. 15; № 2 page 34; by Charles Bagley, follows a brief exploration of symbol forms, in an attempt to create "two new symbols that can stand erect with our ten basic numbers and lend them dignity." This is closely followed by Tom Linton's set. Both sets are examples of rationalized, improvised Least Change symbologies.

Bagley-2:	0 1 2 3 4 5 6 7 8 9 ρ ʃ
Linton:	0 1 2 3 4 5 6 7 8 9 Ↄ ʃ

Creative improvisation. Some authors may find satisfaction with a symbol they simply invented, or which evolved through exercise over a period of time. Dr. Paul Rapoport's symbology, presented in Vol. 2Ӿ; № 2 page 24; in summer 1985, exemplifies an arbitrarily improvised Least Change set. He states, "I decided that I had to create symbols for ten and eleven which would not look like any other numerals or commonly used symbols, nor like any letters of the alphabet". The resultant forms of the numerals, which ended up symmetrical and similar to one another, but distinctive, appear to be completely improvised:

0 1 2 3 4 5 6 7 8 9 ɣ ꝶ

Dudley George's personal symbology, dating from the mid twenties, appears in one of the first Bulletins, Vol. 2 № 2 page 17;. These come complete with a Separate Identity nomenclature (set of number names):

0 1 ∠ ヶ ϑ V Ꝋ Λ Ɛ S ɣ ꝶ

A "creative" system need not stand without order or design. At times there are multiple directives at work which impart forms on numerals which perhaps end up making the underlying systems of order less clear. D. A. Sparrow expressed "A Suggested Series of Notation and Names" in the *Dozenal Newscast*, Year 2, № 1, page ↋;. In his article he explains how he constructs each digit, careful not to produce a new numeral that can be confused with the existing corresponding decimal numeral. He explains, "One has been a straight line for a long time; but this must be changed as .1 would no longer by (sic) the same quantity, that is it will change from [one tenth] to [one twelfth]". For 2 and 3, he continues "Two would be points joined together … Ↄ" and "As a line is not possible, we must have three points joined, and in order to distinguish it from 3, put backward … Ɛ". Thus, Sparrow is attempting to logically produce a system of numerals, the basic sequence {1, 2, 3} constructed by joining points, for instance, but then overlays the need to distinguish these from Hindu Arabic or Roman numerals:

0 ⊽ Ↄ Ɛ ρ Շ Λ ꝶ Ѵ Ɒ Ӽ И

In addition to Ralph Beard's readily-discernable scale of retention of the Hindu Arabic numerals, we can further classify the symbologies according to their source of inspiration for new numerals.

In the next issue, we will explore a set of strategies one can use to produce new numerals. These strategies tend to produce the most vivid "separate identity" symbologies. We'll also visit the DozensOnline forum for some ideas regarding criteria that should be taken into account when devising symbologies.

1 See the original K. Camp and D. George proposals as shown on page 14; the *Duodecimal Newscast*, Year 3, № 2, p. 3, 1165; (1961.) seems to feature a few transcription errors.

2 Though Harry Robert suggested lowercase delta (δ) to represent digit-ten in "Ideas & Opinions", Vol. 2 № 2 page 1Ӿ;, it's unclear whether this is his invention or something inherited from others. The insertion of the epsilon (ε), though not directly specified in Mr. Robert's letter, is supported by at least two others. Mr. John Selfridge penned a letter in Vol. 3 № 3 page 24; supporting delta and epsilon, as well as the practical alternative of "d" and "e" in their stead on typewriters. Mr. George P. Jelliss wrote in Vol. 36 № 2 page 14; supporting delta and epsilon in resonance with these earlier suggestions. Additionally, Mr. Jelliss saw epsilon's shape resonant with the "↋" used by the Dozenal Societies.

3 "New Duodecimal Notations", *Duodecimal Newscast*, Year 2, № 1, page 11;.

4 Retrievable at time of publishing at www.dozenalsociety.org.uk/basicstuff/hammond.htm, part of the official website of the Dozenal Society of Great Britain.

5 Retrieved in early 2010 at http://www.angelfire.com/whittenwords/measure/dozchar.htm, Mr. Niles Whitten's personal website.

the mailbag

Mr. H. K. Baumeister, a Life Member, № 140;, writes:

»Dear Sirs: The following is a teaser that you may find of interest.

Over half a century (How is that idea expressed dozenally?) ago when I started work at my first full time employment at IBM, I was chagrined to think my job did not involve four good wheels and an internal combustion engine. The Research/Development group I was with were developing a personal automatic calculator (PAC) near Columbia University that had a cathode ray tube (CRT) visual output display that easily could be seen from the operating console, a keyboard on a laboratory table, halfway across the small room. They were using the following result of a keyed-in problem to check their calculators first thing in the morning:

12,345,679. × 999,999,999. = ?

While reading your recent *Bulletin* Vol. 4Χ; № 1, especially pp. 9;—Ɛ;, & 25; it occurred to me that some of your readers might be briefly entertained working out both the decimal result, ?., above and the duodecimal result, ?;, below if the machine had been duodecimal:

1,234,567,89E; × EE,EEE,EEE,EEE; = ?

(where E above represents Ɛ; or "el")

Sincerely, H. K. Baumeister, DSA Life Member, № 140; ::::

Notes from "Featured Figures", pages 13; and 14;:

* The reference notation indicates items in the DSA's *Duodecimal Bulletin* (DB) or the DSGB's *Duodecimal Newscast, Duodecimal / Dozenal Review* (NR) in the format Volume·Number·Page. Items deriving from the DSGB's *Dozenal Journal* (DJ) are presented in the format Continuation Number·Page. All figures in the dozenal symbology in use at the time of publication of that issue. Thus, DB 02·1·1Χ refers to Vol. 2 № 1 page one dozen ten; DB 2♯·1·1♯ refers to Volume two-dozen eleven № 1 page one dozen eleven. The publications of the DSGB use the Pitman symbology.

A Items retrievable at time of publishing from the Dozenal Society of Great Britain's official website, specifically, http://www.dozenalsociety.org.uk/basicstuff/digits.htm.

B Retrievable at time of publishing at http://www.dozenalsociety.org.uk/basicstuff/hammond.htm, part of the official website of the Dozenal Society of Great Britain.

C Private communication with the authors.

D P. D. Thomas, (1987). *Modular Counting: A Preview of the Numbers of the 21st Century* (Revised Edition). Clarence Gardens, South Australia. Modular Conversion Bureau.

E These systems are introduced here by the author as examples of his system in the case of "acýlin", and as a plausible example in the case of "Decker".

F Retrievable from the DozensOnline internet forum. George P. Jelliss (2005). Symbols for TEN and ELEVEN? (Internet forum thread in the "On Topic" Forum, "Number Bases" topic) http://z13.invisionfree.com/DozensOnline/index.php?showtopic=11, entry by user "GPJ" at 10:54 am 14 August 2005.

J Retrievable from the DozensOnline internet forum. Shaun Ferguson (2009). Symbols for TEN and ELEVEN? (Internet forum thread in the "On Topic" Forum, "Number Bases" topic) http://z13.invisionfree.com/DozensOnline/index.php?showtopic=11, entry by user "Shaun" at 8:41 am 8 June 2009.

L "Nature's Numbers" or "gravity-generated numbers", retrievable at http://www.earth360.com/math-naturesnumbers.html, Prof. Lauritzen's website.

M *Letters to the editor: On binary notation*, Bruce A. Martin, Associated Universities Inc., Communications of the ACM, Volume 11, Issue 10 (October 1968) Page: 658.

T Retrieved at Traveler Hauptman's wiki at http://www.hauptmech.com/base42/wiki/index.php?title=Main_Page; confirmed through private communication with the author.

Dozenal Mathematical Displays Using LaTeX

by Donald P. Goodman III

Prior to the advent of computers, dozenalists were easily able to use new symbols for ten and eleven simply by putting new symbols on their typewriters. Since computers became standard, however, dozenalists have been increasingly confronted with the fact that computers are designed around interacting with human beings in decimal. The two character encodings in common use today, ASCII and Unicode, both presume the decimal system, providing only ten slots for numeric digits. Since ASCII and the first 127 slots of Unicode are identical, in the future a slight reform of them, perhaps removing some rarely used unprinted characters from the 0-32 range to include digits for ten and eleven, is clearly necessary; until that time, however, the discipline-standard method of typesetting mathematics, LaTeX, provides an easy method for the use of whatever symbols are desired in a way mostly transparent to the author.

TeX and LaTeX 1

For those readers who are not familiar with it, TeX is a typesetting engine designed by famous mathematician and computer scientist Donald E. Knuth. TeX is capable of typesetting arbitrary text; however, Knuth specifically designed it to be good at typesetting mathematics, and since its release it has been a de facto standard among many mathematicians.

LaTeX is a layer of macros running on top of TeX, designed by Leslie Lamport. It automates most of the nitty-gritty of typesetting in TeX; these days, very few people typeset directly in TeX, and most people use LaTeX to provide a layer of abstraction over the powerful but difficult TeX typesetting language. The American Mathematical Society also provides additions to LaTeX to facilitate the typesetting of high-order mathematics; the results produced by LaTeX are generally agreed to be unequalled by other digital typesetters.

The concept of TeX and LaTeX is simple. Most of the work of typesetting can be automated by the computer. Why not, then, let the computer do it, and make the information given to the computer (the user input) as simple and portable as possible? Most computer users these days are accustomed to a "word processing" model; that is, they type their words into a special program which displays them on the screen (theoretically) the same way that they will appear in the final product, saving the words in a usually proprietary binary file that is largely inaccessible to any program but that in which it was written.

TeX and LaTeX, on the other hand, accept the user's input (the document to be typeset) in a plain text file, which includes "markup" which provides instructions to the computer when it is necessary. This provides a file in which the data is always accessible and which can be edited and manipulated by any program capable of dealing with plain text input streams—that is, pretty much any program worth using.

For example, in a word processing program, when the user wants to emphasize a word, he highlights it, selects "italics" from whatever menu it might happen to be in, and then moves on. In LaTeX, however, the author must think not how he wants the word to appear, but rather what he wants it to do. If the italicization is meant to emphasize the word, then, the user will type into his file "\emph{word}", which instructs LaTeX to typeset the word with emphasis. A document class file (programmed by somebody else, and specified by the user) will tell LaTeX how emphasized text should appear. Normally, of course, this will be in italics.

While this system presents a higher learning curve than conventional word processors, it also produces better results and more portable input files. For this reason, LaTeX is extremely common in the academic community. Furthermore, its basis in TeX means that LaTeX, too, is extremely powerful when it comes to typesetting mathematics, bringing it prominence in the mathematical field, as well.

2 Macro Indirection

TeX, and those systems built upon it (like LaTeX), is a macro language, and thus provides an easy way to supply extra symbols for non-decimal number systems mostly transparently. This macro system allows new symbols to be seamlessly entered into sequences of numbers, providing minimal distraction to the author and being produced perfectly by the user:

`\x44 = 1492` yields Ɤ44 = 1492

While still easy to read in the LaTeX code, this markup still produces the appropriate typeset result.

There are two main ways of achieving this result, both extremely simple to implement. The first would be to simply define the macro to be used (in this case, "`\x`") to produce an image, which contains the symbol one wishes to use for that digit. However, this solution presents some problems. Most specifically, it's difficult to make it flexible. Producing the symbol "Ɤ" from "`\x`" in normal text would be fine; however, if the number appears in italic text, or in a different size from normal, the code to implement this solution begins to become overly complicated. While still easy to read in the LaTeX code, this markup still produces the appropriate typeset result.

The other solution is to design special fonts, which contain only two characters each; specifically, the characters DIGIT-TEN and DIGIT-ELEVEN. These characters can, of course, assume whatever shape one desires. Once these fonts have been created, TeX will automatically select the correct size, style, slant, shape, and whatever other attributes have been selected. This solution is so simple that the entirety (not including the font files themselves, of course) can be implemented, with the help of a simple helper package, in a mere six lines.

The advantage of using LaTeX with either of these methods is that any symbol can be substituted for DIGIT-TEN and DIGIT-ELEVEN. So, for example, if one wished to use a simple "χ" and "Ɛ", one simply redefines the macro in order to indicate this. As we shall see shortly, redefinition of the macro is trivial. Thus, we have a truly easy yet powerfully flexible system for producing professionally typeset dozenal documents in the customary professional mathematicians' language.

Implementation: The Dozenal Package 3

The author has used these ideas to produce the dozenal package, a LaTeX package which utilizes a conversion macro written in eTeX by David Kastrup (a coder well-known and respected in the LaTeX community). Naturally, this required the production of font files for the symbols DIGIT-TEN and DIGIT-ELEVEN to match the default LaTeX font, Computer Modern; these files were produced, in both Metafont and Postscript Type 1 forms (the latter produced by computerized tracing of the former). The Metafont fonts are called "`dozchars`"; the Postscript Type 1 fonts, in order to conform to the Berry Type 1 naming scheme, are named "`fdz`". These fonts approximate the Pitman characters {Ɤ Ɛ}, well-known throughout the dozenal community.

The conversion macro designed by Kastrup permits the easy redefinition of all automatic LaTeX counters to produce dozenal output. Therefore, when the package is selected (the user does this simply by including the line:

```
\usepackage{dozenal}
```

in his document's preamble), all the counters will be produced in dozenal. This means that page numbers, footnote numbers, section numbers, chapter numbers, and so on will all be printed in their normal places unchanged, except that they will be printed in dozenal, without the user having to do anything at all.

The characters which will be used for DIGIT-TEN and DIGIT-ELEVEN in those numbers, of course, depend upon the definitions of the macros. By default, the macros will select the "`dozchars`" or "`fdz`" fonts, which are visually identical and differ only relative to internal font definitions, and print them wherever "Ɤ" and "Ɛ" should be printed. However, if the user prefers to use different transdecimal characters (those which symbolize digits greater than or equal to ten), he or she need merely include his or her own font, or select his own symbols, and redefine the macros to use them instead. As mentioned earlier, this process is nearly trivial. To employ the Greek characters chi (χ) for "Ɤ" and epsilon (ε) for "Ɛ", which the author generally did before his Pitman fonts were production-ready, one can simply redefine the macros thus:

```
\renewcommand\x{\ensuremath{\chi}}
\renewcommand\x{\ensuremath{\epsilon}}
```

A better solution would be to use special fonts which contain the characters desired to be used for DIGIT-TEN and DIGIT-ELEVEN. For example, suppose there were a font containing Dwiggins transdecimals { χ Ɛ } designed to blend with the Computer Modern Fonts. Because this solution has greater capability, it also involves greater complication; however, it is still quite simple, and implementing the new symbols is a simple matter of changing a few lines of LaTeX code. Assuming that the font is called "`dwig`" and the symbol for DIGIT-TEN is contained in slot 88 (the slot for uppercase letter "X") and the symbol for DIGIT-ELEVEN is contained in slot 69 (the slot for uppercase letter "E"), and assuming the font definitions (which themselves consist of only nine very simple lines) are already available, all the user need do is enter this into his or her document:

```
\renewcommand\doz[1]{{\fontfamily{dwig}\selectfont #1}}
```

For the rest of the document, "\x" and "\e" can be used to refer to DIGIT-TEN = "X" and DIGIT-ELEVEN = "Ɛ", and the final document will be produced containing the Dwiggins numerals { X Ɛ }, rather than the Pitman numerals { ↊ ↋ } included by default with the dozenal package.

Of course, even this isn't something we should expect the typical LaTeX user to do. Such wizardry should normally be included in a style file, or as an option in the dozenal package itself, and implemented by the user simply by referring to that style file or option. For example, if the Dwiggins characters were defined in a new package ("style file"), the user would simply enter

```
\usepackage{dwiggins}
```

and his solution would be automatically implemented. If, on the other hand, the dozenal package were to include an option for using Dwiggins rather than the default Pitman characters, the user would instead enter

```
\usepackage[dwiggins]{dozenal}
```

and Dwiggins numerals would be used in place of the default Pitman characters throughout the document.

4 Work to Be Done

Naturally, the true solution to this problem is for dozenal characters, agreed upon by the vast majority of the dozenal community, to be included in every font and encoded by default into ASCII and Unicode. However, while this is the perfect solution, it's a solution that remains far in the future. Until then, easy ways of using dozenal characters in typeset works, particularly typeset mathematical works, need to be designed and pursued. The dozenal package for the widespread and popular LaTeX system thus fulfills a longstanding need.

Further work needs to be done, however. Most especially, the author of the dozenal package believes that he has produced credible Pitman characters for the package; however, he is not a type designer, and is well aware that these characters could be considerably improved by a competent font artist. Furthermore, not everyone prefers the Pitman numerals. Unless everyone were to accept the Pitman numerals, a solution the author would admittedly prefer, fonts containing other characters would be appropriate. We have already seen how such solutions might be implemented and how easy they would be for the LaTeX user.

The dozenal package has laid the groundwork for this future work. It is now possible to produce consistently dozenal documents in LaTeX, the typesetting language of mathematics and science. Maintaining and further extending this capability will greatly advance the cause of dozenalism in the world.

problem corner

solution from page Ɛ;

ƐX × Ɛ + 9 = XX
ƐX × Ɛ + 8 = XXX
ƐX9 × Ɛ + 7 = X,XXX
ƐX98 × Ɛ + 6 = XX,XXX
ƐX987 × Ɛ + 5 = XXX,XXX
ƐX9876 × Ɛ + 4 = X,XXX,XXX
ƐX98765 × Ɛ + 3 = XX,XXX,XXX
ƐX987654 × Ɛ + 2 = XXX,XXX,XXX
ƐX9876543 × Ɛ + 1 = X,XXX,XXX,XXX
ƐX98765432 × Ɛ + 0 = XX,XXX,XXX,XXX
ƐX987654321 × Ɛ - 1 = XXX,XXX,XXX,XXX
ƐX9876543210 × Ɛ - 2 = X,XXX,XXX,XXX,XXX

problem
solution in next issue

Skip Scifres proposed the following problem to readers back in 1199; (1989.) We reprint it here, noting that we are yet to receive a solution.

Find a procedure which will generate a set of integers each of which CANNOT be partitioned into four nonzero square numbers.

Example: The year 1199; (1989.) CAN be so partitioned:

$$1199; = 36;^2 + X^2 + X^2 + 5^2$$

The number 2540; (4224.) CANNOT.

Can anyone solve Skip's teaser?

Reprinted from WN 61; VOL. 32; № 1, page 24;.

Dozenal Jottings

We regret to report that Life Member Albert F. Lopez of Bingham, Maine has passed away. Albert was Member № 188;. He was a member of our Society for almost four dozen years, having joined on 1 June 1964.

Pass It Along!

Do you recycle your copies of the *Duodecimal Bulletin* after you've read them, or do they gather dust on a shelf in your attic once you've coded that killer app? Share the love! Pass it along to your local public or school library, share it with a friend. The *Bulletin* makes a stimulating gift for a math teacher. Spark intelligence in your local community: leave one at your local coffeeshop. Forward the electronic version to a friend. You never know where things lead! Help spread the word!

minutes BOARD & MEMBERSHIP 23; JUNE 11Ɛ5;

23; (27.) June 11Ɛ5; (2009.)
Nassau Community College
Garden City, NY 11530

In attendance: Board Chair Jay Schiffman, Board Members: Alice Berridge, Gene Zirkel, and President and Editor Michael De Vlieger; Member Dan Simon, Ms. Jen Seron, Brendan McManus.

The meeting was called to order at 2:00 PM by Board Chair Jay Schiffman in the Old Student Center at the College. (Thanks to Gene for supplying refreshments.) Minutes of the October 11Ɛ4; Board Meeting were accepted as printed in the *Bulletin*. Mike had arranged for Board Member Prof. John Impagliazzo, currently in Qatar, to participate in the meeting electronically via a computer link, but, unfortunately, that did not happen. Everyone introduced themselves: Dan is a nine-year old whiz who is very interested in dozens (see "The Mailbag" of the last issue) who came with his mother, Jen, a science mentor, teacher and tutor in the New York City schools.

Jay announced that he has reached 18; years of involvement with the DSA!

TREASURER Ellen Tufano's half-year financial report was accepted and approved by members. Her report shows that there is a pronounced decline in net worth, that dues membership is down and that the largest expenditure by far is for the printing of *The Bulletin*. Members are grateful for Ellen's work. Readers are reminded to send dues to Ellen's address which is listed in the *Bulletin*.

The NOMINATING COMMITTEE consisting of Alice Berridge, Pat Zirkel and Gene Zirkel proposed the following slate of officers: BOARD CHAIR Jay Schiffman, PRESIDENT Michael De Vlieger, VICE-PRESIDENT John Earnest, SECRETARY Alice Berridge and TREASURER Ellen Tufano. As there were no other nominations, this slate was approved.

Members expressed gratitude for Christina D'Aiello-Scalise's long service to the Society as SECRETARY. The Nominating Committee was reelected for another year.

The next meeting of both the Board and of the Membership was scheduled for Saturday, 22; (26.) June 11Ɛ6; (2010.) at 2 PM in room 309 in the Old Student Center at Nassau Community College. The meeting was adjourned at 2:20 pm.

The meeting was called to order by President De Vlieger at 2:25 pm. Minutes of the last meeting were accepted as printed in the *Bulletin*.

Due to the financial situation and our expenses, our reserves are greatly diminished. It was decided that dues would be raised. Members discussed and agreed on the following:

Members	one dozen six dollars ($18.)
Student Members	three dollars ($3.)
Supporting Members	three dozen dollars ($36.)

In addition, from now on, Members will receive full-color electronic copies of our *Bulletin* via email. Supporting and Life Members will have the option of receiving either the full-color electronic copy or the paper copy as before.

Gene agreed to send a mailing to those on the mailing list informing them of this new plan.

Members discussed options for the next and future meetings. The next Membership meeting will be 22; (26.) June 11Ɛ6; (2010.) at 2 PM at Nassau Community College in Old Student Center room 309. Attempts will be made to conduct part of the meeting as a conference call. Participation would require a fee. Efforts must be made to manage participation carefully so that all will get a chance to speak—this point was raised by those of us who have had experience with conference calls. The Conference Call part of the meeting, timed to occur after business has been conducted, will be geared to particular themes with discussion limited to a specific time slot. Those themes, such as Symbols or Nomenclature (names of numbers) and others would be announced ahead of time in the *Bulletin* and on the website.

Jen Seron suggested that lessons on dozens might be appropriate and that she could expedite those lessons through programs that she conducts with New York City schools as part of teacher training. Gene volunteered himself to teach lessons. Jen suggested that others of us might have suitable classroom materials. Gene mentioned that he has had great success with young students, teaching base 5 using coins.

Members agreed that the latest *Bulletin* is wonderful, that Mike is doing a spectacular job as Editor and that every detail, including the cartoons, is great! It was suggested that a map of members indicating the density of members might be an interesting future feature.

Problems remain with the website. Mike says he is confident that he can fix the website but that he needs time and he is currently very busy at work. A suggestion was made that we approach readers in the next *Bulletin*: "Would you be willing to work in committee, online, to improve the DSA website?"

We considered whether it would be advisable to have a counter for those who access the site, and set up an interactive procedure: "Can you convert your visitor number from base twelve?" Then either "You're right!" or "Let's see how to get the correct answer."

President DeVlieger later noted that Dan had a wonderful time. For example, at one point Mike demonstrated a particular pattern that he had discovered involving repetitive digits found in powers of dek (ten) in base twelve. Young Dan rattled off the next few dozenal powers of dek without pencil or paper! He then noticed a pattern in the length of the repetitions and wondered if this pattern would continue. Dan was quickly shown to be correct, through the use of Mathematica on a laptop present at the meeting. Mike added, "I think it meant a lot to Dan to be here, and being able to contribute and test his mettle was a boon. I do think we'll have the pleasure of seeing him again at the next meeting."

The Nominating Committee proposed that the Board of Directors in the Class of 11Ɛ5; (2009.) Consisting of: John Steigerwald, Carmine DeSanto, Jay Schiffman & Timothy Travis be reelected as the Class of 11Ɛ8; (2012.). In the absence of any other nominations the slate was elected unanimously. Jen pointed out that Dan (with his father's help) might become our Treasurer at some future time. Mike was reappointed Editor of our *Bulletin*, and Gene was appointed Parliamentarian to the Chair by Jay and also to the President by Mike. Members bought attractive Dozenal tee shirts as a special promotion.

Gene mentioned a problem submitted years ago by our recently deceased former Treasurer, "Skip" Scifres". The problem was printed years ago, but no solutions have ever been received. and it was suggested that it be reprinted in our Bulletin.

The Meeting adjourned at 4:30 pm and we adjourned to a nearby restaurant for dinner.

Submitted by Alice Berridge, Secretary

JOIN THE DSA TODAY!

You are invited to join the Dozenal Society of America!
The only requirement is a constructive interest in duodecimals!
Dues include a subscription to the *Duodecimal Bulletin*.

We depend on you! Annual dues are due as of 1 January. Make your checks for only one dozen six dollars ($18.) payable to the Dozenal Society of America and receive an electronic copy of the *Duodecimal Bulletin*, or be a Supporting Member at three dozen dollars ($36) and receive a paper copy of the *Duodecimal Bulletin*. Student dues are $3. A limited number of free memberships are available to students. As you know, our continued work depends very much upon the tax deductible dues and gifts from our Members.

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Students please furnish the information below:

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