

THE *Duodecimal* *Bulletin*

Vol. 4X; № 1; Year 11£5;

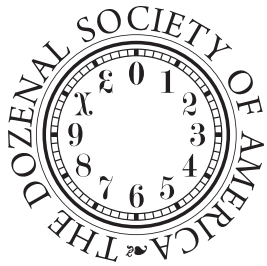


Reflections on the DSGB page 13;

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THE *Duodecimal* Bulletin

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president's message

One of the wonderful things about our being a dozenal society is that we are a group of like-minded individuals. Our Founders, as well as several gross of Members in years past, gathered together in an accessible location (NYC), in part to talk about dozenals, but also to enjoy the company of others who held the same regard for that noble number twelve. From day to day, as a dozenalist, you may find no one else who, just off the cuff, shares your keen appreciation of how the dozen improves life through its efficiencies. You may struggle to identify others who have thought about the way we measure and quantify our world, the way we enumerate things, or arrange items for packaging or sale. But here, on Long Island, as in other years, you can meet up with folks of like mind and discuss the wonders of the arrangement of the world by the dozen.

Please join us this summer, come from far and near. More than ever, one can argue that it is easier and less expensive to stay in New York City than at any other time since perhaps 2001. I'd be delighted to meet anyone in Manhattan between the 25th and the 28th of June to discuss dozenal. I take the Long Island Rail Road to our Saturday meetings, it's always a pleasant 36; minute ride. It would be even more pleasant to roll east, all the while chatting with a fellow dozenalist about our favorite number...the noble dozen! ❀❀❀

~ → We Depend on You ← ~

Annual dues are due as of 1 January. If you forgot, please forward your check for only one dozen dollars to Treasurer Ellen Tufano, 95 Holst Drive West, Huntington NY 11743-3939, USA. Student dues are \$3. As you know, our continued work depends very much upon the tax deductible dues and gifts from our Members. ❀❀❀

~ → Readers: Send in Your Symbols! ← ~

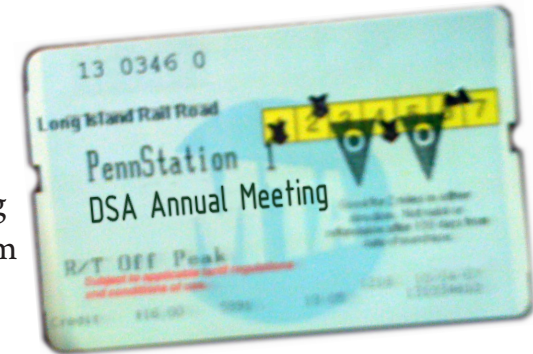
We are compiling a comprehensive study of dozenal symbol sets ("symbolologies") as well as those of other bases. The study will appear in an upcoming issue of the *Duodecimal Bulletin*! Send your symbols to Editor@Dozenal.org today! Here are some examples:

| | |
|---------------------------------|-----------------------------|
| Don Hammond | 0 1 2 3 4 5 6 7 8 9 2 4 |
| Gerard Robert Brost's "Dyhexal" | ⌈ √]] 7 1 ♯ 1 2 [L √ |
| P. D. Thomas' "Modular" | 0 ↓ ↗ ψ † λ × ψ ‡ ∪ ∆ † ❀❀❀ |

Nassau Community College

see <http://www.ncc.edu/About/directions.htm>

2 PM Saturday
25; June 11£5;
(Saturday 27. June 2009.)
Old College Union Building
2nd Floor Conference Room
Room CU309



Come on out and join us!

Items we plan to discuss, among other things:

- Membership Levels · Shall we introduce levels of Membership?
- Communication:
 - ▶ Website Update
 - ▶ Internet Meetings · these would likely present the following effects:
 - Broaden potential attendance
 - International participation, connection with the DSGB
 - Separate the Board and Membership Meetings
 - Maintain a more representative Board
 - Keynote speakers become more feasible
 - ▶ Social Networking
- *The Duodecimal Bulletin*:
 - ▶ Theming: Upcoming Symbols Issue
The *Bulletin* may incorporate "themes" in a limited proportion of upcoming issues. Come discuss what themes you'd love to see.
 - ▶ Topics for upcoming issues (outside of "themes")
 - ▶ Adjustments to the scope of distribution:
 - Traditional Mail: limited color: Shall we restrict distribution to certain Members or offer an option to continue to receive printed *Bulletins*?
 - Electronic: full color: How should these be distributed?
 - Electronic Legacy (back issue) Bulletins: How shall access be granted?



If you have dozenal topics, we'd be happy to discuss these during and after the meetings!

4 October 11£4; (2008.)
Nassau Community College
Garden City, NY 11530

In attendance: Board Chair Jay Schiffman, Treasurer
Ellen Tufano, Board Members: Alice Berridge, Gene
Zirkel, and President and Editor Michael D^e Vlieger.

The meeting was called to order at 2:15 PM by Board Chair Jay Schiffman in the Old Student Center at the College. (Thanks to Gene for supplying refreshments.) Minutes of the October 11£3; Board Meeting were accepted as printed in *The Bulletin*. Gene extended greetings from John Impagliazzo, now residing in Qatar, and from Secretary Christina D'Aiello. Editor Mike gave us a first look at the next issue of *The Bulletin*, WN 97; VOL. 49; Nº 2 (11£4;). Members were extremely impressed with his efforts which will be available soon for mailing to our readers and will also be available digitally on the website. Mike has marvelous ideas and seemingly inexhaustible energy. Gene gave mailing labels to Mike and to Ellen also. During Gene's vacation, Vice President John Earnest has agreed to expedite Dozenal mail from the DSA mailbox at Nassau Community College to Mike or to Ellen. Mike has agreed to respond to inquiries with a supply of materials provided by Gene; these inquiries have been reduced of late – probably as a result of DSA material available on our website www.Dozenal.org and various other websites including:

<http://www.mathworld.wolfram.com> <http://www.Dozenalsociety.org.uk/>
<http://z13.invisionfree.com/DozensOnline> <http://en.wikipedia.org/wiki/Dozenal>

Mike has agreed to amend and improve the DSA website. Mike will investigate whether the website can be fixed or whether it has to be completely overhauled. He will contact Dan Romero who helped design the current website.

Jay announced that he will be speaking at several conferences and intends to spread the Dozenal word.

Jay presented Mike with our Society's Ralph Beard Memorial Award, the text of which appears on page 6. Mike was genuinely thrilled by the presentation.

Treasurer Ellen Tufano's financial report was accepted and approved by members. She pointed out that our net worth has declined and predicts that this loss will be exacerbated in the coming months.

The Nominating Committee recommended that the Directors of the Class of 11£4; (2008.), consisting of Mike D^eVlieger, Christina D'Aiello, Ellen Tufano and Alice Berridge, be re-elected as the Class of 11£7; (2011.) As there were no other nominations, this was approved.

The meeting was adjourned at 3:15 PM.



Prof. Jay Schiffman presents Michael D^e Vlieger the Ralph Beard Annual Award at the 4 October 11£4 (2008.) DSA Meeting at Nassau Community College. The text of the award appears below:

THE RALPH BEARD MEMORIAL AWARD
of the
DOZENAL SOCIETY OF AMERICA

is hereby presented to

MICHAEL D^e VLIEGER
President & Editor

for his zeal and tireless effort for our Society
especially in recognition of his work
in creating and maintaining the master data base
of our historical documents including all of the dozens of past
Duodecimal Bulletins.

Given with gratitude by the Board of Directors on behalf
of all those who will benefit from his untiring efforts.



Mr. & Mrs. Berridge on the occasion of her receiving the Ralph Beard Annual Award at the 16; (18.) October 11XX (2002.) DSA Meeting at Bank Street College.

The Society regrets the recent passing of Mr. Edmund Berridge, dear husband of Professor Alice Berridge on 15 March 2009.

Alice is a long-time faithful member of the Board of Directors of the Dozenal Society of America, a former Treasurer and chair or member of several committees. Edmund had accompanied her to several Dozenal Society meetings.

Contact the Society at editor@dozenal.com for an address to which you may send your condolences. ❧

— → **Symbology & Nomenclature** ← —

The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use Dwigins dek (χ) for ten and his el (\mathcal{E}) for eleven. Whatever symbols are used, the numbers commonly called “ten”, “eleven” and “twelve” are pronounced “dek”, “el” and “dough” in the duodecimal system.

When it is not clear from the context whether a numeral is decimal or dozenal, we use a period as a unit point for base ten and a semicolon, or Humphrey point, as a unit point for base twelve.

Thus $\frac{1}{2} = 0;6 = 0.5$, $2\frac{2}{3} = 2;8 = 2.6666\dots$, $6\frac{3}{8} = 6;46 = 6.375$ ❧



The meeting was called to order by President D°Vlieger at 3:20 PM. Minutes of the last meeting were approved. Mike went into more detail about the mockup of the next *Bulletin*. This issue will include the DSA history, timeline, etc., with numerous photos. Mike is concerned that the cost of 4-color photos will be prohibitive so he may opt for photos on the inside cover with 2-color for the bulk of the text. It is hoped that in the future readers will opt for the digital version in thereby reducing costs.

Mike presented members present with a CD of web-optimized back issues WN 0; through WN 81;. Copies will be given to the NCC Library's Dozenal collection and to the DSA Office at NCC. Interested members in good standing may request a copy. Mike has a master DVD. It will be available from Mike on a limited basis. In the future all of the files will be available on the website.

The Nominating Committee consisting of Alice Berridge, Pat Zirkel, and Gene Zirkel proposed that the five officers of our Society consisting of Board Chair Jay Schiffman, President Mike D°Vlieger, Vice President John Earnest, Secretary Christina D°Aiello and Treasurer Ellen Tufano be re-elected. As there were no other nominations, this was approved. In addition, the Nominating Committee was also re-elected.

Mike was reappointed Editor of our *Bulletin*, and Gene was appointed Parliamentarian to the Chair by Jay and also to the President by Mike.

It was agreed that there would be one meeting next year. We are considering the end of May or in June at Nassau Community College on a Friday or Saturday for our next meeting. We will establish that meeting date early in 11£5; (2009.) when our calendars can be coordinated. Gene will see to this.

—* See page 4 for more information about the June 2009 meeting.

President Mike met with British mathematician, journalist and author Alex Bellos in St. Louis on 20; September this year and made a convincing case to Bellos about the wisdom and ease of using dozens. Mike was able to tie his passion with duodecimals to the use of dozens that he makes in his architectural business on a daily basis. Bellos was impressed with Mike's practical use of dozens as well as his reliance on base twelve to order his daily life (enumerating his projects, tallying years from and to events in his life.)

The meeting adjourned at 4:15 PM and then Board Chair Jay Schiffman delivered a talk: “Exploring Some Popular Problems Involving Patterns in Different Number Bases.” Members were very interested in his premise and the problems he presented.

Those in attendance then retired to a nearby restaurant for dinner.

Respectfully submitted by Alice Berridge for Secretary Christina D°Aiello

A Dozenal Nomenclature

by Owen G. Clayton, Ph.D.

The following is a consideration of the nomenclature necessary to a dozenal counting system in English, which is, by now, pretty much the *lingua franca* of the world. Much of this nomenclature is traditional, with only slight modifications to the cardinal numerals. All numeration herein is dozenal unless otherwise stated. To distinguish dozenal quantities from decimal quantities, they are always quoted accurate to at least one fractional place, with the period as decimal radix point and the semicolon as dozenal radix point. (See facing page.)

Names of Digits: All the digit-names are reducible to monosyllables. In the case of zero, we simply adopt the British “null”; “sem” and “lem” are shortened forms of seven and eleven respectively, and the well-known linguistic process of lazy-mouth will help insure that these two new words will be adopted.

There is nothing particularly problematic in retaining ten as a digit-name. Twelve, on the other hand, while it is monosyllabic, derives from “two left over (ten),” and the use of the word dozen for it is common enough to admit that word or a shortened form thereof; hence, doz.

The names for base-multiples are fairly straightforward; “sidoz” for six-dozen and “eighdoz” for eight-dozen are logical simplifications. Names for the base-powers, however, present a problem because the first grocers were not high-volume dealers, and seem to me not to have commonly used “great gross” to mean “twelve cubed.” I have chosen to resolve this problem by adopting two terms commonly used in commercial inventory: batch for the third and lot for the sixth power of the dozen.

To give some examples: there are “three batch eighdoz” feet (3080;0 = 5280.0) in a statute mile. The population of the United States is over “eighdoz four lot, five gross semdoz sem batch, one gross fordoz” (84 577 140;0 = 300,000,000.0).

Symbology: My preference for the digit-symbols for ten and lem is based on their representability in a typical 7-segment LCD/LED grid. The symbol for ten would resemble the symbol 5, but reversed on its vertical axis, and minus its top. The symbol for lem would be the symbol “F” reversed on its horizontal axis (upside-down), which is quite close to one of the more preferred suggestions for “lem.”

Editor’s Note: The Editor added a table at the lower right which summarizes the author’s suggested symbology. The symbols are similar to Mr. Don Hammond’s proposal available at <http://www.dozenalsociety.org.uk/basicstuff/hammond.html>.

~ → Got something dozenal to say? ← ~

Send your proposals and dozenal doings to Editor@Dozenal.org. We’re always delighted to hear from our Members. Include a description or drawing of your symbols, and we’ll attempt to set them to print within your article.

The Owen G. Clayton System of Dozenal Nomenclature and Symbology

NOMENCLATURE OF THE COUNTING SYSTEM

| Number | Cardinal | Ordinal | Partative | Multiplicative | Iterative |
|--------|----------|---------|-----------|----------------|-----------|
| 0 | ∅ | null | nullth | | |
| 1 | 1 | one | first | whole | single |
| 2 | 2 | two | second | half | double |
| 3 | 3 | three | third | | triple |
| 4 | 4 | four | fourth | | quadruple |
| 5 | 5 | five | fifth | | quintuple |
| 6 | 6 | six | sixth | | sextuple |
| 7 | 7 | sem | semth | | septuple |
| 8 | 8 | eight | eighth | | octuple |
| 9 | 9 | nine | ninth | | nonuple |
| ∂ | ∂ | ten | tenth | | tenuple |
| ε | ε | lem | lemth | | lemptuple |
| 10 | 10 | doz | dozth | | dozuple |

NOMENCLATURE OF THE BASE-MULTIPLES, ETC.

| Multiple | Cardinal | Ordinal/Partative |
|----------|----------|-------------------|
| 20 | 20 | twodoz |
| 30 | 30 | thridoz |
| 40 | 40 | fordoz |
| 50 | 50 | fivdoz |
| 60 | 60 | sidoz |
| 70 | 70 | semdoz |
| 80 | 80 | eighdoz |
| 90 | 90 | nindo |
| ∂0 | ∂0 | tendoz |
| ε0 | ε0 | lemdoz |
| 100 | 100 | gross |

NOMENCLATURE OF THE BASE POWERS

| Power | Cardinal | Ordinal/Partative |
|-------|----------|-------------------|
| 1 | 1 | doz(en) |
| 2 | 2 | gross |
| 3 | 3 | batch |
| 4 | 4 | dozen batch |
| 5 | 5 | gross batch |
| 6 | 6 | lot |
| ... | ... | ... |
| 9 | 9 | bilot |
| ... | ... | ... |
| 10 | 10 | trilot |

| SYMBOLY | | |
|---------|---------------------|-----|
| | Clayton/ Hammond | DSA |
| null | 0 | 0 |
| one | 1 | 1 |
| two | 2 | 2 |
| three | 3 | 3 |
| four | 4 | 4 |
| five | 5 | 5 |
| six | 6 | 6 |
| sem | 7 | 7 |
| eight | 8 | 8 |
| nine | 9 | 9 |
| ten | ∂ | χ |
| lem | ε | ε |



problem from last issue:

In our last issue we presented the following beautiful mathematical symmetry, asking, “Can you find a similar pattern in base twelve? Can you generalize it to work in any base?” ❄❄❄

$$\begin{aligned}
 1 \times 1 &= 1 \\
 11 \times 11 &= 121 \\
 111 \times 111 &= 12321 \\
 1111 \times 1111 &= 1234321 \\
 11111 \times 11111 &= 123454321 \\
 111111 \times 111111 &= 12345654321 \\
 1111111 \times 1111111 &= 1234567654321 \\
 11111111 \times 11111111 &= 123456787654321 \\
 111111111 \times 111111111 &= 12345678987654321
 \end{aligned}$$

❄ CONTINUED ON PAGE 25;

❄ **Metric Silliness Continues** ❄

Jean Kelly

Many people under the European metric dictatorship still want to use the convenient British Imperial units of measurement in place of the awkward decimal metric units.

The European Parliament is still trying to enforce rules such as you cannot order a pound of bananas in Great Britain or a pint of ale in a pub.

After much discussion, they magnanimously consented to allow a few exceptions last December. The Irish may still post speed limits in the easily understood mile per hour, and the English may order their pint of ale. However the pound of bananas may only be priced in pounds if it is also priced in metric.

Since these exceptions are temporary, we will hear more of this idiocy in the future.

“The government may be eager to scrap the pound as our currency, but at least we can say we have saved it indefinitely, as a measurement at any rate. Under this law, shoppers will be able to continue using the measurement they prefer.”

— Giles Chichester, a British member of the European Parliament.

— Adapted from the *New York Times*

See “Metric Martyr’ Loses Historic Case”, the related story of the man convicted of selling bananas by the pound, in our *Bulletin*, WN 85; VOL. 42; Nº 2 pages 4 – 9. ❄❄❄

❄ **Our British Associates** ❄

The Yardstick, published by the
BRITISH WEIGHTS & MEASURES ASSOCIATION

users.aol.com/footrule

£; ELEVEN

THE DUODECIMAL BULLETIN

the mailbag



Mr. **Charles Dale**, Member number 193; writes:

» Dear Mr. D^eVlieger,

The last issue (WN 97; VOL. 49; Nº 2, p. 15;) asks for comments on your society timeline. In the summer of 1965, I was an undergraduate at Kent State University and read an article in *Scientific American* (perhaps by Martin Gardner?) that mentioned the society. I still remember vividly a statement like, “There is even a Duodecimal Society of America, and they have a duodecimal slide rule.” I was very impressed and I tracked the society down, which wasn’t easy long before the Internet. At one time I owned a duodecimal slide rule, and it was fascinating and instructive to learn how to use it. I also joined the society then, and I’ve been a member ever since.

Bottom line: It would be interesting to discuss more about the history of the duodecimal slide rule, and even the mention of the society in *Scientific American* if you can track it down. *Scientific American* was enormously influential at that time, and I suspect that even that one mention might have brought in more new members than just me.

❄ Best regards,

Charles Dale 193;

Gaithersburg, MD ❄❄❄

» Mr. Dale,

I’ve performed a brief search on the terms “gardner”, “scientific american”, “slide rule”, “duodecimal”, and haven’t really been able to delve into all that I’ve turned up. It seems a Martin Gardner is heavily into slide rules. The SciAm website features back content limited to the mid nineties, nothing is available before around 1996. Unless I find a resource where the back issues of *Scientific American* are actually physically stored, I don’t think I’d be able to track down the article to which you refer. It is interesting, however, that duodecimal instruments were described in such a top-drawer publication. (I’ve been a big fan of SciAm since my own boyhood in the 1970s.)

There are several articles about the duodecimal slide rule in the *Duodecimal Bulletin* itself:

| | | |
|---------------------------------|------------------------|------------------------|
| VOL. 4 Nº 2 pp. 7 & 24; | VOL. 13; Nº 1 Page 3 | VOL. 2X; Nº 1 Page 5 |
| VOL. 4 Nº 3 Page 5 | VOL. 17; Nº 1 Page 10; | VOL. 32; Nº 3 Page 21; |
| VOL. 5 Nº 1 pp. 8, £;, 18;, 20; | VOL. 18; Nº 1 Page 3 | VOL. 33; Nº 1 Page 1H; |
| VOL. 6 Nº 1 Page 17; | VOL. 27; Nº 1 Page 4 | VOL. 34; Nº 1 Page 19; |
| VOL. 12; Nº 1 Page 4 | VOL. 28; Nº 2 Page 4 | |

I haven’t read all the above references but am sure that, before the advent of the electronic calculator and the capability of scripting code to process dozenal mathematics on computers, the slide rule was of intense interest among dozenalists.

If you could find that article that would be wonderful. Perhaps we can write a brief article about the history of dozenal computation devices in general. This would take in the slide rule, its development within the DSA by Camp, Humphrey, etc. Then we could describe the impact of the calculator in the 1980s, and software nowadays that crunch dozenal figures. Hope to see you at the coming June meeting on Long Island!

❄ Michael T. D^e Vlieger, Editor, *Duodecimal Bulletin* ❄❄❄

VOLUME 4X; NUMBER 1; WHOLE NUMBER 98;

ONE DOZEN 10;



Brian and Jean Bishop toast you from their sunny home. Cheers from the DSGB!

Reflections on the DSGB

compiled by Gene Zirkel

Editor's Note: We continue Professor Zirkel's review of the history of the Dozenal Societies of America and Great Britain. The following material was included in an appendix to the article "A History of the DSA" which appeared in the last issue.

↪ See our Bulletin WN97; VOL. 49; № 2.

Presenting some thoughts compiled from notes from Brian Bishop, Shaun Ferguson, and Robert Carnaghan.



Shaun Ferguson, at the helm of the DSGB's website

A few early members of the DOZENAL SOCIETY OF GREAT BRITAIN:

Brian Bishop was DSGB's first Secretary and Editor. He was followed by **Shaun Ferguson** who handed the reins over to **Don Hammond**. Hammond improved the style of the magazine and had plenty of useful ideas; he died of an asthma attack.

Since Hammond passed away the DSGB's growth has slowed. Under the direction of Shaun Ferguson and with the coming of the Internet, the DSGB managed to take on a new lease on life. They seem to be attracting more people, especially with the new website Shaun Ferguson set up at www.Dozenalsociety.org.uk.

For example, **Brian Parry** is a student and very keen; lots of ideas and energy. He has set up the Dozenal Forum at <http://z13.invisionfree.com/DozensOnline>, which seems to be attracting some attention. Long may it continue!

Lou Loynes, a little chap, very cheerful, was an artist and devised and published a system for distinguishing colours based on duodecimals. I believe some DSGB meetings were held in his office.

Arthur Whillock worked at the government building research establishment near Wallingford. He devised systems of measurement and time based on duodecimals, which he explained on his visit to the DSA.

Fred Ruston, a friend of Brian's, was one of the early members of DSGB. He lives with his wife Elizabeth in a hamlet in Stambridge.

On one occasion Brian met **Professor A. C. Aitken**, from New Zealand, of Edinburgh University. He recalled, "I think Robert Carnaghan enrolled him for us. He asked for my telephone number. I looked for a piece of paper to write it on. What an insult. He at once recited its cube root. He produced a pamphlet against metrication."

Robert Carnaghan comments that it was the kindly Professor Aitken who mentioned the DSGB to me when I was a student and not the other way around. I took the liberty of speaking to him (on something quite different, but he evidently recognised an idealist) at the end of one of his lectures to first-year students.

Carnaghan remembered attending some of DSGB's annual meetings in London, probably in the late 1960s or possibly the early 1970s. "At one, **Sir Iain Moncreiffe** of that ilk and his wife came, and I met them. As with Aitken, only years later did I learn from occasional articles or obituaries what a knowledgeable man Sir Iain was, and how much he had done in his life, particularly in the study of genealogy and history."

Shaun Ferguson was another keen member of DSGB in its early days, and is now one of the few active members left.

Bruce Moon came from New Zealand several decades ago and spent a year in the department of computing of University College, London. He wrote for DSGB a good basic introduction to duodecimal arithmetic for British readers called "Dozens Arithmetic for Everyman". (It's now out of date because it refers to the shillings of twelve pence which were then part of Britain's currency.) Like the DSGB's magazine in those days, it was typed onto wax stencils and reproduced on a duplicator.



Brian Parry, founder of the DozensOnline forum



Arthur Whillock in the States, attending an Annual Meeting of the DSA

A Merger

A strand in the history of DSGB was the founding some years but not many years later, quite independently, of a DUODECIMAL ASSOCIATION. That association had not been in existence for long when the two societies discovered each other and agreed to merge.

Carnaghan also recalled, “I don’t remember attending any DSGB meetings at **Louis Loynes’** office in Monmouth Street in central London, but I remember Brian introduced me to him there, and I met him several times”. He favoured base eight or one dozen four, but said that when he devised his system for numbering colours (he wrote a book about colour theory and produced it himself), he found that for that specific purpose the duodecimal system was best because of there being three primary colours. He called his colour notation the BYRAZ system, these letters representing blue, yellow, red, white and black.

On one occasion, probably at Loynes’ office, I was introduced to another supporter of base eight or one dozen four, **Douglas Blacklock**, who on the first meeting seemed very clever. As Brian and I both worked in central London at the time, we met him for lunch on a number of occasions when Blacklock (who must have been retired) was in London, and we got to know him fairly well.

Blacklock saw himself as an inventor, and he did seem to have an imaginative mind. He was a pleasant enthusiast if you had some time to spare!

Apparently he had once written a book on accounting for management (he was then an accountant) before the war. In discussion he would jump around disconcertingly from one idea or subject to another; it was reasonably interesting or entertaining once one had got used to it.

Blacklock in turn introduced Brian and me to a man called, I think, **Carr-Carme**, who had a small office in Shaftesbury Avenue in central London, and who had devised a means of making music easier to read music by use of colour. I don’t know whether anyone took up his idea. Doubtless he could have implemented and demonstrated it much more easily if cheap ink-jet printers had then been available. But here I’m wandering way off course.

Arthur Whillock worked as a professional engineer at the (then government-owned) well-known hydrological research establishment at Wallingford. He used to write good articles about the virtues of duodecimals and about the failings of the metric system. He became Information Secretary of DSGB. After Don Hammond’s untimely death he became Secretary as well, in the absence of any other volunteer. Arthur was also the editor of the *Dozenal Review-cum-Dozenal Journal*.

I spoke to Don Hammond once by telephone but never met him. Unfortunately, by the time he became active, as Secretary and Editor, DSGB had altogether ceased its annual meetings. There was some justification for this, inasmuch as members lived far apart (in British terms). However, it would have been possible

to compromise and have a meeting every few years, so as to give members a chance to meet one another and develop a social side that does so much to bring into the fold those who have a passing interest but who often move on in the absence of social contact.

Several societies I’ve been a member of have single-mindedly concentrated on higher thoughts and neglected the social side, with the result that they have also withered. To some extent the Internet now provides an alternative through the use of forums. A forum could allow anyone interested to put forward ideas, proposals, questions and so on, and with luck to get reactions.

One simple thing I learned years ago was how to count up to a dozen on one hand, using the thumb as a pointer and the joints of the four fingers as positions, and then to count dozens similarly on the other hand. Occasionally, when the chance arises and I remember, I show someone, usually a child, how to do this, in the hope of sparking an interest at this level, and sometimes there is interest. One way of making it more “concrete” is to refer to months and years. It’s not much, but it’s a way of trying to pass on something few have come across.

There’s not much more I can do now, with so much else left to do, but neither can I completely let go after all these years. In particular, I must find a good home sooner or later for the books and magazines that I have or have been entrusted with. I believe that Arthur Whillock would also like to find a home for his collection; he would like them to be kept together.

So there are some reminiscences and thoughts, for what they are worth. ❖❖❖

— → **Our British Associates** ← —

THE DOZENAL SOCIETY OF GREAT BRITAIN

www.dozenalsociety.org.uk

❖ ❖ ❖ ❖ ❖ ❖ ❖ ❖ ❖ ❖

ONE HAND IS BETTER THAN TWO...

It has been claimed that the only reason we still count by tens instead of dozens which are easier to learn, easier to use, and more compact, is the fact that we have ten fingers.

However, it takes two hands to count to ten on your fingers while you can easily count to a dozen on only one hand using your thumb to point to the twelve phalanges (or bones) in your four fingers.

SOME NOTES ON THE HISTORY AND DESIRABILITY OF USING ALTERNATE NUMBER BASES

IN ARITHMETIC

by Christopher J. Osburn

We all do arithmetic. We do it in the supermarket when providing for our families. We do it on the highway when comparing our speed with the posted limits (sometimes). We do it in the restaurant when determining how much of a tip to leave on the table, or whether we'll have to wash dishes to pay for the meal. Arithmetic, and mathematics as a whole, is always around us from the most mundane tasks to the most embarrassing and profound situations.

With some minor exceptions we perform arithmetic operations in base ten. But is base ten really the best way to do arithmetic? Are calculations easier to perform in some other base, say twelve or sixteen? Let us take a brief look at some of the inherent advantages and disadvantages of the use of alternate number bases in arithmetic, starting with our tried and true friend, base ten.

The primary advantage that base ten gives us is that we're accustomed to it. The most popular explanation is that we have ten fingers on our hands. We are able to match our fingers to up to ten of some other object. Counting the number of times we can do this before we run out of whatever we were counting allows us to use numbers greater than ten. Some cultures, for similar reasons, have used number systems of five and twenty (the latter by calling the toes into play).¹ There are some disadvantages that are immediately apparent in these bases. Base five, for instance, is a fairly small base, which leads to long strings of numerals even for small values. (Compare one hand and three fingers (13_{five}) to eight fingers (8_{ten}) Base twenty has the problem that very few of us can bend our toes independently of the others. Peoples in colder climes may have to remove shoes of moccasins to do any such counting. Whatever the reason, our familiar decimal system predominates.

There are some notable exceptions. Ancient Sumerians and Babylonians used a sexagesimal (base sixty) system of enumeration in connection with the place-value system. Each sexagesimal place, however, was constructed of cuneiform symbols giving the number of tens and units for that place.² Some Northern European societies had a quantity known as a "great hundred" made up of ten dozens (decimal 120.), reflecting the rudiments of a duodecimal (base twelve) counting system.³ The Romans, even though they used base ten for their integer counting, had a system of duodecimal fractions. It is believed they chose this because of easy divisibility in so many different ways.⁴

Despite our use of the decimal system for many millennia, there is something that requires us to consider non-decimal enumerating: the electronic digital computer.

Computers, at their lowest levels of operation, know only whether a current is flowing through a transistor or not. This off/on choice leads us to the binary (base two) system of numeration. But while computers have little problem working in binary, for humans it can be a bit cumbersome. For example:

$$842_{\text{ten}} = 1101001010_{\text{two}}$$

As you can see, relatively small numbers in decimal produce some real monster-sized

binary numbers. To cope, we have developed some convenient shortcuts. By converting binary numbers into octal (base eight) or hexadecimal (base sixteen) numbers, we make binary numbers more manageable for humans. This is actually quite easy. Taking our example from above,

$$1101001010_{\text{two}}$$

we divide the number up three places at a time from the right,

$$1\ 101\ 001\ 010$$

and then convert each group of three into single octal digits by finding the values that correspond to each place:

$$\begin{array}{cccc} 1 & 101 & 001 & 010 \\ 1 & 5 & 1 & 2 \end{array}$$

This gives us:

$$842_{\text{ten}} = 1512_{\text{eight}}$$

The process for converting binary into hexadecimal is similar; start by dividing the number into four digit groups:

$$11\ 0100\ 1010$$

and insert the appropriate values:

$$\begin{array}{ccc} 11 & 0100 & 1010 \\ 3 & 4 & 10 \end{array}$$

This leads to a bit of a problem. How do we squeeze that ten into a single digit? The current usage in the computer industry is to represent the values ten through fifteen by the letters "A" through "F": "A" equals ten, "B" equals eleven, etc. Our conversion from above then becomes:

$$\begin{array}{ccc} 11 & 0100 & 1010 \\ 3 & 4 & A \end{array}$$

giving us:

$$842_{\text{ten}} = 34A_{\text{sixteen}}$$

In some bases, identifying prime numbers *greater than 2* and perfect squares (or at least ruling them out) is fairly easy, in others it is more difficult. A good test is to check the final digit in the number. For example, in base ten we know there are no prime numbers ending with the numeral 4 and there are no perfect squares that end with a 7. How many of the available numerals in a given base can terminate a prime number? How many will terminate a perfect square? It is also useful to compare that number with the total available. If, for instance, a prime number can end with any digit at all, that test becomes useless.

We have so far come across several different numbering systems, which we can categorize as follows:

1. The "Finger" Bases: five, ten, twenty;
2. The Binary Bases: two, eight, sixteen;
3. Other Bases: twelve and sixty.

TABLE 1: RULES OF DIVISIBILITY FOR SELECTED BASES

- Base 2:
 2: A number is even if it ends in 0, odd if it ends in 1
- Base 5:
 2: Any number whose digits add to a multiple of 2
 4: Any number whose digits add to a multiple of 4
- Base 8:
 2: Any number ending in an even digit
 4: Any number ending in 0 or 4
 7: Any number whose digits add to a multiple of 7
- Base 10:
 2: Any number ending in an even digit
 3: Any number whose digits add to a multiple of 3
 5: Any number ending in 0 or 5
 6: Any even number whose digits add to a multiple of 3
 9: Any number whose digits add to a multiple of 9
- Base 12:
 2: Any number ending in an even units place
 3: Any number ending in 0, 3, 6, 9
 4: Any number ending in 0, 4, 8
 6: Any number ending in 0, 6
 11: Any number whose digits add to a multiple of 11
- Base 16:
 2: Any number ending in an even units place
 3: Any number whose digits add to a multiple of 3
 4: Any number ending in 0, 4, 8, 12
 5: Any number whose digits add to a multiple of 5
 6: Any even number whose digits add to a multiple of 6
 8: Any number ending in 0 or 8
 10: Any even number whose digits add to a multiple of 5
 15: Any number whose digits add to a multiple of 15
- Base 20:
 2: Any number ending in an even units place
 4: Any number ending in 0, 4, 8, 12, 16
 5: Any number ending in 0, 5, 10, 15
 10: Any number ending in 0, 10
 19: Any number whose digits add to a multiple of 19
- Base 60:
 2: Any number ending in an even units place
 3: Any number whose units place is a multiple of 3
 4: Any number whose units place is a multiple of 4
 5: Any number whose units place is a multiple of 5
 6: Any number whose units place is a multiple of 6
 10: Any number ending in 0, 10, 20, 30, 40, 50
 12: Any number ending in 0, 12, 24, 36, 48
 15: Any number ending in 0, 15, 30, 45
 20: Any number ending in 0, 20, 40
 30: Any number ending in 0, 30
 59: Any number whose digits add to a multiple of 59

| Base | Prime Number End Digits | % | Perfect Square End Digits | % |
|------|--|-----|--|-----|
| 2 | 1 | 100 | 0, 1 | 100 |
| 5 | 1, 2, 3, 4 | 200 | 0, 1, 4 | 60 |
| 8 | 1, 3, 5, 7 | 100 | 0, 1, 4 | 38 |
| 10 | 1, 3, 7, 9 | 80 | 0, 1, 4, 5, 6, 9 | 60 |
| 12 | 1, 5, 7, 11 | 67 | 0, 1, 4, 9 | 33 |
| 16 | 1, 3, 5, 7, 9, 11, 13, 15 | 100 | 0, 1, 4, 9 | 25 |
| 20 | 1, 3, 7, 9, 11, 13, 17, 19 | 80 | 0, 1, 4, 5, 9, 16 | 30 |
| 60 | 1, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 49, 53, 59 | 53 | 0, 1, 4, 9, 16, 21, 24, 25, 36, 40, 45, 49 | 20 |

TABLE 2: PRIME NUMBER END DIGITS (TOTATIVES) AND PERFECT SQUARE END DIGITS.

We have also seen that some bases are good for humans while others are good only for computers. Is there some way we can quantify the usefulness of these systems (for humans) so that we can compare them? Which of these bases is really the best for counting and arithmetic for humans?

One way of comparing number bases is to compare some of their divisibility indicators. For example, a divisibility indicator in base ten would be the fact that all numbers divisible by five end in a zero or a five digit. Easy rules like this are one way we make counting and arithmetic easy on ourselves. George Terry, in his book *Duodecimal Arithmetic*, suggests tests to help identify prime numbers and perfect squares.⁵

DIVISIBILITY RULES.

Let us take a quick look at the divisibility rules first. We will concentrate on the “easy” rules (hard rules aren’t that valuable to humans). We’ll restrict ourselves to numbers less than the base number itself (except for base two). Table 1 on page 19; shows when a number in the given base is divisible by the digit in the left hand column.

END DIGITS OF PRIME AND SQUARE NUMBERS.

Note: the columns marked “%” in Table 2 refer to the percentage of digits that appear against the given base. The percentages given after the prime digit column refer to the number of odd digits that appear. Base 5 reads 200% in this column, as numbers ending in even digits can also be prime.

REGULARITY OF DIGITS.

Table 3 on page 1£; lists “regular numbers” along with a “regularity index” for each base. A regular number is a number, in base sixty, the reciprocal of which has a finite number of places. We can extend this concept to any other base and say a regular number has a terminating fractional part in that base. For example, $\frac{1}{3}$ is a terminating fraction in base twelve (0.4_{twelve}) but it is not a terminating fraction in decimal ($0.333 \dots_{\text{ten}}$). So, three is a regular number in base twelve but not in base ten. This is a good alternative to counting the divisibility rules presented in Table 1. If we look at every single-digit number greater than 1 in each base we can see what portion of them are regular. We call that portion the “regularity index” and express it as a percentage.

| Base | Regular Numbers | Regularity Index (%) |
|------|---|----------------------|
| 2 | [none] | 0 |
| 5 | [none] | 0 |
| 8 | 2, 4 | 33 |
| 10 | 2, 4, 5, 8 | 50 |
| 12 | 2, 3, 4, 6, 8, 9 | 60 |
| 16 | 2, 4, 8 | 21 |
| 20 | 2, 4, 5, 8, 10, 16 | 33 |
| 60 | 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, 25, 27, 30, 32, 36, 40, 45, 48, 50, 54 | 41 |

TABLE 3: REGULAR NUMBERS FOR SELECTED BASES. *Editor's Note: this table includes regular numbers by the Author's definition which are less than the base given in the leftmost column. Such positive integers lesser than the base would thus be single digits in that base.*

COMPARING THE BASES.

We have quite a lot of data to digest. Let's look at how we might combine our indices and percentages into something we can use for comparisons. This will be somewhat subjective since we're really trying to quantify how a human will feel about each number base while counting and doing arithmetic.

We should give a positive consideration to the regularity index, since we'd like to avoid infinite fractions. We'll give a smaller positive consideration to the fact that a larger base yields a more compact notation; the length of a numeral gets shorter or remains unchanged as the log of its base increases. (We use the natural logarithm to avoid showing preference to any integer base.)

Negative consideration should be given for the number of different digits that are found at the ends of prime and square numbers (fewer is better). And we'll consider the size of the multiplication table. A bigger base has a larger table to learn and we should think of the school kids. Combining all these influences gives us the following relation:

$$I = \frac{R \times \ln b}{P \times S \times b}$$

where:

b is the base in question

R is the regularity index

P is the percentage of odd digits found at the ends of prime numbers

S is the percentage of all digits found at the ends of perfect squares

and the percentages P , R and S are expressed as fractionals.

This yields the data shown in Table 4.

This table indicates that base twelve is, by far, a much more logical base to do arithmetic in. Bases eight, ten and

| Base b | Index I |
|----------|-----------|
| 2 | .000 |
| 5 | .000 |
| 8 | .231 |
| 10 | .240 |
| 12 | .559 |
| 16 | .149 |
| 20 | .208 |
| 60 | .265 |

Table 4: Compare the bases.

sixty seem to have fared about equally well (even with base sixty's enormous multiplication table). It would seem that base twelve ranked so much higher because it combines good divisibility patterns (noted by the regularity index) with a fairly small set of operation tables.

On the other hand, note bases two and five bringing up the rear. For base five, there are no terminating decimal fractions. Also, as an odd-numbered base, we have more difficulty finding odd and even numbers in base five. A base five prime number may end in any digit. For example: 31_{five} has an odd last digit, but is equal to 16_{ten} , an even number. Base two fails mainly because it is so cumbersome to work with, and that it's more difficult to guess whether a number might be prime or square (shown by high values of P and S). The regularity index of base two, zero, may be merely a problem in defining the regularity index. There are simply no integers between 1 and 1. Arbitrarily setting the regularity index to 50% gives a final index value of .173. This is still quite low, but seems more appropriate.

Of the bases we haven't considered, does anything else compare to base 12? Base 6 does with an index of .504. These are the only two bases that come in above .500 and, in fact, the only two coming in above .400. (Base 4 came in third at .347.) The top ten are:

12, 6, 4, 24, 30, 18, 60, 10, 36, 8.

Should we convert to base twelve? Re-educating several billion people seems like a daunting task, so we might begin by teaching duodecimals in parallel with decimal math to children just entering school. In my fifth year at elementary school, I volunteered to teach octal arithmetic to the class. My classmates reacted positively, having fun playing with slightly altered arithmetic rules and viewing the world through the eyes of an eight-fingered creature. Today I carry out counting tasks in parallel with decimal and dozenal, which provides me a reality check of sorts. Giving people another lens thorough which to see the world will do no harm and may well be of great benefit. ❖❖❖

Notes

¹ Eves, Howard; *An Introduction to the History of Mathematics*, fifth edition, Philadelphia: Saunders College, 1982; p. 4

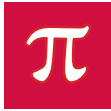
² Ibid, p. 10

³ Menninger, Karl; *Number Words and Number Symbols*, English translation, Cambridge, Massachusetts: The MIT Press, 1969; pp. 154ff.

⁴ Ibid, pp. 158ff.

⁵ Terry, George S.; *Duodecimal Arithmetic*, London: Longmans Green and Co., 1938

Editor's note: The text of this article can be retrieved at <http://www.ubergeek.org/~chris/random/base12.html> ❖❖❖



Here we present the first great gross digits of π , as requested by Mr. Benedikt Jahnel in February 2009. This is a truncation of his request for 2,000; digits.

featured figures

The entire output was generated by Mathematica 7.0 on a common "Wintel" laptop in a fraction of a second. Compare this convenience of our modern world to what the best and brightest could muster in the 1950s!

~ See "Eniac π " in VOL. 6 № 2 pp. 3X; of our *Bulletin*. ☒

NEGATIVE EXPONENT OF FINAL DIGIT IN ROW

40 3;184 809 493 491 866 457 3X6 211 441 515 51X 057 292 90X 780 9X4
 80 927 421 40X 60X 552 56X 066 1X0 375 3X3 XX5 480 564 688 018 1X3
 100 683 083 272 444 X0X 370 412 265 529 X82 890 344 425 648 403 759
 140 X71 626 48X 546 876 218 494 849 X82 256 164 442 796 X31 737 422
 180 942 391 489 853 943 487 637 256 164 472 364 027 X42 1XX 17X 384
 200 52X 18X 838 401 514 X51 144 X23 315 X30 09X 890 646 148 448 X62
 240 253 X88 X50 X43 4X0 944 572 315 933 664 476 43X X44 775 839 751
 280 206 835 264 754 462 060 440 344 325 519 137 727 29X 214 755 353
 300 179 384 8X0 402 499 945 058 535 374 465 X68 806 716 644 039 539
 340 X84 319 351 985 274 939 941 129 90X 440 383 410 764 542 457 7X5
 380 160 143 624 X88 47X 676 X39 929 121 21X 213 887 492 873 946 X61
 400 332 242 217 XX7 354 115 357 744 939 112 602 4X4 488 881 8X3 269
 440 222 452 848 774 783 999 4X4 223 465 487 626 954 228 226 694 X00
 480 X58 609 784 2X5 175 036 207 345 X76 836 342 144 X9 7X4 X19 444
 500 774 939 980 492 217 5X0 68X 467 394 619 90X 206 544 0X3 044 X47
 540 024 X58 541 X84 428 195 489 784 X07 X33 1X7 40X 157 456 543 734
 580 054 03X 5X8 0X1 3X4 878 577 346 799 855 58X 537 317 8X7 428 271
 600 992 X38 94X 577 608 508 349 423 842 220 542 462 888 641 X24 X48
 640 430 83X 449 659 172 X31 247 851 865 449 4X0 686 625 86X 181 835
 680 X64 440 429 70X 122 813 975 898 815 367 208 905 801 032 881 449
 700 223 841 428 763 329 617 531 239 49X 657 405 584 014 534 390 458
 740 762 560 644 809 237 959 444 437 57X 431 403 955 628 297 8X6 X49
 780 590 553 490 4X1 844 947 175 637 X90 824 745 012 772 246 444 138
 800 0X8 524 084 745 813 019 447 0X6 766 344 265 654 340 698 844 761
 840 321 933 444 X55 X21 28X 038 389 746 064 851 429 793 21X 408 067
 880 225 X5X X44 346 4X1 X17 473 595 333 909 X49 127 079 655 431 644
 900 684 942 8X9 481 8X2 20X 025 X40 934 203 995 47X 62X 7XX 739 355
 940 340 539 4X3 182 905 419 390 560 3X4 346 604 942 6X9 229 469 714
 980 4X8 96X 542 339 358 442 472 944 489 635 407 1X6 351 211 360 482
 X00 041 882 X48 433 454 757 487 X37 328 441 4X1 82X 103 264 764 369
 X40 X4X 636 545 848 018 994 441 525 567 654 75X 704 449 446 42X 394
 X80 589 71X 849 051 278 645 029 404 818 644 323 552 916 170 43X 447
 400 363 496 427 408 846 872 5X6 857 004 061 794 928 907 742 780 69X
 440 094 559 324 48X 668 284 405 494 029 606 542 300 330 592 569 X74
 480 764 924 X12 935 854 6X9 460 456 7X0 901 362 856 373 444 568 979
 1000 462 564 417 241 450 474 351 364 749 X33 996 X81 4X8 847 347 X84

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↪ **The MAILBAG**, CONTINUED FROM PAGE 12;

P.S. Note the 18; fold increase in price from 3/- to £3- which I paid for it, but considerably cheaper than the £15 I paid for the 2nd edition including postage from the USA.

£15.72 = 15 pounds + 72 pence. Given a decimal pound = a dozenal pound, then that would be £13; . The 72 pence converted to dozenal is another matter. If we take $72/100$ of 144 dozenal pennies and that £1 = 10; shillings of 10; pennies each, that would be $72/100 \times 144 =$ of 12 shillings and that would be $72/100 \times 144 = 103.68$ or 87;81£ rounded pennies and thus 8s 8d; rounded. So the result is £13 8s 8d; rounded. ::::

Mr. **Daniel Dault**, Member number 396; writes:

» Hello, I am interested in joining the DSA, and would like information on how to go about doing so. I am an electrical engineering student pursuing my second undergrad at Michigan State University, and the more I learn about the dozenal system, the more impressed I become with its utility in both engineering and everyday practical calculations.

Another question—have any local chapters of the organization been started? I would possibly be interested in forming a local group at MSU and pursuing the cause of dozenal mathematics in higher education.

↪ Regards,
Dan Dault ::::

» Welcome Dan!

The membership blank on the website is being revised, but it is certainly good enough as is. Student dues are only \$3 a year.

Many years ago a group of high school students formed a chapter in New Jersey. There are none at present, but we are certainly open to the formation of a new one.

Again, welcome. You will become member number 396; Happy New Year, and may 11£5; bring you happiness and prosperity. Perhaps you and your friends may wish to join us at our next Annual Meeting in New York City.

↪ Prof. Gene Zirkel, Board Member, Board Chair Emeritus ::::

» Very good Gene, thank you for your reply. I will send in my membership application and fee. I doubt a meeting in New York is in the cards for me this year (student budget), but I would certainly be interested in reading the proceedings.

I will sound out some of my colleagues in the engineering school and perhaps the math department to see if a local chapter would be feasible.

↪ Happy New Year,
Dan Dault ::::

The young Mr. **Dan Simon**, Member number 395; writes (via his mother):

» Dear Dozenal Friends,

My young son Dan did his science project last year on base twelve. Dan is self taught and has a gift for patterns and math, among other things. He has studied many different bases and has come to the conclusion that we all should switch to dozenal. I found your society last year while helping Dan research his topic.

For our history fair next week he wants to go as the person who first discovered dozenal. If you have any links you could forward that would be great—I'm going to be doing an internet search now but any additional info would be great.

We printed the *Atlantic Monthly* article but Dan says he doesn't want to go as Charles X of Sweden. Rather, he wants to go as the *originator* of the dozenal idea—Simon Stevin in 1585.

Your next meeting: I think he might enjoy going to meetings and talking with other people who share his interest in math, bases, etc. I very much appreciate that you exist. Please let us know about your next meeting time and place.

↪ Sincerely, Dan's mom ::::

» Hello,

I would love to receive a copy of Dan's report. Two very important figures for Dan to consider are F. Emerson Andrews and Ralph Beard. They were the key figures in the modern dozenal movement. You will be able to read about them in our upcoming *Bulletin*.

↪ Wishing you both a gross of good luck,
Prof. Gene Zirkel, Board Member, Board Chair Emeritus ::::

» Dear Gene,

It seems to me that the only references Simon Stevin made to the duodecimal system were in his writing on music. It is in the earlier and later versions of *Vande Spiegheling der Singconst* that he uses a dozenal system and 10,000-to tune a lute. It is frustrating because Simon Stevin appears not to have made the leap from the utility of dozenal to anything but music. Rather, he seems to be the one who popularized the use of decimals versus unwieldy fractions.

I'm sure Dan would greatly appreciate getting the *Bulletin*! He really needs a community of like-minded people with whom to interact. Dan loves to think and talk about math, codes, riddles, etc. Someone gave him a *Schoolhouse Rock!* video for his third birthday and within a month he'd taught himself double-digit multiplication. After that he was all for finding patterns in numbers—especially the primes, Fibonacci, and Lucius, alternate base systems like the Mayan one, base seventeen, and especially now, base twelve. When Dan was still three he said to his dad: "If we can find an answer to pi we'll be famous". He's in the middle of primary school now and has come up with his own set of numbers which he calls "Important Numbers".

On another topic, we came across a problem with Simon Stevin which seems to be under contention by many scholars. It seems a Chinese scholar at the time published, one year prior to Stevin, in 1584 his own version of dozenal as useful to creating "equal temperament". It seems that the Chinese version is entirely different than the European version but still, it appears that Zhu Zaiyu was first. I am going to try and find an English translation of the paper by Zhu Zaiyu to make sure he does indeed promote a duodecimal system.

We put the membership application in the mail last week. Dan addressed and filled in the application on his own. I hope whoever is in Babylon, NY will take Dan's application seriously because his skill at writing in no way reflects his skill at math. ☺ He pulled twelve dollars in cash out of his own piggy bank because he wanted to pay for the membership himself.

↪ Sincerely, Dan's mom ::::

↪ Dozenal Jottings ↪

We welcome our latest Members: **Cole Spooner** 393; and **Vanessa Sampson** 394; both of Rochester, NY. New Members **Dan Simon** 395; and **Michael A. Leach** 396; hail from New York City and Portland, OR, respectively. ::::



solution from page 8

$$\begin{aligned}
 1 \times 1 &= 1 \\
 11 \times 11 &= 121 \\
 111 \times 111 &= 12321 \\
 1111 \times 1111 &= 1234321 \\
 11111 \times 11111 &= 123454321 \\
 111111 \times 111111 &= 12345654321 \\
 1111111 \times 1111111 &= 1234567654321 \\
 11111111 \times 11111111 &= 123456787654321 \\
 111111111 \times 111111111 &= 12345678987654321 \\
 1111111111 \times 1111111111 &= 12345678987654321
 \end{aligned}$$

In fact, the general pattern for any base b is:

$$\begin{aligned}
 1 \times 1 &= 1 \\
 11 \times 11 &= 121 \\
 111 \times 111 &= 12321 \\
 &\dots \\
 111 \dots 1 \times 111 \dots 1 &= 123 \dots [b-2] [b-1] [b-2] \dots 321
 \end{aligned}$$

whose k^{th} row is the square of k 1s, namely:

$$111 \dots 1 \times 111 \dots 1 = 123 \dots [k-1] [k] [k-1] \dots 321$$

problem solution in next issue

Here's a beautiful example of mathematical symmetry. Can you find a similar pattern in base twelve?

$$\begin{aligned}
 9 \times 9 + 7 &= 88 \\
 98 \times 9 + 6 &= 888 \\
 987 \times 9 + 5 &= 8,888 \\
 9876 \times 9 + 4 &= 88,888 \\
 98765 \times 9 + 3 &= 888,888 \\
 987654 \times 9 + 2 &= 8,888,888 \\
 9876543 \times 9 + 1 &= 88,888,888 \\
 98765432 \times 9 + 0 &= 888,888,888 \\
 987654321 \times 9 - 1 &= 8,888,888,888
 \end{aligned}$$

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