✧Spring Meeting – See page 14;✧

FOUNDED 1160;(1944.)

THE DOZENAL SOCIETY OF AMERICA
c/o Math Department
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is a voluntary, non profit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in calculations, mathematics, weights & measures, & other branches of pure & applied science.

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THE DUODECIMAL BULLETIN

Whole Number Nine Dozen Six

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DSA

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IN THIS ISSUE

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A Plea for Help

We have no more copies of several early Bulletins. If you have any old copies why not consider donating them to our DSA archives? Your help is greatly appreciated. *-The Editors*

PRESIDENT'S MESSAGE

The Dozenal Society of America is proud to announce two general meetings this year, both in New York City. We welcome all members to join us for dozenal discussion and fun in April and in October.

As editor to-be and President of the DSA, I am excited about the worldwide interest in dozenal as demonstrated here and on the internet. Our Dozenal Bulletin is a mathematical journal, your mouthpiece to fellow dozenal enthusiasts. It represents more than six dozen years of thought, covering number bases, nomenclature and symbology, systems of metrication, puzzles and games, and musings of our fathers in dozenal thought. Gene Zirkel's work since the 1970s, from reviving the Bulletin to leading the organization, is a gift to all of us. The Bulletin is a crossroads of the generations of folks who've marveled at the compact versatility of the number twelve. You can be a part of it by joining and submitting your thoughts to the Bulletin.

The Bulletin is only one benefit of membership. The DSA's annual meetings are a great way to meet others that think the same way you do. The DSA website is an opportunity for you to help spread the word about the power of twelve and related topics. We have a sister organization in the Dozenal Society of Great Britain. In the future, the DSA may be able to offer more benefits; the power is in your hands. For twelve dollars, you can join and participate.

I am honored to have been elected President of such a fine organization. Many good people have served and continue to be involved in the DSA's operations. I believe dozenal is the optimum base for general human computation. As a registered architect, I use dozenal daily in my work. Like many of you, I enjoy pondering alternative number bases and cleaner systems of measure. Let's share the wonder and joy of one of nature's most versatile numbers – twelve

Michael DeVleiger, *President* ✱

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The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (✱) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and a semi-colon, or Humphrey point, as a unit point for base twelve. Thus ½ = 0;6 = 0.5 ✱

NUMBERS: CHEAPER BY THE DOZEN?

Part 2

[Part 1 appeared in our previous issue]

A Senior Project by Addie Andromeda Evans

A MATHEMATICAL EXAMINATION OF DIFFERENT NUMBER SYSTEMS

The choice in number base can make a significant difference in the use of numbers. There are a variety of ways in which numbers are used in daily life: calculations, applied to scientific endeavors, and in the abstractions of thought such as number theory.

Before we get into number bases, I will briefly discuss that we are examining number bases within the Arabic numeral system, which is abstract place notation. Although the Babylonians had a sexagesimal system, the notation consisted of symbols for tens and ones only. The Arabic numerals are much more convenient for complex computations than the Babylonian notation or Roman numerals.

"The number base of ten has no sanction but long habit and the physiological accident of ten fingers." -Andrews[1]

Did base ten survive because it is the most convenient number base? The reasons for the convenience of a number base lies mainly in divisibility, which was another concept that now seems simple but was hard for human consciousness to develop. By the time that division, fractions, and fractionals were well understood, base ten was already well established. To this day, fractions and fractionals still remain much more difficult for the mind to grasp than the integers.

So what would make a convenient number base? What should a number base be evaluated for? One reason is convenience in measurement and thus division. If a basic unit of measurement — time, money, volume, weight — can be divided well into multiple fractional parts, than it is convenient.

"Sumerians and Latins, alike, have drawn from metrology the division of the abstract unit. But while the Latins have adopted the division through twelve without regard to their system of numeration, which was built upon the base of ten, the Sumerians have divided the unit by the same number which

[1] A list of works cited appears at the end of Part 2. -Ed.

formed the base of their numeration." -Thureau-Dangin

Half the argument for a good number base already lies in issues of measurement. The factors of the base make for convenient fractional representation of those factors as denominators. So for a convenient representation of one half and one third, two and three would need to be factors of the base. So we want to choose a number system with factors that are conducive to the needs of daily measurement. When that is established, a measurement system of the same base should be established as well.

"Because of our two eyes and two hands we prefer to halve and double things. Therefore we should use measures and arithmetic containing only even factors." -Tingley

A case has been made for the Octal system, base eight, in the past. Eight has the advantage of being divisible by 2 three times, which would work well for smaller and smaller increments of units such as money or volume for example, as the size of the step is always the same. The fact that 2 is the only divisor makes the octal system a poor choice, even if two is the divisor three times over. Octal has been taught in schools to children to help them understand numbers, which may be a good exercise but, does not win the case for the octal system.

"Arithmeticians and educators wrongly favor the scale of twelve containing in the place of five the equally offensive odd and prime three." -Tingley

In spite of Tingley, neither I nor most mathematicians find odd or prime numbers offensive as factors of a number base. As stated before, the more the factors in a number base, the greater the chance of terminating fractionals. In any number base the sets of rational, irrational, terminating and non-terminating numbers will all be infinite. However, in a base with more factors than, for example base ten, there is a higher probability of terminating fractionals then there is in base ten.

"In any decimal-form system, those denominator-numbers can be expressed as whole numbers whose prime factors are all factors of the base of the system; but denominators containing a factor which is not a factor of the number base, resolve themselves into endlessly repeating 'decimals'." -Andrews

In that case, it would be nice to have a number base that contained as many prime numbers as possible. Base sixty is nice in that almost all numbers up to twelve are either a factor of sixty or are the product of two factors of sixty. For example, sixty is not divisible by 8, but it is divisible by its factors: 2 and 4.

Thus, dividing by 8 in a base sixty will yield finite fractionals. The only numbers below twelve that would yield unending fractionals in base sixty are the primes 7 and 11, because they are not factors of the base and are not products of factors of the base.

Do we really need as many primes as possible? Three is the most important odd prime, along with the equally important and only even prime 2. The primes other than 3, being 5, 7, 11, and so on, are not particularly useful in terms of mensuration. Five is only still perceived as useful because it is half the current number base. However, 6 would be a much better half number of the number system, because it is the product of the two most important primes 2 and 3. That number system is base twelve.

A number base should also be limited to a realistic number of symbols. Of course, it doesn't sound too hard to remember sixty symbols, especially in comparison to how many kanji characters the Japanese use in their daily life. However, the problem here is in memorizing the multiplication table. To memorize 3600 products at a young age would be a daunting task. And it is truly important to know the whole multiplication table of the number base, as it reveals the number patterns of the entire system.

In choosing a number base, we can already rule out prime numbers, since we know that for practical purposes we want as many factors as possible. Prime bases are mathematically intriguing in that their fractionals are all irreducible. There are many fractionals in base ten that have multiple fractional representations. For example, 0.36 can be represented as 36/100 or 18/50 or 9/25. In a prime base, neither 0.36 nor any other fractional can be reduced in fraction representation. This is because we would represent only numerators that are not powers of the base and denominators that are always powers of the base. There is no common divisor here and thus no simplification. Nor will there ever be a common divisor. There will rarely be non-terminating fractionals in a prime base because, as probability indicates, most fractions will not have a denominator that is either the base number or one of its powers. Thus prime bases are out of the question for a number base of common use.

We also would not want a number base to be too small. Quinary, or base five, would have too few symbols and representations of large numbers would get too large too fast (quinary is also not a desirable base because it is a prime number). Thus, the number system needs to be a good medium size. What size is medium? According to Menninger, "If the size of 20 intervals is too large for convenience in counting, the intervals of five are two small." This point was made in relation to five, ten, and twenty being numbers corresponding to human digits, which have been the main basis for human systems of numeration. The table on the next page shows the numbers 5 through 20 and their factors.

As the table displays, twelve, eighteen, and twenty are the medium sized number with the most factors. Since each has four factors, we could go ahead and pick twelve as our base because it is the smallest base with the most factors. But even better than that, we can pick base twelve because its four factors — 2, 3, 4 and 6 — are the most convenient for mensuration.

No.	number of factors			
	1	2	3	4
5				
6	2	3		
7				
8	2	4		
9	3			
10	2	5		
11				
12	2	3	4	6
13				
14	2	7		
15	3	5		
16	2	4	8	
17				
18	2	3	6	9
19				
20	2	4	5	10

Base twelve, also called the duodecimal or dozenal system, is the favored number system in modern mathematics literature. Georges Buffon (1707-88) was the first to advocate base twelve, and since then many mathematicians have followed suit. LaPlace, a man of impressive mathematical ingenuity, thought that base twelve would be best and convinced Napoleon of this as well. LaPlace also thought that binary, base 2, which uses only 1 and 0 was the number system of the gods.

"He imagined that Unity represented God, and zero the void; that the Supreme Being drew all the beings from the void, just as unity and zero express all numbers in his system of numeration." -Ellis

There is a certain beauty in the simplicity of zeros and ones. To create all numbers out of two is a fascinating thought. It would be difficult for humans to think in binary as well as perform daily mathematical operations in binary. Since there are only two symbols, the numbers get unnecessarily large quite quickly, for example 1111111111000000000110000011111101. It is also harder for humans to distinguish differences in the numbers when there are only two symbols. Binary is the necessary language of computers, due to the two possibilities inherent in electrical circuits; on and off. Binary, however, can end up taking an enormous number of digits, and therefore binary information is compacted with the octal (base eight) and hexadecimal (base sixteen) systems. To compact information of base two, it makes sense that bases that are powers

of two were chosen for this job, eight being two to the third power and sixteen being two to the fourth power. Binary clearly has its place in the computer world. We discussed octal above, and for the same reasons, hexadecimal is also not a practical base.

THE DOZENAL SYSTEM

As we have reviewed before, the dozenal system is a popular choice for a base because of its large number of divisors and its convenient size. The power of this is seen in the multiplication table, where there are very easy and convenient patterns for all the divisors and their multiples. Before looking at the multiplication table, let us examine some facts about the dozenal system. To reinforce the positive attributes of the dozenal system, some facts about the decimal system are relayed first, followed by facts about the dozenal system.

DECIMAL SYSTEM FACTS

1. All even numbers must be divisible by 2.
2. All numbers ending in 0 must be divisible by 10.
3. All numbers ending in 0 must be divisible by 5.
4. All numbers ending in 5 must be divisible by 5.

DOZENAL SYSTEM FACTS

1; All even numbers must be divisible by 2.
2; All numbers ending in 0 must be divisible by 10.
3; All numbers ending in 0 must be divisible by 6.
4; All numbers ending in 0 must be divisible by 4.
5; All numbers ending in 0 must be divisible by 3.
6; All numbers ending in 0 must be divisible by 2.
7; All numbers ending in 9 must be divisible by 3.
8; All numbers ending in 8 must be divisible by 4.
9; All numbers ending in 8 must be divisible by 2.
✕; All numbers ending in 6 must be divisible by 6.
#; All numbers ending in 6 must be divisible by 3.
10; All numbers ending in 6 must be divisible by 2.
11; All numbers ending in 4 must be divisible by 4.
12; All numbers ending in 4 must be divisible by 2.
13; All numbers ending in 3 must be divisible by 3.

Anyone who has studied number theory will immediately see how much more convenient the dozenal system is for finding prime numbers. In the decimal system, the only hint we get when looking for large prime numbers and divisibility is that even numbers and numbers ending in 5 are not prime. But in the dozenal system we know that, besides even numbers, numbers ending in 3 and 9 are not prime. When dealing with numbers higher than 144 in the decimal system, we have no reason to know whether they have divisors unless they end in 2, 4, 5, 6, 8 or zero. The only other method is to take the square root of the number, which will usually not be an integer, and then try to divide the original number by all primes less than or equal to the square root. For some numbers that can be time consuming. In the dozenal system, there are more indicators of divisibility and thus one would have to resort to the square root method less frequently.

All numbers that share a factor with base twelve have easy to remember and very convenient multiplication tables. The only numbers that this does not include is 5, 7, and 11. The number $\#_{twelve}$ or 11_{ten}, however, has an easy to remember multiplication table because it is one less than the base, just as it is the case with 9_{ten}. These facts demonstrate that daily calculations will be easier to remember and thus compute in one's head, if in the dozenal system. The patterns in the dozenal system are more common throughout than in the decimal system.

DOZENAL MULTIPLICATION TABLE

	1's	*2's*	*3's*	*4's*	*5's*	*6's*	*7's*	*8's*	*9's*	**'s*	*#'s*	*10's*
1's	1	2	3	4	5	6	7	8	9	*	#	10
2's	2	4	6	8	*	10	12	14	16	18	1*	20
3'	3	6	9	10	13	16	19	20	23	26	29	30
4's	4	8	10	14	18	20	24	28	30	34	38	40
5's	5	*	13	18	21	26	2#	34	39	42	47	50
6's	6	10	16	20	26	30	36	40	46	50	56	60
7's	7	12	19	24	2#	36	41	48	53	5*	65	70
8's	8	14	20	28	34	40	48	54	60	68	74	80
9's	9	16	23	30	39	46	53	60	69	76	83	90
**'s*	*	18	26	34	42	50	5*	68	76	84	92	*0
#'s	#	1*	29	38	47	56	65	74	83	92	*1	#0
10's .	10	20	30	40	50	60	70	80	90	*0	#0	100

Of great importance is the world of fractionals, which is made much more convenient. As F. Emerson Andrews writes in *New Numbers*: "Carried out to

one place only, duodecimals represent closer values for one half of all the possible fractions, decimals represent better values for one-third of all possible fractions; the remaining sixth are expressed equally closely in either system." The table on the next page displays this fact.

It should also be noted that dozenals are also more accurate per dozenal fractional place because the positions represent one twelfth, one hundred forty fourth, and so on. This means more bang for your buck with your dozenal fractions.

OPERATIONS IN BASES

One can add, subtract, or multiply any two integers and still obtain an integer, negative or positive. When dividing a random integer by another random integer, the quotient will usually result in an integer part and a fractional part – sometimes one that is a non-terminating fractional.

FRACTIONS	DECIMAL	DOZENAL
one	1	*1*
one half	0.5	*0;6*
one third	0.333333…	*0;4*
one forth	0.25	*0;3*
one fifth	0.2	*;249724249724…*
one sixth	0.166666…	*0;2*
one seventh	0.142857142857…	*0;186*35186*35…*
one eighth	0.125	*0;16*
one ninth	0.111111…	*0;14*
one tenth	0.1	*0;124972497…*
one eleventh	0.090909…	*0;111111…*
one twelfth	0.083333…	*0;1*

As an exercise for this topic, I wrote a computer program that "seemingly" does operations in the dozenal system. For the program, I had to make mathematical conversions from dozenal to decimal and back again, although it should be noted that the computer actually thinks in binary. The QBasic program, as with most computer software, is designed to let us think

A Computer Program

in decimal while it actually does its work in binary. In fractionals, the non-terminating fractions are truncated by the program. So there is no exact accuracy for any number base because of this. For practical purposes, however, only a few fractional places are necessary.

The program takes two dozenal numbers, converts them to decimal, performs an operation on them and then converts the result from decimal to dozenal again. The conversion from dozenal to decimal is simple. Just multiply the coefficient by the power of twelve corresponding with its place. Then you'll have a decimal number which you can operate on. For example:

$$276\#;X01 = 2(12^3) + 7(12^2) + 6(12) + 11 + 10(12^{-1}) + 0(12^{-2}) + 1(12^{-3})$$

is 4547.83391… or approximately 4547.8339

Once the operations are finished, the conversion back to dozenal is more complicated. To do this, I separated the integer part from the fractional part. For example separate 4547.8339 into 4547 and 0.8339

To deal with the integer, I used the division algorithm. The division algorithm takes a number, the dividend, and sees how many multiples of the divisor goes into it, leaving a remainder that is less than the divisor. If we take a number n in base ten and want to convert it to base twelve then we take n and divide it successively by twelve. The remainders are then the digits in base twelve. Let's take 4547 in base ten and convert it to base twelve.

4547/12 = 378 with a remainder of 11
378/12 = 31 with a remainder of 6
31/12 =2 with a remainder of 7
7/12=0 with a remainder of 2

Reading the remainders in reverse order the division algorithm gives $4547_{ten} = 276E_{twelve}$. The subscripts are used to indicate the different bases.

To deal with fractions I used the multiplication algorithm, successively multiplying the fractional part by twelve and removing the integer parts which are the dozenal digits. See the example on the following page.

Note that I separated the integer and fractional parts. I dealt with the integer part with the division algorithm. I dealt with the fractional part with a multiplication algorithm. The fractional part is multiplied by twelve and then the integer part of that product is the coefficient. The fractional part of that product becomes the new number to go through the algorithm in the exact same manner.

For example 0.8339:

0	.8339
	× 12
10	.0068
	× 12
0	.0816
	× 12
0	.9792
	× 12
11	.7504

Thus 0.8339 is 0;X00#… or approximately 0;X01.

Pseudo code for a program to make this conversion follows:

```
        PROGRAM Decimal to Dozenal Conversion Program
BEGIN
        REMARK: Input a decimal number
        INPUT Decimal integer DecValue
        SET DecInteger = Integer part of DecValue
        SET DecFraction = DecValue - DecInteger
        CALL SUBROUTINE Convert Integer
        CALL SUBROUTINE Convert Fraction
        CALL SUBROUTINE Output Dozenal Integer
END PROGRAM Decimal to Dozenal Conversion Program

SUBROUTINE Convert Integer
BEGIN
        REMARK: Repeatedly divide DecInteger by twelve and store the
                remainder in the array, DozRmainder.
        Set SaveCount = 0
        IF DecInteger = 0 THEN
                SET SaveCount = 1
                SET DecInteger = 0
                ELSE
                    LOOP WHILE DecInteger ≠ 0
                        SaveCount = SaveCount + 1
                        DozRmainder[SaveCount] = DecInteger MOD twelve
                        DecInteger = Integer part of (DecInteger / twelve)
                    END LOOP WHILE
        END IF
END SUBROUTINE Convert integer
```

```
SUBROUTINE Convert Fraction
BEGIN
        REMARK: Change fractional part to dozenals. Repeatedly multiply
                DecFraction by twelve & store the integer part in the array
                DozFraction
        LOOP FOR Count = 1 TO 7
                DecFraction = DecFraction * twelve
                DozFraction[Count] = Integer part of (DecFraction * twelve)
                DecFraction = DecFraction - DozFraction[Count]
        END FOR LOOP
END SUBROUTINE Convert Fraction

SUBROUTINE Output Dozenal Integer
BEGIN
        REMARK: Print all on one line
        PRINT "The dozenal equivalent of", DecValue, "is",
        LOOP FOR Count = SaveCount DOWN TO 1
                IF (DozInteger[Count ] = 10) THEN
                        THEN PRINT "*"
                ELSE IF (DozInteger[Count ] = 11) THEN
                        THEN PRINT "#"
                ELSE
                        PRINT DozInteger[Count ]
                END IF
        END FOR LOOP
        PRINT ";"
        LOOP FOR Count = 1 TO 7
                IF (DozFraction[Count ] = 10) THEN
                        THEN PRINT "*"
                ELSE IF (DozFraction[Count ] = 11) THEN
                        THEN PRINT "#"
                ELSE
                        PRINT DozFraction r[Count ]
                END IF
        END FOR LOOP
END SUBROUTINE Output Dozenal Integer
```

[Editor's Note: Addie also provided code for another program to do:
dozenal to decimal conversion
dozenal addition, subtraction, multiplication or division
decimal to dozenal conversion]

CONCLUSION

For the future world, a dozenal number system and dozenal system of units would be a vast improvement, especially if adopted across the western paradigm. The shift to a different number system and system of units, might be complicated but the end result would be worth the shift, as would be seen in the sciences, commerce and all aspects of daily life. The opposition will be great in the transitional generations. It is easier to relate in terms of the first system learned, and thus many will cling to the old system.

However, there may be an even deeper level of the number system that once understood could lead to more elegant mathematics. I wanted to explore the possibility that pi or e could reveal a pattern in a different number base. I determined this to be impossible. As well, the square root of two will always be irrational in any number base. For an answer to the mystery of pi and the square root of two, the answer must be sought elsewhere.

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[Continued on p 1]*

SPRING MEETING& SOCIAL

Our Spring meeting will be held on Saturday afternoon, 5 April 2008 in the conference room, room 309 in the College Union Building at Nassau Community College. It will be followed by dinner and a social. For further information email us at *Contact@Dozenal.org* or call 631 669 0273. Reservations for the social are required before 15 March 2008.

△ △ △ △

PROBLEM CORNER 1

Here is a beautiful example of mathematical symmetry. Can you find a similar pattern in base twelve?

$1 \times 8 + 1 = 9$
$12 \times 8 + 2 = 98$
$123 \times 8 + 3 = 987$
$1234 \times 8 + 4 = 9876$
$12345 \times 8 + 5 = 98765$
$123456 \times 8 + 6 = 987654$
$1234567 \times 8 + 7 = 9876543$
$12345678 \times 8 + 8 = 98765432$
$123456789 \times 8 + 9 = 987654321$

For the solution see page17;

□ □ □

From our newest member, Chris Osburn 342;

I drove a bus for several years. I was curious how many passengers I was picking up, so I counted over a year's time. My fellow drivers were quick to note that I was crazy. Then I told them I counted in base twelve. They asked why and I had to tell them it was cheaper that way. "Huh? Cheaper?" they'd ask. "Yup. Everything's cheaper by the dozen!"
(Chris had 31,863; passengers.)

BINARY CODED DIGITS - AN UPDATE

by Gene Zirkel 67;

[This article arose out of discussion by those attending our Annual Meeting last October. My thanks to them for their suggestions.*]*

1½ dozen years ago I wrote an article showing how dozenal digits could be expressed in a 7-segment display. It appeared in Whole Number 66; vol. 33; no. 3 pp. 8 thru 18; of this *Bulletin*.

Basically it expanded upon Bill Schumacher's ideas that we consider the 4 vertical lines in the 7-segment display to indicate 4 binary digits. Starting with the lower right vertical segment and moving in a clockwise direction we obtain:

the 4-bit → |_| ← the 8-bit
the 2-bit → |‾| ← the 1-bit

Where the central horizontal segment represented 0.

Thus, ignoring for the moment the diagonal strokes, we had:

Expanded Hindu-Arabic	Binary	Schumacher[1]
0	0000	-
1	0001	[illegible]
2	0010	[illegible]
3	0011	[illegible]
4	0100	[illegible]
5	0101	[illegible]
6	0110	[illegible]
7	0111	[illegible]
8	1000	[illegible]
9	1001	[illegible]
X	1010	[illegible]
#	1011	[illegible]

Clearly, this table might easily be extend to include digits up to 13;(15.) or H.

[1]This font was created by Mike DeVlieger 37#;

and could be used for bases up to 14;(16.).

In the original article — unmindful of the ramifications — I stated that for bases larger than hexadecimal we could consider the bottom horizontal segment to be the 14;(16.)-bit and the upper horizontal segment the 28;(32.)-bit. This would allow us to create digits up to 53;(63.) And handle any base up to 54;(64.). including sexagesimals.

However, I had never actually tried working with any bases larger than twelve. When our President Mike DeVlieger got me interested in working in sexagesimals I discovered some problems.

First, several of the digits were disconnected such as: ⌊ and ⊔ which represent 18;(20.) and 24;(28.) respectively. They looked awkward representing a single digit each.

The second difficulty was more serious. Some pairs of digits were identical except for the fact that one was entirely in the upper half of the display while the other was in the lower half. For example

[and [

are different, the former representing 30;(36.) and the latter representing 16;(18.). Altho it would be clear enough which is which on a 7-segment calculator display, as hand written symbols they could easily be confused with one another.

We can eliminate both of these problems by drawing one diagonal extending from the upper right hand corner to the lower left hand corner of the 7-segment rectangle wherever it is needed. This way both of these problems are eliminated with one fell swoop (or one falling diagonal). Now the four examples above appear as ⊻, ⊻, ⁊ and ⍷.

Another issue which arose was the zero digit. It is the same as a minus sign and the subtraction operator. Thus does '⁄--' represent '100 ' or '1-0 '?. To eliminate this problem we have decided to use 0 (the 6 outer segments) to represent zero. Thus we would have '⁄00' to represent '100 ' and '⁄-0' to represent '1-0'.

Did you know that our mailing list covers more over
3 dozen States and more than a dozen foreign countries?

PROBLEM CORNER 2

Solution to the Problem Corner from page14;

1 × ✱ + 1 = #
12 × ✱ + 2 = #✱
123 × ✱ + 3 = #✱9
1234 × ✱ + 4 = #✱98
12345 × ✱ + 5 = #✱987
123456 × ✱ + 6 = #✱9876
1234567 × ✱ + 7 = #✱98765
12345678 × ✱ + 8 = #✱987654
123456789 × ✱ + 9 = #✱9876543
123456789✱ × ✱ + 9 = #✱98765432
123456789✱# × ✱ + 9 = #✱987654321

In fact, the general pattern for any base is:

{123...n} × (b-2) + n + {αβ6δ...(b-2)} where
b = the base
α = b-1, β = b-2, 6 = b-3, δ = b-4, etc.
and {...rst} = the number $t + sb + rb^2 + ...$, when rst is a string of digits, and n varies from 1 to (b-1)

This can be proved by using mathematical induction.

<u>*ANOTHER PATTERN*</u>

Here is another pattern. Can you find a similar pattern in base twelve? Can you generalize it to work in any base?

1 × 9 + 2 = 11
12 × 9 + 3 = 111
123 × 9 + 4 = 1111
1234 × 9 + 5 = 11111
12345 × 9 + 6 = 111111
123456 × 9 + 7 = 1111111
1234567 × 9 + 8 = 11111111
12345678 × 9 + 9 = 111111111
123456789 × 9 +10= 1111111111

The Solution will be given in the next issue of this *Bulletin*.

Book Revu

by Gene Kelly

Perfect Figures: The Lore of Numbers and How we Learn to Count
by Bunny Crumpacker, Thomas Dunne Books, NY, 2007

This interesting book reveals many interesting tidbits about those things called numbers, their idiosyncrasies, their history, their good and bad aspects. Most books are divided into chapters labeled with the ordinal numbers: 1^{st}, 2^{nd}, 3^{rd}, ..., n^{th}. This tome is divided into 17 sections labeled: 1, 2, 3, ... 9, 0, 10, 11, 12*, 100, 1,000, 1,000,000, And One More.

Thus right from the start we know that we are in for something different, and different it is. (You will have to read it to discover why zero isn't first.)

I bought the book because I wanted to read about twelve, but I enjoyed reading all the sections and you will too if you are someone who enjoys fooling around with numbers. In section eleven, for example, we learn about Fibonacci Numbers' eleven quirk, and in the penultimate section we learn how big a billion really is.

Crumpacker(?) tells us that twelve is both advanced and perfect. She also claims that the main purpose of eleven is to get us from ten to eleven..

I only spotted one error, but in a book with this many facts one shouldn't expect perfection. In section Zero we read "you can't divide zero", and it is not just a misprint.

To add to the enjoyment the author has packed the pages with interesting quotes such as these two: "Are twelve wise men more wise than one? or will twelve fools, put together, make one sage? Are twelve honest men more honest than one?" —Herman Melville, *Mardi*, and "Ninety percent of this game is half mental." —Yogi Berra

Try it, you'll like it.

*Subtitled: *The Last Basic Number Name.*

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REMEMBER — your gift to the DSA tax deductible

MAIL BAG

To: The Dozenal Society of America, Shalom from Israel.

I am a Dozenalist.

Before I heard of your society I believed that the Dozenal way is the better way. Your web site is great and clear!

You wrote on your web site that the symbol "Ӿ" is very often used for ten and "#" is used for eleven. The "#" is good, but why do you use these symbols? What are your reasons?

Wouldn't "*" be better for ten?

I believe that the symbols "#" and "*" are better, and that ten should be symbolize with "*".

Please respond,

N.S.

Our Response:

Welcome N.S., and thanks for your input regarding symbols.

The original choice for the X with a horizontal line through it and the octothorpe (#) was because the phone company put these two symbols on the telephone when they expanded from ten touch pad keys to a dozen such keys.

The thinking at that time was that everyone would soon become familiar with them.

Later, one of our members added the clever idea that the Roman Numeral for ten crossed out symbolized 'not ten'. Similarly the octothorpe could be thought of as eleven crossed out twice and thus symbolized 'not eleven'.

Many people have different ideas about symbols. Our official attitude is that for convenience and simplicity, our official publications will use one standard set of symbols except in the case where someone — such as you — is discussing symbols. So feel free to privately use and advocate for any symbols.

Originally the founders of our Society had a famous type face designer named Dwiggins design a special cursive X and E for our use. There has recently been some talk of reverting to that usage. No matter what symbols are used, they are usually referred to as dek and el.

NUMBERS: CHEAPER BY THE DOZEN? [Continued from p 13;]

Menninger, Karl. *Zahlwort und Ziffer. (Number Words and Number Symbols: A Cultural History of Numbers.)* Cambridge; M.I.T. Press, 1969.
Shlain, Leonard. *The Alphabet Versus the Goddess.* Viking. New York, 1998.
Terry, George S. *Duodecimal Arithmetic.* NY, Longmans, Green and Co., 1938.
Zebrowski, Jr., Ernest. *A History of the Circle.* Rutgers University Press. New Brunswick, New Jersey; 1999.

Articles:

Powell, M.A. *The Antecedents of Old Babylonian Place Notation.* Historia Mathematica, 1976.
Powell, M.A. *The Origin of the Sexagesimal System: The Interaction of Language and Writing.* Visible Language, 1972. Vol. 6, Issue 1. Pages 5-18.
Thureau-Dangin, F. *Sketch of a History of the Sexagesimal System.* Osiris, 1939 Volume 7, Pages 95-141.
Tingley, E.M. *Calculate by Eights, Not by Tens.* School Science and Mathematics; April, 1934.

You *Can* Fight City Hall!

The European Parliament in Brussels voted in November 2007 to allow Britain and Ireland to keep some old imperial measurements so pubs can still serve pints and road signs can still show miles.

European Union rules aimed to phase out the imperial measures by 2009 but the EU's executive body, under pressure, reversed course last September.

"This is good news for British and Irish citizens, who are used to their traditions of miles and pints," said a EU official.

Minutes of Our Annual Meetings

DOZENAL SOCIETY ANNUAL MEETINGS
October 6; 11#3;(2007.)
Nassau Community College
Garden City, NY 11530

Annual Board Meeting

Attendance: Gene Zirkel, Jay Schiffman, Ellen Tufano, Alice Berridge, Michael DeVlieger. Other Board Members were unable to attend due to other commitments.

Board Chair Gene Zirkel called the meeting to order at 10:00 in Room 309 in the Old Student Center at the College.

1. Minutes of the 11#2;(2006.) Board Meeting were approved.

2. The Nominating Committee's slate for officers for the new year: Jay Schiffman- *Board Chair*, Michael DeVlieger– *President*, John Earnest – *Vice-President*, Christina D'Aiello – *Secretary*; and Ellen Tufano – *Treasurer* was approved by the Board. Gene Zirkel was given the honorary title of *Board Chair Emeritus*.

3. Gene supplied refreshments for the meeting.

4. Gene Zirkel was appointed as Parliamentarian to the Chair.

5. Board Chair's Report - the major thrust of the report was the status of our *Bulletin*. Mike will be the new Editor beginning with the issue in Fall 11#4;(2008.) - Gene will layout the next issue. Mike will produce the issue in PDF format. He is able to make fonts of any kind, and do layout very efficiently which should make the task easier. This summer there has been extensive emailing between Board Members re *Bulletin* issues.

Beginning with Mike's first issue we will "step back in history" and revert to the historical symbols that appear on the DSA seal on the cover of this issue, the script dek and el designed by the noted typesetter William Addison Dwiggins. In addition several changes to the cover were discussed and it was agreed that they would be made when the new editor takes over. Mike says that this will be "intelligent use of sizzle."

Mike will present more of his format ideas to us within the next few months. All agreed that our *Bulletin* is the lifeline of the DSA.

In addition Mike will convert the mailing list to Excel in an effort to reduce the labor involved in mailing

Letters will be sent to all people on our mailing list informing them of the fact that we will be producing an e-Bulletin in addition to the paper version.

At present there are about 178 viable members of which 23 are Life Members. Our plan is to produce an accurate email address file and that many of those listed will receive our *Bulletin* on line.

Gene saw an anti-decimal metric segment on the national TV show "The Colbert Report". He wondered if we should try to get a guest appearance on the show. He felt up to the challenge of the usual combative nature of these interviews and felt that this exposure would be of benefit to the Society and that it might be fun. Colbert makes an effort to ridicule and challenge each person on the show. It was agreed that he would try.

Alice mentioned an interesting book *Twelve Fingers* by Jose Soares which she thought might amuse members. Also, there is interesting information about polydactyllism on Google.com which might make an interesting "Jottings" item.

There was no other business. The dates of our next meetings will be April 5 and October 4, 11#4;(2008) at X;(10.) AM at Nassau Community College. An effort will be made to plan a social meeting with spouses on the Friday evening preceding the meeting. The meeting was adjourned at #;(11) AM.

Annual Membership Meeting

President Jay Schiffman presided.

1. Minutes of the 11#2;(2006.) Membership Meeting were approved.

2. The Nominating Committee proposed the members of the Board of Directors of the Class of 11#3;(2007) –Dr. John Impagliazzo, Rob Roy McPherson, Gene Zirkel and John Earnest be re-elected as the class of 11#6;(2010). The slate was elected unanimously.

A Nominating Committee consisting of Alice Berridge, Pat Zirkel and Gene Zirkel was also elected.

3. Gene was appointed as Parliamentarian to the President.

4. President's Report – Jay said he was very happy with the student article by Scott Proctor and also with Addie Evans' article "Numbers: Cheaper by the Dozen?" The second part of which will be printed in the next issue of the *Bulletin*. He was gratified by student interest in dozens. He praised Gene for his work on the *Bulletin* and gave kudos to Ellen Tufano for her work, acknowledging that both the Editor and the Treasurer are particularly crucial to the success of the Society.

Jay mentioned that it is critical to update the DSA index which Fred Newhall had begun. Fred's work goes up to 11#0;(1992.) and after discussion of what seems to be a formidable task it was decided that if the job were done issue by issue it might not be so bad. Mike agreed to begin the chore using Fred's format with Excel. Ellen suggested that we each might take a segment to ease this big job.

Jay wants to make a call for member referees/reviewers in the next issue. He also would like to make a call for problems – especially problems that are aimed at younger students.

Mike discussed "Zirkel Numbers," which he came across at dozensonline.com where there has been extensive 'chatting' with persons interested in digital coding. Mike Punter is part of that group and someone cited Gene's "Binary Coded Digits" which appeared in our Fall 119X;(1990.) issue, whole number 66;(78.). We were all very pleased to be come aware of this.

Jay expressed thanks to John Impagliazzo and Christina D'Aiello for their extensive work for the Society and the web site. Gene mentioned that John is very interested in music and wrote an article many years ago about Dozens and music. It was suggested the all the "music" articles be published in a special issue. Jay thanked Mike for his extensive work for the Society for his infectious enthusiasm and support of Dozens.

5. Treasurer's Report – Ellen Reported that because of a positive Stock Market that our Net Worth has increased by about $3,000. It was agreed that dues should remain at $12.00. Ellen brought some letters that she had recently received: past dues from Allen Paqutte, a note from Tony Scordato and another from Courtney Owen. Court has been sending monthly contributions to the Society. Mike had asked to be apprised of our status with IRS form 1023 relative to a 501 (c) 3 corporation. It was suggested that we might be able to use our tax ID number and that because our gross receipts are less than $25,000 a "blank" Form 990 could be filed. (Details are available from www.rurdev.usda.gov/rbs/ezec/Toolbox/501c3factsheet.html)

6. There was no other business; the dates of the next meetings were set to coincide with the Board Meetings mentioned above.

Gene gave a brief report on "Zirkel Numbers." He gave us a printout of the numbers.

Mike brought our attention to the book *Perfect Figures* by Bunny Crumpacker and a section on p.166;(222.) which might be interesting to include in the *Bulletin*.

Mike gave a presentation to the group using a booklet he had prepared: *The Reciprocal Divisor Method for Abbreviation of Multiplication Tables*. We were all very impressed with his talk and his examples, and found the publication to be A plus! It is a splendid document and the talk was enlightening.

The meeting was adjourned at 1:00.
Respectfully submitted by Alice Berridge for Secretary Christina D'Aiello ✱

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"Each one teach one" - Ralph Beard, *A DSA Founder*

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition--("Who needs a symbol for nothing?")--the new notation did come into popular use. Released from the drag of Roman notation, people's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance. Then, in a related development, people awoke to the fact that different number bases could be used.

A parallel to today seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, the most advantageous base should be used, the one best suited to the task at hand. (Similar to computer scientists use of binary, hexadecimal or octal - whichever is most convenient.) If this were done, duodecimals would progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited

✹

DETACH--HERE--OR--PHOTOCOPY

YOU ARE INVITED TO JOIN THE DOZENAL SOCIETY OF AMERICA

The only requirement is a constructive interest in duodecimals

Name ______________________________ / / ___

Last First Middle Date

Mailing Address (including full 9 digit ZIP code)

____

____

____

Phone: Home______________ Business______________

Fax ______________ E-mail ______________

Business or Profession ______________

Annual Dues .... Twelve Dollars (US)

Student (Enter data below) .... Three Dollars (US)

(A limited number of free memberships are available to students)

School ______________

Address ______________

Year & Math Class ______________

Instructor ______________ Dept. ______

College Degrees ______________

Other Society Memberships ______________

To facilitate communication do you grant permission for your name, address & phones to be furnished to other members of our Society?

Yes: ___ No:___

Please include on a separate sheet your particular duodecimal interests, comments, and other suggestions.

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City LI NY 11530-6793