THE DUODECIMAL BULLETIN

The Duodecimal Bulletin

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c/o Math Department
Nassau Community College
Garden City, New York 11530-6793

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✧ Annual Meeting - See Page 1 Dozen 5 ✧

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, non profit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in calculations, mathematics, weights & measures, & other branches of pure & applied science.

Membership dues are $12 (US) for one calendar year. Student Membership is $3 (US) per year.

The Duodecimal Bulletin is an official publication of the DOZENAL SOCIETY OF AMERICA, INC., % Math Department, Nassau Community College, Garden City, LI, NY 11530-6793.

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DSA

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IN THIS ISSUE

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The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and a semi-colon, or Humphrey point, as a unit point for base twelve. Thus ½ = 0;6 = 0.5

PRESIDENT'S MESSAGE

On Saturday, 04/19;/11#3; (April 21, 2007.), our Board of Directors convened at Nassau Community College / SUNY in Garden City, LI, NY to discuss issues of dozenal interest in an informal meeting setting. Among the topics discussed were improvements to our Website, the latest issue of *The Dozenal Journal* which represents the publication of our sister society The Dozenal Society of Great Britain (DSGB) and our goal of convening Mathematics and Computer Science clubs at local colleges to help foster our mission.

The latest issue of *The Dozenal Journal* honored two prime advocates for duodecimals during their lifetime - **Arthur Whillock** from the DSGB and our own **Fred Newhall**. Fred compiled a fabulous index of our *Duodecimal Bulletin* capturing the authors, titles, subjects, and key words for every article published in the DSA through 11*0;(1992.). His work was exemplary and he is sorely missed by all of us not only for his tireless energies devoted to DSA causes but as a dear friend. It is hoped that the latest issues of our *Bulletin* will soon become available on-line which will serve as a second venue to clearly delineate who we are and what we represent. Having our *Bulletins* on-line as well as classic articles throughout the rich history of the DSA will serve to defray printing costs as well

All Board members present heartily agreed that we should meet semiannually. In addition to our Annual Fall Meeting in October, we will meet every spring as well. The meeting concluded with a nice discussion on The Babylonian System of Numeration which utilizes two wedge shaped symbols to represent numerals, one to represent one and the other to represent five dozen. Five dozen was the base of choice for the Babylonians. For additional information on our special spring meeting, please view our "Jottings" on page 1 dozen 8;(20.) In this issue of our *Bulletin*, the first of an excellent two-part article delineating some of the advantages of base twelve is furnished by **Addie Evans** as part of her undergraduate senior thesis. I am confident that our readers will thoroughly enjoy Addie's contribution. We encourage students, faculty, and all others enamored with number bases to submit articles to our *Bulletin* as well as to provide us with 'jottings' of interest. Why not consider joining us? Our dues are very reasonable when contrasted with other professional mathematics and science organizations. Our Annual Meeting will take place on Saturday, ⚹/06/11#3;(October 6, 2007.) at the same venue as our spring meeting. [See page 1 dozen 5;(17.)]. You are cordially invited to attend and will miss a great & enjoyable experience if you skip this year's Meeting. My best wishes as we continue to educate all on the advantages and beauty of duodecimals.

Jay L. Schiffman, President

NUMBERS: CHEAPER BY THE DOZEN?

Part 1

[Part 2 will appear in our next issue]

A Senior Project by Addie Andromeda Evans

ADDIE, WHO EARNED A B.S. IN MATHEMATICS IN 2004,
HAS BEEN ACCEPTED BY SAN FRANCISCO STATE UNIVERSITY.

INTRODUCTION

Mathematics is often referred to as the language of the universe. Mathematics, however, is also the language of science; both mathematics and science being disciplines created by humans to interpret and manipulate the universe. The concept of "number" is often viewed in western society as a natural property of the universe, and human understanding of mathematics inevitably unraveling. The extent to which mathematics is the construction of the human mind goes mostly unrecognized.

The history of mathematics dates back 5000 years, with most of its development occurring in the last few hundred years. The development of mathematical concepts is so long and drawn out, it displays just how unnatural mathematics has been to human existence.

> "One of the most fundamental processes ever invented, counting, which consists of enumerating number names in successive increments of one unit, was to become the cornerstone of all subsequent human progress." -Gazalé[1]

This reference to counting as an invention suggests that counting is not inherent in the existence of intelligent life. Though the extent of human intellectual capacity is debatable, humans have developed number systems and come to utilize them within a particular framework where almost the only system that is taught is base ten, the decimal system. Even when we do learn other number systems, we are already thinking in the system that we initially learned. Unlike word languages, in the modern world, there is one number language that people learn, the decimal system. The exceptions are computer number language, base two, which is called the binary system, as well as the octal (base eight) and hexadecimal (base sixteen) systems that are used in conjunction with binary. These all use the same place notation that our decimal system uses, and therefore, exist in the same class of number systems as the decimal system.

[1] A list of works cited appears at the end of Part 2. -Ed.

"Decimal arithmetic is a contrivance of man for computing numbers and is not a property of time, space or matter." -Andrews

Since the creation of number systems, humans have used them in many ways. Numbers both have common and academic uses, the latter being where they receive the fancy name of "mathematics." Everyone uses numbers and, in essence, they "do math", but not all people would consider themselves mathematicians. Mathematicians are the people who not only use math, but keep grappling with concepts until coming to a deeper understanding of numbers. It is this desire for a better understanding of math, rather than a necessity to use math, that keeps improving the number invention.

The fact that a thing so simple and useful as one-third can only be represented approximately as a decimal fractional[2] has always struck me as an indicator of the imperfection of our number invention. I also had grand ideas that π— the ratio of a circle's circumference to its diameter — which can only be mathematically approximated in the decimal system in the future, could be "cleaned up." Approximations of numbers work well enough for practical and mechanical reasons. On the other hand, the search for the most beautiful and simple mathematics is ongoing. Several mathematicians and their computer counterparts have spent years looking for patterns in π in our decimal system, hoping that this pattern would give some answers to the mystery of this special number. But perhaps we were not using the right system to represent π. What if a different number system would unravel a pattern for pi, e and other such numerical curiosities? Perhaps our current system of mathematics is not serving as the best interpreter of the natural world.

NUMERATION, SYSTEMS OF NUMBERS

"1 2 3 4 5 6 7 8 9 and 0 — these ten symbols which today all peoples use to record numbers, symbolize the worldwide victory of an idea." -Menninger

Since we frequently use numbers to explain the world around us, in modern western society we have begun to think that numbers are inherent in nature. Numbers are a human invention. Besides numbers as a whole being an invention, more specifically, there is nothing in nature that inherently points

[2]People who use only one base use the words 'decimals' or 'decimal fractions' to refer to *fractionals* such as 0.5 for one half. Such expressions are valid in every base and are not limited to base ten. Thus 0;6 represents one half in base twelve as does 0.1 in base two.

towards the use of ten symbols that many have accepted and many more have had imposed on them around the globe. In fact if we consider nature to be the entire universe, we could speculate that there are extraterrestrials out there with eight or twelve "fingers" and if they have a number system, do they use ten digits? Probably not.

In fact, the system that we have today was slow in developing. The most infamous old number system of antiquity for the western world is that of the Roman numerals. We all have learned that for Roman numerals, there are symbols for 1, 5, 10, 50, 100, 500, 1000 etc. We still see Roman numerals in print to this day, for example, when used as book chapters. Since we never see them in arithmetic, we don't see large numbers in Roman numerals anymore. But the seemingly simple Roman numerals actually become quite bulky and hard to read when dealing with large numbers.

The table on the next page displays Roman Numerals, Named Place Notation and Abstract Place Notation, which is the current number notation system.

The Chinese had an interesting number notation system somewhere in-between Roman numerals and our modern fractional system, which is referred to as Named Place Notation (as seen below). They had symbols for the powers, 10, 100, etc. They also had symbols for 1-9, which they used as coefficients for the powers. The symbols used in the table example below are not the ones the Chinese used, but the Chinese system is represented by Roman Numeral symbols for the powers of ten and the modern decimal symbols for the nine unit symbols and the zero. So for 300, instead of writing the symbol for one hundred three times, the Chinese wrote the symbol for three to the left of the symbol for one hundred. This notation was much more compact than Roman numerals, but was not as compact as modern place notation. If there were no multiples of a given power of ten as part of the number to be expressed, then that power was just not written.

As we can see in Roman numerals, we only have symbols for a few numbers: some powers of ten, including the unit (one), as well as some powers multiplied by five, or divided in half, which gives the same set of symbols either way you look at it. If we have to represent multiples of any of these powers, we do so by repeating the symbol that many times. The only aspect of this number system that makes it a little more compact is the rule that the symbol of lesser value placed before the symbol of immediate greater value means subtracting the lesser from the greater. Thus IX means one unit less than ten, which equals 9, and XL means ten less than fifty, which equals 40.

In Roman numerals, large numbers become very cumbersome to write. As Georges Ifrah wrote in his book *The Universal History of Numbers*: "The writing of the Roman numerals as well as its simultaneous use of the contradictory principles of addition and subtraction, are the vestiges of a distant past before logical thought was fully developed." While mathematics is known to be a logical process, the collective knowledge of society is only ever at a certain level. The Romans did not have the knowledge that could lead them to logically simplify their number notation problem. While the lack of proper notation held back the development of mathematics, the lack of complicated mathematics in

Roman Numerals	Named Place Notation	Abstract Place Notation
I	1	1
II	2	2
III	3	3
IV	4	4
V	5	5
VI	6	6
IX	9	9
X	X	10
XI	X1	11
XIV	X4l	14
XV	X51	15
XVI	X61	16
XIX	X91	19
XL	4X	40
L	5X	50
LX	6X	60
C	C	100
D	5C	500
M	M	1000
MDCLXVI	M6C6X6	1666

Roman society did not necessitate a better system of numbers. Ultimately it was probably a lack of desire that held the Romans back from making breakthroughs.

If we view the addition problems on the following page, we can see that not only are the Roman numerals cumbersome but that the addition by columns does not apply here, and there is not a single reason for which the Roman numeral system is more advantageous than our current system.

Since some powers were just absent in the Roman Numerals and in Named Place Notation, addition or other operations by column was still not possible, and therefore these operations were not streamlined the way they are with modern place notation. The zero that we use positionally was not needed for this named place notation. The Chinese did not start to use zero until 1200 AD.

Abstract Place Notation	Named Place Notation	Roman Numerals
1023	M2X3	MXXIII
405	4C5	CDV
+ 67	+ 6X7	+LXVII
1495	M4C9X5	MCDXCV

"It was the invention of the zero, not its attaching to any particular number base, which at one leap raised numbers from mere symbols for quantities to symbols capable of use in intricate calculations." -Andrews

The place notation that we have today was not the first place notation developed. The Babylonians, who we will discuss later, also had developed place notation, although still not as complete as the one used today, Abstract Place Notation. First of all, the Babylonians did not truly develop a concept of zero until almost the time of Christ, although they eventually developed a symbol for the empty column. Their number system was also complex, for reasons which will be discussed further in the text. While place notation did develop before the birth of Christ, the simplicity of the modern Abstract Place Notation was slow in developing. As Karl Menninger mentions in his book *Number Words and Number Symbols: A Cultural History of Numbers*: "Medieval Europe never achieved a logical 'named' or expressed place-value notation...but only the Chinese succeeded in representing them in 'written' form in a named place-value notation, and only the Indians in an abstract place-value notation." The abstract part of the place notation that we have today is that the powers of ten are implied by place or position. The number 345 implies 3 times one hundred plus 4 times ten plus 5 times one. In abstract place notation, no matter what number you have, you can always line up corresponding columns to execute a number of operations more efficiently than in the other systems.

While the Romans, Chinese, and Hindus each had a different method of notation for numbers, they still had the number base ten in common. Counting starts with a series of numeric symbols that have a specific order to represent higher and higher amounts. In abstract place notation, for a given number base n, there are n numeric symbols. After the first series is finished, the numerals repeat themselves, in a positional manner. For our number base of ten, also called the decimal system, we have the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. To represent higher numbers we then use the above mentioned abstract place notation. The symbols 0 through 9 represent unit numbers. The next number we represent as 10, which says that we have 1 ten and 0 units.

"Our decimal system, which is based in the principle of local value, is said to be positional meaning that each symbol's contribution is equal to the value assigned to that symbol multiplied by the power of 10 conferred by its localization, or position." -Gazalé

In the decimal system, it is the power of ten because we are in base ten. We have ten symbols including zero and thus powers of ten. We could have this same system with any number, 2, 5, 7, 8, 12, 16, 20 or even sixty. If we had a base of eight, the number 23_{eight} in base eight would equal, in base ten, $2\times8^1+3\times8^0$, which is 19_{ten}. The essence of the positional system is that the number written represents the coefficient to be multiplied by the power of the base as is indicated by the position of that coefficient.

Let's say we have a number that is 3×10^4 and 2×10^3 and 6×10^2 but there are no multiples of ten to the first power or ten to the zero power to be added. Do we write this number 326? No, we certainly do not. We write this number 32600. However, this development of the use of zero in the written expression of numbers was slow in development.

"This invention of something to represent nothing is a stroke of genius which can scarcely be over praised. Upon the insignificant zero, symbol for nothing, rests the whole of mathematical science." -Andrews

A BRIEF HISTORY OF NUMBER SYSTEMS

In over 5000 years of mathematics history, it has only been within the last few hundred years that most of what the western world knows as mathematics was developed. And by the time humans got around to laying out these logical deductions called mathematics, the base ten fractional system was already well established, albeit, incomplete. Our polished number system, with invention of

the fraction (or decimal) point as the cherry on the top, is less than half the age of Christianity.

The earliest notable mathematical developments in evidence go back to around 3000 BC in the Mesopotamian region that is now historically referred to as Babylon. Babylon was not always the center of Mesopotamia, but it has been the custom to refer to the region of that period as "Babylonian". It was first evidenced that the Babylonians used a sexagesimal system, or base sixty, which they wrote with an incomplete place notation. In fact, the usefulness of their place notation led to the academic continuation of the sexagesimal system up until the dawn of Christianity. Remnants of the system are still used today in the measurement of time and division of the circle.

The sexagesimal system, however, was first developed without the understanding of the concept of zero, and was not a true positional system until that time. The absence of the zero concept gave rise to some confusion to the rank of the number. If we did not use zeros in our number system, for example, 34 could mean exactly what it means today or it could mean $3\times100+4\times10$ or $3\times100+4$, or another combination of different powers of ten. Critics say there is too much ambiguity in this system. On the other hand, according to M.A. Powell, this "system of notation functioned without a sign for zero: the sexagesimal numerals are arranged in quite clear columns according to their proper power." Babylonians did not fully develop the zero concept until 300 BC, but their place notation was quite useful as it stood.

The Babylonian sexagesimal system was also more complex than our place notation today because it was not purely base sixty. They had only symbols for the unit, and the "decade" – which is the name for ten. The digit 25 was represented decimally by two decades on the left and five units on the right.

It is clear that this sexagesimal system was developed after a decimal system was already in use. Exactly why it developed can only be conjectured from limited and unclear evidence. It was once thought that the sexagesimal system was a hybrid of the decimal system, which one group of people used, and a base six system, which another group of people used. It is even possible that the Sumerians, the ancestors of Babylon, had already fully developed the common use of the sexagesimal system before 2000 BC.

Although the origin of base sixty is not clear, the advantages of this system are easily understood. In mensuration — a practice used at all levels of life — to halve and to third are the basic manners of division. To be able to evenly halve and third something, it must be divisible by six, a common multiple of these two fractions. Besides being divisible by 10 and 6, sixty is also divisible by a fine array of useful numbers: 2, 3, 4, 5, 6, 10, 12, 15, 20, and 30. This makes base sixty suitable for combining the common denominator 6 with the base ten system and a very convenient system for mensuration.

The sexagesimal system eventually fell out of daily use and the Babylonians returned to base ten. Since the sexagesimal system applied itself so conveniently to astronomy, it continued to be used in this academic field. Because it was the Babylonians who passed the sexagesimal system on, the system was often referred to as Babylonian, although its origin is in Sumerian society.

Egyptian mathematics was once thought to be the earliest of impressive developments. That was due to the discovery of the key to their hieroglyphics before the discovery of the key to Babylonian cuneiform. The Egyptians, while employing many clever mathematical tricks for their time, had a very inconvenient system for using fractions. Babylonians had to resort to tables for their fractions, however, they were very convenient tables due to their development of place notation for powers of the base both as multiples and as divisors.

While the sexagesimal system does have its advantages, it was the advantage of the place notation for fractions that carried the sexagesimal system. The Greeks, for division purposes in astronomy, found the Babylonian system very useful. It was mainly for the ease of fractions in the Babylonian place notation that the Greeks took the system as a whole for astronomy purposes. They did not need to use the base sixty element of the system, just the place notation.

The Greeks did little to advance algebra and more complex expressions of numbers and were held back by a lack of interest. Thus their number system hardly developed, and they used their alphabet to symbolize numbers; the problems therein exasperating the numerical stagnation. They only really cared for the use of numbers as applied to astronomy and geometry, which the Greeks held in very high regard.

It has been said sarcastically that the only contribution the Romans made to the history of mathematics was the slaughter of the great Archimedes, the great Greek scientist and mathematician. It is interesting to note that although the Romans used the decimal system as far as conceptualizing numbers, they used a base twelve system of mensuration. Every subunit of measurement was a 12th of the unit before. As convenient as this system had potential to be, the clunky

Roman numerals were not equipped to do complicated fractions and thus resorted to rounding their divisions. And since twelve divisions were used within a base ten number system, this also led to rounding fractions.

"Many people have assumed, without really thinking it through, that it is the base ten in our number system which gives it these mathematical advantages...The Romans had a number system based on ten, and we have seen how mathematically useless it was. The early Egyptians had a number system based on ten, and it embodied grouped symbols which were nearly the equivalent of separate numerals; but they made the fatal mistake of having a separate new symbol for ten instead of zero, and were able to represent two tens only by repeating this symbol." -Andrews

"The Greeks, who knew that [Babylonian] system from astronomy, were well aware of its great advantages, but they did not seem to have measured the whole range of its importance. They mutilated it by retaining from it the division of the unit only. By the juxtaposition of the sexagesimal division and the decimal numeration, they associated two heterogeneous elements. Finally, as it seems, they were unable to appreciate the value of the principle of position. When the Hindu-Arabic system, based like the Babylonian system upon the same principle, became first known in the Occident, it made the effect of a great and astonishing innovation." -Thureau-Dangin

In the year 662 AD an abbot of a Syrian monastery had been quoted (by Thureau-Dangin) in amazement at the Hindu-Arabic "fluent method of calculation and at their art of computation that surpasses description, namely that which is done by the means of nine signs." The Hindu-Arabic numerals were brought westward by way of the Arabs, and are often misattributed as being Arabic. Although it was this influence that set Western mathematics in motion, the Arabs had adopted the sexagesimal division from the Greeks. We can credit the continued use of the sixty minute hour to the Arabs. Still, it was not until the 14th century that Europe began to use positional notation for fractions.

"The idea of applying to the fractions the same progression as to the integers, but in decreasing order, has not been realized in our system of numeration prior to the dawn of the modern times." -Thureau-Dangin

In the last few hundred years BC, the base ten system of counting was settling in for the next few millennia. It was accepted in India by 500 BC, and possibly earlier in China. It was used by Egyptians and Greeks around this time as well. As Keith Ellis wrote in his book *Number Power*, base ten has had "no serious rivals since."

One of the most interesting cases of mathematical development is the impressive achievements of Mayans. At the time of the European invasion of the Americas, the Mayans had already developed a very impressive system of mathematics which was mostly motivated by the society's use of astronomy for all facets of life. The level of Mayan mathematics was certainly impressive in comparison to the European invaders.

The Mayan number system is notable because it was a vigesimal system (base twenty), although not purely vigesimal. There were elements of the number naming that were decimal, however, the notation of their numbers was vigesimal. There is evidence that this system did not generate from common use, but that it was developed for purposes of astronomy. The Mayan system may have originally been base ten but later adapted to base twenty.

The Aztecs also used a combination of decimal and vigesimal, but their system appears to have developed more naturally and with less studied effort. It is exactly that studied effort that led the Mayans to develop much more impressive mathematics. The Mayans knew about zero one thousand years before Europe learned of it and 500 years before China. The Mayans used place notation while the Romans were still using their gargantuan numerals.

According to Menninger, there appears to be no purely vigesimal number system ever developed by any people. There is also evidence of some peoples using elements of a quinary number system (base five), however this is never the full system and is always found in conjunction with base ten. So it seems even when there is an element of quinary, vigesimal, or sexagesimal, there is always the decimal system tied up somewhere in it. There is abundant evidence that many early societies chose number base systems that corresponded with the human anatomical digits — fingers and sometimes toes as well. Thus, for most of human history numerical systems have been in base five, base ten and base twenty.

THE CASE FOR A NEW SYSTEM

"The origin of the decimal system is clear. If this system of numeration has been adopted by the immense majority of the human race, it is because man originally counted upon his ten fingers. In reality, however, the number ten is a very defective basis, because of its insufficient divisibility. A system founded upon a number divisible by 2 and 3 would certainly be more practical. In fact, one may observe a general tendency to conceive of the number 6 or 12 as of a new unity." -Thureau-Dangin

If we are to agree with Mr. F. Thureau-Dangin, then the question remains, should we change to a base six or base twelve? The even more complicated question that follows is, could society make such a switch at this point? According to E.M. Tingley in 1934, it would take three generations for a complete switch over. With the exponential growth of population and technology, it could possibly take even more time. As Carl B. Boyer lamented in his book *A History of Mathematics*: "From a mathematical point of view it is somewhat inconvenient that Cro-Magnon man and his descendants did not have either four or six fingers on his hand." While to early humans, counting with ten fingers was very handy, this is now mathematically outdated.

"The reason for the present method of counting, based on the number of fingers on the human hand, is physiological. It was convenient for keeping track of small things. It is much less convenient for dividing things into groups. The interesting thing is that the system has never seemed quite satisfactory. Had it been so, there would have been no reason for the British shilling, the linear foot, the present calendar and clock, or the universal habit of buying eggs by the dozen." -Terry

It was not until the 19th century that a system of mensuration was adopted to match our system of numeration. That system — the decimal metric system — has not been adopted by the entire population in use of the decimal system. It was first proposed in 1791 but it was not until 1850 that a movement of scientists and businessmen pushed for a more widespread use of the metric system. In day to day life, people find the need for different subunits of mensuration than the metric system accommodates, thirds for example.

Subunits of twelve parts have been used, and are still used today for inches, hours, and dozens of eggs. It is interesting to note how well this division of twelve has lasted, within decimal numeration. Unfortunately, the lack of corresponding mensuration and numeration leaves us with far too many messy fractions.

The metric system makes sense only as mensuration designed to match numeration. Calculations are streamlined and easier to compute mentally, rather than resorting to paper or calculator. It would be best if the system of mensuration, made to match the system of numeration, was actually designed for a convenient system of numeration.

[The conclusion of Addie's paper will appear in the next issue of our Bulletin.*]*

I NEVER THOUGHT ABOUT THAT

by Jean Kelly

Duodecimal or Dozenal?

Recently we were asked by Ian Cabell when did we change our name from the Duodecimal Society of America to the Dozenal Society of America.

A little research showed that the minutes of our annual meetings bear the heading *Duodecimal Society of America* for the Dec 29-30, 1977 meeting and *Dozenal Society of America* for the Jun 2-4, 1978 meeting. Neither document refers to the name change. Hence, it must have happened somewhere in between those meetings.

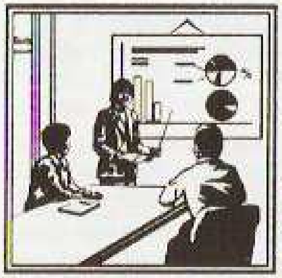
When I asked the inquirer why he asked I received this response:

> I'm an editor at Wikipedia (as you may know, anyone can be :)
>
> Recently someone added the phrase:
>
> "According to the DVD liner notes, "Little Twelvetoes" is dedicated to the Duodecimal Society of America, though no such group exists. However, the Dozenal Society of America is a real organization which promotes the base-12 system."
>
> I did some research on that, since it caught my eye, and found your group. I've since changed the phrase to say:
> "According to the DVD liner notes, "Little Twelvetoes" is dedicated to the Duodecimal Society of America, which later changed its name to the Dozenal Society of America, an organization which promotes the base-12 number system."
>
> And in my research, I wondered when the name change took place.
>
> You can see the full Wikipedia page here: Schoolhouse Rock!
>
> Thanks very much for your help!
> --Ian

Following up on the links he provided I learned that one of the reasons for the change was that the word 'duodecimal' is a *base ten*(!) word from the Latin for two plus ten. I never thought about that.

OUR ANNUAL MEETING

We will Gather at Dek (Ten) am on Saturday, 6 October at Nassau Community College in the conference room on the 2nd floor of the old Student Center.



Once again, all members are invited, so come and bring a friend.

As is our custom, coffee and donuts will be served, and we will go out to lunch together after the meeting. For info you can reach us at Contact@Dozens.org or call 631 669 0273

O O

PROBLEM CORNER

A. What is the probability that the product of 3 consecutive positive integers will be divisible by dek?

B. What is the probability that the product of 3 consecutive positive integers will be divisible by do?

(See page 20; for solutions)

Δ Δ Δ Δ

REMEMBER — *your gift to the DSA is tax deductible!*

JOTTINGS

Board Member Honored

John Impagliazzo, a professor of computer science at Hofstra University and chair of the university's Computer Science Department, received a Lifetime Service Award from the Association for Computing Machinery.

ACM is a premier organization of computing professionals and academics. Its Lifetime Service Award honors an individual who has a long history of volunteer service to computer science education.

We applaud the ACM's recognition of John's many valuable contributions to the field of computer science education, Generations of students have benefitted from his knowledge, international experience and continuing passion in his field.

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Several Officers and Board Members gathered for an informal meeting on April 19th;[21th.] at Nassau Community College. They included Gene Zirkel, Christina D'Aiello, Ellen Tufano, Jay Schiffman and Alice Berridge. The meeting room was arranged by VP John Earnest.

Copies of the *Dozenal Journal* of Great Britain were distributed to all. This is a "catch-up" issue since the death of Arthur Whillock had resulted in delay in publishing the *Journal*. It is full of interesting articles, and also a tribute to our own Fred Newhall.

Gene had printed a dues letter which members folded and labeled for mailing to our membership. Ellen is working on updating our membership list.

Christiana is going to scan the latest issue of our *Bulletin* for possible posting onto our website. It is hoped that eventually all the *Bulletin*s will be posted on the web.

She is also looking into the possibility of emailing it to members or interested parties. Hopefully we can generate an email list and then send out electronic issues of our *Bulletin*.

Ellen suggested that when that happens we ask via the website and the *Bulletin* the question: "Do you want to receive the *Bulletin* via the web or in hard copy?"

Christina also took copies of the several VCR tapes which our members had

made over the years for possible posting on our website. It would be wonderful if Jim Malone's excellent talk, "Eggsactly" were to be available on our website.

Ellen gave us an overview of our current finances.

In the interest of maintaining DSA continuity Gene felt it incumbent for members to be apprised of DSA materials he has at home in his computer foldei '1dsa', his file cabinets and on his bookshelves. In addition he mentioned sever: listings that post information about the DSA such as www.gale.com, The World of Learning, Bowker, and MathWorld.wolfram.com

Gene drew our attention to *Bead Arithmetic* by Kwa Tak Ming. It is an instruction booklet in how to use an abacus including methods for finding squar roots and cube roots. The Chinese abacus using 2 and 5 beads was adapted by the Japanese who eliminated the redundant extra beads for base ten, and so it evolved it into the soroban which has 1 and 4 beads. Hence we can use the Chinese version to do dozenal arithmetic. (Later, Alice did a search of www.Amazon.com and found copies available for purchase starting at $3.34.)

The next meeting of our Society is October 6, 2007. We discussed generating interest, and attendance at the meeting, by contacting clubs and faculty at variou local colleges.

Everyone thought that this gathering was worthwhile, and it was suggested that we have 2 meetings every year. This will be taken up at our October meeting.

Finally, Gene discussed Babylonian notation for base sixty. Their system was a combination of a modern base counting system (including fractionals!) With a base ten — Roman Numeral-like — system for indicating the digits from 1 to 5(He was inspired to look into this by a paper submitted by Addie Evans (see pag 5).

"Each One Teach One."

-Ralph Beard, a founder of the DS

MAIL BAG

Can anyone help Ric?

From: Ric Lantz - Madison Rep
To: contact@dozens.org

Hello,
I have duodecimal unit question I was hoping you could help me with. Is there a term for a gross of gross? and if so what is it? This is mostly to satisfy my curiosity as I once knew the term and can't remember it now. Any help you can give would be much appreciated.

Thank you,
ric lantz

(If you know the answer to Ric's question please send it to us at Contact@Dozens.org or call 631 669 0273. - Ed.)

***

Symbols

You guys should sell stuff! I want a dozenal clock!!!

I just wanted to comment on the symbols. I think the idea to use other symbols that are already on the keyboard and phones is a great idea. And it's also great that you're trying to pick symbols that people will accept more readily.

I don't like the fact that * and # do not resemble other numbers, but that we can deal with. I think there are however two main problems.

First of all both of these symbols are already used with numbers to mean other things. You wouldn't want somebody to think #3 means "number three." Also, * is already widely used on computers and calculators (including those on cell phones) to denote multiplication. This could lead to many more problems than the convenience is worth.

In addition, * is simply not a reasonable character to use. It's small, superscript and hardly even resembles a character (much less a number).

I notice that on the site you generally use an X with a line through it. The problem with this is that it entirely defeats the purpose of having a common, easy

to use symbol! There is no key that resembles this on a standard keyboard and, in most cases on a computer, using an alternative symbol would be just as easy (not to mention more visually acceptable).

I think that you're main concern in choosing symbols should be focused on ease of use with a computer considering that the dozenal system's popularity is primarily spread over the internet. The existence of * and # on telephones will have little to no impact in this.

In my opinion X and E are both sufficient. Other symbols that more closely resemble numbers and the original handwritten ones used would be nice, but these do just fine. Neither of these is already used alone in writing as a word and so they easily stand out as numbers. And while x is commonly used in algebra, a capital X can still clearly represent a number. And as I said, the nicest thing is that they're both simple to use on the computer.

I don't suppose I can really change your mind about it, but I thought I'd throw in my two cents worth.

But again, I think it's great that you are trying to encourage unity in the choice of symbols.

Joshua Harkey

Atlanta, GA 30084

Editor's Note:

The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (✱) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

We see merit in the idea that the asterisk is too small, the crossed out X too much work and the octothorpe has the meaning of number. Ideally we would want two symbols that are found on the computer keyboard and that are not already used in mathematics. That eliminates both the English and Greek alphabets along with many other symbols such as % or !,

We receive many proposals for symbols and are slow to change. We await the day when two wonderful symbols arrive in the mail. ✱

A Dialog

From: Maxime
To: Contact@Dozenal.org
Subject: dozenal system

I agree fully with having a dozenal numerical system, but in my opinion it would be impossible to make the transition. 10 has been imprinted in my brain and I wouldn't be able to assign another value to it other than the one I have been taught since grade 1.

Our Response: Thanks for writing. I agree progress is difficult. Speed limits for autos were once 10 mph in most cities. Some even demanded that a man with a lantern had to precede an auto so as to warn teamsters that their horses might get frightened.

However common sense did ultimately prevail.

I have seen youngsters in grammar school work in binary when learning about computers. I have seen an 8th grader construct a dozenal multiplication table.

Negative thinking has never produced positive changes. Try to think positively.

From: Maxime

Do the other two digits have names?

I was thinking if we were to put a new number system, each digit should have completely different symbols to the ones we have today, as not to cause confusion with our current one.

Lastly, is there a plan on how to convince schools to teach a dozenal numerical system?

Our Response: Various authors have used a variety of symbols and names. To reduce confusion our Society uses an asterisk (✱) for ten and the octothorpe (#) for eleven. We call them dek and el and then proceed to

call 10 do (pronounced as doe or dough). We then continue with do-one, do-two etc.

Referring to dek and el, a previous Editor of our Bulletin referred to them as not ten (an X crossed out) and not eleven (11 crossed out twice).

You can find more information on our website: http://www.dozenal.org or I can mail you some of our free literature if you provide me with your address.

From: Maxime

This is very cool, thank you for replying so fast and so informatively. One last thing I was wondering was, what is so bad about having 0.333333333333? Does being able to write more fractions as perfect decimals help man-kind advance any faster than our current rate?

Our Response:

It is more accurate, for example 1/3 of 1000 = $333.33333333333...
approximations give 0.3 x $1000 = $300
0.33 x $1000 = $330
0.333 x $1000 = $333
etc.
$1000. decimal = $6#4; dozenal
0;4 x 6#4 = $239;40 exactly

Accuracy is also very important in science and engineering, stress tests on airplanes, building bridges etc.

Rounding off when one needs to we note that 0.1 (1/10) has an error of ±0.05 while 0;1 (1/12) has an error of ±0;06 or 0.41666...
Compare 2 places 0.01 with an error of = ±0.005 with 0;01 which has an error of only ±0;006 or 0.00347222...
3 places 0.001 with an error of = ±0.0005 with 0;001 which has an error of only ±0;0006 or 0.00028935185...
4 places 0.0001 with an error of = ±0.00005 with 0;0001 which has an error of only ±0;00006 or 0.00002411265...
etc.

From: Maxime

Yes but if you just express it as a third, then you keep all accuracy. But then again you are right because with money its not the same. All right I'm convinced. Thanks for all the help regarding the dozenal system. Viva la Society, viva!

Our Response:

I'm am happy that you are convinced, and I thank you for your good wishes for our Society. Why not join us. It's only a dozen dollars a year and only $3 for students.

My offer to send you some free literature still holds. Simply send me your mailing address.

Solution to Problem Corner

from page 20;

A) Consider the product (n-1)(n)(n+1). One of these integers must be divisible by 2. In addition, each of these 3 integers has a 1/5 chance of being divisible by 5. Hence $^3/_5$ of these products are divisible by dek.

B) Consider the product (n-1)(n)(n+1). One of these integers must be divisible by three. If n is odd then one integer is divisible by 4. Since n is odd ½ of the time we have ½ of the products are divisible by do. When n is even then ½ of the time it is divisible by 4. Hence another ¼ of these products are divisible by do. Therefore altogether ¾ of the products are divisible by do.

Thus 3/5 or 60% is only about 0;725 that is approximately 72;5 $^{\circ}/_{g}$(per gross) while 3/4 is 0;9 or 90;$^{\circ}/_{g}$. That an increase of a little more than 19;7 $^{\circ}/_{g}$.

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition--("Who needs a symbol for nothing?")--the new notation did come into popular use. Released from the drag of Roman notation, people's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance. Then, in a related development, people awoke to the fact that different number bases could be used.

A parallel to today seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, the most advantageous base should be used, the one best suited to the task at hand. (Similar to computer scientists use of binary, hexadecimal or octal - whichever is most convenient.) If this were done, duodecimals would progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited

DETACH—HERE—OR—PHOTOCOPY

YOU ARE INVITED TO JOIN THE DOZENAL SOCIETY OF AMERICA

The only requirement is a constructive interest in duodecimals

Name ____________________ / /
Last First Middle Date

Mailing Address (including full 9 digit ZIP code)

____

____

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Phone: Home__________ Business__________

Fax __________ E-mail __________

Business or Profession __________

Annual Dues .... Twelve Dollars (US)
Student (Enter data below) .... Three Dollars (US)
(A limited number of free memberships are available to students)

School __________

Address __________

Year & Math Class __________

Instructor __________ Dept. ______

College Degrees __________

Other Society Memberships __________

To facilitate communication do you grant permission for your name, address & phones to be furnished to other members of our Society?
Yes: ___ No:___

Please include on a separate sheet your particular duodecimal interests, comments, and other suggestions.

Mail to: Dozenal Society of America
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