THE DUODECIMAL BULLETIN

DOZENAL SOCIETY OF AMERICA

Whole Number: 89; 8 Dozen 9
Volume: 44; 4 Dozen 4
Number: 2; 2
Year: 11X#; 1 Great Gross 1 Gross Ten Dozen Eleven

ISSN 0046-0826

Vol. 44;(52.), No. 2, Year 11X#;(2003.)

THE DOZENAL SOCIETY OF AMERICA
c/o Mathematics Department
Nassau Community College
Garden City, New York 11530-6793

FOUNDED 1160;(1944.)

= Annual Meeting - October 4th @Rowan U. in NJ - See Page 4 =

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, non profit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights & measures, & other branches of pure & applied science.

Membership dues are $12 (US) for one calendar year. Student Membership is $3 (US) per year, and a life Membership is $144 (US).

The Duodecimal Bulletin is an official publication of the DOZENAL SOCIETY OF AMERICA, INC., % Math Department, Nassau Community College, Garden City, LI, NY 11530-6793.

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Whole Number Eight Dozen Nine

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IN THIS ISSUE

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The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use Cap X with strikeout (✗) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are"dek", "el" and "do" (pronounced *dough*) in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and a semi-colon, or Humphrey point, as a unit point for base twelve.

Thus ½ = 0.5 = 0;6

Our duodecimal party will take its message to the picturesque borough of Glassboro, NJ, the site of Rowan University's main campus. We will convene on Saturday, October 4, 11*#; (2003.) at 10:30 A.M. The meeting will take place in the Dewey Faculty Lounge located on the First Floor of the Robinson Academic Building, which houses the Department of Mathematics on the Third Floor. I look forward to invite interested department faculty to witness a unique mathematical experience. Currently four of our faculty's expertise lies in the field of mathematics education. Two of our courses (Structures of Mathematics for pre-service teachers and Discrete Structures serving as a foundations course for Computer Science) cover the topic of number bases in some detail. I am hoping to have a panel discussion in addition to other duodecimals topics of interest.

The historic borough of Glassboro was home to The Summit of The Century in June 117#;(1967.) between the leaders of the United States (President Lyndon B. Johnson) and the Soviet Union (Premier Alexi Koseigyn). The site was strategic. It lies midway between The United Nations in New York and The White House in Washington, DC. Here is a query: Did you know that Glassboro State College lies south of the Mason-Dixon line? In 11*0;(1992.), philanthropist Henry Rowan of the South Jersey firm Inductotherm bequeathed the college with what is believed to be the second largest endowment in the history of a college or University ($100,000,000) to initiate a first class engineering program. In September 1992, the name Glassboro State College became Rowan College of New Jersey in his honor. The college subsequently applied for university status. With the hiring of numerous outstanding professionals in the 1990's and an enhanced curriculum, the college successfully was granted university status in 1997. The College of Engineering is now a thriving component of Rowan University. A unique doctoral program in Educational Leadership likewise has added a great deal of prestige. Rowan certainly has come a long way from the days of being Glassboro Normal School with the main focus on the training of future teachers. While teaching and the training of future teachers remains a prioritized initiative, most faculty are actively engaged in some aspect of research. The Department of Mathematics has members whose research interests are diverse. Current fields of research include number theory, mathematics education, applications of mathematics to technology, analysis, algebra, biomathematics, finite and discrete mathematics, operations research, and probability and statistics. The Department sponsors seminars and colloquia on a regular basis, and is the host to The South Jersey Mathematics Alliance dealing with current trends in mathematics education that are of interest to the high school, two year and four year college community. Rowan is also the host to the Southern New Jersey AMTNJ Spring Regional Conference (The Association of Mathematics Teachers of New Jersey). It is consequently an

exciting time to be a faculty member at the university. Moreover, this will be The Dozenal Society of America's second visit to Rowan. On October 12; 11*3; (October 14, 1995), we convened on a very rainy day in the Mathematics Department Conference Room.

Rowan University is easily reached by automobile. Take the New Jersey Turnpike to Exit 2 and proceed East approximately ten miles until you reach the intersection of Bowe Boulevard and Route 322. Further down 322 (past the WAWA Food Market) on the left will be the Campbell Library, the Student Center and University Bookstore. Just beyond is a parking attendant who works for The Department of Public Safety who will direct you to the visitor lot. Tell him that you need to be close to Robinson Hall. Please plan on joining us, especially if you are a member residing in the tri-state area of Pennsylvania, New Jersey, or Delaware. If the weather permits, members would be interested in taking a scenic and historic campus tour. The meeting will be followed by dinner at a nearby restaurant. A number are within 5 minutes of the meeting site. You will be extremely disappointed if you miss our Annual Meeting.

That's Saturday, October 4, 11*#; (2003.) at 10:30 A.M. *See you there!*

Mail & Fone Bag

Just wanted to tell you that I really enjoyed the Duodecimal Bulletin and that I wish I could attend some of the lectures you have at your meetings.

Also, I wanted to mention that Harvey Kramer Hawkes' calculator has been a great teaching tool in the classroom. In particular, one of the things that one of them saw right away was that entering 1+1 and holding enter causes it to count up--point being, that is where he really solidified his sense for how this counting works. Thanks!
Andrew Kirk

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Thomas Goodman, life member number #6, foned: He said that our recent *Bulletin* was (most interesting and spectacular(. He especially mentioned Peter Seymour(s lead article, *Drastic Measures*, which he said was (outstanding(, and our standard, *Why Change*? He was very pleased with our exchange with the British.

[THE BRITISH WEIGHTS AND MEASURES SOCIETY SENT THE FOLLOWING]

US DEPARTMENT OF COMMERCE AGENCY TO ABOLISH USE OF LB/OZ AND PINT MEASURES

Under US government policy, transition to the metric system is **voluntary** in the USA. The law requires labelling for most packaged goods to show both US and metric systems so that consumers can **choose** which system they prefer.

Warning from England: DON'T DO IT!

However, the National Institute of Standards and Technology (NIST), an agency within the US Department of Commerce, is to propose a bill to Congress that will end the use of inch-pound units for packaged goods. Hints of NIST's intention appeared in November 2002 when it held a forum to:

"...identify areas of work needed to ensure the effective voluntary transition to the use of metric units in all commercial transactions."

To "ensure" something that is "voluntary" is a contradiction in terms! The NIST proposal is now available on the internet. How does it get round US government policy that metric is voluntary? NIST has developed a **form of words** that describes its proposal as **"permissible metric-only labelling"**, and which appears to offer a choice for business:

Metric System to Be COMPULSORY For Retail Transactions

A. Metric and US customary; or
B. Metric

IT'S A TRICK! The two labelling obligations proposed by NIST cannot lawfully co-exist. It is impossible for the law to require *both* systems to be displayed while also stating that only metric need be shown.

Accordingly, the only requirement under the NIST proposal is that packaged goods show metric. Producers *may* print lb/oz/pint equivalents, but such information is **surplus** to the legal requirement. Decoded, the phrase "permissible metric-only labelling" means compulsory metric labelling, since the word "permissible" actually refers to inch-pound.

The NIST proposal, if implemented, will mean the end of US measures as **trading units** for most packaged goods. It will be legal to describe a carton of milk as "473mL" but ILLEGAL as "one pint". The upheaval and costs to business will be huge, since systems and processes will have to change to accommodate metric.

BRITAIN'S DISASTROUS EXPERIENCE OF METRIC

Britain knows all about compulsory metric conversion. Since 2000, metric measures, invented in France in 1790, have been made compulsory by the European Commission. In 2001, trader Steven Thoburn was dubbed the **"Metric Martyr"** after being convicted and fined for selling bananas in pounds and ounces. Packaged goods are meanwhile "downsized" on conversion from English to metric quantities - *with no decrease in price.* Surveys show 85% of British people prefer feet and inches, pounds and ounces.

Metric Martyr fined for selling in pounds

The archetype kilogram is stored in a vault near Paris and the USA requires permission from the French government to examine it. **Thomas Jefferson** said: "If other nations adopt this unit, they must take the word of the French mathematicians for its length...So there is an end to it!"

If Americans want to defend fair play in the marketplace and freedom to use customary measures then they must wake up to moves now developing to force them to use metric.

Visit website www.bwmaOnline.com for the following information:

- The exact wording of the NIST proposal
- American consumers beware - The Great Metric Rip-Off
- Is NIST breaking the law?
- The story of the English metric martyr traders

BWMA is a non-profit body that promotes equality in law between inch-pound and metric units. It enjoys support from across Britain's political spectrum, from all manner of businesses and the general public.

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MEMBERS, Your 11#0;(2004.) dues are due as of 1 January.
And THANKS to those who have already paid and/or sent gifts.

ENGLISH MEASURES: Their True Scientific Importance.

Michael O'Halloran

PART 1: WHY ARE ENGLISH MEASURES SO IMPORTANT?

English measures are of great antiquity; they have hardly changed for thousands of years; they were in active use in both ancient Sumeria and ancient Egypt. It is not their great age, however, which makes English measures so significant. They are important because they are the keys which can unlock for us the immense treasures of ancient science.

Yes; the ancients did have a science, as detailed and as sophisticated as our own these days; perhaps even more so. There is so much about the ancient world that we do not understand; and so much has been lost; but a surprising amount remains. The problem is that we have largely forgotten how to interpret the material which is still sitting there in front of us.

The keys to that interpretation are provided by English measures. With the aid of English measures we find that the ancient system remains a thing of great beauty; that it hangs together with a seamless integrity and elegant simplicity.

Two English measures are of the utmost importance : the English rod; and what became known after 1824 AD as the *British Imperial* foot/inch.

In order to bring the two onto the same level it is necessary to convert the English rod into a foot/inch measure. It is important to realize that the foot/inch derived from the English rod is not the same as the British Imperial foot/inch. There is not a lot of difference between them in mere size; but the difference is significant.

The difference is significant

The English rod is said to be 16.5 feet; and its modular double is 33 feet (the basis of 33 degrees in Freemasonry). But what sort of feet? Different foot measures have different lengths; unless we specify what sort of feet are being referred to, a great deal of confusion can result.

The foot/inch which derives from the English rod needs a specific name which distinguished it from the British Imperial foot/inch. I call it the *English Winchester* foot/inch, because it is the measure found most commonly in the design and layout of buildings in the medieval English city of Winchester. Nonetheless, the measure itself is much older than medieval times.

To be brief, the English rod measures 5.04 meters; and the foot/inch which derives from the rod measures 305.4545mm and 25.4545mm respectively. (5040mm ÷ 16.5 = 305.4545... which ÷ 12 = 25.4545...)

So the English Winchester foot/inch measures 305.45mm/25.45mm; whereas the British Imperial foot/inch measures 304.8mm/25.4mm. Not much difference, is there? But hang onto your hat, because this is where it becomes most exciting!

What has become known as the British Imperial foot/inch is not primarily nor originally a measurement of length!

British Imperial foot/inch is not primarily nor originally a measurement of length!

The original and primary measurement of length is performed via the English Winchester foot/inch! By contrast, the British Imperial foot/inch is primarily a measure of capacity.

In the next section I explain the difference.

PART 2: THE ENGLISH WINCHESTER FOOT

This measure, derived from the English rod, is the oldest of the very old English measures. To understand its significance, it is important to recognize from the start that there is nothing arbitrary about measures. Measures in the ancient world were at the heart of their science, they were not something dreamed up. The ancients possessed a science as detailed and as sophisticated as our own. Measures embody the fundamentals of their scientific understanding of the physical world.

There is another ancient measure which goes hand-in-hand with the English Winchester foot: the ancient Etruscan foot. During the renaissance the Etruscan foot also became known as the Venetian foot; they are one and the same. The Etruscans were an ancient civilization which settled in northern Italy, between the Alps and the River Tiber, roughly corresponding to the modern Tuscany, pre-dating the Romans who eventually overthrew their hegemony. Their architecture has Egyptian affinity.

The famous architect Inigo Jones (1573-1652) was also convinced of the great antiquity of English measures. He was one of the first people in modern times to do a systematic study of the ruins of Stonehenge in England. He never published his findings; his apprentice and relative John Webb gave an account of his researches after Jones' death. There was much uproar and controversy at the time, and Jones' supposed findings were, and still are, treated with derision. In Webb's account Jones is reported as arguing that Stonehenge was a Roman temple; and this is regarded as a colossal blunder on the part of the great architect.

On the other hand, what Inigo Jones did discover, as his drawings of Stonehenge make clear, was that Stonehenge was designed and built on the basis of the Etruscan foot; and this aspect has been ignored ever since. Inigo Jones appreciated the great significance of this finding; and he pondered its significance for the rest of his life.

The measurement of the Etruscan/Venetian foot is quite precise: it is 357.14285mm; and it measures the horizon. Going hand-in-hand with it, in this task of measuring the horizon, is the English rod and the foot derived from it.

What do I mean by 'the horizon'? The word literally means a boundary; and specifically the boundary of our vision, affected by the curvature of the earth. That is what I mean by horizon here : the boundary of our individual human visual apparatus. What the ancient Etruscan foot measures is the radius of our vision; that radius which describes the horizon of our sight. The radius of our vision is in fact 10,000 Etruscan feet, or 3,571.4285 meters. A light disappears on the horizon at a distance of 10,000 feet from the observer; because of the curvature of the earth.

In practice, we can see only a hemisphere of horizon at any given time (the rest of the circle is behind us). Since the earth is curved, we actually see further than the direct line implies. Thus if the direct line, in what seems to be a straight line from point to point, is 3,571 metres, then the rounded version, following the curvature of the earth, is longer; it is in fact 3,600 metres. This is the basis of the sexagesimal (60x60) system of measures which was adopted in Sumeria, for example. The English rod and its foot derives directly from this sexagesimal measure derived from the radius of our vision.

How does this come about? Since the curvature of the earth is a factor included in these standards of measure — the Etruscan foot and the English foot — then the earth itself can be measured in their terms to a good degree of accuracy.

Without going into the details, consider the sphere of earth divided into 4 quadrants. It so happens, firstly, that each quadrant measures 2800 times 10,000 Etruscan feet; or 10 million meters (2800 x 3571.4285); and secondly, that each quadrant measures 2 million English rods, or 10,080,000 meters (2800 x 3600m).

That is, the earth approximates a nice round multiple of the ancient measure derived from the direct line of the radius of our vision (the Etruscan foot) on the one hand; and approximates a nice round multiple of the equally ancient measure derived from the rounded version of the radius of our vision (the English rod) on the other hand.

Remember that the English rod measures 5.04 meters; and the English Winchester foot measures 0.3054545 meter. Therefore the rounded version of the earth derived from the radius of our vision is 33 million such feet x 4.

There are some startling implications from all this, some of which we shall pursue in the next section.

<u>PART 3: THE ANCIENT METER</u>

The meter measure was not invented by the French in the 18th Century. The meter is in fact an ancient measure, whose length is *slightly different* from the French meter. The true ancient meter, like the ancient cubit, derives from the ancients' assessment of the cycles of the planets and the stars.

When the French scholars went to such elaborate lengths to measure the line of meridian from the north pole to the equator, they were trying to recapture the ancient knowledge upon which the ancient meter measure was based. As you know, the French scholars made errors in their measurement exercise. The current metric measure which is being enforced around the world is in fact constructed upon an erroneous base, since the current metric system still reflects those errors. But it is not just the measuring which is wrong (that can be fixed); the very basis of their quest for a unit standard is wrong.

In fact it is a French archeologist and mathematician, Charles Funk-Hellet, who insists that the meter was a measure well-known to the ancients. He also suggests how the ancients arrived at the calculation of the meter; and he is very close to the truth of the matter.

The idea that the meter is a recent human invention, illustrating the triumph of reason after ages benighted by superstition and ignorance is a conceit which has no basis in fact.

> *A conceit which has no basis in fact*

So the French meter, and the metric system based upon it, is wrong. The French scholars were aware that the meter should be a cosmos-commensurate measure, and that it related to the spherical shape of the earth. However, the French were looking in the wrong spot when they looked to the measurement of the meridien as sufficient and compelling basis for the re-construction of the (ancient) meter.

The meter is a cosmos-commensurate measure; but it relates to the radius of our vision, affected by the curvature of the earth. The earth is a factor in the determination of the radius of our vision; but strictly speaking the earth is not measured directly nor precisely thereby.

The French were looking for a length which contained 10 million units; they knew that such a length was related somehow to the spherical curvature of the earth; they thought that they had found it in the measurement of the meridien; but they did not appreciate the finer points of the ancient science upon which the ancient meter (which they did know about) was based.

Why didn(t the French scholars, who were a formidably learned lot, appreciate these finer points of ancient science? Because the real basis of ancient science had been actively and passionately suppressed by the political/scientific establishment for over three thousand years.

The next section will deal with these issues.

PART 4: THE FOUNDATIONS OF ANCIENT SCIENCE

So far in this article we have considered the English rod, and Winchester foot derived from it, as related to the measurement of the radius of our vision.

It is accepted that the ancients were intensely interested in studies of light. All the ancient stories of beginnings are concerned with the phenomenon of light. It is not generally appreciated, however, that in their studies of light the ancients were really concerned with radiation. Light is simply a perceived effect of radiation.

Ancient science is founded upon an acute and sophisticated understanding of radiation.

It is only when we realize this that the ancient stories, and the ancient measures, finally make sense.

The truth of the matter is that the radius of our vision inescapably refers to our visual perception and interpretation of radiation.

The history of the human race is pitifully short. The human race itself is nonetheless of enormous antiquity; it is the researched story of our forebears (which is what history is) which is so limited. Over all those aeons of human existence, the human brain has hardly changed. It is utterly inconceivable that our modern generations are the only ones which have used the human brain to significant scientific effect. Our overweening conceit in these matters is breathtaking.

Yet from the earliest of our histories our forebears were utterly convinced that they were the inheritors of a vast body of science. The elements of that science are embodied in the ancient stories which have been handed down to us; but by and large we have forgotten how to interpret them; their language remains foreign to us.

5,000 years ago the ancient Egyptians were convinced that they were the inheritors of a detailed and sophisticated science; from an era which we lump vaguely into a prior age which is called pre-dynastic Egypt. The task of the Egyptian state was accepted to be that of ensuring the integrity and vitality of this inheritance, which they called ma'at. From time to time there was a resurgence of interest in the renaissance of this ancient knowledge, aimed at recapturing the vitality of a bygone era.

The most recent, most celebrated, most tumultuous, and most misunderstood resurgence of interest in the integrity of ma'at by the Egyptian state took place some 3,400 years ago, during the so-called Amarna period, when Amenhotep III and Amenhotep IV (Akhenaten) represented the Egyptian people.

What Akhenaten and his wife Nefertiti did was reaffirm and reassert the significance of the ancient measures, explicitly derived from and rooted in the science of radiation. In effect, they reasserted the primacy of the hydrogen atom in the scientific determination of all substance, and as the true basis of measures.

This is not a notion which sits well with us today. We remain completely and unshakeably convinced, do we not, that the ancients could have had no knowledge of the measurement of the hydrogen atom, nor of its significance in the ordering of the stable chemical elements; etc? Yet the facts, embodied in the measures, belie this, and contradict our conviction.

And so we return to the Etruscan foot and the English rod. 10,000 Etruscan feet are related to 2 million English rods via the radius of our visual perception of radiation. The horizon of our vision, described thereby, relates to the spherical shape and dimensions of our earth by a factor of 28 x 100 x 4; as we have seen.

However, the radius of the hydrogen atom is $\sqrt{28} \times 10^{-12}$ (or 52.9 times 10^{-12}); that is, 52.9 picometers. With this as radius, the unit spread of the radiation is contained within a 'foursquare' of this dimension.

The number 28 was enormously important in ancient science, as a *scale* representation equally of the very tiny and of the enormously large - such as the dimensions of the hydrogen atom and of our Earth.

Number 28 enormously important as a scale

Our human ability to perceive radiation; the radiation-spread of the hydrogen atom; the very dimensions of the hydrogen atom itself; the Etruscan foot and the English rod; all are thus explicitly and inextricably connected.

This is what Akhenaten and Nefertiti wanted to reaffirm. This is the structural basis of their new capital city. This is the science which is expressed in the Giza pyramids, which their new capital city was designed to mimic, as the paradigm of the structure of the universe itself, from the celestial world to the underworld. At the heart of it all lie the measures, rooted in the structure of the hydrogen atom.

The seemingly radical change evident in the art and literature and science of the Amarna period mentioned above was really nothing new; it entailed the restoration of an old order. The old order restored was nothing other than the focus of all Utopian stories and movements before and since: there are no grounds for the subordination of any substance; all substance is one.

Such a doctrine does not appeal to those of authoritarian or totalitarian inclination. The Egyptian state was no exception; the military elite overthrew the Amarna family and systematically tried to obliterate their memory.

Isaac Newton found the key to his studies of universal gravitation in the ancient Egyptian measures; and he wrestled long and hard with the Amarna doctrine. He agreed that ancient science rightly asserted and affirmed the unity and integrity of all physical substance; but ultimately he could not accept the central tenet that all substance is one. Isaac Newton thought he had to accept the radical subordination of the creature to the creator. The resultant dualism still bedevils our science. And for more than 200 years his literary and scientific executors have suppressed Isaac Newton's writings on this subject.

Given that the English rod and the foot/inch which issues from it is rooted in the science of radiation, what can be said about the British Imperial foot/inch? In an earlier article I said that this is not primarily a measurement of length but is rather a measurement of capacity. This is the issue which we will turn to next.

PART 5: MEASURES OF LENGTH AND MEASURES OF CAPACITY

Length in the ancient world was always conceived as a 'volumetric'; that is, it is seen as a 'bulk' measurement, with volume implied in the length. It is only since Euclid's systematization of Greek work in geometry that the ancient notions have been attenuated into abstractly defined 'lines' without dimension other than length. In the ancient world a 'line' is juxtaposed units of 1 x 1 x 1, in 3 dimensions; so that a length of 12 units is (1x1x1) a dozen times; or 12x1x1. So a length has 'bulk'; as does a square, such that 12 squared is a 'bulk' of 12x12x1.

These ancient 'bulk' measures of length form the basis for measures of capacity. Hence a standard 'bulk' unit of length provides the side of a cube which then contains a standard measure of capacity. So measures of length and measures of capacity are directly related, in the official standards. Capacity implies density, so there is more than volume entailed in measures of capacity.

Capacity implies density

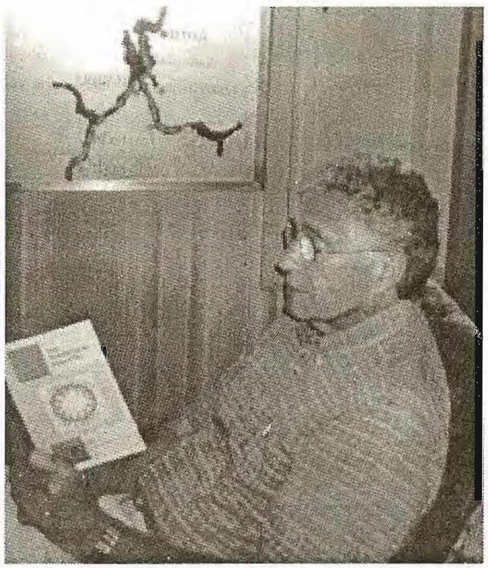
In English measures, the unit of capacity based upon the 'bulk' length of 1 foot (1'x1'x1') is called a firkin. In other words, a 'firkin' measures the capacity which is enclosed in a cube with sides of 12x12x12 inches. This is the standard which vessels of varying shape had to fit. The word 'firkin' comes from an old word meaning 'a fourth'; specifically a quarter of a barrel of ale, which contains 36 gallons. Some texts will tell you that a firkin is a cask of indeterminate or variable size; however, it is really quite precise so far as its capacity is concerned; because of the density of its contents. This the real reason why a barrel of ale is different from a barrel of beer!

Vessels such as the firkin measure the void within. But the void is contained by a structure which has definite dimensions and 'bulk'.

The 'bulk' structure which contains a void is very important in ancient measures. Much of ancient science is incomprehensible if we forget that the void within has different dimensions from those of the 'bulk' structure which contains, or gives

[Continued on page 16;]

DSA on a Russian River Cruise



On July 2003 a trio of DSA members: Edmund & Alice Berridge, Herb (and Iris) Plaker sailed together on a Russian River cruise. It was the 210;th (300.th) anniversary of the founding of St. Petersburg and a good time to visit Russia. We enjoyed many wonderful adventures aboard the MS Maxim Litvinov along the Czar's Waterways from the start in Moscow to the spectacular climax in St. Petersburg. But a special highpoint of the cruise involved the DSA!

Herb brought a copy of our *Bulletin* which Alice and he discussed aboard ship. During the cruise our professor/Lecturer, Dr. Sev Mariniov presented interesting talks such as: "Russia in Search of Democracy", "President Putin on the Way to Reelection" & "Russians Gone with the Wind". In conversations with Dr. Marinov we told about the DSA. *We were delighted to see that he* was vary interested in our activities and in our *Bulletin* and thrilled with the opportunity to spread the word about dozens.

The photos show Herb reading our *Bulletin*, the 4 travelers relaxing on deck, Dr. Marinov and Herb discussing the *Bulletin* and Dr. Marinov with our *Bulletin*.

[Continued from page 13;]

boundaries to, the void. We miss this if we consider that a 'line' has no breadth nor depth, and hence no volume. This is part of the reason why the ancient use of pi is so much misunderstood.

In English measures, the primary base for the firkin is the Winchester foot/inch.

What about the British Imperial foot/inch? In my previous article I stated that the BI foot/inch is not primarily nor originally a measure of length. The British Imperial foot/inch is first and foremost a measure of capacity.

Measures of capacity such as the firkin measure a void; not an empty space, but a gas-filled space in the 3 dimensional world. (In the 4 D world the void is a number-filled space; but that is a different issue.) It is well-established in modern science that equal volumes of gases under the same conditions of temperature and pressure, contain the same number of molecules or entities. The number is known as Avogadro's Number; and it is a constant.

The British Imperial foot/inch originates as the scalar expression of the raw number of entities contained within a vessel of standard size. In other words, the British Imperial foot/inch is conceived as a scale model of the constant number of molecular entities contained in a void of standard size, under the same conditions.

The number is very large: these days it is said to be 6.096×10^{23} entities in a gram-atom of any elementary substance. So there are a little more than 6 hundred thousand billion billion numbers of atoms in 1 gram of hydrogen, or in the gram-equivalent of the atomic weight of any element. The ancients were not quite so fascinated with 'counting' as we seem to be today; they were more interested in the proportionality of things. This is expressed in the enduring principle of ancient science: 'as above, so below', which emphasizes that all phenomena stand in a proportional relationship, no matter how large nor how small they may be. All substance is one; and the same proportionality endures throughout. That proportionality is expressed in the raw numbers - the significant digits - (rather than the sheer number of zeros after or before) as *scale* representations of a larger truth

As usual, however, the true ancient scientific significance of measures is obscured by (French) metric reference. The touchstone in the ancient world for this number was not the (French) meter, but cubits, or English feet, or ancient meters.

It would seem that the English measure which we now know as the British Imperial standard was crafted specifically as a scalar expression of the number

254. Taking 254 as the unit of scale, you will notice that a scored dozen of such units provides Avogadro's Number. (A scored dozen is 240, or 20 x 12; so 254 x 20 x 12 = 60960.)

In fact many significant measures cluster around this 254 unit of scale; including those measures based upon the dimensions of the hydrogen atom.

On the other hand, don't let the (French) metric reference obscure the principal issue: the scalar unit is 1. For the ancients what we call Avogadro's Number is simply a scored dozen units, in the appropriate scale; where the unit corresponds to what we now call 1 British Imperial inch (which happens to be the equivalent of 25.4mm in length).

Many people will tell you that the ancients could have known nothing about the dimensions of atoms; could not have known anything about such things as Avogadro's Constant. But they would be wrong.

Avogadro's Number emerges quite clearly in the most important chapter of perhaps the most significant surviving text from the ancient world: Chapter LXIV of the Egyptian Book of the Dead — when you know how to look.

That introduces other issues in ancient science which we cannot hope to deal with here; if you want to know more, cf. The Foundations of Ancient Science at http://goulburnbrewerypublishers.netfirms.com

English measures are important. They remain as unchanged relics of the ancient world; and therein lies their true significance. They are not mere curios. So much has been lost from the ancient world; so much has been deliberately destroyed. The very fact that English measures survive intact from ancient times means that through them we have still the keys to the vast depository of ancient knowledge. Without English measures, what other keys remain? Treasure and safeguard the English measures, by all means; but, above all, use them; as they were always intended to be used, as vital living expressions of *ma'at*, of that ancient wisdom of our forefathers which affirms that there are no grounds for the subordination of any substance; that all substance is one.

Unchanged relics of the ancient world

In the meantime, if you want to know more, see The Foundations of Ancient Science at http://goulburnbrewerypublishers.netfirms

IRREGULAR DOZENAL DIGITS

Brian Dean

Preface

In this article, I will discuss irregular digits, a numbering system wherein the radix is a positive integer but the digits have different values than in the regular form of that base. For example, you can have a dozenal base with digits {0, 1/5, 2/5, 3/5, 4/5, 1, 6/5, 7/5, 8/5, 9/5, 2, #/5}. I discuss this particular example below.

Digits can be positive, zero or negative. In what follows we will write negative digits as underlined italicized numbers. Thus we would write the two symbols for the single digit -1, -2, -3, ... as the single symbols $\underline{1}$, $\underline{2}$, $\underline{3}$, ...

We do this because -1 is deemed to be a single symbol representing one digit, not two adjacent symbols. {J Halcro Johnson in his seminal work on negative digits used a digit with a bar over to signify a negative digit. Thus :

$\bar{6}\ \bar{5}\ \bar{4}\ \bar{3}\ \bar{2}\ \bar{1}$ 0 1 2 3 4 5 6 -Ed.}

Thus in base 3 the digits {$\underline{1}$, 0, 1}, can express 2 as the numeral 1$\underline{1}$, since in base 3

1$\underline{1}$ is equivalent to 1 x 3 + (-1) x 1 = 3 - 1 = 2

Definition A number is said to be representable in a place-value number system with digits {d_0, d_1, d_2, ... d_n} if and only if it can be expressed either finitely as $d_a d_b d_c \dots d_d . d_e d_f d_g \dots d_h$ or infinitely as $d_a d_b d_c \dots d_d . d_e d_f d_g \dots$, where a, b, c, ... h belong to {0, 1, 2, ... n}. Note this excludes the minus sign (-).

Complete Bases There are some bases which are what I call *complete* bases and others that are not complete. In order for a base to be complete it must follow one of the three criteria

1. All of the positive real numbers are representable, or
2. All of the negative reals are representable, or
3. All reals both positive and negative are representable.

For example, the dozenal base with no positive digits, {$\underline{\#}$, $\underline{X}$, $\underline{9}$, ..., $\underline{1}$, 0} behaves in the same way that regular dozenal does except that only negative numbers are representable. Therefore this base is complete.

Base Conversions To see how such bases work, I will discuss in detail the

particular dozenal base {$\underline{5}$, $\underline{4}$, $\underline{3}$, $\underline{2}$, $\underline{1}$, 0, 1, 2, 3, 4, 5, 6}. I will call this base ***Semi-Dozenal.***

Recall how one converts a decimal numeral into duodecimals by repeated division. For example to change the decimal 3851 to ordinary dozenals you divide as follows:

```
12 )3851
12  )320  11 as the remainder
12  )26   8
12  )2    2
     0    2
```

And so reading the remainders we have decimal 3851 is dozenal 228#.

Now to convert a decimal to a Semi-Dozenal, you proceed in the same way except when you get a remainder which is not in your set of digits. For example, if the remainder is 7, you subtract the divisor (12.) obtaining the negative digit "$\underline{5}$" and you add $\underline{1}$ to the quotient. With a remainder of $\underline{8}$ you use the positive digit 4 and add "$\underline{1}$" to the quotient, etc. In general, the way you decide this is to consider which numbers have the same class modulo the base as the ordinary remainders. For example the *negative* digit, $\underline{8}$ has the same modular class as 4 because 10; + $\underline{8}$ = 4. Or, another way of putting it is, if the remainder falls outside of the range {$\underline{5}$, $\underline{4}$, $\underline{3}$, ..., 4, 5, 6} then you repeatedly either add or subtract 10; until you obtain a number that is in the range {$\underline{5}$, $\underline{4}$, $\underline{3}$, ..., 4, 5, 6}. Thus, since $\underline{8}$ falls outside of that range, you add 10; to $\underline{8}$ and you get a number that does fall in that range (i.e. 4) and so $\underline{8}$ is in the same class as 4 modulo twelve.

For later comparison purposes, let us rewrite the repeated division we performed above as follows:

3851 = 12(320) + 11
320= 12(26) + 8
26 = 12(2) + 2
2 = 2(0) + 2

Now in Semi-Dozenal the calculation looks like this:

```
12 )3851
    320  11
```

But since the remainder of 11 is not in our set of digits which range in value from -5 to +6 we adjust the division by *subtracting* 10; from the remainder of 11 obtaining -1 and *adding* 1 to the quotient obtaining

12)3851
321 -1

This is justified since
3851 = 12(320) + 11 = 12(320) + 12 + 11 - 12 = 12(320 + 1) + (-1)

In the same way
12)321
26 9
becomes
12)321
27 -3
completing the division we obtain

12)3851
12)321 -1
12)27 -3
12)2 3
0 2
or

3851 = 12(320) + 11 = 12(321) + (-1)
321 = 12(26) + 9 = 12(27) + (-3)
27 = 12(2) + 3
2 = 2(0) + 2

So decimal 3851 in Semi-Dozenal is 23$\underline{31}$. You can check this by verifying that 3851 = (2 x 12^3) + (3 x 12^2) + (-3 x 12) + (- 1).

CONVERTING A NEGATIVE NUMBER The algorithm to do this is similar, but you have to be careful about your modular classes. So to convert - 3851 the calculation would look like this

12)- 3851
- 320 -11

This time, instead of subtracting 12 from the remainder, we *add* 12 in order to obtain a remainder within our set of digits which range in value from -5 to +6. To balance this we now *subtract* 1 from the quotient. This gives us

12)- 3851
-321 +1
Since -3851 = 12(-320) - 11 = 12(-320) - 12 + (-11) + 12 = 12(-320 - 1) + 1

Continuing in this fashion we have

12)- 3851
12)- 321 + 1
12)- 27 + 3
12)- 2 -3
0 -2

So decimal -3851 would be $\underline{23}$31 in Semi-Dozenals.

Alternatively, one could first convert + 3851 into 23$\underline{31}$ as above and then -3851 becomes -23$\underline{31}$ = $\underline{23}$31.

MULTIPLICATION The algorithms for addition, subtraction, and multiplication are very much the same in Semi-Dozenal as in regular dozenal. If you are careful, all of the carry rules for addition and multiplication are the same. For example, (2$\underline{3}$ x 3$\underline{5}$) would be done in the following way

2$\underline{3}$
x 3$\underline{5}$
$\underline{1}$33 (-5 x -3 = +13; write 3 and carry +1, -5 x 2 + 1 = - 9 or $\underline{1}$3)
53 (3 x -3 = -9 or $\underline{1}$3, write 3 and carry -1, 3 x 2 = 6 + -1 = 5
463

SUBTRACTION Subtraction takes a bit of care because altho the principle is the same as in regular dozenal, except when the result is not a single digit you borrow. Thus
2$\underline{3}$
- 3$\underline{5}$
$\underline{1}$2
is pretty straightforward since -3 - (-5) = 2 and 2 - 3 = -1.

However,
2$\underline{3}$
- 35
$\underline{2}$4
is not as straightforward because -3 - 5 = -8 or $\underline{1}$4. But that is not a single digit. So you need to borrow and get 1$\underline{3}$ - 5 = 4. Then 1 - 3 = -2 or $\underline{2}$.

This is actually no different than subtracting in regular dozenal except that in regular dozenal, something like 5 - 7 would not be representable without invoking a negative sign. But another way of looking at that is to consider that a negative number can't be represented in regular dozenal by a single digit.

You could also proceed as follows:
$\underline{3}$ - 5 = $\underline{1}$4. Write down the 4 and carry the $\underline{1}$. Then 2 - 3 - 1 is -2 or $\underline{2}$.

Alternatively, one could change subtraction into addition as follows:
2$\underline{3}$ - 35 = 2$\underline{3}$ + (-35) = 2$\underline{3}$ + $\underline{35}$ and then

$$\begin{array}{r} 2\underline{3} \\ +\ \underline{35} \\ \hline \underline{2}4 \end{array}$$

(-8 = $\underline{1}$4, write 4 and carry -1, and 2 - 3 - 1 = -2)

Division is rather difficult in Semi-Dozenal and I know of no easy algorithms for division. Perhaps readers would like to experiment to see if they can come up with an easy algorithm for division in Semi-Dozenal.

SEQUENTIAL BASES A ***sequential base*** is one where all of the digits follow an arithmetic sequence and at least one of the digits is zero. All sequential bases are complete. For example, the dozenal base with digits {0, 1/5, 2/5, 3/5, 4/5, 1, 6/5, 7/5, 8/5, 9/5, 2, #/5} mentioned above is a sequential base. For clarity, let us use {0, a, b, c, d, 1, e, f, g, h, 2, i} respectively as *single* symbols for these digits.

Now, in order to express the next number in the sequence after #/5, proceed as follows. The next number would be decimal 12/5 but that is equal to 1/5(12) + 0 = a(12) + 0 = a0. Thus we have 12/5 is a0, 13/5 is aa, 14/5 is ab, 15/5 = 3 is ac, etc.

For convenience, let's call this base ***One-Fifth-Dozenal***. The reader will note, after working with this base, that all of the algorithms for addition, subtraction and multiplication described above, work in this base.

To convert a decimal number to One-Fifth-Dozenal do the following. Since all of the digits in One-Fifth-Dozenal are the same as regular dozenal times 1/5 divide the number you want to convert by 1/5 and apply the same algorithm.

Thus since 3851 / (1/5) = 19255 and dividing this by 12 yields

12)19255
1604 7

We write
12)19255/5
1604/5 f

Continuing in this way we obtain

12)19255/5
12)1604/5 f
12)133/5 g
12)11/5 a
0 i

So 3851 in One-Fifth-Dozenal is iagf. Verifying this we have

$(11/5)(12)^3 + (1/5)(12)^2 + (8/5)(12) + 7/5 = 3851.$

Alternatively, we could convert decimal 3851 to 228# and then multiply this by 5 obtaining #187. Finally, dividing each digit by 5 we obtain iagf.

In fact, in general, if a base is complete using digits {a, b, c, ...} then it is also complete with digits {a/r, b/r, c/r, d/r, ...} where r is any real number not equal to zero.

NON-SEQUENTIAL BASES All other bases fall under the category of ***non-sequential bases***. In general, these bases are not complete. However, there are enough non-sequential bases that are complete to make looking at them an interesting exercise.

An example of an incomplete non-sequential dozenal base would be {0, 1, a, b, 2, c, d, e, f, 3, g, h} where a = √2, b = √3, c = √5, etc. Since h = √11 it is clear that the largest number which is less than twelve which is representable would be h.hhhhh... But this is equal to √11 x (12/11) which is 3.618....
So there exists a range of numbers between 3.618.... and 12 which are not representable in this base

I don't know very much about which non-sequential bases are complete and which are not except to say that if:

1. All of the digits are integers (with zero being one of your digits) and
2. Let L = the Largest of the absolute values of all the digits. Then, if all of the integers which between -L and +L inclusive are FINITELY representable, then the base is complete.

The proof of this is beyond the scope of this article. The interested reader may e-mail me at bridean@ureach.com if he wishes to see it. This proof was originated by Steven Gunhouse.

In other words, if I have base 3 with digits {0, 1, a} where a = -10 then all I have to do is test to see if all of the integers between 5 and -5 are finitely representable and if so then this base is complete. This particular base appears to fail the test since 5 does not seem to be finitely representable (it is however, infinitely representable as 101.aaaaaaa...) Note that it is possible that 5 is represented some other way.

However, the case of base 3 with digits {0, 1, a} where a = -7 is complete. So is the case where a = -25 or a = -31.

I have also found that all of the digits have to be in different modular classes with respect to the base. So for example, base 3 with digits {0, 2, a} with a = -1 would not be complete since -1 is in the same class as 2 modulo 3.

Assuming that all of your digits are in different modular classes you would convert a decimal number into a non-sequential base in the following manner.

Let's take base 3 with digits {0, 1, a} where a = -7 (I have checked and verified that this is a complete base). To convert 3851 into this base the algorithm looks like this

3)3851
1283 2 (2 is not one of our digits, but it is congruent to a = -7 modulo 3)

Thus we obtain 3851 = 3(1283) + 9 + 2 - 9 = 3(1283 + 3) - 7.

Proceeding in the same manner we have:

3)3851		
3)1286	a	
3)431	a	(1286 = (3851 + 7)/3)
3)146	a	
3)51	a	
3)17	0	
3)8	a	
3)5	a	
3)4	a	
3)1	1	
0	1	

So 3851 in this base would be 11aaa0aaaa.

If the base is not complete (for example base 3 with digits {0, 1, a} where a = -10) you get an infinite loop for numbers that are not finitely representable. For example, 5 in base 3 with digits {0, 1, a = -10} will give you an infinite string of (a)s.

As far as I know, all of the usual carry rules for addition and multiplication apply for non-sequential bases but they are extremely difficult to deal with. Subtraction and division are nigh on impossible.

For the interested reader, analyzing to see what criteria must be satisfied in order for a non-sequential base to be complete would be a challenge. The first significant base to look at would be base 3. This is because in base 2, if you have digits {0, a} then a can be any real number that you want and the base will be complete (it is merely one of the sequential bases). However, base 3 gives you the opportunity to look at incomplete bases.

In conclusion, there are many interesting bases out there waiting to be discovered. I hope that this article will give the reader an even broader outlook on what sorts of bases are possible.

Problem Corner - See previous article

P89-1. To show that {0, 1, -7} base 3 is complete, we need to show that the integers from -3 to +3 can be represented with these three digits.

a) Justify the above statement.

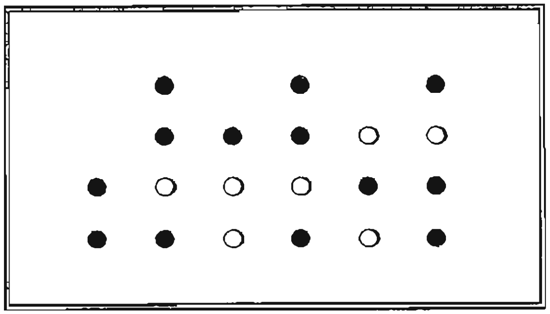
b) Represent these 7 integers in this base. {Hints: -1 = 27 - 28 = 27 - 21 - 7 becomes 10aa; -3 = -1(3) in base 3; and -2 = -3 + 1, etc.}

P89-2. Show that {0, 1, -25} is complete. {Hints: If n is known then n(3), n(3) + 1, and n(3) - 25 are also known. In addition -4 = -7 + 3; 8 = 2 (4); -10 = 81 + 9 - 75 - 25}

Answers, see page 26;

Solutions to Previous Problems:

Problem P88-1. An illustration in an advertisement for a "Powers of 2 Clock" shows a face as in this diagram:



The ad reads that "it's an eye-catching light show that changes every second. But its true purpose is to tell the time — in binary code, the zeros-and-ones language of computers. Techies love this clock and can read it right away." (Hint: It runs in 10; or 20; hour mode.)

1a. Can you tell what time the 7 lit bulbs (O) are displaying?

1b. Can you explain why the lights are arranged in the manner pictured?

Solution by Jean Kelly

1a. Each column is a vertical binary representation of a decimal digit reading down from the top. Reading from the left they are binary 00, 0010, 011, 0010, 101 and 0100. These are the decimal digits 0, 2, 3, 2, 5 and 4 respectively, and they represent 02 hours, 32 minutes and 54 seconds. Thus it is 2:32:54 o'clock.

1b. Since the clock can run in either one dozen or two dozen hour mode, it must be able to represent time from 01:00:00 to 24:59:59. Thus we need the digits indicated in the table below.

column	1	2	3	4	5	6
base ten	0-2	0-9	0-5	0-9	0-5	0-9
base two	00-10	0000-1001	000-101	See col. 2	See col. 3	See col. 2

Thus we need 2, 4, 3, 4, 3 and 4 binary digits (and therefore) bulbs in columns 1 to 6 respectively.

Proposal: P89-1a. Can you show that a similar display of the time in dozenal digits for a *2 dozen hours, 5 dozen minutes, 5 dozen seconds clock* would require 2 fewer bulbs?

b. Can you show that a similar display of the time in dozenal digits for a 1 dozen *duors* (twelfths of a day), 1 dozen *temins* (twelfths of a dour), 1 dozen *minettes* (twelfths of a temin) clock would require 5 fewer bulbs?

Problem P88-2. Pythagorean triples can be generated as (x,y,z) by where x = m2 - n2, y = 2mn and z = m2 + n2.

Can you generate the first dozen *primitive* Pythagorean triples with both odd components prime? A primitive triplet is one in which there is no common factor dividing x, y and z.

Solution by Jay Schiffman

m	n	(x,y,z)
2	1	(3, 4, 5)
3	2	(5, 10, 11)
6	5	(#; 50, 51)
Ӿ	9	(17, 130, 131)
13	12	(25, 2#0, 2#1)
26	25	(4#, 1010, 1011)
27	26	(51, 10#0, 10#1)
30	2#	(5#, 1560, 1561)
34	33	(67, 1980, 1981)
43	42	(85, 2#50, 2#51)
56	55	(Ӿ#, 4#70, 4#71)
5Ӿ	59	(#7, 5710, 5711)

Proposal: P89-2a. Note that m > n, m and n are of opposite parity (in the sense that one integer is even and the other is odd) and m and n are relatively prime. Is this always the case?

b. What appears interesting here is that y and z differ by one in the sense that z = y + 1 as do m and n; for m = n + 1 for the initial dozen outcomes where x and z are both prime components in the primitive Pythagorean triplet. Is this always the case?

□ □ □

Answers from page 23;

1a. (i) 0, 1 and -7 are all integers and 0 is included
(ii) ½L = ½|-7| = 3.5 and -3, -2, -1, 0 , 1, 2, 3 comprise all of the integers between -3.5 and + 3.5

1b. 0 = 0 1 = 1 3 = 10	2 = 9 - 7 = 10a	-1 = 27 - 21 - 7 = 10aa -3 = -1(3) = 10aa0 -2 = -3 + 1 = 10aa1
2. 0 = 0 1 = 1 3 = 10 4 = 11 9 = 100 10 = 101 12 = 110	5 = 10(3) - 25 = 101a 11 = 12(3) - 25 = 110a 2 = 27 - 25 = 100a 6 = 3(2) = 100a0 7 = 6 + 1 = 100a1 -7 = 6(3) -25 = 100a0a -4 = -7 + 3 = 100a1a 8 = 2 (4) = 110aa -1 = 8(3) -25 = 110aaa	-3 = -1(3) = 110aaa0 -2 = -3 + 1 = 110aaa1 -6 = -2(3) = 110aaa10 -5 = -6 + 1 = 110aaa11 -9 = -3(3) = 110aaa00 -8 = -9 +1 = 110aaa01 -12 = -4(3) = 100a1a0 -11 = -12 + 1 = 100a1a -10 = 81 + 9 - 75 - 25 = 101aa

○○

Do you know of a friend who would appreciate a sample copy of our Bulletin? Just send us his or her name and address and we'll be happy to oblige.

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition--("Who needs a symbol for nothing?")--the new notation did come into popular use. Released from the drag of Roman notation, people's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance. Then, in a related development, people awoke to the fact that different number bases could be used.

A parallel to today seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, the most advantageous base should be used, the one best suited to the task at hand. (Similar to computer scientists use of binary, hexadecimal or octal - whichever is most convenient.) If this were done, duodecimals would progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited