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THE DUODECIMAL BULLETIN

7*;(94.)

HOW METRIC IS EUROPE?

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IN MEMORIAM: ANTHONY H SARNO

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Volume 3#
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*11*6 (1998.)*

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, non profit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

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THE DUODECIMAL BULLETIN

Whole Number 7; (Seven Dozen Dek)*

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*11*6;*

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IN THIS ISSUE

MINUTES OF THE ANNUAL MEETING

Saturday, October 16; 11*5 (1997)

Nassau Community College, Garden City, NY

FEATURED SPEAKERS

Attendance: Prof. Alice Berridge, Prof. Rafael Marino, Prof. Jay Schiffman, Prof. Gene Zirkel

The meeting opened at 10:30 AM.

1. The group watched the video "Numbers of the Future - Mr. L" featuring Bill Lauritzen. He described ancient number systems and explained that these systems and our current system are out-of-date He used spheres to show that 12 has high stability and high symmetry and demonstrated his dozenal number system which uses circular face values:

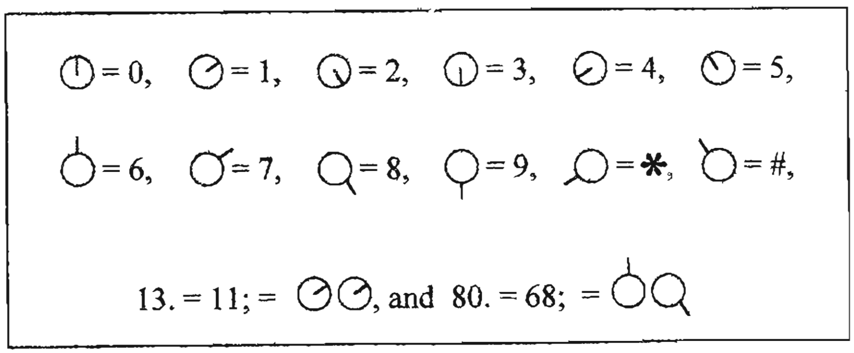

Figure 1: Lauritzen's Numerals

He said that he has lectured to 2600 children and feels that dozens as circular numbers are more visual and are easier to learn and to use than our current number system.. Members were very interested in Lauritzen's presentation.

Professor James Malone's video at last year's meeting was also enjoyable for members and it was suggested that members with other interesting ideas might be willing to videotape their own presentation for use at a future meeting and for the Dozenal Society archives.

2. President Jay Schiffman of Rowan College, Camden Campus presented "Another Set of Puzzles" for the attendees to solve. He guided us through a very interesting analysis of each puzzle and provided "A Multiplicative Duodecimal Magic Square" and "Two Geometric Magic Squares" for discussion. We were very absorbed by these provocative puzzles.

3. Professor Gene Zirkel of Nassau Community College discussed the different ways that he brings dozens into the classroom. In particular in courses which include number bases he regularly numbers his handouts, review sheets and tests in different bases and requires students to determine the number base used. He installs the Dozenal Clock in the classroom and waits for students curiosity to click in. After an explanation he asks: "Now what will the clock read when class is over?" In computer classes he makes programming assignments to change decimal to binary, decimal to duodecimal, etc. He has lectured effectively to elementary school teachers on percentages taught as per"gross"ages and has found the duodecimal history to be of interest to general audiences.

BOARD OF DIRECTORS MEETING

1. President Jay Schiffman convened the meeting at 1:00 PM. The following Board members were present: Alice Berridge, Rafael Marino, Jay Schiffman, Gene Zirkel
2. The minutes of the meeting of October 18; 11*4 (10/20/96) were approved as published in The Bulletin.
3. The Nominating Committee (A. Berridge, J. Schiffman, R. Marino) presented the following slate of officers which was elected unanimously:

Board Chair:	Rafael Marino
President:	Jay Schiffman
Vice President:	Gene Zirkel
Secretary:	Christina Scalise
Treasurer:	Alice Berridge

4. Appointments were made to the following DSA Committees:

Annual Meeting Committee: Rafael Marino and Gene Zirkel. Awards Committee: Gene Zirkel, Patricia Zirkel, Rafael Marino. Volunteers to these committees are welcome at any time.

5. The following appointments were made:
Editor of the Duodecimal Bulletin: Jay Schiffman.
Parliamentarian to the Board Chair: Dr. Patricia Zirkel.

6. Other Business of the Board:

The next Board Meeting will be held during the third weekend in October 11*6 - probably on October 15; (October 17, 1998). Details will appear in the next Bulletin.

The Board Meeting was adjourned at 1:30 PM.

ANNUAL MEMBERSHIP MEETING

1. President Jay Schiffman gavelled the meeting to order at 1:30 PM.

2. The minutes of the meeting of October 18; 11*4 (10/20/96) were approved as published in The Bulletin.

President's Appointment: Parliamentarian to the President: Dr. Patricia Zirkel

Treasurer's Report - Alice Berridge

Alice presented Income Statements for the years 11*5 and 11*4 for comparison. She reported that stock market increases have increased the Society's Net Worth. There are two new Life Members. Extra Life Members' contributions amounted to $192. Regular Members contributed $249.12.

Editor's Report - Jay Schiffman

Jay reported that articles by Ian Patten and Robert McGehee will be published and that he intends to contact Lauritizen. He said that he has a lot of good material. He has already incorporated a number of the ideas suggested by Court Owen in his recent package to the Board in the latest *Bulletin*, Whole Number 79;. All agreed that the new layout is very effective. The typesetter, Joan Firester, has made some striking changes to her work which will save the Society money. We are very grateful to Joan.

Annual Meeting Committee - Alice Berridge

The next Annual Meeting will take place during the third weekend in October, 11*5 following the meeting of the Board of Directors. Rafael and Gene will work together to plan this meeting. Rafael will contact a Manhattan college as a possible site.

Nominating Committee - Alice Berridge

The Committee presented the following slate for the Class of 11*8;(2000):
John Hansen, Jr., Rafael Marino, Jay Schiffman, and Timothy Travis

The slate was elected unanimously. Alice Berridge, Jay Schiffman and Rafael Marino were proposed as the Nominating Committee for the coming year. They were elected unanimously.

Other Business:

Gene announced that Arthur Whillock said the DSGB will be publishing another issue of *The Dozenal Journal* which we are looking forward to reading. Whillock sent an article "Imperial v Metric" for members to read. Gene drew our attention to a recent correspondence from Dr. Sebastian Frobenius of Japan. The group considered the packet of materials from Court Owen and Gene and Jay mentioned that a number of his suggestions had already been put to use. Underwood Dudley has suggested that we submit articles on Dozenal history and other topics for possible publication in *The College Mathematics Journal.* It was suggested that Jay's puzzles might also be submitted there or to another journal so as to generate interest and spread the word about dozens. Raphael said that he has received inquiries about the Society on his e-mail address as well as greetings from John Hansen. Gene said that the new DSA office at Nassau Community College is in disarray and that the boxes and materials are not yet moved into the office. Members considered a packet of materials sent by Bob McPherson which they found very interesting.

The meeting was adjourned at 1:50 PM.

Respectfully submitted,

Alice Berridge ☐

In Memoriam - Professor Anthony H. Sarno
RIP "Mister St. John's"

Gene Zirkel

Professor Emeritus Anthony H. Sarno, Member number 26#, passed away in February at the age of 86. He had served as a Major in the Marine Corps during WW II, and as a Commander of a Veterans of Foreign Wars Post which he had co-founded. He was an advisor to the NY State Veteran's Affairs Commission for Governor Mario Cuomo, his former student.

Tony, long-time Chair of the Math Department at St. John's University, also served as Dean of Men and as Assistant to the President. He initiated the intramural program at St. John's and served as Director of Intramural Sports.

In the early 1950's, Tony instilled a love for mathematics into our current Vice President. In a course in higher algebra, which included number bases, he mentioned to his class the existence of the DSA, and that led to Gene's interest in duodecimals. Soon afterward, Gene submitted an article entitled "I'm a Dozener" to the math club. It was published in their *Mathazine*, and subsequently our editor, Ralph Beard, reprinted it in this *Bulletin*. (*Editor's Note*: In memory of Tony, we reprint Gene's article in this issue. Please see P.8.)

In addition, President Jay Schiffman was also a student at St. John's and knew Tony well.

We are indebted to Tony, and we'll miss him, as he joins the *Twelve* Apostles. □

Errata

In Bulletin 79; (93.), on Page 13; (15.), the entry in Table 4, Part B should read 5432 = **2**x7x471, not 5432 = x7x471 as is printed. We apologize for this error and appreciate our readers bringing any errors they see to our attention. □

HOW METRIC IS EUROPE?

John Gardner

The following article appeared in The Yardstick, *Journal of the British Weights and Measures Association, Number 3, November 1996 on Pages 2-3 and is reprinted with permission.*

Although most of Europe has been metric officially for over a century, British visitors to the Continent are often surprised to discover an ongoing use of non-metric measures, much as in the same way Americans visiting Britain are surprised to learn that the UK uses miles for road signs instead of kilometres. So, while the Department of Trade and Industry (DTI) has convinced itself that adopting various metric units is necessary to bring Britain into line with Europe, its programme of compulsory metrication is effectively pushing Britain *out* of line with Europe.

The metric system was invented in the aftermath of the French Revolution and was intended to be as unlike previous weights and measures as possible. When the French government imposed metrication in 1837, the outlawing of customary units such as the *livre* (pound) was widely resented. Instead of abandoning the livre, French people adjusted it from its pre-revolution weight of (what was to become) 490 grams (17.3 ounces) to its modern weight of 500 grams (17.6 ounces). This adjustment made the livre compatible with the metric system by providing an easy conversion between kilograms and livres: two livres to the kilo. Modern French shoppers do not say "1.5 kilos" or "1,500 grams" but ask for "three livres." The standard traditional French loaf of bread, whilst weighing one kilogram, is referred to as "pain de deux livres." For smaller quantities of food such as butter, French traders and shoppers divide the livre into a *demi-livre* and a *quart de livre*, traditional divisions based on a half and a quarter that cut across the livre's internal metric division of 500 grams. Thus, a French quarter pound (0.125 kilo) is 4.4 ounces and close to the UK/US quarter pound.

The survival of avoirdupois is seen in numerous other European countries: the Danes buy using the *pund*, the Swiss the *livre* and *pfund*, the Germans the *pfund* and so on. As in France, it is acceptable in Holland and Switzerland to refer to quarter and half pounds. The reason for the pound's ongoing use in Europe is not merely tradition but that the pound is a more appropriate weight for food markets than the kilogram. A pound is only half the weight of a kilo and so allows a wider choice of weights to be dealt with as whole numbers and simple fractions. For items such as fruit, vegetables, meat, cheese and butter, the pound is the right *measure*.

Pounds are not restricted to use in food markets. In France, the livre is used by wholesalers. In Germany, people may give their own weight in pfunds, or use pfunds to describe the weight of a new-born baby. The use of pounds on the Continent reveals the lack of knowledge by Britain's metric lobby on how metrication is actually applied in Europe. Historically, pound weights varied across Europe: the German pfund was the equivalent to 16.5oz/468 grams, the Dutch *pond* 17.4oz/494 grams, the Danish pund 16.6oz/471 grams and so on. France had several versions of the livre in different cities and regions. The effect of metrication has not been to abolish the pound but to standardise it across frontiers.

The use of pounds across Europe has ensured the survival of the hundredweight. In Germany and Switzerland, the *zentner*, in like manner to the British cwt, is one-twentieth of a tonne and is used by industry, commerce and agriculture for weighing machinery, wood, farm produce and coal. The zentner is equal to 100 pfunds or 50 kilos, weights within 2% of the British hundredweight and has been adopted instead of the two nearest metric alternatives,the myriagram (a hundredth of a tonne) and the quintal (a tenth of a tonne). In Germany, the metric tonne has thus been cut up to recreate customary measures and illustrates a departure from the metric principle of units building up in multiples of 10, 100, 1000: thus, 500 grams = 1 pfund; 2 pfunds = 1 kilogram; 50 kilograms = 1 zentner; 20 zentners = 1 tonne. No German unit of weight is ten times the previous unit. Those metric units needed to complete the decimal structure - the dekagram, hectogram, myriagram and quintal are not used in Germany.

The practice of reducing metric weights to customary sizes extends to volumes of liquid. In Italy, for instance, it is common to refer to "quarter-litres." In the Flemish-speaking part of Belgium, a quarter-litre makes a *pintje* (small pint), again defying the metric rule that units are divided throughout only by ten. Having converted to the metric system, Europe is returning to the very use of fractions which the metric system was supposed to eliminate, and giving these fractions new identities based on traditional human-scale systems.

The inch is another customary unit that is retained in Europe. In Belgium, DIY shops describe tools in inches; thus, the teeth of a saw is described as the "number of points to the inch." In Swedish tool shops, equipment is described in terms of *tums*, and in Norway, *tommes*, alongside metric. Swedes also

describe the sizes of electronic wafer boards in tums. Inches are often used on the continent for plumbing. Whereas British plumbers converted to metric some years ago, necessitating the use of adaptors where metric pipes do not fit existing installations, in Belgium all plumbing equipment, whether fittings or pipes, continues to be supplied in customary measures. Inches are used in Italy to describe plumbing pipe threads, in Iceland for radiator circuits, and in Germany the inch (*zoll*) is used to describe diameters of pipes, taps and washers (for instance, 1/4" or 3/4"). Germans also use inches for producing firearms, as illustrated by .202 and .303 rifles. Most, if not all, European publishers use the inch for measuring type size and ink density because global computer and printer technology is now inch-based.

Customary units sometimes exist in name but not in practice, as in the Netherlands where the "ounce" referred to by shoppers is in fact a hectogram (100 grams). Conversely, customary units exist in practice but not in name, as with the 300-millimetre "unit" used on the Continent for design work (the foot in disguise).

Shipping and aviation are two areas where customary units are used frequently. In aviation, feet are used by European aircraft for measuring altitude. In shipping, the nautical mile and the knot (*mille marin* and *noeud* in French) are used for distance and speed. France and Spain also use nautical leagues. Sweden uses feet (*fots*) for the construction and description of boats. For land distance, road signs in Sweden are specified in kilometres but, when speaking, Swedes prefer *mils*, a traditional measure some ten times the length of a kilometre. A Swede glancing at a metric road sign stating "150 kilometres" will say "15 mils."

Non-metric units for measuring land are common. Sweden uses the *tunnland* which is the equivalent of 1.2 British acres and Austria has the *joch* (1.4 acres). Denmark and Belgium use their own variations on the acre and France uses the *perche*. For fields and farmland, Italy uses the *acro* (equal to 4046 square metres), and Germany the morgen (0.6 acre). These units are alongside the metric hectare.

Human-based measures exist in Europe for approximations. In parts of Spain, people use the *palmo* as a rough measure of distances that can be covered by the span of a hand, such as the gap between two cars when parking, or the length of a small table. In Italy, the *spanna* is used in the same way. Italians also use the *dito* (finger-width; plural: *dita*) to give an impression of the length of nails and screws, or the depth of a hole in a wall when drilling. In France, horses are measured in standardised hands or *paumes*.

In Britain's quest to imitate Europe, metric measures have been adopted for uses which do not apply in Europe. Spirits, for example, are now served in Britain using optics of 25 and 35 mililitres, yet no Continental country uses optics. Spirit measures in Europe are necessarily approximate because they are poured freehand from the bottle. In Italy, a customer requesting a spirit might ask for two *dita*. Nor do most European countries use metric units for the serving of draft beer. Beer, unless bottled, is sold simply by the glass, and the size of the glass can vary from outlet to outlet. Drink servings on the Continent are seldom standardised as in Britain and Ireland. It is therefore a paradox of European "standardisation" that Europe's only standardised spirit measure, the gill, has been banned outright, and one of the very few routinely used standard measures of draft alcohol, the pint, has been restricted in its use.

Having enforced metric measures where they do not apply in Europe, the British government has failed to implement metrication in several areas where it does apply. The government has exempted two areas where the same set of metric units applies consistently throughout the Continent: kilometres and metres for road-signs and centimetres for clothing sizes. In some cases, the British government has adopted different metric units to those in use on the Continent. In the year 2000, Britain will use the gram for food markets whereas other countries such as Italy, Sweden and the Netherlands use the hectogram (100 grams). The millimetre has been accepted for a vast range of uses in Britain that would be unheard of in other countries due to its tiny size. Centimetres are much more common in Germany.

While the DTI has decreed compulsory metrication in Britain on the grounds that metric units are adopted all over the world for comparison and compatibility, a vast range of manufactured goods in Europe are based on non-metric specifications for the same reason. Computer disks are produced in Europe based on the 3 1/2-inch standard and are sold as such, and the number of tracks on a computer disc are expressed as 48 or 96 tracks per inch (tpi). Video tape is universally half an inch wide; tape used within a sound cassette is an eighth of an inch wide. Car tyre diameters are specified in inches throughout Europe as are most sizes of bicycle tyre and wheel diameters. Shops in Europe frequently express the width of television screens and electric fans in inches (*pollici* in Italy, *pulgadas* in Spain, *pouces* in France). European manufacturers willingly produce goods designed to imperial specifications specifically for export to non-metric markets. Examples of this are Belgian carpets and rugs, and French glasses in pint and half-pint sizes.

Although Britain's supporters of metrication regard Europe as a decimal role-model, there is a wide range of other systems on the Continent which are based on customary divisions of twelve and sixteen. Clocks and calendars, with twelve hours on a clock face and twelve months in a year, have yet to adopt a recommendation of 1875 that they be decimalised. There are still (except to some extent in France) 360 degrees in an angle and each degree is divided into 60 minutes and 60 seconds. The dozen (*dutzend* in German, *douzaine* in French) is widely used by European industry as a quantity for packaging. Pure gold is specified as 24-carat. Compasses all round the world are marked off with sixteen points and music remains binary; one semi-breve consists of two minims, four crotchets, eight quavers and sixteen semi-quavers (not to mention twelve notes in an octave). Similarly, European paper sizes adopt non-decimal divisions whereby one sheet of A1 equals two sheets of A2, four of A3, eight of A4 and sixteen of A5. It is contradictory that metricators regard sixteen ounces in a pound as random and archaic while accepting the self evidence of sixteen sheets of A5 in a sheet of A1. Even the Euro '96 football finals started with sixteen teams, divided into four groups of four to enable the progression of semi-finals and quarter-finals.

The metric system is firmly established and widely used in Europe, but not in the form imagined by its supporters in Britain. Many metric units have long since been obsolete. Other metric units have been cut up to reproduce measures based on customary sizes and some trades never converted to metric in the first place. Customary units exist in general conversation and for convenient approximations. Many products made in Europe are based on pound/foot specifications. Europe uses the metric system but is not obsessed by it; only in Britain has metrication become a measure of whether one is "European."

Thanks to: *J Ödemark, Natacha Tual, Justin Brooke, Monica Marin, Joan Pontius, Maria Tsatazoni, Jonathan Rogers.* ☐

Each One Teach One - Ralph Beard, founding editor of this Bulletin

I'M A DOZENER

Gene Zirkel

(*Editor's Note:* This article originally appeared in Volume 9, Number 1 of *The Bulletin* in April 1169; (1953) and is a tribute to the late Tony Sarno, Gene's Professor who introduced him to duodecimals during his sophomore year in college.)

$5 \times 4 = 18$ $\quad 14 \div 2 = 8$ $\quad 8 + 7 = 13$ $\quad 169 - 92 = 97$

$7^2 = 41$ $\quad 69^{1/2} = \pm 9$ $\quad 3^3 = 23$ $\quad 54^{1/3} = 4$

What grade would you give to a student who turned in a paper with the above problems in arithmetic? Zero? I'd give him a perfect mark. All his calculations are correct, it's just that he's working with twelve symbols instead of our ordinary ten. He's counting in the duodecimal system, a number system that counts by dozens rather than by tens. His numbers proceed as follows:

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

10	12	13	
do-one	do-two	do-three	... etc.

If you now count off four groups of units with five units in each group you will see that five multiplied by four equals do-eight (see the first problem above). All the other problems above can also be verified in this way. In fact this is the way your multiplication tables were originally constructed. However, there is a simpler way to check these problems (or do some others).

Any series of digits merely means a sum of a power series where the digits are the coefficients of a power of the base of the number system. e.g.

A) 123 in a system of 5 symbols $1 \cdot 5^2 + 2 \cdot 5^1 + 3 \cdot 5^0$ or decimal 38

B) 123 in a system of 10 symbols $1 \cdot 10^2 + 2 \cdot 10^1 + 3 \cdot 10^0$ or decimal 123

C) 123 in a system of 12 symbols $1 \cdot 12^2 + 2 \cdot 12^1 + 3 \cdot 12^0$ or decimal 171

Thus the problem above, 169 - 92, becomes:

$$
\begin{array}{l}
1 \cdot 12^2 + 6 \cdot 12 + 9 \\
\underline{\qquad - 9 \cdot 12 - 2}
\end{array}
$$

$1 \cdot 12^2 - 3 \cdot 12 + 7 = 12(12 - 3) + 7 = 9 \cdot 12 + 7 = 97$ in the scale of twelve.

We have now seen how to change a number from the scale of twelve to the scale of ten (C above). In the reverse process we can change any number in the scale ten to the scale twelve by dividing that number by twelve, the remainders being the new digits. Thus 437 in the ten scale is changed to the scale as follows:

$$
\begin{array}{l}
12\underline{)437} \\
\quad\underline{)36} + 5 \\
\qquad 3 + 0
\end{array}
$$

hence 437 in the scale of ten is 305 in the scale of twelve.

What does all this amount to? What is the practical value of a new number system? Why should we change when our system of ten symbols is apparently just as good?

The answer to these questions lies in the word *apparently.* Have you ever studied any other system or for that matter even your own? You may have noticed that you have ten fingers and ten symbols in your counting system. This is no coincidence. The first counting was done on fingers and when people ran out of fingers they started over again, saying one ten fingers and one etc. until they got to two ten fingers. Someone started a symbolism of vertical lines so that we had /, //, ///, ////, /////, but this became too unwieldy and so a symbol for five was invented, namely V. Twice five became two V's one inverted under the other as

$$\overset{\mathrm{V}}{\Lambda} \quad \text{or X.}$$

Thus the system of Roman Numerals came into existence. Following this we had the invention of individual symbols 1, 2, 3, 4, 5, 6, 7, 8, 9 and so our counting became

no ten fingers & none	1 ten fingers & none	2 ten fingers & none
no ten fingers & 1	1 ten fingers & 1	2 ten fingers & 1
no ten fingers & 2	1 ten fingers & 2	2 ten fingers & 2
no ten fingers & 3	1 ten fingers & 3	2 ten fingers & 3
⋮	⋮	⋮

which soon became

none	1 & none	2 & none
1	11	21
2	12	22
3	13	23
⋮	⋮	⋮

This was a convenient symbolism for all values save one in each ten which led to the development of the zero.

Thus our number system today is a combination of nine digits and a zero for place. The symbols and zero were derived by necessity but the base ten was purely accidental and most inconvenient. Most of our measures which were derived for practicality, use twelve as a base. Thus we have twelve inches in a foot, twelve months in a year, twelve objects in a dozen, and twelve ounces in a pound (Troy), just to name a very few. But the base of our system of counting which was not derived by practical use is ten. Why did grocers (the word comes from the same root as gross) sell things in dozens and why did carpenters put twelve divisions in a foot? Simply to facilitate the use of the common fractions $^1/_2$, $^1/_3$, and $^1/_4$. *So by experience it was learned it was easier to count by twelve's.*

Let's look at the advantages of the duodecimal system.

1. In the duodecimal system we count 143 units in only two digits, 44 more than in the decimal system; and in general all numbers have less digits in the duodecimal system.

2. The multiplication table is easier to learn in the new system with more repetition than in the decimal system. The table has only one three-digit number in the duodecimal system but eleven three-digit numbers in the decimal system.

3. The base of the duodecimal system has twice as many factors as the base of the decimal system. That is $^1/_2$, $^1/_3$, $^1/_4$, and $^1/_6$ of twelve are all whole numbers while only $^1/_2$ and $^1/_5$ of ten are whole numbers.

4. Corresponding to the decimal point we have a *more* convenient duodecimal point which gives an exact value for $^1/_3$ and $^1/_9$ which were repeating decimals in the former system. It also simplifies $^1/_4$ from 0.25_{ten} to 0.3_{twelve} and $^1/_8$ from 0.125_{ten} to 0.16_{twelve}.

5. Many practical problems are simplified, e.g.

A) Find the area of a rectangle 4'3" long and 6'7" wide.

decimal	duodecimal
4 • 12 + 3 = 51" 6 • 12 + 7 = 79"	4.3' • 6.7' = 23.#9 ft^2 or 23 ft^2 9 in^2
51" • 79" = 4029 in^2 4029 ÷ 144 = 27 ft^2 141 in^2	
4 Steps	1 Step

B) Add the following:

3 years + 10 months
2 years + 5 months
6 years + 9 months
5 years + 8 months

decimal	duodecimal
3 years + 10 months	3.* years
2 years + 5 months	2.5 years
6 years + 9 months	6.9 years
5 years + 8 months	5.8 years
16 years + 32 months	16.8 years or 16 years & 8 months
16 years + 2 years + 8 months	
18 years + 8 months	
3 Steps	1 Step

In conclusion then the duodecimal system is less complex in both learning and application. It has many advantages and only one so-called disadvantage, namely it is a change and many people don't want to change. But then the current cumbersome denary system was opposed by narrow minded people who used the Roman Numerals and who were too lazy to improve themselves. The better system eventually won out and SO WILL DUODECIMALS. □

THREE RECURSIVE SEQUENCES IN THREE DIFFERENT NUMBER BASES

Jay Schiffman
Rowan University, Camden Campus

Recursive sequences play an essential role in mathematics and computer science. Two famous sequences in the study include the Fibonacci and Lucas sequences. The Fibonacci sequence plays a role in such diverse disciplines as architecture, art, poetry, music, botany, psychology, and education in addition to the mathematical sciences. A number of unique patterns, relationships, and identities are in no small measure the reason for the Fibonacci sequence's appeal. In what follows, I am pleased to present the first three dozen elements of the Fibonacci, Lucas, and Tribonacci Sequences in the Decimal, Duodecimal and Hexadecimal number bases. These sequences are recursive in the sense that we assign values to the initial two terms of the Fibonacci and Lucas sequences and the first three terms of the Tribonacci sequence. A rule is then given to generate all succeeding elements of the sequence. For the Fibonacci sequence $\{F_n\}$, define $F_1 = F_2 = 1$ and $F_n = F_{n-2} + F_{n-1}$ for $n \geq 3$. For the Lucas sequence $\{L_n\}$, define $L_1 = 1$, $L_2 = 3$, and $L_n = L_{n-2} + L_{n-1}$ for $n \geq 3$. Finally for the Tribonacci Sequence $\{TRI_n\}$, define $TRI_1 = TRI_2 = TRI_3 = 1$, and $TRI_n = TRI_{n-3} + TRI_{n-2} + TRI_{n-1}$ for $n \geq 4$.

We begin with the Fibonacci Sequence, $\{F_n\}$ found in Table 1 below. Our next goal is to form the four-column table for the Lucas Sequence, $\{L_n\}$ found in Table 2. In Table 3 we provide the four-column chart dealing with the Tribonacci Sequence, $\{TRI_n\}$.

We conclude by noting that the terminal digits of the Lucas Sequence in base dek form a cyclical period of length one dozen; namely 1 3 4 7 1 8 9 7 6 3 9 2 1 3 4 7 1 8 9 7 6 3 9 2 1 .... By virtue of this, no Lucas number is divisible by either 5 or ∗. The reader is invited to detect other interesting patterns associated with these sequences. □

Table 1. The Fibonacci Sequence, $\{F_n\}$

Number	Decimal	Duodecimal	Hexadecimal
1	1	1	1
2	1	1	1
3	2	2	2
4	3	3	3
5	5	5	5
6	8	8	8
7	13	11	D
8	21	19	15
9	34	2∗	22
∗;	55	47	37
#;	89	75	59
10;	144	100	90
11;	233	175	E9
12;	377	275	179
13;	610	42∗	262
14;	987	6∗3	3DB
15;	1597	#11	63D
16;	2584	15#4	A18
17;	4181	2505	1055
18;	6765	3∗#9	1A6D
19;	10946	6402	2AC2
1∗;	17711	∗2##	452F
1#;	28657	14701	6FF1
20;	46368	22∗00	B520

21;	75025	37501	12511
22;	121393	5*301	1DA31
23;	196418	95802	2FF42
24;	317811	133#03	4D973
25;	514229	209705	7D8B5
26;	832040	341608	CB228
27;	1346269	54#111	148ADD
28;	2178309	890719	213D05
29;	3524578	121#82*	35C7E2
2*;	5702887	1*#0347	5704E7
2#;	9227465	310##75	8CCCC9
30;	14930352	5000300	16BDB0

Table 2. The Lucas Sequence, $\{L_n\}$

Number	Decimal	Duodecimal	Hexadecimal
1	1	1	1
2	3	3	3
3	4	4	4
4	7	7	7
5	11	#	B
6	18	16	12
7	29	25	1D
8	47	3#	2F
9	76	64	4C

*;	123	*3	7B
#;	199	147	C7
10;	322	22*	142
11;	521	375	209
12;	843	5*3	34B
13;	1364	958	554
14;	2207	133#	89F
15;	3571	2097	DF3
16;	5778	3416	1692
17;	9349	54#1	2485
18;	15127	8907	3B17
19;	24476	121#8	5F9C
1*;	39603	1*#03	9AB3
1#;	64079	310##	FA4F
20;	103682	50002	19502
21;	167761	81101	28F51
22;	271443	111103	42453
23;	439204	192204	6B3A4
24;	710647	2*3307	AD7F7
25;	1149851	47550#	118B9B
26;	1860498	758816	1C6392
27;	3010349	1012125	2DEF2D

28;	4870847	176*93#	4A52BF
29;	7881196	2780*64	7841EC
2*;	12752043	432#7*3	C294AB
2#;	20633239	6*#0647	13AD697
30;	33385282	#22022*	1FD6B42

Table 3. The Tribonacci Sequence, $\{TRI_n\}$

Number	Decimal	Duodecimal	Hexadecimal
1	1	1	1
2	1	1	1
3	1	1	1
4	3	3	3
5	5	5	5
6	9	9	9
7	17	15	11
8	31	27	1F
9	57	49	39
*;	105	89	69
#;	193	141	C1
10;	355	257	163
11;	653	465	28D
12;	1201	841	4B1
13;	2209	1341	8A1
14;	4063	2427	FDF
15;	7473	43*9	1D31
16;	13745	7#55	35B1
17;	25281	12769	62C1
18;	46499	22**#	B5A3
19;	85525	415#1	14E15
1*;	157305	77049	26679
1#;	289329	11#529	46A31
20;	532159	217#67	81EBF
21;	978793	3#2521	EEF69
22;	1800281	7299#5	1B7859
23;	3311233	1138281	328681
24;	6090307	2058597	5CEE43
25;	11201821	3902651	AAED1D
26;	20603361	6*972*9	13A61E1
27;	37895489	10836315	2423D41
28;	69700671	1#414053	4278C3F
29;	128199521	36#25655	7A42B61
2*;	235795681	66#73*01	E0DF4E1
2#;	433695873	1012#14*9	19D9AC81
30;	797691075	1*318*943	2F8BCCC3

FROM THE EDITOR

Jay Schiffman

Due to a rather hectic academic year, I was unable to keep my usual schedule of having the first issue of The Bulletin for 11*6 (1998) in time for the spring. With the conclusion of my academic year, I finally completed the editing of the first issue which is now in your hands. The second issue is being worked on as you read this and should be received by our members in early September. This will enable you to plan for our Annual Meeting in October. The meeting will take place on Saturday, 15 October 11*6 at half past dek in the morning (17) October 1998 at 10:30 AM) on Long Island. For further information, please call (516) 669-0273. Thank you for your patience and please continue to spread the word- **Twelve Is Best!** ☐

JOTTINGS

From Members and Friends

Bill Holdorf, member number 359, has a contribution in the March issue of *The Yardstick*, the excellent and interesting Journal of the British Weights and Measures Association. To subscribe write to BWMA/45 Montgomery Street, Edinburgh EH75JX/Great Britain [http://users.aol.com/footrule]. We recommend it highly & it is only 10£ a year.

Vice President Gene Zirkel spoke about the advantages of dozenals to two groups of students in grades 5 thru 8 from five schools. The talk entitled "Don't Cut Off Your Toes" was part of the *Student Symposium in Mathematics*, sponsored each year by the Nassau County Association of Mathematics Supervisors and the Nassau County Mathematics Teachers Association. While at the symposium, Gene spoke with another volunteer speaker, Marianne Goudreau of the Goudreau Math Museum. She was amenable to obtaining some of our literature for students who visit the museum.

President Jay Schiffman presented a paper entitled "The Fibonacci and Lucas Sequences - A Comparative Study" at **The Metropolitan New York Section of The Mathematical Association of America** on Sunday, May 3, 1998.

Charles F. Marschner writes to us from Florida and thanks us for our contributions and advises us to keep up the good work! As an aside he contributes the following:

Have your foreign travelers noticed that all foreign countries package most bottled and canned goods in twelves? A 4x3 arrangement being more efficient & stackable than a 5x2! Who needs a 5 pack?

Also clocks are still in twelves - even in France & circles have 260; (360) degrees (30 x 12).

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The husband of our faithful layout artist needs surgery. At the last minute a volunteer tried his best to fill in for Joan Firester. This emergency added to the tardiness of this issue, and accounts for any errors or design flaws. We pray for Joan's husband and wish him a quick recovery. We are grateful for our volunteer who wishes to remain nameless. ☐

I recently taught a student whose surname is 'Dekel', or as we might say, 'Teneleven'.

-GZ

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition--("Who needs a symbol for nothing?")--the new notation did come into popular use. Released from the drag of Roman notation, people's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, people awoke to the fact that different number bases could be used.

A parallel to today seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, the most advantageous base should be used, the one best suited to the task at hand. (Similar to computer scientists use of binary, hexadecimal or octal - whichever is most convenient.) If this were done, duodecimals would progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited. □

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The only requirement is a constructive interest in duodecimals

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