DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

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THE DUODECIMAL BULLETIN 76;

IN MEMORIAM

CHARLES BAGLEY

(See page 8;)

Volume 39;
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1996
11*4.

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00 (US) for one calendar year. Student Membership is $3.00 per year, and a Life Membership is $144.00 (US).

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THE DUODECIMAL BULLETIN

Whole Number Seven Dozen Six

Volume 39; Number 1;

11✻4;

IN THIS ISSUE **FOUNDED 1944**

DOZENAL SOCIETY OF AMERICA
MINUTES OF THE ANNUAL MEETING 11*3 (1995)

*Saturday, October 12; 11*3 (1995)*
Rowan College of New Jersey
Glassboro, NJ 08028-1701

I. BOARD OF DIRECTORS MEETING

1. Dr. Patricia Zirkel, Board Chair, convened the meeting at 11:15 AM. The following Board members were present:

Alice Berridge	Jay Schiffman
Jamison Handy Jr.	Gene Zirkel
Rafael Marino	Patricia Zirkel

Members Vera Handy and Edmund Berridge were also in attendance.

2. The minutes of the meeting of October 13; 11*2 were approved as published in the Bulletin.

3. The Nominating Committee (A. Berridge, J. Schiffman, R. Marino) presented the following slate of officers:

Board Chair:	Dr. Patricia Zirkel
President:	Jay Schiffman
Vice President:	Rafael Marino
Secretary:	Alice Berridge
Treasurer:	Alice Berridge

The slate was elected unanimously.

4. Appointments were made to the following DSA Committees:

Annual Meeting Committee:	Alice Berridge
Awards Committee:	Gene Zirkel, Chair
	Patricia Zirkel
	Rafael Marino

Volunteers to these committees are welcome at any time.

5. The following appointments were made:

Editor of The *Duodecimal Bulletin:* Jay Schiffman
Parliamentarian to the Board Chair: Jamison Handy Jr.

(Continued)

6. Other Business of the Board:

Gene mentioned that there have been about eight dozen items of correspondence this year. He said that he routinely asks students who request information from the Society to submit to us a copy of their report. There are two student reports which we may publish in the *Bulletin.* Members discussed the origins of the Society and reminisced about the "old days". Gene has on hand George S. Terry's volume *Duodecimal Arithmetic* which Terry had published in 1156;. Members enjoyed examining this work.

The next Board Meeting will be on October 17; 11*4;. We expect the site to be Hofstra University in Hempstead, LI, NY.

The Board Meeting was adjourned at 11:35 AM.

II. ANNUAL MEMBERSHIP MEETING

1. DSA President Jay Schiffman gavelled the meeting to order at 11:35 AM. Jay welcomed us to the main campus of Rowan College and mentioned that this was the first time the Society had met outside of New York State since 1186;.

2. Gene Zirkel moved to accept the minutes of the meeting of October 13; 11*2;. So approved.

3. President's Report - Jay Schiffman

Jay said he has been pleased with this year's progress of the Society. He expressed disappointment by the lack of New Jersey attendees at this meeting; the late issuance of the *Bulletin* may have been the reason.

Members discussed the feasibility of writing text materials for the younger grades as Vera Handy suggested.

President's Appointment:

Parliamentarian to the President: Dr. Patricia Zirkel

4. Treasurer's Report - Alice Berridge

Alice reported that there are 25; Life Members and that four of those are new. Six Life Members made extra contributions to the Society. There are five new Regular Members and seven Student Members, two of whom are new. Extra contributions (17; Regular Members and one student member) made a big difference this year to our financial status. It was agreed to send the dues solicitation letter earlier than usual in the hope of prompting our long-time supporters to re-institute their membership. For this year the letter will be sent

(Continued)

ASAP; in the future the letter will be sent in September. The extra contributions and the current improved status of the stock market increased the Society's net worth to $18,956.04.

Members extended praise and thanks to Alice for her work as treasurer.

5. **Editor's Report** - Jay Schiffman

Jay reported that he is pleased with the variety of articles, especially the ones from students. He finds the management of new technology challenging.

Members expressed praise and thanks to Jay for his fine work.

6. **Annual Meeting Committee** - Alice Berridge

The next annual meeting will take place on Saturday, October 17; 11*4 (1996), following the meeting of The Board of Directors.

7. **Awards Committee** - Gene Zirkel

The Ralph Beard Memorial Award was presented to Arthur F. Whillock by Board Chair Dr. Patricia Zirkel in Wallington, England on July 16; 11*3; in recognition of "his outstanding service and devotion as an advocate of Dozenal Counting and Metrics and for his years of service to The Dozenal Journal."

8. **Nominating Committee** - Alice Berridge

The Committee presented the following slate for the Class of 11*6;:

Alice Berridge (Secretary-Treasurer)
Dr. John Impagliazzo
Robert R. McPherson
Gene Zirkel

The slate was elected unanimously.

The names of Alice Berridge, Jay Schiffman, & Rafael Marino were presented as our Nominating Committee for the coming year. They were elected unanimously.

9. **Other Business**

Gene reported that Bob McPherson sent a comic strip which he adapted to promote use of dozens. Members commented about the fine job done by

(Continued)

typesetter Joan Firester for the *Bulletin*, especially with charts and tables. A letter of commendation will be sent to her.

Gene reported that at the prompting of Arthur Whillock he has approached the Art Department of Nassau Community College soliciting student design for dek and el.

The National Council of Teachers of Mathematics (NCTM) has recently published an eight-page bulletin on numeration with half of the pages devoted to metrics. Gene has written a rebuttal. Gene has been submitting input to the NCTM daily puzzles feature using the occasion to advocate dozens.

Gene displayed a dozenal tee shirt which he purchased at a crafts fair. A photo of Gene wearing the shirt will be printed in the Bulletin.

On July 16; 11*3;, National Public Radio (NPR) broadcast an interview with Gene and Robert Siegel. Gene is writing to gain approval for us to print a transcript. This radio talk was prompted by a lengthy article in The Washington Post and other newspapers. The article was picked up by the wire services and was reprinted in Iowa and Boston to name a few locales. Readers who have noticed it elsewhere are invited to inform us.

The US Post Office has issued a panda stamp. There is a red panda at the Bronx Zoo in New York City.

Jamison reported about the status of multiple telephone lines. He believes that a switch to dozenals would be expeditious.

The meeting was adjourned at 12:30 PM.

III. FEATURED SPEAKERS

1. Professor Gene Zirkel of Nassau Community College presented a talk on how to convert fractions from the decimal base to the duodecimal base and vice versa employing calculator technology with the degrees-minutes-seconds (DMS) key on the TI-85 graphics calculator.

2. President Jay Schiffman of Rowan College, Camden Campus spoke on the topic "Hexadecimally Speaking" which focused on Hexadecimal number theory including a calculator component employing the BASE key on the TI-85 graphics calculator.

Respectfully submitted,

Alice Berridge

□

IN MEMORIAM: CHARLES S. BAGLEY

Gene Zirkel

Miriam Bagley, former member of our Board of Directors and member number 243 informs us of the death of her husband Charles, member number *3. Charlie, a Bishop of the Latter Day Saints, was a member of our Board of Directors for two dozen years from 1959 to 1983. He served as President from 1961 to 1972 and then as Board Chair from 1972 to 1983 when we reluctantly accepted his resignation. Like many others, Charlie was an advocate of dozenal counting and measuring before he discovered our Society. When he joined us in 1957 he had already authored a very complete system of duodecimals in a paper titled "Redivivus Reckoning". This was printed in this *Bulletin* in 1958, and has been referred to by many other authors in footnotes over the years. Charlie wrote several other articles, one of which, "Duodecimal Society Urges Use of Base Twelve in Science" also appeared in the *Monthly New Bulletin of the Holloman Section, American Rocket Society.*

A Geodesist with the Air Force Missile Development Center at the Holloman-White Sands Range before he retired, Charlie received the Annual Award of the DSA in 1982. The words said of him then remained true until he died.

> Those who work for the cause of spreading dozenal arithmetic and measurement owe Charlie a great debt of gratitude for his efforts on their behalf and for his leadership over the years. ... It is fitting that we thank him in this small way for his dedication and service to the Society.

Charlie never forgot the Society. He was generous with his time and his money. He appeared on the radio, wrote for this *Bulletin,* and encouraged the rest of us. In 1990 he wrote

> It pleases me to know that qualified, worthy people are still carrying on the traditions of the Dozenal Society. I will never regret my association with it and the companionship of men like Terry, Beard, Humphrey, Andrews, Linton, Churchman, Handy and others ...

Life Member Charlie was eight dozen and nine years young and his bride of five dozen and six years, also a Life Member, is one year younger.

I first met Charlie when he was installed as President at our Annual Meeting in 1961. His opening words were said with a copy of F. Emerson Andrew's book, *New Numbers* in his hands. The publisher had placed a paper cover around it which contained an advertisement for another book. Charlie proceeded to read the ad which said, in part, "Special emphasis is placed on weights and measures (including the metric)". He then dropped the book onto the floor. The resounding noise startled us, and so Charlie had our complete attention for In the rest of his remarks about the dozenal metrics he advocated and the directions in which he hoped to lead us. I never forgot that moment. I shall never forget the leadership Charlie inspired in every member of the Society.

(Continued)

CALCULATOR BASE CONVERSIONS

USING ANGLE CONVERSION FUNCTIONS TO CONVERT TO (AND FROM) DOZENAL FRACTIONS

Gene Zirkel

Some calculators provide access to binary, octal and hexadecimal base conversion. This is easily done because the machines work in base two, and conversion between base two and any base which is a power of two is extremely simple.

It would be wonderful if we could likewise convert between base dek (or ten) and dozenals.

Most scientific calculators provide functions which can convert between degrees-minutes-seconds (DMS) and decimal expressions of degrees. For example they will convert back and forth between 0.5° and 0°30'0". This is essentially a conversion between base ten and base sixty. Notice that 0°9'9" = 0.1525° is simply another way to say that 0.99_{sixty} = decimal 0.1525.

We can use these functions to convert fractions such as 0.5 into 0;6 and vice versa.

PART I - Decimals to Dozenals

An Example

We can use the fact that 0.5° = 0°30' to easily change 0.5 into its dozenal equivalent since

$$0.5 = \frac{5}{10} = \frac{30}{60} = \frac{30/5}{12} = \frac{6}{12} = 0;6$$

We obtain this result on a calculator in the following three steps:

1. Enter the decimal fraction 0.5
2. Use the DMS function to change this to 0°30'
3. Since 60 is 12(5), *divide the minutes by 5*, obtaining the desired result 0.5 = 0;6

A Second Example

Convert 1/3 to dozenals.

1. Key in 1/3 =, obtaining 0.333...
2. Press the DMS function changing this into 0°20'
3. Divide 20 by 5 obtaining 4. Therefore 1/3 = 0;4"

(Continued)

In Memorium: Charles S. Bagley *(Concluded)*

As we go to press, we have received word of the death of two stalwarts of our Society, former Treasurer Jim Malone and former Vice President Dudley George. Tributes to them will appear in our next issue.

□

Now let's try obtaining more than one dozenal digit by converting 0.3125 to base twelve.

1. Key in 0.3125
2. Press DMS to obtain 0°18'45"
3. Divide the minutes by 5:

$$\frac{18}{5} = 3.6$$

obtaining a quotient of 3 and a decimal remainder of 0.6. Hence the first digit of our result is a 3.

4. Calculate the integer remainder by multiplying by 5. This gives (0.6)(5) = 3

4. Multiplying this by 60 and adding it to the seconds gives 3(60) + 45 = 225

5. *Divide this result by* 5^2, obtaining the second digit, **9**. Hence 0.3125 = 0;39

This follows from the fact that

$$0.3125 = \frac{18}{60} + \frac{45}{60^2}$$

$$0.3125 = \frac{18/5}{12} + \frac{45}{60^2}$$

$$0.3125 = \frac{3 + \frac{3}{5}}{12} + \frac{45}{60^2}$$

$$0.3125 = \frac{3}{12} + \frac{3}{60} + \frac{45}{60^2}$$

$$0.3125 = \frac{3}{12} + \frac{3(60) + 45}{60^2}$$

$$0.3125 = \frac{3}{12} + \frac{[3(60) + 45]/25}{12^2}$$

$$0.3125 = \frac{3}{12} + \frac{9}{12^2}$$

Hence 0.3125 = 0;39

(Continued)

Of course the two multiplications by 5 and by 60 can be combined with the division by 25 so that we multiply the decimal remainder by 12 and add 1/25 of the seconds.

Thus our calculations from 0°18'45" are simply:

first digit = integer part of 18/5 = 3
second digit = fraction part of 18/5 times 12 + 45/25
= (0.6)(12) + 1.8 = 9

A Final Example

Convert 0.15 to dozenals.

1. 0.15° = 0°9'0"
2. Dividing 9 by 5 we obtain 1.8, thus we see that the first digit is 1
3. The second digit is 0.8(12) + 0/25 = 9.6 ≈ ✻

Thus we have 0.15 ≈ 0;1✻

In theory, one can convert decimals with more than 2 places by simply repeating the process on the remaining digits since 0;abcd = 0;ab + 0;cd/100 in any base. Of course, the precision of the result will be machine dependent.

For example, on my Sharp EL-509G nine digits are displayed and two more are apparently used internally. This results in approximately four or five place precision in dozenal digits. My TI-85 displays a dozen digits and uses two additional digits internally. This results in about five or six place precision in a dozenal fraction.[1]

For example, converting 10/11 = 0.90909090... which equals 0;✻✻✻✻...

I obtained 0;✻✻✻# on my Sharp and 0;✻✻✻ ✻✻# on my TI-85.

The former machine used 54'32.73", the latter 54'32.727". It appears as if the Sharp actually calculates with this number while the TI-85 merely displays it, but uses 0.909 090 909 090 91.[2]

Calculation of the above on the Sharp resulted in:

10/11 = 0.909090...
DMS 54'32.73"
54/5 = 10.8, hence first digit is ✻
0.8(12) + 32.73/25 = 10.9092 (and not 10.9090...), hence next digit is ✻

(Continued)

repeating the process using the fraction 0.9092 we obtain
DMS 54'33.12
54/5 = 10.8, hence the next digit is ✻
0.8(12) + 33.12/25 = 10.9248 ≈ # for the fourth digit.

Hence 0.909090... ≈ 0;✻✻✻#

PART II - Dozenals to Decimals

As a curiosity, it is possible to reverse this process, but it is unnecessary to do so. Conversions from dozenals to decimals are much easier. Simply divide each duodecimal digit by the appropriate power of twelve. Thus

$$0;39 = 3/12 + 9/12^2 = 0.25 + 0.0625 = 0.3125$$

Conclusion

Now you can easily convert fractions between duodecimal and decimal notation. Gratefully, the several multiplications and divisions can easily be done on the machine. At the very least,this method it enables us to check arithmetic done by hand.

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[1]Recall that dozenal fractions are, in general, more precise than their decimal equivalents. $12^{-4} \approx 0.000\ 05$, and $12^{-5} \approx 0.000\ 004$.

[2]Of course, the machines work in base two, not in decimals.

Editor's Note: This article was presented at our 1995 Annual Meeting at Rowan College of New Jersey on October 12; 11✻3.

□

The DSA does **NOT** endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus 1/2 = 0.5 = 0;6.

DOZENAL NUMERATION SYSTEM

George Faulkner III, 32#;

[This was a portion of George's report for his 7th grade math class]

In 1600 A.D., Simon Stevin originated the decimal point to separate fractions from whole numbers. He is also credited with the concept of the number base. Fifteen years earlier, he suggested that the base twelve system of numeration offered superiority to the base ten system.

How does base twelve (called the dozenal system) operate? The place value is changed from ten to twelve; numbers are expressed in successive powers of twelve as dozens, grosses, and great-grosses, etc., instead of tens, hundreds, thousands, etc. Expanding to base twelve would require two additional number symbols for the numbers ten and eleven. The symbol ✻, called dek, is then used for ten, and the symbol #, called el, is used for eleven. 10; is now employed to signify one dozen and is called do.

In light of the fact that a foot consists of twelve inches, we can have a whole number of inches in 1/2, 1/3, 1/4, or 1/6 of a foot. A yard (36 inches) or (12 x 3) divides evenly into two, three, four, six, nine, twelve, or eighteen parts. In looking at other ways base twelve already exists in our lives, we can examine our year which has twelve months, or our day which is divided into two twelve hour periods. Even when we shop, eggs, oranges, and rolls are sold in dozens.

In dividing our hour and minute, we combine the superior system of twelve with the bad system of ten..."sixty" is the smallest number at which ten and twelve meet. [Such a number which is the smallest common multiple of both ten and twelve is called the *least common multiple*. - Ed.] Our degree system is another compromise. The first part, thirty-six, is divisible by the four directions north, south, east, and west, and the last part is divisible by ten.

Learning to count the dozenal way is like learning another language. Once learned, it would make life easier in many ways:

1	2	3	4	5	6
one	two	three	four	five	six
7	8	9	✻	#	10
seven	eight	nine	dek	el	do

□

WHAT EVER HAPPENED TO THE METRIC SYSTEM?

Gene Zirkel

An article with the above title appeared in the October 1995 issue of the National Council of Teachers of Mathematics (NCTM) *News Bulletin*. It occupies about one-half of the eight page newsletter.

It is one of the most biased articles I have ever read. Many 'facts' are twisted so that they appear to favor one of the awkward decimal metric systems.

The article states that "a hush comes over the... room" when metrication is mentioned. That is definitely not my experience. Many people voice strong opinions both for and against the awkward decimal metric systems. Fortunately for the USA, so far the majority oppose this halfway measure.

The author asks why, and offers three possible reasons. Not included as a possible answer to the question is the simple idea that most people do not want to be forced to convert.

The article goes on to state that our government 'inexplicably' seemed to back away from an aggressive conversion campaign. What is so inexplicable about recognizing that the majority of voters are unhappy about being forced to use an unwieldy system which cannot easily handle the everyday fraction .

It cites Ralph Richter, an engineer at the National Institute for Standards and Measure (NISM) who says "...the U.S. government made the metric system voluntary instead of mandatory. I think that was the mistake"! Like most proponents of one of the decimal metric systems, he urges that we be forced to accept his minority way of doing things, because he and they cannot persuade us to adopt an artificial measuring system.

It is interesting to note that

> not once—never—in the course of history has any society, anywhere, ever voluntarily adopted one of the unfortunate decimal metric systems. Why is it that in every country where one is required today, it had to be *forced* upon an unwilling populace by law with the threat of fines and/or imprisonment? Are all of us everywhere so ignorant of what is good for us that a few *Big Brothers* in government must tell us how we must sell butter and rugs to one another? I don't think so. I think that common people have resisted and rejected this accident in favor of simple ordinary fractions because they know which is really more convenient.
>
> Good ideas are often resisted when they are first presented. For example, some localities passed laws that a person holding a lantern was required to walk in front of an automobile lest these new-fangled, frivolous toys frighten horses which were needed for commerce and industry. Of course, eventually good ideas do win out.[1]

(Continued)

Richter goes on to admit that "Many people were—and still are—against the national implementation of the metric system", using the singular, when an expert from the NISM knows full well that there are many such systems, that there are variations from country to country between and among the various decimal metric systems. He cites the opposition of some labor unions and offers some reasons for this, omitting the possibility that people simply do not want his idea shoved down their throats.

He is "shocked" that some teachers are not using this unwieldy system, and that publishers either do not include it in texts or relegate it to a minor section that can be easily omitted. Is he so out of touch with Americans: students, parents, teachers, school boards, that he cannot hear what they are loudly and clearly saying: "GO AWAY"? Publishers are in business to make money. They print what they can sell at a profit. (It is usually one of the arguments of decimal metric advocates that we must change over for the economic advantages it supposedly would bring if we did so. Now, when a publisher seeks economic advantages, he is shocked.)

One sub head asks "Is it any wonder Teachers are confused?" Are they? Isn't it possible that they very clearly see the disadvantages of losing our natural measurements with their convenient halves, thirds and quarters?

In quoting William Aldridge, the executive director of the National Science Teachers Association (NSTA), who had several negative things to say about metrication, the author inserts a snide parenthetical "soon-to-retire" remark, hoping to imply that Aldridge is somehow not to be taken too seriously.

In this context, NSTA's President Shelley Fisher first states "I can't speak for him", but then goes on to claim that he was quoted out of context. Well, Shelley, which is it? Or does any excuse get offered in defense of the overwhelming facts? People don't want it! (But of course, NSTA knows what is good for us, even if we don't.)

Aldridge was spreading what the pro-metric bunch would deem heresies, such as *the dollar cost of industry converting to the metric is just too high*, and *I have seen little evidence that there is a major switch to metric system usage by U.S. companies.*

Lorelle Young, president of the U.S. Metric Association states that IF (emphasis added) organizations such as the NCTM would take a leadership role in metrication it would happen. Young knows full well that the NCTM has been trying to force metrics on its members since 1986, wasting our dues money on a dubious program that very few want. The NCTM has taken such a leadership role, and guess what? Voluntary metrication has not happened.

We are all aware of the fiasco of gasoline companies who lost money switching to liters, and then lost customers who switched to the dealer across the street who retained gallons. We are also aware of the money wasted in putting up metric highway signs and then taking them down again.

(Continued)

At least Fisher is honest enough to admit that students should still be taught the current popular measuring system. Young doesn't even want that taught to our youngsters!

According to Young, only the USA, Burma and Liberia are not completely metric, but, the article goes on to say, that is not true, giving Canada as one example of a country that is not completely metric. This was pointed out by Rol Fesden, director of inventory control at L. L. Bean, which has clothing produced all over the world. Fesden says, "If a particular manufacturer needs a conversion, it's already been done by the computer. Everything is computerized now, so it's a non-issue for us."

In fairness, the article did give some opposing views such as: according to the *Engineering Times* James Rutherford American Association for the Advancement of Science spokesperson does not see the need to teach metrics to school children, in fact, he states, it could turn them off to science. He was supported in this opinion by his colleague Andrew Ahlgren.

However, the article is filled with half truths aimed at misleading the reader who quickly skims the material. Statements such as "Engineers, scientists, and mathematicians represent a small sector of the U.S. population. They use the metric system daily." This gives one the erroneous impression that all, or almost all of them do, when in fact a very small portion of this very small sector do so. As Aldridge said, "If the metric system was taught in engineering schools, graduates would not be prepared for jobs which exist now."

There is more, but you get the idea.

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[1]See "A Brief Introduction to Dozenal Counting", this *Bulletin*, volume 38; number 2; pp 6-9.

□

Book Review *(Concluded)*

I wasn't sure if reference was to the Romans' package of products or to the Roman counting basis. For this reason I would prefer "base" where appropriate and reserve "packing" for its ordinary meaning. If one must change the word, then "core" is clearer than "packing"

Finally, I didn't understand the references to gravity at all. They simply confused me.

Writing this book was obviously a labor of love, and all in all it is a worthwhile addition to any dodekaphile's library.

□

Book Review

Jean Kelly

Nature's Numbers, by William G Lauritzen, Grassroots Press, 809-D East Garfield, Glendale CA 91205

It is always exciting to see another book advocating the use of dozenals. This was no exception. The two outstanding features of this book are Lauritzen's new symbols and his historical references and anecdotes.

The symbols that he has created are clever and easy to learn, for they contain a simple pattern: a line rotating about a circle. Because of this, once they are understood, they can easily be recalled even if one has not used them for a while.

In addition each page is numbered in both these new symbols and the usual base ten numerals we are all accustomed to. This double numbering helps the neophyte become accustomed to the new numerals.

I particularly enjoyed the anecdotal material: The mention of the city of Florence, of Napoleon, and of the power of words are right on the mark. The last sentence on page 23 referring to the decimal metric system is excellent, as is the history of numerical data, and the other historical references throughout the text. These are the kinds of things that first time readers, as well experts, can tap into.

The diagrams and charts are clear and easy to follow, and I found only a half dozen minor typographical errors in the text.

On the down side, parts of the layout of the book were confusing. Pages of text have bars on top and bottom, pages of ancillary material do not. This is just the opposite of the usual convention. The side-bar material on page 2 was clearly shaded. Unfortunately this was not continued thruout the text.

Figures do not appear near their references, for example figure 5 is referred to on page 15, and appears on page 20, without any indication as to where it is to be found. This makes for a lot of unnecessary page turning. A frustrated novice plowing thru this new material might be tempted to give up.

The text ending on page 15 is continued on page 21, but there is no notice to the reader to this effect. At first I thought that some material had been left out.

Lauritzen suggests that "core, package, packing, or packed" be used instead of "base." If one must use something else (and I don't see why since the term "base" is taught to students in schools and understood by many people) then only one term should be used. I know this material very well, and at times even I was confused. Particularly on page 14.

(Continued on page 14)

ADVANTAGES OF BASE TWELVE

Millie Chu 351;

[This is a summary of an oral report delivered by our newest member for her college math class at Wellesley College, MA. - Ed.]

We seem to use twelves a lot in our everyday lives. (There are twelve inches in a foot, twelve times two hours in a day, twelve times 2;6 dozen degrees in a circle, eggs sold in dozens, etc.) Despite this, our numeration system is based on ten. In order to avoid working with these awkward tens, we should convert our number system to base twelve. Then we would have 10; months in a year - a nice round number which leads to easier calculations. In light of the fact that we are in duodecimals, it is easily divisible by 2, 3, 4, and 6.

Metrics and Fractions. Note that if we converted to the cumbersome decimal metric system, one-third of a meter is 33 1/3 cm (Yuck fractions!), but if we instead employed base twelve, one-third of a yard is simply 10; inches. We lose divisibility by five when we move from base ten to base twelve, but thirds, fourths, and sixths are much more useful.

Converting into Base Twelve. A less well known method[1] of converting from base ten to base twelve which I demonstrated in class involves filling columns from right to left using mod (or remainders). For example, to convert 249_{ten} to base twelve, the following steps are taken:

$$249 \text{ (mod twelve)} = 9 \rightarrow 249 = 12\,(20) + 9 \quad (1)$$
$$20 \text{ (mod twelve)} = 8 \rightarrow 20 = 12\,(1) + 8 \quad (2)$$
$$1 \text{ (mod twelve)} = 1 \rightarrow 1 = 12\,(0) + 1 \quad (3)$$

Substituting equation (3) into (2), and then (2) into (1), we obtain

$$20 = 12[12(0) + 1] + 8 = 12^2(0) + 12(1) + 8$$
$$249 = 12[12^2(0) + 12(1) + 8] + 9$$
$$= 12^3(0) + 12^2(1) + 12(8) + 9$$
$$= 189;$$

We thus obtain $249_{ten} = 189_{twelve}$

[Millie's professor wrote in the margin at this point, 'You illustrated this really well in class.' -Ed.]

New Symbols. Two extra digits are needed in base twelve. The symbol used to represent ten in the units column is , or ✻ and is pronounced "dek". An eleven in the units column is written , or # and is pronounced "el".[2]

THE DSA. The Dozenal Society (formerly The Duodecimal Society) of America was founded in 1160_{twelve} (or 1944) and pursued the idea of this logical conversion. They publish a journal, *The Duodecimal Bulletin.*

(Continued)

Other Points of Interest for Base Twelve

The *Divisibility Tests* for Base Twelve are as follows. A number is divisible by:

2 if the last digit is even.
3 if the last digit is 0, 3, 6, or 9.
4 if the last digit is 0, 4, or 8.
6 if the last digit is 0 or 6.
8 if the last two digits are divisible by 8.
9 if the last two digits are divisible by 9.
\# if the sum of the digits is divisible by #.
10 if the last digit is 0.

As determined by George Terry in 1941[3]:

Mersenne Primes,[4] (2^p-1 where p is a prime) greater than 7 when written in base twelve end in either *27* or *✻7*.

For example, $2^5-1 = 31_{ten} = 27_{twelve}$

and $2^7-1 = 127_{ten} = ✻7_{twelve}$.

F. Emerson Andrews[5], who promoted the duodecimal system, commented, "We shall continue counting on our fingers in the logical silly system of ten to the end of our days".[6]

[Here another marginal comment from Millie's professor stated, "Fun Topic. I enjoyed your presentation. I'm ready to grow two more fingers". We hoped she earned an A. She richly deserved to. -Ed.)]

End Notes

1. William H. Beyer, *Handbook of Mathematical Sciences, 6th Ed.,* Boca Raton, CRC Press, Inc., 1987, P. 91.

2. Underwood Dudley, *Mathematical Cranks,* The Mathematical Association of America, Washington DC, 1972, P. 23.

3. Dudley, P. 25.

4. Dudley, P. 25.

5. Dudley, P. 23[b].

6. Dudley, P. 23.

□

SOLUTION TO A DUODECIMAL NUMBER THEORETIC PROBLEM

Jay L. Schiffman
Rowan College of New Jersey, Camden Campus

In the latest issue of the Duodecimal Bulletin, I posed a query regarding the terminal digit of all duodecimal even perfect numbers greater than the initial one which is 6. After conjecturing via the second through seventh even perfect numbers in base twelve, I devised a formal proof. My contention is that every even perfect duodecimal integer terminates in the digit 4, save the first. The proof is contingent on several easily deduced elementary number theoretic facts.

PROOF OF THE CONTENTION: Euclid proved that every even perfect integer n is of the form $(2^{p-1})(2^p - 1)$ where both p and $2^p - 1$ are prime. It can be shown that if $2^p - 1$ is prime, then p is likewise prime. If p = 2, then n = 6. Consider p greater than 2. Every prime exceeding 2 is of the form 4m + 1 or 4m + 3. (Test a few of them to witness the plausibility.) We consider these two cases separately. If p is of the form 4m + 1, then n = $2^{4m}(2^{4m+1} - 1) = 14^m[2(14^m) - 1]$ where $m \geq 1$. One can easily demonstrate that every positive integer power of 14 ends in 4. Then $[2(14^m) - 1]$ ends in 7, and n terminates in 4.

In a similar manner, if p is of the form 4m + 3, then n = $(2^{4m+2})(2^{4m+3} - 1) = 4(14^m)[8(14^m) - 1]$ where $m \geq 0$. In view of the fact that 14^m terminates in 4, $[8(14^m) - 1]$ terminates in 7 and hence n ends in 4. We deduce that every even perfect number ends in 4.

We conclude by enumerating the initial seven perfect numbers in both the decimal and duodecimal bases:

Decimal Perfect Number	*Duodecimal* Perfect Number
6	6
28	24
496	354
8,128	4,854
33,550,336	#,29#,854
8,589,869,056	1,7#8,891,054
137,438,691,328	22,777,#33,854

A second solution was obtained by *Charles Ashbacher* of Hiawatha, IA and is printed below:

The answer to the above problem is 4.

Proof: If we examine the powers of 2 modulo 10, we view a simple pattern.

(Continued)

Exponent	Value	Value modulo 10
0	1	1
1	2	2
2	4	4
3	8	8
4	14	4
5	28	8
6	54	4

and the pattern is clear: 8 if the exponent is odd and 4 if even. We next employ the given expression for an even perfect number N that exceeds 6. In such case, the prime p in question must be of odd parity. Therefore, 2^{p-1} must be congruent to 4 modulo 10. This in turn forces $2^p - 1$ to be congruent to 7 modulo 10. The product of these two factors is then 4 modulo 10.

□

HALF DOZENS

The Bassari of eastern Senegal have invented an extraordinary headdress which commonly contain a scaling series of hexagons in the mask's center. The number 6 is a prominent feature of Bassari mathematics. The traditional calendar has 6 months per year, each having two and one half dozen (6x5) days, with an initiation about every dozen (6x2) years, to a total of nine initiations. The important rite of passage to adulthood last six days. Their string tallies, traditionally used for recording various counts, often grouped knots in half dozens. A popular game using pebbles is played on two axes with six holes in each line. In divination, shells are cast six times.

Adapted from "Scaling hexagons in a Bassari initiation mask" by Ron Eglash, *The Mathematics Teacher,* October 1995, Volume, Number 7, pp. 618 & 620.

□

ADVICE TO AUTHORS

The following guidelines are given in order to increase the likelihood of your article or letter getting published.

- If possible, submit your article **both** on disk in **WordPerfect 5.1** and also in hard copy format. (If you **must** use another word processing program please indicate the name and the version of the program.) The width of the page should be 4.5 inches. (The length can vary, since our compositor will cut and paste when laying out the issue. For your own information and to estimate the number of pages, our pages usually run about 7 or less inches of copy.)
- Unless the article is about symbolism, us the asterisk (*) for dek and the octothorpe (#) for el. Use a semicolon (;) for the fraction point. If you have several fonts, try to use a six pointed asterisk.
- Do not double space between sentences.
- Do not indent new paragraphs.
- If there are nonstandard symbols or figures in your article, draw them on **separate** sheets in black ink, ready for photographic reproduction. If the entire article is permeated with such symbols, try to submit the hard copy ready to print.
- In text avoid using the ambiguous symbol 10. Write out 'ten', 'twelve' or 'do' as the case may be.
- An article needs an introduction giving its intent, and a closing. Brief items making one or two points are preferred. Decide what your point is and stick to it. If you wish to address several diverse issues, send several submissions.
- Keep in mind that more than half of our *Bulletins* go to first time readers including elementary school pupils and non-mathematicians. Be clear!

□

Do you keep a copy of our DSA brochure or of Andrews' *Excursion* at home and in the car? You never know when you might want to give one to a friend. Be sure to always have one on hand.

NEW LETTER

The following is the text of a letter sent to Board Member Gene Zirkel from Guy Murdoch of Consumer's Research Magazine:

28 September 1995

Dear Gene,

Thank you for sending me the information on the Dozenal Society. I enjoyed it thoroughly. In particular, the reprint of *An Excursion In Numbers* by F. Emerson Andrews was fascinating.

I thought I would drop a line with some things that struck me as I was reading your materials.

(1). Andrews remarks that primitive people never used twelve as a base because no part of the body served well for counting by twelves. While this may have been true, his note that a people in Brazil counted by threes on the joints of their fingers points to a body-based visualization for the duodecimal system: Use the joints on the four fingers of the hands. This not only represents a dozen but is superior to a ten-based system in that you obtain *two* dozens on your hands rather than *one* set of ten.

(2). The new figures should be as clean/simple as possible-you shouldn't have to lift your pen more than once to make a figure. Therefore, I feel the symbols Andrews uses are better than the ones in your brochure. Also, it seems to me, to continue using the old zero is very confusing. Instead you could perhaps utilize the "p" from printers' measure of picas and points (based on twelve). For illustrative purposes, a pica consists of twelve points and is represented by "1p", or one dozen points. Continuing, 2p equals two dozen points, 3p three dozen points, and so on. As you see, here "p" has a function similar to zero.

Best of luck in your efforts.

Regards,

Guy Murdoch

□

THINK 12 . . .twelve . . .10; . . .do . . . .

FROM THE EDITOR

William H. Guyer of Cambridge, MA recently wrote an interesting letter to Board Member Gene Zirkel concerning duodecimals as well as a short piece. We gladly print the text of his letter together with his short article below.

Dear Professor Zirkel,

In The Boston Globe of July 10, 1995, there was an article on the Metric System including mention of you and the Dozenal Society of America.

A few years ago, I had some amateur thoughts concerning the use of the number twelve as a base for measurement, and I wrote the enclosed list of my comments.

I am happy to support your views on the subject.

Sincerely yours,
William H. Guyer

(See page 21)

□

NEW MEMBERS

The Dozenal Society of America is proud to introduce several new members this year. The new members are:

345	Ian M. Shea
346	John R. Tyler
347	Peter A. Smith
348	Gilberto Diaz-Santos
349	Robert Judson McGehee
34✱	Daniel A. Gabriel
34#	William Herriman Guyer
350	David Carson Berry
351	Millie Chu

FANTASY OF NUMBERS

William H. Guyer
September 1991

A duodecimal system of numbers with units of twelve would have the following advantages over the present decimal system with units of ten.

1. Twelve is divisible by six numbers (1, 2, 3, 4, 6, 10;) while ten is divisible by only four numbers (1, 2, 5, ✱). This would be particularly useful for dividing the unit into three parts.

2. Large numbers would be expressed with slightly fewer digits.

3. Twelve is consistent with the present uses of

hours of ante meridian
hours of post meridian
months of the year
signs of the zodiac
numbers on a clock
inches in a foot
ounces in a pound (troy weight)
pence in a schilling (British money)
dozen (eggs for example)
number of apostles of Christ

The duodecimal system would require two additional characters that we will tentatively call ✱ and #. Numbers would then be
1, 2, 3, 4, 5, 6, 7, 8, 9, ✱, #, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 1✱, 1#, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 2✱, 2#, 30, etc.

A dollar might be divided into two dozen cents with coins of one, four, eight, and twelve cents.

□

Do you have an idea to share with our members? Why not submit an article to the *Bulletin*?

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition—("Who needs a symbol for nothing?")—the new notation did come into popular use. Released from the drag of Roman notation, our thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, people awoke to the fact that different number bases could be used.

A parallel to today seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4), and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in our Society may prove mutually profitable, and is most cordially invited.

□

We extend an invitation to membership in our society.
Dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______________________________
LAST FIRST MIDDLE

Mailing Address (For DSA items) ______________________________

____
(See below for alternate address)

Telephone: Home ______________ Business ______________

Date & Place of Birth ______________________________

College Degrees ______________________________

Business or Profession ______________________________

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______________________________

Address ______________________________

Year & Math Class ______________________________

Instructor ______________ Dept. ______________

Other Society Memberships ______________________________

____

Alternate Address (Indicate whether home, office, school, other)

____

____

____

Signed ______________ Date ______________

My interest in duodecimals arose from ______________________________

____

____

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
%Math Department
Nassau Community College
Garden City, LI, NY 11530

DETACH HERE-- OR-- PHOTOCOPY