DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

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THE DUODECIMAL BULLETIN 75;

On the banks of the Thames near Oxford Dr. Patricia Zirkel, Chair of the Board of Directors of the DSA, presents the Ralph Beard Memorial Award to Arthur Whillock, Information Secretary of the DSGB, while Mrs. Ruby Whillock and Gene Zirkel, Member of the Board of Directors look on.

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THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00 (US) for one calendar year. Student Membership is $3.00 per year, and a Life Membership is $144.00 (US).

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THE DUODECIMAL BULLETIN

Whole Number Seven Dozen Five

Volume 38; Number 2;

11*3;

IN THIS ISSUE

FOUNDED 1944

ANNOUNCEMENT OF OUR ANNUAL MEETING

For the first time in nearly two dozen years, the setting for our Annual Meeting will not be Long Island. Our Duodecimal party will take its educational message and convene in the picturesque Borough of Glassboro, New Jersey come Saturday, October 14, 1995 at 10:30 A.M. Rowan College of New Jersey, the home school of our President and Editor, has graciously provided The Conference Room in The Department of Mathematics for our gathering. It is hoped that this locale will attract the dozen or so members who reside in the tri-state area comprising Pennsylvania, New Jersey, and Delaware.

The Borough of Glassboro has been the proud home to higher education in South Jersey since 1923. *Glassboro State College* has long been known for its outstanding programs in education and has furnished the region with numerous excellent primary and secondary teachers. Over the years, the college has blossomed into a multifaceted institution of higher learning with very highly regarded programs in Arts and Sciences, Business, and Fine and Performing Arts to mention a few. In 1992, the very generous gift of *Henry and Betty Rowan* enabled the school to initiate work on The School of Engineering which will commence with its initial class within a year. The Board of Trustees voted to rename the school *Rowan College of New Jersey* as of September 1, 1992.

In addition, the college has applied for University status and is planning its first Doctoral Program in Educational Leadership. Pending state approval, Rowan College of New Jersey (which has received numerous accolades for outstanding teaching and currently has the 1994 New Jersey Mathematics Teacher of The Year among its distinguished faculty) will be able to further serve this region's educational needs. With a most gracious new state of the art library dedicated February 1, 1995, it is obvious RCNJ is in the forefront as we venture into the twenty-first century and that this is a dynamic time for one to be associated with the college which comprises two campuses.

Rowan College also serves the greater Philadelphia and Camden region with a branch campus in downtown Camden which it leases from the county college. On February 24, 1994 in Camden, we celebrated a quarter of a century of education at this location. Our Camden Campus brochure summarizes succinctly our mission statement and is aptly expressed by our dean as follows: "Here in Camden, Rowan College is journeying into the future when minorities will become majorities and diversity will be the norm."

DIRECTIONS

The main campus of Rowan College of New Jersey is located in Southern New Jersey in scenic Gloucester County, thirteen miles Southeast of Philadelphia, one of America's finest historic cities, at the intersection of Routes 47 (Delsea Drive) and 322, east of Exit 2 on The New Jersey Turnpike. The Mathematics Department is located on the third floor of Robinson Hall in the southeast corner of the building.

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The meeting will commence with the usual Board of Directors Meeting followed by The Annual Membership Meeting. We expect several presentations from our active members. It has been confirmed that our active Board Member, *Professor Gene Zirkel* of Nassau Community College will present a talk on how to convert fractions from the decimal base to the duodecimal base and vice versa employing calculator technology. *Professor Jay Schiffman* of the host institution in Camden and President of the Society will speak to us on the topic *Hexadecimally Speaking* which will focus on Hexadecimal number theory and will likewise include a calculator component employing the BASE key on the TI-85 graphics calculator. It is hoped that a late luncheon or early dinner will follow at one of Philadelphia's fine restaurants. For a unique experience in a dynamic setting, please mark your calendar for a day that promises to be fulfilling both educationally and socially. You will be extremely disappointed if you miss our Annual Meeting.

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Problem Corner

DUODECIMAL PERFECT NUMBERS

Jay L. Schiffman
Rowan College of New Jersey
Camden Campus.

It is indeed a rare find for a positive integer to coincide with the sum of all its proper divisors. An integer is styled *perfect* when this unusual situation occurs. In the decimal base, the initial seven even perfect numbers are 6; 28; 496; 8,128; 33,550,336; 8,589,869,056; and 137,438,691,328. It can be shown via elementary number-theoretic considerations that every even perfect number terminates in 6 or 8 (in fact 28). It has been proven by Euclid and other prominent mathematicians that a characterization (necessary and sufficient condition) for an even positive integer N to be perfect is that $N = (2^{p-1})(2^p - 1)$ where both p and $2^p - 1$ are prime numbers, the latter known as a *Mersenne Prime*. Our question is as follows: In the duodecimal base, in what digit does every even perfect number > 6 terminate? (Conjecture with the second through seventh perfect numbers and then try to formulate a general proof.) Solutions are invited and will be presented in the future.

A BRIEF INTRODUCTION TO DOZENAL COUNTING

by Gene Zirkel

Most of the world evolved *a counting system based on ten*, but *a system of weights and measures based on twelve*. Why?

Origins For the most part, our ancestors counted on their fingers. In a world where communication was limited, most societies independently developed a ten-based counting system. Of course there were exceptions. A few barefoot tribes counted in twenties, the Babylonians used sixty, and one tribe in South America counted in threes. Can you guess upon what parts of their bodies they counted? (Answer given below.)

At the same time, practical people measured in dozens. Once again, people throughout the world independently arrived at the same conclusion. Thus:

- the baker sold donuts in collections of twelve
- the carpenter divided the ruler into twelve subdivisions
- the *grocer* dealt in dozens and in dozens of dozens or *grosses*
- the druggist and the jeweler still use the twelve ounce pound
- the minters divided the shilling into twelve pence, etc.

Why? Counting in tens is a biological *accident*. If only we had been born with twelve fingers how much simpler all this would be. But measuring was not accidental. It was devised by practical people who used the fractions: 1/2, 1/3, and 1/4. That is why merchants and tradespeople chose to divide their units of weights and measures into twelve parts. Simply put, by choosing twelve subdivisions, they could have their cake and eat it too. They could use the three most common fractions without having to actually employ fractional notation. For 1/2, 1/3, and 1/4 of a foot are 6, 4, and 3 inches respectively — whole numbers, not fractions!

Thus using the period (.) for the fraction point in base ten and the semicolon (;) in base twelve, we obtain the following

	In Base Ten			In Base Twelve		
1/2	5 tenths	*0.5*	1 significant digit	6 twelfths	0;6	1 significant digit
1/3	**about** 3	**about** *0.333333...*	infinitely many significant	4 twelfths	0;4	
1/4	2 ½	*0.25*	2 significant	3 twelfths	0;3	

(Continued)

Solutions Over the course of time, many people suggested that we try to align our counting and our measurements. Proposals were made advocating various bases. For example base eight was offered as a solution to this dilemma, since in base eight halves, quarters and eighths are simplified. Computer scientists use a similar idea when they switch between bases two and sixteen.

The desirability of aligning our counting — which is based on a biological accident — with our measuring — which was devised by pragmatic people — was well understood at the time of the French Revolution. It was evident that either counting should be changed to base twelve or that measuring should be changed to base ten so as to be in agreement with one another. The French blundered into changing the wrong one. Maladroitly, they decided to keep the accidental and to change the practical. It is analogous to cutting off one's toes instead of obtaining a larger shoe.

Human Progress Good ideas are often resisted when they are first presented. For example, some localities passed laws that a person holding a lantern was required to walk in front of an automobile lest these new-fangled, frivolous toys frighten horses which were needed for commerce and industry. Of course, eventually good ideas do win out.

But not once — never — in the course of history has any society, anywhere, ever voluntarily adopted the unfortunate decimal metric system. Why is it that in every country where it is required today, it had to be *forced* upon an unwilling populace by law with the threat of fines and/or imprisonment? Are all of us everywhere so ignorant of what is good for us that a few *Big Brothers* in government must tell us how we must sell butter and rugs to one another? I don't think so. I think that common people have resisted and rejected this accident in favor of simple ordinary fractions because they know which is really more convenient.

In the United States, every pupil in science class is taught the so-called advantages of the abominable decimal metric system. Metric measuring devices are available. Yet when given a chance to measure something for their own use, a chance to use whatever measure they prefer, they use dozenal measures because fractional parts of units are easier to handle.

A Misconception Some people wrongly believe that the ability to multiply and divide by powers of the base by simply moving the fraction point is an advantage special to base ten. But such is **not** the case. It is not "*ten*-ness" that gives this property (after all it wouldn't work with ten based Roman Numerals). No, **this advantage exists in every base**, for it is a property of the *place value notation* we use for expressing numbers along with *a symbol for zero*. Thus we see that

$$\mathbf{110.11 \div 10^2 = 1.1011}$$

is always true, **no matter what base one is using**.

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Counting In base ten counting we use ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. The numeral 342 represents 3×(ten^2) + 4×(ten) + 2. In dozenal counting we use twelve symbols, adding two digits to represent ten and eleven since **10** still represents the base. The numeral **342** represents **3×(twelve^2) + 4×(twelve) + 2**. Thus **342** in dozenals represents *482* in the familiar decimal base.

The Base Twelve Multiplication Table

x	1	2	3	4	5	6	7	8	9	*	#	10
1	1	2	3	4	5	6	7	8	9	*	#	10
2	2	4	6	8	*	10	12	14	16	18	1*	20
3	3	6	9	10	13	16	19	20	23	26	29	30
4	4	8	10	14	18	20	24	28	30	34	38	40
5	5	*	13	18	21	26	2#	34	39	42	47	50
6	6	10	16	20	26	30	36	40	46	50	56	60
7	7	12	19	24	2#	36	41	48	53	5*	65	70
8	8	14	20	28	34	40	48	54	60	68	74	80
9	9	16	23	30	39	46	53	60	69	76	83	90
*	*	18	26	34	42	50	5*	68	76	84	92	*0
#	#	1*	29	38	47	56	65	74	83	92	*1	#0
10	10	20	30	40	50	60	70	80	90	*0	#0	100

(Continued)

Many people use either * and # (or else ↊ and ↋ — a rotated 2 and 3) to represent the digits for ten and eleven. They are pronounced **dek** and **el**. Counting proceeds as in the accompanying base twelve multiplication table.

Conclusion The above are some of the reasons why thinking people advocate a gradual change to dozenal counting. Because of the prevalence of computers, many students at present are being taught about base two and base sixteen counting. It would be simple to teach children both a dozenal metric system and the ill-advised decimal metric system, and then allow them to freely use the one they prefer. In one generation awkward systems would go the same way ancient Roman Numerals have gone — relegated to clocks, cornerstones and other curiosities. Remember, until the Crusaders brought what are called the Hindu-Arabic numerals to the West, all of European commerce was dependent upon Roman Numerals, and many people were convinced that they would never be changed.

In answer to the earlier question: The South Americans mentioned above counted on the segments of the fingers. If one uses the thumb as a pointer, one can easily count to twelve on one hand. Incidently, whereas the basis of almost every system of counting was the result of biology, the Babylonians were the one civilization which intelligently developed a number base — base sixty. If twelve has the advantage of the factors 2, 3, 4, and 6, sixty has *2, 3, 4, 5, 6, 10, 12, 15, 20,* and *30.*

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Do you keep a copy of our DSA brochure or of Andrews' *Excursion* at home and in the car? You never know when you might want to give one to a friend. Be sure to always have one on hand.

ANNUAL AWARD

The Ralph Beard memorial award for 1995 was given to Arthur F. Whillock by Board Chair Dr. Patricia Zirkel in a ceremony preceding lunch in the Shillington Arms Hotel on the banks of the River Thames in Wallington, England. Also in attendance were Mrs. Ruby Whillock and Board member Gene Zirkel.

Upon receiving the award, Arthur turned to his wife and said, You have earned this award just as much as I have.

For years Arthur has been the mainstay of the DSGB and Editor of their *Dozenal Journal.*

The text of the plaque reads as follows:

The Board of Directors

of the

Dozenal Society of America

present

The Ralph Beard Memorial Award

to

Arthur F. Whillock

Information Secretary

of the

Dozenal Society of Great Britain

for his outstanding service
and devotion as an advocate of
Dozenal Counting and Metrics
and for his years of service to
The Dozenal Journal

11*3; 1995.

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IF WE ONLY HAD TWELVE FINGERS

Rafael Marino
Nassau Community College
Garden City, NY

Some of the ideas presented in this article are the result of thinking about ways of combining the conveniences of the English system of measurements with the obvious advantages of the metric system and also of reflections on Roman and Mayan numerals.

We write numbers in base ten, the *decimal numeral system*, due to the anatomical fact that we have ten fingers. If we had twelve fingers, we would probably have adopted a *duodecimal numeral system*, a number system in base twelve, and mathematics would have been a little bit easier. Twelve is a more divisible number than ten. Twelve is divisible by 2, 3, 4 and 6, while ten is only divisible by 2 and 5. Because of this, twelve or multiples of twelve (like 24, 60, and 360) have been used by many people as bases for measurement, especially of time. For instance, we divide one day into 24 hours, one hour into 60 minutes, one minute into 60 seconds; and we say that an angle that makes a complete circle has 360 degrees. (Why don't we say that a complete circle has 400 or 100 degrees?) Additionally, we count many things by dozens: coke is sold in twelve-packs, and one foot has twelve inches.

The discrepancy between these facts and our use of a decimal numeral system has created difficulties. The obvious solution to these problems would be to adopt a duodecimal numeral system which would have all of the advantages, and none of the disadvantages, of the decimal system. I also propose a way of dividing time that is less cumbersome and confusing than the one that we presently use, a way in which we define hours and design our clocks so that they simulate the apparent motion of the sun during the day. While presenting my point, I will illustrate that the fact that the way we write mathematics is much more crucial than most people suspect. Before presenting my ideas, however, we must clarify some technical details.

First of all, let's distiguish between *numbers*, which are abstract mathematical entities, and *numerals*, the symbols that different civilizations have used to denote numbers. Most civilizations developed *non-positional* notations of numbers, and only a few - such as the Indians and the Mayas - came up with a *positional* notation. The number notation that we use - the one invented by the Indians - is positional. This means that in a number like 2,507, the positions in which the digits 2, 5, 0, and 7 are written tell us the value of the number. More precisely, 2,507 corresponds to 2 thousands, 5 hundreds, 0 tens, and 7 units or

$$2{,}507 = 2\times10^3 + 5\times10^2 + 0\times10^1 + 7\times10^0$$

We have expressed our number as a sum of powers of ten. Because we use powers of ten rather than powers of some other number, we say that our notation of numbers is *decimal*. The Romans would have written MMDVII instead of our 2,507. The Roman number notation is not positional, that is, the fact that the letter D is in the third position does not change anything about its value, 500.

What most people do not realize is that any number could have been used as a base for the notation of numbers. For example, in the city of Bombay, we find traces of 5 as a base in the way that some merchants use their fingers to count.

On the other hand, the Mayas, Aztecs, and Celts counted by 20s. In fact, modern French still say *quatre-vingt* (four twenties) for eighty.

Numbers in Dozenland

Imagine a place, that I call Dozenland, where people have twelve fingers. (They are believed to be descendants of pandas. It is also believed that Dozlandians had contact with the Mayas from whom they borrowed several ideas for their notation of numbers and their calendar.) In Dozenland, they write their first 11 numbers like this:

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

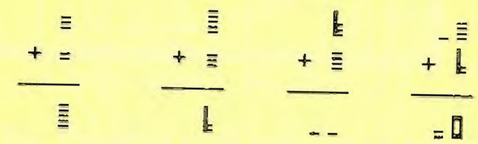
A dash (–) represents one of their six fingers and a bar (|) a hand. The next number (our number twelve) is written –0 . 0 is their symbol for zero. Dozlandians write numbers like =≣0L , which for those of us accustomed to our decimal notation, looks like so many meaningless squiggles. You may want to read it "two five zero seven." Let's do some translation, converting it into a decimal number:

$$\begin{aligned} =\!\equiv\!0L &= 2\times12^3 + 5\times12^2 + 0\times12^1 + 7\times12^0 \\ &= 2\times1{,}728 + 5\times144 + 0\times12 + 7 \\ &= 3456 + 720 + 7 \\ &= 4183 \end{aligned}$$

How do we interpret Dozlandians digits to the right of the duodecimal point? The same way that 0.3 means $\frac{3}{10}$, 0.≡ would mean (for us) $\frac{3}{12}$ or $\frac{1}{4}$. By the way, in Dozenland 1/3 is exactly equal to 0.≣, not the infinite decimal expansion 0.333... .

(Continued)

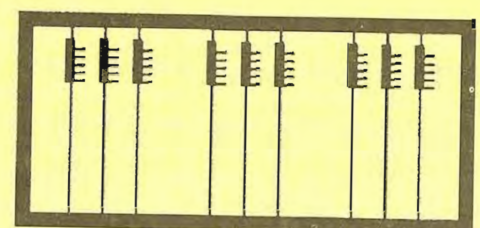
Additionally, Dozenlandian children learn the basic arithmetic operations very easily with abaci that, after being used, turn out looking like the numbers they represent. Addition, for instance, becomes a simple exercise of adding dashes, remembering that six dashes become a bar, and two bars become a dash one position to the left. Operations become dynamic and pictorial. Here are some examples:



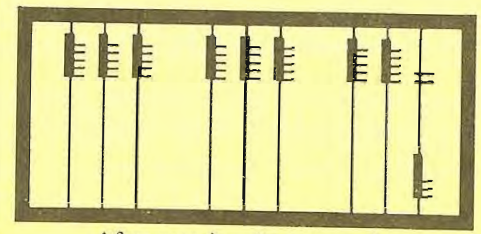
which corresponds to our

$$\begin{array}{r} 3 \\ +\ 2 \\ \hline 5 \end{array} \qquad \begin{array}{r} 5 \\ +\ 3 \\ \hline 8 \end{array} \qquad \begin{array}{r} 9 \\ +\ 4 \\ \hline 13 \end{array} \qquad \begin{array}{r} 16 \\ +\ 8 \\ \hline 24 \end{array}$$

An abacus in Dozenland does the third addition as follows: (Readers who are not familiar with abaci may want to skip this.)



Before starting to add



After entering the top number

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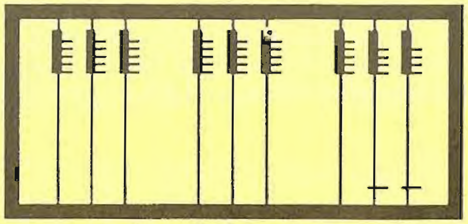

At the end of the operation

In the same way that it is very easy for us to multiply and divide by ten or by one-hundred, it is very easy for Dozlandians to multinly and divide by _▯ or by _▯▯. For instance, =≣▯L divided by _▯ is =≣▯.L

Time in Dozenland

In Dozenland, people divide the day into _▯ (twelve) hours, each of which is equal to two of our hours. Each hour is composed of _▯ (twelve) minutes, and each minute is composed of _▯ (twelve) seconds. This way, one hour in Dozenland has _▯▯ (one hundred and forty four) seconds. Their scientists continue dividing time into even (one 12th) shorter periods of time.

The hands of clocks in Dozenland move in the same direction as our clocks, but the hour hand is at the bottom of the clock at midnight and at the top at noon; it goes around the clock once in a day. The hour hand imitates the motion of the sun, and Dozenlandians - especially the ones that live near the Equator - can easily approximate what time of day it is by looking at the sun. Dozenlandians don't have to keep track of A.M. and P.M. Instead, by looking at their clocks, they know if it is morning or afternoon. Additionally, the numbers on the face of their clocks not only indicate hours but also minutes and seconds. What would correspond to midnight is denoted by the "0" hour. When Dozenlandians want to refer to "3 minutes after midnight" they write ▯.≡, not our confusing "12:30 A.M." Additionally, the two outer rings of their clocks tell the day of the week and the day of the month. The following figure shows how a Dozenlandian clock looks like at ⱡ.≡ o'clock.

The reader could observe, in the following drawing, that ⱡ.≡ (8.3) (duodecimal) hours correspond to $16\frac{6}{12}$ (decimal) hours, 1630 military time, or 4:30 P.M.

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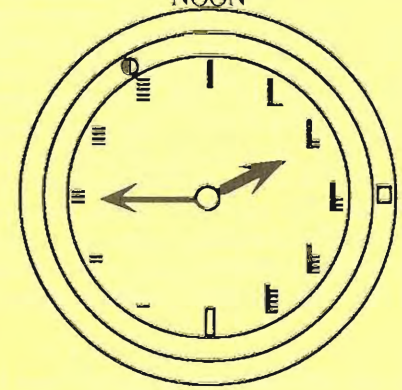
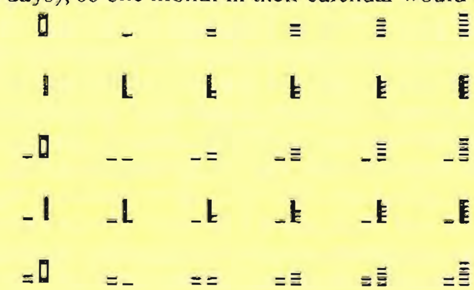


One week in Dozenland has 6 days: 4 days of work and 2 of rest. One month has 5 weeks (30 days), so one month in their calendar would look like this:



However, they very seldom use calendars since they are so simple that everybody has memorized them. The pattern is very easy to learn. Day 15 of any month, for instance, always falls on a third day of the week (our Tuesday). College professors and other people who have to plan schedules for activities that happen on the same days of every week have an easier time doing this than we do with our system of months having 30, 31, or even 28 days, and months not having an integral numbers of weeks.

One year has _▯ (twelve) months plus $5\frac{1}{4}$ "lucky" days at the end of the year when they have a carnival. This way their year has $365\frac{1}{4}$ days. Clocks in Dozenland have a special mechanism so that at the end of every year, they automatically go backwards $\frac{1}{4}$ of a day.

(Continued)

Examining further the clocks in Dozenland, we observe that they have two outer rings along which a small disk and a small square rotate. The innermost disk, called the day disk, goes around $2\frac{1}{2}$ times in a month. The first time around, the day disk is just a small circle (○). The second time, half of the interior of the circle has been blackened (◐). The third time, the whole circle is black (●). At the end of each month, a special mechanism automatically moves the day disk 1/2 of a circle so that it is in position 0 to start a new month. This way, the position of the day disk indicates the day of the week and the day of the month. The outermost square, the month square, rotates once in one year, and it is always a simple square (□). The clock in the above drawing, for instance, is saying that the time is 8.3 o'clock, that it is the 5th day of the third week (the 17th day of the month) and that it is the 9th month of the year.

Dozenlandians do not get confused saying that the year 1492 is in the 15th century, nor they will have the silly discussion that we will have when some people will celebrate December 31 of 1999 as the last day of the 20th century while others (the perfectionist who are right, but so what?) will celebrate it on December 31 of 2000. (Of course many people - the wise and happy - will celebrate both days.) This is why such confusions do not exist among Dozenlandians: One century in Dozenland is composed of -▯▯ years. The first century is called century zero, not one, and the first year of a century similarly is year zero, not one. Similarly as the first second, minute, and day are numbered zero. This way the year ≡E= is in the ≡ century, and the year =≡▯L is in the =≡ century. The last year of any century in Dozeland are the years that end in EE.

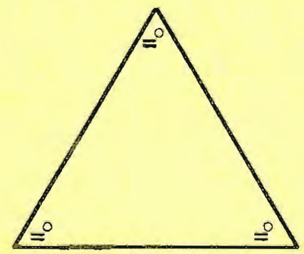
By the way, not everybody is wise in Dozenland. Some of them are saying that the world will end in the year =▯▯▯. But then they did not know what to say and became very confused when one of their mathematicians informed them that down here, where we poor ten-fingered creatures live, some of us are saying that the year to be be concerned about is the year 2000. But =▯▯▯ equals 3456, so now they do not know if doomsday is in the year 2000 or in the year 3456.

Geometry in Dozenland

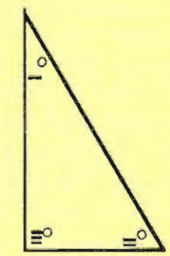
Geometry is also much easier in Dozenland, where a whole circle has -▯° (12 degrees) and a right triangle has ≡° (3 degrees). Each degree is divided into -▯ (12) minutes and each minute into -▯ (12) seconds. So one degree, like one hour, has -▯▯ seconds (that is, 144 seconds). Their Euclidean geometers claim that the sum of the interior angles of any triangle is 6 degrees (that is, 2 right

(Continued)

angles) and that each angle of an equilateral triangle has 2 degrees. Their books of trigonometry pay special attention to angles with degrees: 1, 1.5, 2 and 3 (Our 30°, 45°, 60°, 90°). It so happens that angles that are $\frac{1}{3}$, $\frac{1}{2}$ and $\frac{2}{3}$ of a right angle are very common (here and there) and that, unfortunately (for us), 100 is not divisible by 3. That is the reason why we don't say that a right angle has 100° - or some other more convenient number, but instead we say it has 90°. If we said that a right angle has 100°, we would have to say that one-third of a right angle (an important angle) would have 33.333...°. Dozlandians do not have this problem.



An equilateral triangle



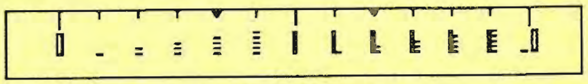
Half of the triangle on the left

Some figures in a text of geometry in Dozenland

For these reasons, it is easier for Dozlandians students to learn geometry and trigonometry than it is for our students. They find the measurement of angles similar to the motion of the sun and the motion of the hands in their clocks. For them, a right angle is related to the time between sunrise and noon; an equilateral triangle is related to the lapse of 2 hours. Also, they do not have to worry about the difference between our notation of angles with degrees, minutes, and seconds, and our corresponding decimal notation. Our angle 60° 30' 30", when written in decimal form, becomes the awkward 60.508333...°. In Dozenland, there is only one kind of notation: the duodecimal notation. They just simply write, for instance, L.≡=° (8 degrees, 4 minutes and two seconds). The kind of nuisances that our students have to deal with do not help much in their understanding of mathematics. Instead, they become distracting inconveniences.

Finally, the basic unit of length in Dozenland is the meter composed of -▯▯ (144) egrometers (similar to our centimeters). Their carpenters have the convenience (like our carpenters who use a decimal metric system) of easy conversion between units. Additionally, when they have to divide one meter in 3 equal parts, each part will measure exactly ≡▯ (48) egrometers (similar to the convenience enjoyed by our carpenters who use the English system); not the awkward 33.33... centimeters.

(Continued)



A ruler in Dozenland

The numerals that I have presented above are a variation of the Roman numerals. Instead of the Roman numeral for one " I " (clearly representing a finger) I propose a dash " – ". Romans used a "V" representing a hand or rather the five fingers in a hand. I propose a more simple bar " I " representing a (six-fingered) hand. My idea of numbering the seconds, minutes, hours, days, weeks, months, years, and centuries starting with zero, rather than with one, was inspired in the Mayan custom of numbering the days of their months (in their civil calendar) starting with zero; an idea that is often used in modern computer science.

I first began to put in writing these ideas about four years ago. At that time I did not even know that there was a Dozenal Society of America. A few month ago I came to teach at Nassau Community College and much to my surprise and joy I found out that many of my ideas are shared by other people and that many of the details of what I propose coincide of what these other people propose, which I find encouraging. If we independently came to similar results, they probably make some sense.

The clock face that I propose is very similar to clock faces that can be found in different issues of *The Duodecimal Bulletin*, and that are suggested in the logo of the Society. At least as early as 1953, Gene Zircle in "I' am a Dozener" Volume 9 , Number 1) presents such a clock face. All these clock faces however keep the zero on top. I have retained the words "hour" and "meter" to keep my exposition as simple as possible, but different terms have been proposed to denote units of measurement in a duodecimal system. The reader can find these terms in the *Manual of the Dozen System* published by the Society. ◻

FROM THE EDITOR

The following is the text of a letter to our Board Member Gene Zirkel from one of our newest members, Robert J. McGehee of Arizona, which presents some very dynamic ideas concerning the base twelve system of numeration:

Dear Gene,

I just received your reply and must say that I am very thankful to you for keeping me abreast of the current status of work on the dozenal system. I am eager to indulge in the work of the dozenal society as soon as possible. I would be interested in obtaining back issues of the dozenal bulletin/newsletter, particularly because I desire to see the kinds of dozenal mathematical tables that have already been developed and which ones still need to be developed so that I will not be re-inventing the wheel or duplicating past efforts. In other words, what tasks are most essential at this time? My other concern is copyrights. Does the dozenal society automatically receive the the copyright for articles published in the journal? I am only asking because when I write a piece for the dozenal society, I might wish to have it reprinted in some other publication at some future time.

Turning to my credentials, I must admit that I only have moderate expertise in mathematics. I have mastered basic algebra and elementary function theory but am weak in calculus and higher branches of analysis. On the other hand, I do possesss some specialized knowledge in the realm of number theory, particularly in algebraic coding and cryptography, and Have done considerable work with the binary system as well as the dozenal system. At present, my computer knowledge is somewhat limited, but I am beginning to master spreadsheet programs such as Excel and Lotus. I am also fortunate to have a younger brother who is now a professional engineer and computer programmer, and who has worked with me on various projects in the past. Although he is usually very busy, he may be able to devote some time to perform some jobs for the dozenal society if he is given a specific project to work on. I myself have some knowledge of graphics programs such as Superpaint and Macdraw, and I have been able to design different kinds of mathematical symbols with these programs, although I have found this to be a rather laborious process. I am aware that there is a program available for the Macintosh called *Fontographer* that is supposed to make the job of modifying and creating new fonts very easy, and perhaps a program like this will prove helpful if one needs to develop any new symbols for working in the dozenal system. Moreover, my brother is capable of designing new fonts by manipulating the pixels in graphics programs directly, without the aid of any special programs. With these preliminaries, I would now like to take the opportunity to offer some comments concerning the materials you have sent me thus far.

I notice that the dozenal society does not hold any position concerning what symbols are to be formally adopted to replace the decimal numbers ten and eleven. The asterisk * and octothorpe # do share the advantage of being on the same line of the typewriter keyboard as the other numerical symbols, though they must be accessed by the shift key. On the other hand, these symbols will not occur often enough in dozenal numerals for this to count as a significant handicap. Moreover, these symbols already occur on telephone dials, even if

(Continued)

they are utilized for purposes other than those for which they would be best suited! Therefore, I raise no objection to conforming to the present practice of the duodecimal bulletin when using dozenal numbers in my writings for the society.

I should point out that the usual convention for representing numbers larger than nine in number bases larger than ten is to employ the letters of the alphabet for the extra symbols needed, as is now done in the case of base sixteen (although without some controversy- Editor's Note). These letters can be found on any calculator (such as the TI-85) that furnishes hexadecimal conversions. Furthermore, but on the other hand, perhaps the most self explanatory symbols for ten and eleven, at least for someone in the English speaking world, are the letters "t" and "e". These symbols are perfectly easy to remember, and may be spontaneously employed any time it is necessary to perform dozenal calculations on paper, as will often be the case for those whose job already requires them to work with dozens and grosses, or for those at home who may wish to represent thirds and quarters of a cup in dozenal fractions (fractionals) for the purpose of doubling or tripling recipies--in short, for anyone who occasionally finds it more convenient to jot down numbers in dozenal form, regardless of whether they are already committed to converting our entire society to the dozenal base; self explanatory symbols such as t and e require no justification when dealing with either supervisors or family members!

I only bring this matter up because you have mentioned that so far attempts to manufacture a dozenal calculator have not been successful. Naturally if someone asked for a calculator manufacturer such as Texas Instruments or Casio to research, design, and exclusively manufacture an *exclusively* dozenal calculator I can well imagine that they would turn down this proposal as unprofitable! Of course, common sense would dictate that one would simply ask for a calculator to have an extra provision to the dozenal base; already most scientific calculators (including the TI-85) allow conversion between and among the binary, octal, decimal, and hexadecimal systems, so that getting the manufacturers to add the dozenal base would be a trivial matter, and the calculator would still allow all of the usual operations in base ten as before. In the dozenal setting, the first two letter keys A and B which serve for the hexadecimals would serve equally well for the dozenals. Moreover, a dozenal setting would have real value at the present time for the wholesale and packaging industries where dozens, grosses, and great grosses are still very much in use. Incidentally, since the degree/minute/second and decimal degree conversion functions on scientific calculators already possess the same circuitry as would be employed in a dozenal conversion setting, the cost of designing the dozenal feature would be minimal. The one drawback of such a calculator is that it probably would not allow one to access a great deal of higher mathematical functions when it was in the dozenal mode, but this is the same handicap that users of the hexadecimal and octal settings have to face. In any case, this inconvenience would be but a small price to pay for the opportunity to have a calculator that would at least permit basic calculations in the dozenal base!

Another possibility is to obtain a programmable calculator that would allow base conversions in any base from, let's say, binary to hexadecimal. Such a set-up would not only satisfy the dozenal community but would be of interest to number theory enthusiasts as well. Finally, regardless of the possibility of actually marketing a physical pocket

(Continued on page 19)

DOZENAL JOTTINGS

From Members and Friends

The Dozenal Society of America is delighted to announce that *Professor Gene Zirkel*, Professor of Mathematics at Nassau Community College and one of our most energetic Board Members is the recipient of *The Distinguished Faculty Award* for mathematics based upon his outstanding teaching, work as The Catholic Campus Chaplain, and two scholarly articles recently published in the *Bulletin*. (Please see his articles entitled *Proving Identities by a Change of Base* and *Significant Digits and Factorials*.) These articles were published in the *Bulletin* during the past two years.

The Society is delighted that such an outstanding individual who has furthered the cause of both education and duodecimals received his just due. Kudos to Gene! The Distinguished Faculty Award was conveyed to Gene and several other colleagues in various academic disciplines this spring.

The picturesque setting of Glassboro NJ, the home of the 1967 (117#;) *Summit at Hollybush* between The United States and The Soviet Union, will be the 1995 home for our Annual Meeting on Saturday, October 14, 1995. (Please see our related article entitled Announcement of our Annual Meeting.) *Rowan College of New Jersey* is the home school of our Society President *Jay L. Schiffman*. President Schiffman would like to express his appreciation to *Dr. Gary S. Itzkowitz*, Chairperson of The Mathematics Department for his generous hospitality. We cordially invite the departmental faculty and interested students as well as our membership who reside in the tri-state (PA, NJ, and DE).

(Continued on page 1✱)

From The Editor (Concluded)

calculator, one can always develop a "virtual calculator" entirely through software, as is the case with the little calculator that appears on the screen of your computer. Certainly such a program would not be very hard to write, and someone with the computer programming experience of my brother could easily manage it. Therefore, either one way or the other, it should be possible to design some sort of dozenal calculator available to members of the dozenal society and subsequently to the general public.

I will look forward to hearing from you.

Sincerely,

Robert J. McGehee

Editor's Note: The thoughtful ideas elicited by Robert are very much appreciated and embrace numerous ideas that have been discussed at both our Annual and Board Meetings as well as in our Bulletin. We look forward to much fruitful work in the future. It is indeed members like Robert that help to spread the gospel: *Twelve Is Best!*

□

On July 17th, NPR (National Public Radio) is scheduled to air a five minute interview regarding duodecimals by *Gene Zirkel* in its "All Things Considered" program airing from 5 PM to 7 PM in NYC, on FM 93.9 and AM 820 (WNYC), and in other cities throughout the country.

Dr. John Impagliazzo, (Member Number 27#;), Professor of Computer Science at Hofstra University in Hempstead, NY, recently published a new book which furnishes a novel approach to the first computer science course. John co-authored this work with his Hofstra colleague *Dr. Paul Nagin*. The book is entitled COMPUTER SCIENCE: A Breadth-First Approach With C and is published by John Wiley and Sons of New York. John is a Board Member of our society and was extremely active in society affairs during the 1980's. He presented a number of well received talks at our Annual Meetings. Kudos to John and much success with his new book! John spoke about this and another of his new books recently at *The Metropolitan New York Section of The Mathematical Association of America* held at Hunter College (CUNY) in New York City.

John D. Hansen, Jr (Member Number 30#;) of Vista, CA writes our treasurer Alice Berridge the following warm note with his dues and donation to DSA:

Dear Alice,

Warmest greetings to a fellow twelve-lover. Thanks for keeping up the good work. Auspiciously, 11✻3; (1995 dozenally) is divisible by four of the first seven odd primes. (3, 5, 7, and 17;)

Yours truly,

John D. Hansen, Jr

Gene Zirkel, Member of the Board and past President of the Dozenal Society of America was interviewed by Staff Writer *Guy Gugliotta* of The Washington Post in his Capital Notebook feature on Tuesday, June 20, 1995. The column went out over the wire services and was represented in several places including Colorado and Alabama. □

Errata.

Whole Number 74 was Volume 38, Number 1, (it mistakenly read Number 3). Sorry about that.

Remember – your gift to the DSA is tax deductible.

NEW LETTER

We recently received a very interesting letter from *John R. Porter* of Shizuoka City, Japan. The text of his letter follows:

Dear members of the Dozenal Society:

I wrote an essay about twenty years ago which among other things advocated the use of base twelve. I just recently became aware of the existence of the Dozenal Society. I would very much like to know more about your organization, to read its literature, and to be cognizant of its activities and history. I would also desire to acquire knowledge of additional organizations related to such causes if they exist.

I am enclosing a much shorter essay which I wrote recently. It is copyrighted for the purpose of proving my authorship, not to prevent unauthorized printing. On the contrary, I welcome the spread of these ideas. You are welcome to print this essay in your journal if you wish. I have also written a number of essays concerning proposals unrelated to mathematics.

I wish you good luck with your work.

Yours truly,

John R. Porter

Editor's Note: We are delighted to receive interest in our work and reprint John's fine article below.

Defects in Conventions of Communication
John R. Porter

The conventions of society are the result of tradition, not of research to determine what is most practical. There are often obviously better alternatives.

Conventions entail the following seven categories:

A. alphabet
B. planned language
C. Number base
D. units of measure
E. manner of expressing qualities
F. manner of expressing areas and volumes.
G. standard time

(Continued)

A. alphabet
The conventions of writing most languages are far from ideal. The twenty six letters of the Roman alphabet are insufficient in number for a phonetic alphabet. The shapes of symbols employed in writing are often too similar to other symbols. The distinction between capital and lower-case letters is unnecessary.

B. planned language
A carefully planned or "artificial" language should be designed as an international language. This was the purpose of Esperanto. I am not convinced that Esperanto was well planned, although I am not well familiar with it. I suppose that before such an international language can be accepted, a much greater number of people must recognize its advantages and must have the confidence in their own ability to learn it. I would favor a language which is uninflected, employs particles to indicate case, and allows omission of information about number and tense.

C. Number base
Society should abandon base ten as a number system. Bases two, four, six, eight, twelve, and sixteen are plausible alternatives. Bases six and twelve are convenient for division by two or three. Base two digits written in groups of four can easily be read as base four or base sixteen. Bases two, four, and sixteen are very easy to convert into one another (Just convert individual digits-Editor).

D. units of measure
The units of measure used for time and angles are not the most conceivable units. The most easily grapsed unit of angular measure appears to be one revolution. The most easily grapsed unit of spherical surface appears to be the spherical surface of one complete sphere. The only units of time necessary in the lives of most people are years and days and occasionally cycles of the moon. They are separate independent cycles and they should be treated as such. The calendar should not make each year fit into a whole number of days. Units of measure to express fractions of these basic units are unnecessary.

E. manner of expressing quantities
Rather than insignificant zeros, exponents of the number system base should be utilized. For example, use $2 \times B^9$ instead of 2,000,000,000. (In the case of base ten, "B" would equal ten.) This would mean that units of measure would not be changed merely to prevent numbers from becoming large or small. By the present conventions, we use different units of measure, such as millimeters and kilometers, to express distances which are greatly different from each other. Only one units of measure for each physical concept is necessary.

F. Manner of expressing areas and volumes
The cube root of a volume is easier to visualize rather than an actual volume. For example, $(2m)^3$ is easier to visualize than $8m^3$.
The square root of an area is easier to visualize than an actual area. For example, $(3m)^2$ is easier to visualize than $9m^2$.

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ANTI METRIC PROTEST SUCCEEDS!

Jean Kelly
New York, NY

The Clinton Administration backed down on its plan to *force* us to put undesirable decimal metric signs on our highways.

More than 1140; (2200.) people wrote the Federal Highway Administration opposing the idea. *Newsday* referred to this action as "part of a national metric resistance."Several members of congress have introduced bills to abolish the action of changing our signs to the maladroit decimal metric system.

Never in the course of history has any country — not even France where this abomination started — willingly converted to awkward decimal metrics. It has always been under the force of law with the threat of penalties.

When will advocates of the second-best decimal metric system stop trying to impose their limited views on those whom they cannot convince with their impoverished logic?

□

New Letter *(Concluded)*

G. standard time
Time zones should be abolished and universal time should be employed universally. It would not be difficult for people who do not travel to get accustomed to. For the people who do travel, it would be an advantage.

Final Comments

These changes would involve some inconvenience, and it is debatable whether making the changes is economically beneficial. On the other hand, I believe it would be worth the inconvenience. The changes are likely to produce a spirit of improving society rather than just letting it drift and take its own course, surrendering to the force of tradition. This spirit is difficult to evaluate economically. Many people would feel better about living in a society that is attempting to improve.

Note: This article was reprinted save for minor editing from *Conventions* 94.09.18 with permission from the author.

□

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition--("Who needs a symbol for nothing?")--the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used.

A parallel to today seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

□