DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

ISSN 0046-0826

THE DUODECIMAL BULLETIN 73;

DUODECIMAL CHRONOLOGY

- See Page 4

1994

1994 ANNUAL MEETING

October 15, 1994

- See Page *;.

Volume 37;
Number 2;
1994
11*2;

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00 (US) for one calendar year. Student Membership is $3.00 per year, and a Life Membership is $144.00 (US).

The Duodecimal Bulletin is an official publication of the DOZENAL SOCIETY OF AMERICA, INC., c/o Math Department, Nassau Community College, Garden City, LI, NY 11530.

BOARD OF DIRECTORS OF THE DOZENAL SOCIETY OF AMERICA

Editorial Office:
923 Spruce Street
Philadelphia, PA 19107
(215) 922-3082

THE DUODECIMAL BULLETIN

Whole Number Seven Dozen Three

Volume 37; Number 2;

11*2;

IN THIS ISSUE

FOUNDED 1944

HISTORY OF THE DUO-DECIMAL, BASE 12, DOZENAL IDEA, CHRONOGICALLY

Fred Newhall
Smithtown, NY

Ever since the ancient beginnings of arithmetic, I am certain many philosophers have considered other bases than ten, and have realized the efficiency of Base 12. So I'd like to list chronologically actual written excerpts of references to duo-decimals by date, author, book and quotation:

2500-606BC Babylonians, see Encyclopaedia Britannica V3, 107d

1585 Simon Stevin - L'Arithmeticue
[But see this Bulletin, Whole Number 52; p 17; -Ed.]

1665 Blaise Pascal - De Numeriis Multiplicibus

1670 Bishop Joannis Caramuel - Mathesis Biceps, Vetus et Nova (bases to 12)

1682-1718 King Charles XII of Sweden

1687 Jordaine 2 books, see Books About Duo-Decimals

1740 Christopher Frideric Vellnagel - Numerandi Methodi (bases 2-12)

1747 Johann Albert Berchenkampf - Leges Numerandi Univerales (bases 2013, 15, 24, 30)

1760 Georges Louis Leclerc, Compte de Buffon - Essai d'Aritmetique Morale

1784 Encyclopedie Methodique - chapter on Echelles Arithmetique

1790 Pierre Laplace - Systeme du Monde "In truth, our arithmetical scale is not divisible by three and four, two divisions whose simplicity render them very common. The addition of two new characters is sufficient to procure this advantage, but a change so important would have been infallibly rejected with the system of measures which was attached to it. Moreover, the duodecimal scale has the inconvenience of requiring that we retain the products of twelve numbers which surpasses the ordinary length of memory to which the decimal scale is proportionate."

1799 J.F. Montucla - Histoirie des Mathematiques (bases include duodecuple)

1801 Haser - Anleitung zum Rechnen nach dem Duodezimal- system

1808 Garnier-Deschenes - Recherches sur L'origine du Calcul Peter Barlow - Duodecimale

(Continued)

1810 Zehner - Die Zwolfsysteme zum Zahlen und Rechnen
Baron von Humboldt -

1822 John Playfair - Base du Systeme Metrique Decimal

1844 Pujals de la Bastida - Filosofia de la Numeracion - - Napoleon Bonaparte-

1849 Breithaupt - Das Duodecimal-system Fischer, Cassel

1855 Sir Isaac Pitman -

1858 Cautier - 3 books, see Books About Duo-Decimals.

1866 Thomas Leech - Dozens versus Tens pages, Robert Hardwiche, London (only copy at Columbia University cannot be loaned out).

1871 Horstig - Das Arithmetische Duodecimal-system, Leipzig

1875 Nystrom - 2 books, see Books Mentioning Duo-Decimals

1884 Perry - The American System of Mathematics

1884 Grunwald - Saggio di Arithmetica, Verona

1884 Totten - An Important Question in Metrology

1897 Herbert Spencer - (provision in his will to promote duodecimals) Against the Metric System, Appleton, NY

1911 Encyclopaedia Britannica V3 107d, V18 137a

1913 Elbrow - The New English System of Money, Weights, etc

(Continued)

The DSA does **NOT** endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus 1/2 = 0.5 = 0;6.

1926	D'Autremont - The Duodecimal Perpetual Calendar
1934	F. Emerson Andrews - Atlantic Magazine "Excursion - -" Many articles and Books.
1935	Norland - The Twecimal System
1937	J. Halcro Johnston - The Reverse Notation
1938	Terry - 3 books, see Books About Duodecimals
	F. Howard Seely
	Chas. Q. DeFrance
1944	Meeting to incorporate D-D Society under laws of NYS.
1944	George Bernard Shaw - Letter to London Times "Basic English is a natural growth which has been investigated and civilized by the Orthological Institute on the initiative of Mr. C. K. Ogden, whose years of tedious toil deserve a peerage and a princely pension. The only job comparable to it is that of an American George S. Terry, who has given us the tables of duodecimal logarithms."
1949	GBS letter to a music publisher "I am greatly taken by Mr. G's (Godjevatz) plan. - - - Its adoption - - - would teach people to count duodecimally with two new digits: - - - as duodecimal arithmetic is a coming reform. - -"
1944-	All the officers and contributors to DSA and to DSGB and the Editors of their periodicals!
1955	Essig - Douze: Notre Dix Futur
1985	Pendlebury and Thomas booklets; Dixon - Reciprocals.

I would like to add to or correct this chronology with exact quotes and accurate dates, so I welcome any suggestion.

□

A SURPRISING CYCLE OF LENGTH 12

Monte J. Zerger
Adams State College
Alamosa, CO 81102

Let the first four terms of an infinite sequence be *any* four real numbers x_1, x_2, x_3, and x_4, such that $x_2 \neq x_1 x_3$. Define all other terms recursively by

$$x_n = \frac{x_{n-4}\, x_{n-1}}{x_{n-4}\, x_{n-2} - x_{n-3}}, \quad n = 5, 6, 7, \ldots$$

This sequence will endlessly repeat a cycle of 12 numbers. That is, $x_k = x_{k+12}$ for k = 1, 2, 3, ...

For example, if one begins with the first four natural numbers, then the 5th term is ((1)(4))/((1)(3) - 2) = 4, and the sequence formed is

1, 2, 3, 4, 4, 8/5, 3/5, 1, 5, 8, 12/5, 4/5, 1, 2, 3, 4, . . .

It is easy to verify that this cycle is independent of the original four numbers chosen, if one does not mind the necessary algebra. I took the easy route and enlisted the aid of the mathematical software package, DERIVE, to show that if x_1, x_2, x_3, and x_4 are a, b, c, and d, then

$$x_5 = \frac{ad}{ac - b}$$

$$x_6 = \frac{abd}{(ac - b)(bd - c)}$$

$$x_7 = \frac{ac}{bd - c}$$

$$x_8 = ac - b$$

$$x_9 = bd - c$$

$$x_{10} = \frac{bd}{ac - b}$$

(Continued)

$$x_{11} = \frac{acd}{(ac - b)(bd - c)}$$

$$x_{12} = \frac{ad}{bd - c}$$

$x_{13} = a$, $x_{14} = b$, $x_{15} = c$, $x_{16} = d$, ...

According to Martin Gardner's *The Magic Numbers of Dr. Matrix* (p. 305), this fact was first discovered by J. H. Conway.

□

ERRATA

Oops! We goofed.

(a) In Volume 35; Number 1; page 15; line 3 of the first paragraph should read F.C.S.

(b) In Volume 35; Number 2; page 15; Bill Crosby points out that he worked at *Darthmouth College*. Incidently, Bill is the author of the item on "Unicals" on page 9; Volume 1; Number 2; of this *Bulletin* printed back in 1945!

(c) In Volume 35; Number 2; page 6;, the last line sum of 7 and 36 should, of course, read 43 and not 36. (We are in the decimal base here.) Similarly, four lines above, the reference to 'the sum (38)' should likewise be 43.

Incidently, does 7 + 36 = 38 in any base? How about in modular addition?

The above two questions were posed by Gene Kelly whose resolution is printed below:

(i) There is no base in which 7 + 36 = 38.
(ii) $7 + 36 \equiv 38$ (modulo 5).

(d) In Bulletin 72; page 2, Class of 1996, our third member of the board is *Tony Scordato.*

(e) The cover of Bulletin 72; should be *In Memoriam: A Duodecimal Legend.*

(f) In Bulletin 72;, the title of Jay L. Schiffman's paper should be *Duodecimal Combinatorics.* The top heading on pages 9;-13; is incorrectly spelled.

MECHANICAL SIMULATION

Ed Nu

All your life you've been running on 2 cylinders, of a powerful 4-cylinder mathematical engine! One and two divide into ten evenly, but your third and fourth cylinders would work fine if your engine were tuned to base twelve. One, two, three, and four divide into twelve evenly.

You could own a shiny limousine if you would trade in that sickly old-fashioned worn-out base 10 heap! We are selling a Dozenberg with a powerful engine; the car of the future; guaranteed to outrun any Infiniti! It is the equivalent of a 12-cylinder engine at only a fraction of the cost in learning and teaching. Take a trial run in our demonstration model and you'll prove for yourself the smooth-handling comfort of a luxury mathematical base. You will agree that your low priced effort will be worth the value in the long run. Discover Cards accepted.

Contact Dozenal Society of America at our convenient showroom in Garden City. Nassau Community College Library has an Archives, Room 314, where you can learn all about our model mathematical base. Our salesmen are available at your service to provide brochures and friendly information concerning our worldwide membership. When in England, visit the Dozenal Society of Great Britain in Moulsford, Oxford; Denmead, Hampshire; Leigh-On-Sea, Essex; and others. We have "sales reps." (members) in Diamond Harbor, New Zealand; Armdale, Australia; Sokoto, Nigeria; Bombay, India; Beijing, China; Espoo, Finland; Paris; Sao Paulo, Brazil; Lisse, Holland; and other Xotic places. Of course, we have many in Canada, Alaska, Hawaii, and interstate USA.

Our older model, the XE, even though it still excels any other base, is being replaced by our phone-compatible model, the *#. You will enjoy driving any other of our custom-built models, as well, since they all have better eX-El-eration than any other number systems on the road.

Dozen it make sense to promote the best? Just fill out the sales slip on the back of any of our publications and include the easy payment in an envelope to:

DOZENAL SOCIETY OF AMERICA
Nassau Community College, Math Dept.
Garden City, L.I., New York 11530

□

Do you know of a friend who would appreciate a sample copy of our *Bulletin*? Just send us his or her name and address and we'll be happy to oblige.

ANNOUNCEMENT OF OUR ANNUAL MEETING

Saturday, October 15, 1994
Six Brancatelli Court
West Islip, LI, NY
10:30 AM

The 1994 Annual Meeting of The Dozenal Society of America will take place on Saturday, October 15, 1994 at the home of Gene & Pat Zirkel, active board members of The Dozenal Society. This location will constitute a change of venue from our traditional setting at Nassau Community College in Garden City, LI, NY.

As in previous years, we will commence with The Board of Directors Meeting to be immediately followed by The Annual Membership Meeting.

We are looking forward to two presentations in the afternoon session. Rafael Marino, a mathematics professor at Nassau Community College who is Vice President of the Society will present his paper entitled *"IF WE ONLY HAD TWELVE FINGERS"* which promises to be illuminating. Jay L. Schiffman, President of the Society and a mathematics faculty member at Rowan College of New Jersey's Camden Campus will speak on the topic entitled *"THE PERSONALITY OF DUODECIMAL INTEGERS FROM ONE TO ONE GROSS."*

For a good time both educationally as well as socially, be sure to mark this date on your calendar. We will be extremely disappointed if you miss our Annual Meeting. □

THE FOLLOWING ARE AVAILABLE FROM THE SOCIETY

1. Our brochure. (Free)
2. "An Excursion in Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, October 1934. (Free)
3. *Manual of the Dozen System* by George S. Terry. ($1;00)
4. *New Numbers* by F. Emerson Andrews. ($10;00)
5. Dozenal Slide Rule, designed by Tom Linton. ($3;00)
6. Back issues of the *Duodecimal Bulletin*, as available, 1944 to present. ($4;00 each)
7. *TGM: A Coherent Dozenal Metrology*by T. Pendlebury ($1;00)
8. *Modular Counting* by P.D. Thomas ($1;00)
9. *The Modular System* by P.D. Thomas ($1;00)

LETTER TO THE STONY BROOK ARCHIVES

The following is the text from a letter written by our late President Fred Newhall regarding a very dynamic work on Dozenals:

Wednesday, March 24, 1993
Mr. Evert Volkersz, Archivist, Special Collections
Main Library
State University of New York at Stony Brook
Stony Brook, N.Y. 11790
re: Thomas Leech "Dozens vs Tens, etc." 1866
and Dozenal Index

Dear Mr. Volkersz,

This is the oldest book in English published on this important fundamental idea of Base 12 Arithmetic. The only copy in this area was carefully preserved in the Columbia University Library Archives.

We were fortunate in having one of our members in The Dozenal Society of Great Britain duplicate a copy from the Cambridge University Library. He sent us the copy which I have had made into several new books. I sent one to Columbia University , and we have one in our Dozenal Office at Nassau Community College.

Although the book was written 125 years ago, the ideas expressed in the book never get old, so that our Society feels that the book was an important achievement in the history of mathematics. Please register this gratis copy in your library for general circulation.

As you may remember, I have a 3-volume looseleaf book "12 Is Best" in your Special Collections Department. As a supplementary volume to it, I should like to donate this copy of my Dozenal Index, also in looseleaf form so that it also can be updated occasionally.

This is a Comprehensive Index to all of our 149 publications by both the American and Great Britain Dozenal Societies since the year 1945, so is a valuable reference for research on Base 12 mathematics.

We appreciate your generous cooperation.

Fred Newhall

□

Do you have an idea to share with our members? Why not submit an article to the *Bulletin*?

THE NON-UNIQUENESS OF NON-INTEGRAL BASES

Brian M. Dean
Ohio

Most readers of the Duodecimal Bulletin are very familiar with integral number bases (bases like 2, 3, 4, etc.) Depending on our mathematical skills, we have little trouble understanding and/or using such bases.

Some time ago, Gene Zirkel wrote a series of articles entitled "Strange Bases"[1] which dealt with bases that are not positive integers greater than 1. I will not go quite as far as Gene did in that I won't consider bases that are negative, or complex. I will be restricting my paper to a discussion of bases that are positive, but don't fall into the category of being integral. The most interesting property of such bases is that any number can be represented in at least two totally different ways and this will be the main emphasis of the paper.

In 1984, when I was a student at Bowling Green State University, a good friend of mine, Steven Gunhouse, brought to my attention that the base

$\frac{\sqrt{5}+1}{2}$ (the well known GOLDEN MEAN)

had the property that 1 = 0.11 in the base. This signifies that this base, which we will rename as base b,

$(b = \frac{\sqrt{5}+1}{2})$

has the property

$\frac{1}{b} + \frac{1}{b^2} = 1.$

I will let the reader verify this. Also, since 1 = .11, it would make sense that .11 = .1011; for .11 = .1 + .01 and .01 = .01 x .11 (because 1 = .11).

.01 x .11 = .0011, so therefore .11 = .1 + .0011 = .1011. You can then verify that .1011 = .101011 using the same process and in fact you can carry this process out as many times as you desire and you will conclude that 1 = $.1\overline{10}$

[1]The Duodecimal Bulletin, Vol. 2*, No.2, p. 6 (Part I of article); Vol. 2*, No. 3, p. 10 (Part II); and Vol. 2#, No. 1, P. * (Part III).

(Continued)

One can also verify that $.1\overline{10} = .01\overline{11}$. I leave the proof of this up to the reader. This shows that there are an infinite number of ways in which 1 can be represented in base

$\frac{\sqrt{5}+1}{2}$

Now I will expand this idea to include any positive non-integral base. The idea of the same number being represented in more than one way should not be totally surprising. Even in our favorite base, 1 could be represented as the repeating fraction 0;###. However, if you use any non-integer base the different representations are usually quite strange.

Let us examine the base b = #;6. (I will use a period (.) for a fraction point in base #;6, and a semicolon (;) if the number is in our favorite base.) If you observe a typical two digit number in this base (call it ab), then both a and b could be represented by any of the digits ranging from 0 to #. This generates 10; different possibilities for each digit and therefore ab could possess 100; different representations.

If we examine the number $##_b$ it is #5;6 in our favorite base, then we notice this is less than 100;. Now $(#;6)^2$ = #0;3. Therefore in base b^2#;6, 100_b < ##;. Since the value of 100_b lies somewhere between 0 and ##, it is reasonable to assume that there is some numeral on that interval that equals 100_b even though we might need some sort of expansion for it (such as ab.cde... where the ... connotes that the expansion continues past what is shown (for example, $\sqrt{2}$ = 1;4#...)). In fact if you toy with this base enough, you will find that 100 = #5.872191*23...

I will next briefly discuss a method of obtaining diverse representations of different numbers given a non-integral base. First, I will discuss a procedure concerning how I usually go about converting normal things (like base dek to base 10;). Most of you are probably familiar with doing this, but I would like to go through it so that the reader might more easily modify my methods to suit his or her own needs or interests.

First of all, if I am given a number such as 56.34 in base dek to convert to our favorite base, I treat the 56 and the .34 separately. I convert 56. to 48; in the usual way, but the way I treat .34 might be slightly different so I will explain it. The idea is that we want to move the dozenal place over one place to look at each integer. So if you imagine .34 symbolically as a dozenal number, the way you would look at the first digit after the dozenal point is to multiply by 10;. Since we don't have an easy algorithm to multiply a base ten number by a base twelve number we are stuck with using 12. instead of 10;.

Now .34 * 12 = 4.08, so our first digit after the dozenal point is 4. This gives us 56.34 = 48;4... Now we subtract the 4 to get .08 and repeat the process again to get the next digit. Since .08 * 12 = 0.96, we obtain 0 as our next digit. We then repeat the process In light of the fact that .96 * 12 = 11.52, our next dozenal digit is #. You can repeat this process to yield as many dozenal digits as desired. (56.34 = 48;40#62*68...)

(Continued)

Let us next consider the base $\sqrt{2}$. Since $\sqrt{2}$ = 1;4#... we could consider 1 as a fraction instead of an integer and look at the first number after the . (in base $\sqrt{2}$) by multiplying and getting 1;4#... and then continuing the process to generate additional numbers. If you continue this for a while you will eventually obtain .1001000001001000000001000001... and this is equal to 1 if you are employing base $\sqrt{2}$.

If we were to decide not to subtract the 1 right away and multiply by $\sqrt{2}$ again we have 2, but we can treat this as a 1 + decimal and subtract 1 to get a 1 remaining. Since we delayed subtracting 1, our first number after the point is 0 and if we continue the process we get .011001000001001000000001000001...

The question is then the following: Can we go indefinitely before we subtract 1? The answer is no, because if we go too far we will obtain such a large number that after we subtract 1 and then multiply by $\sqrt{2}$ again, we will arrive at a larger number than the one we initiated, and this will yield an incorrect response.

Thus the largest number we can use in this algorithm is the solution to the equation

$\sqrt{2}\,(x - 1) < x.$

Thus we could go as high as 3. Then if we subtract 1 to yield 2 and then multiply by $\sqrt{2}$ we will end up with something less than 3. If we continue the process we will obtain something that will converge and give us a correct answer. If you try this with 1 and go as high as you can before subtracting 1 all the time you will get .00110111110110111111110111101111101110...

One of the things that interested me about Gene's articles, "Strange Bases", is the fact that you can devise an algorithm for adding, subtracting, multiplying, and dividing in a base such as $\sqrt{n}$. For those of you who have forgotten, or for those of you who have not read the article, the manner in which you add in base $\sqrt{n}$ is exactly the way you would add in base n, except that if there is a carry, you carry the number over two places instead of one.

What I have been experimenting with is taking two different representations of the same number in a certain base and using the algorithms for adding and dividing to come up with a different representation.

(Continued)

For example, we could add two different representations of 1 in base $\sqrt{2}$ and then divide by 100 (which is the same as 2 in this base and has the effect of moving the point over two places, just as in any other base). If we try this with 1 and .1001000001001... we arrive at

$$\frac{1 + .1001000001001...}{100} = .011001000001001...$$

which is a number that we obtained earlier. If you use the algorithms given in the article, some representations for 1 might be easier to derive than using the method above.

In closing, I would like to say that perhaps the best way to see different representations of different numbers in certain bases is to experiment on your own. I hope that these different representations will make these strange bases more attractive rather than less so (but perhaps this is too much to hope for!)

Most people, I feel, would find representations such as 1 = .1001 unattractive. This is because they might feel either that the answer which they produce is wrong, or conversly, that any possible answer would be correct (both of these feelings are unfounded). I hope that at the very least, this paper will stir some interest in studying strange bases in the future.

[1]The Duodecimal Bulletin, Vol. 2*, No.2, p. 6 (Part I of article); Vol. 2*, No. 3, p. 10 (Part II); and Vol. 2#, No. 1, P. * (Part III).

□

JURASSIC VIDEO

Mark Schubin

NEED A DATE FOR THE ORIGIN OF DIGITAL? How about 395,000,000 years ago? That's about 205 million years earlier than the beginning of the Jurassic Period. *Crossopopterygii*, lobe-finned fish, predate not only mathematicians but also dinosaurs.

Sometime in the Devonian period, it seems, a fish was born with bony fins that eventually evolved into paws, feet, and hands. Able to walk on land, the descendants of that fish also developed lungs, which is why the modern loalach of the opening paragraph of this column, a fish with lungs but no gills, will drown if kept under-water.

The fin bones of *Crossopterygii* included those that would eventually form what the ancient Romans would call *digiti*, fingers and toes. There were certainly many genera that evolved between the Devonian *Crossopterygii* and the Roman *Homo*, but while experiments have proven that modern animals have a sense of number, there's no evidence that they can count. Human's can, often assisting themselves by touching parts of their bodies.

There have been cultures that have counted knees, elbows, and other readily identifiable body parts. Today, we usually count only on fingers, but the French *quatre-vingts* and the English "score" indicate that it wasn't all that long ago that we stopped using our toes, too.

Ten fingers led to the base-ten number system, but, other than its human-hands orientation, there's not much to recommend it. Near Bombay, the base-five Maharashtra number system can be counted on one hand. The ancient Sumerians used a base-60 (sexagesimal) system, vestiges of which remain in the way we tell time and use a compass. Sixty has the advantage of being divisible by all six of the first positive integers.

In the Eighteenth Century, the French mathematician Joseph Lagrange proposed a number system based on primes, indivisible by anything but themselves and one (but was also responsible for the metric system's decimal base). His contemporary, naturalist George Buffon, argued in favor of shifting to base-12 (duodecimal) to retain at least divisibility by all positive integers through four; the Duodecimal Society of America keeps the latter idea alive with its numerals from zero through nine plus X and E (the last two pronounced "dek" and "el").

Buffon and Lagrange both had the advantage of printed numerals, said to be Arabic, though, in fact, the symbols we use for four and five were developed in Europe, our zero came from India, and, can be traced to other non-Arabic cultures. About the only thing Arabic numerals have in common with each other is that they are not finger-based, like Roman numerals (I is the shape of a finger, V is a hand with the thumb spread, and X is two hands wrist-to-wrist.)

(Continued)

The term *Arabic numerals* is simply one of many we use to honor abu-Ja'far Mohammed ibn-Musa al-Khowarizmi, an early-Ninth-Century mathematician in the court of Mamun in Baghdad. Latin translations of his works brought not only Arabic numerals but also much of medieval Europe's mathematical knowledge. The first word of his book Al-jabr w'al-muqa-balah became what we now call "algebra"; the last part of his name is familiar to those working in digital video as "algorithm."

Even today, however, machines cannot always read hand-written Arabic numbers accurately. A modern European might write a one in a manner looking remarkably like an American's seven. Accordingly, Europeans place a dash in their sevens, but Americans don't. Computers are easily confused by those variations and others. Is a dash very close to a seven part of a numeral, or is it a minus sign? Is a strange-looking cross a crudely drawn four or should it be a plus sign? The only number system clearly distinguishable to a machine is one consisting only of two states: on or off, yes or no, present or absent, one or zero.

No one can say who first considered a base-two (binary) number system. The concept appeared in Khowarizmi's work, but it had certainly also been known to earlier thinkers. Long before the first electronic computer graphics or music devices, machines, operating on binary mathematical principles, created both graphics and music. One such music machine, an ancient form of organ, was excavated at a Roman archeological site just north of Budapest.

It wasn't until the Twentieth Century, however, that binary mathematics was applied to the recording, transmission, or manipulation of sounds or pictures. The problem was largely technological. First, there weren't even electronic sounds (let alone pictures) to record, transmit, or manipulate until the end of the Nineteenth Century. Second, the circuitry required to digitalize even a simple telephone call didn't exist until shortly before that feat was achieved, in 1939 (it was the analog-to-digital converters that slowed things down; digitally generated speech and music predated the digitilization of sound, just as computer graphics predated the digital video timebase corrector and international standards converter).

Scientists working on digital signals weren't even able to use today's common term for the little pieces of information they dealt with until 1948. In July of that year, in the *Bell System Technical Journal*, Claude Shannon, considered by many the creator of information theory, credited J.W. Tukey with suggesting a contraction of the words *binary digit* into *bit*. But the word *digital* definitely comes from the Latin *digitus*, which means fingers and toes, and they all came from a fish that lived hundreds of millions of years ago.

Editor's Note: This article has been reprinted with permission from both the author and the magazine VIDEOGRAPHY SEPTEMBER 1993

□

PUZZLE SOLUTIONS

Charles Ashbacher
Hiawatha, IA 52233

The following is a proposed solution to the base number puzzle that appeared in Bulletin 69;:

In every base b, there is always a three digit number which is equal to the sum of its digits multiplied by (b+1). In base two the number is 110 since
(1 + 1 + 0) * (11) = 10 * 11 = 110.

Can you find the three digit numbers with this property for all the bases from three through do-one?

Here is a proposed solution:
The numbers are:

Base	Number
2	110
3	121
4	132
5	143
6	154
7	165
8	176
9	187
*	198
#	1*9
10;	1#*
11;	1 10; #

This pattern represents an algebraic identity. If b is the base, then

1 (b-1) (b-2) = b^2 + b(b-1) + b - 2 = $2b^2$ - 2

1 + (b-1) + (b-2) = (2b - 2)

and (2b-2)(b+1) = $2b^2$ - 2b + 2b - 2 = $2b^2$ - 2.

Editor's Note: Gene Zirkel also provided a fine solution in *Bulletin* 6*;. □

Remember – your gift to the DSA is tax deductible.

PUZZLE CORNER

The Fall 1195; issue of the *Bulletin* has an interesting problem:
find dozenal numbers (the Society prefers "dozenal" to "duodecimal") that are exactly twice as large as their decimal conterparts. One answer is 11788. Quite right:

$(1)(12^4) + (1)(12^3) + (7)(12^2) + (8)(12) + 8 = 23{,}576 = (2)(11{,}788)$.

Other solutions, the *Bulletin* says, are

11790, 11818, 11820, 12298, 12328, 12330, 24658, 24660, 25168, 25170, 25200, 36988, 36990, 37528, 37530, 38038, 38040, 49858, 49860.

Are those *all* of the solutions? What about dozenals that are three of four or five times as large as base-ten numbers with the same digits? Dozenals can raise interesting questions.

Solution by Charles Ashbacher of Cedar Rapids, IA.:

On Page 27 of the book *Mathematical Cranks*, Underwood Dudley gives the problem

Find dozenal numbers that are integral multiples of their base 10 counterparts.

He provides a list of solutions where the multiplying factor is 2.

I have found the additional solution

$$34798_{12} = (2)(37498_{10})$$

for a multiplying factor of 2 and the two solutions

$$158662568_{12} = (4)(158662568_{10})$$

$$159006368_{12} = (4)(159006368_{10})$$

where the factor is four.

(Continued on page 19;)

"Each one teach one."
-- Ralph Beard, Founder of the DSA

SOME IDEAS FOR THE READER'S PERUSAL

Gene Zirkel,
Nassau Community College,
Garden City, LI, NY 11530

The concept of weights and measures is vital in various situations including governmental studies. To cite an example, many cities incorporate a bureau of weights and measures as part of their operation. The following brain twisters will serve to test the interested reader's knowledge of such ideas:

A WEIGHTY QUESTION:

Which weighs more: an ounce of feathers, an ounce of silver, or an ounce of pearls?

Answer to A WEIGHTY QUESTION:

A pound of feathers is heavier than an ounce of silver which, in turn, is heavier than an ounce of pearls! (If you can't figure out why this is so, see below.)

WHICH IS HEAVIER?

Feathers are weighed using the Avoirdupois scale, silver is weighed using the Troy scale, and pearls are weighed using the Diamond and Pearl scale, and

1 ounce Troy	= 340; (480) grains Troy
1 ounce Avoirdupois	= 305;6 (437½) grains Troy
1 ounce Diamonds and Pearl	= 288 $^{39/112}$ (428$^{39/112}$) grains Troy

And now, which is heavier: a pound of feathers or a pound of silver?

Answer to WHICH IS HEAVIER?

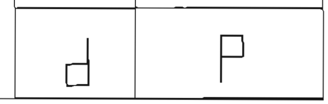
Since a Troy pound (one dozen Troy ounces) is only 3400; (5760) grains Troy, while an Avoirdupois pound (1¼ dozen Avoirdupois ounces) is 4074; (7000) grains Troy, the answer is the reverse of the former answer. This time the silver is heavier than the feathers.

□

SYMBOLS FOR DEK AND EL

Recently Paul Schumacher sent us the dozenal papers of his deceased father, William Schumacher. Among the papers was some correspondence about symbols for dek and el. Bill proposed a reversed 6 and a reversed 9. They are similar in appearance to our familiar numerals and they easily lead to 7-segment calculator displays.

In response to Bill's suggestion, Don Hammond of England wrote that Bill's idea of a reversed 6 for dek was very close to the Dozenal Society of Great Britain's proposal of a modified rotated 2, and the reversed 9 was somewhat similar to their rotated 3. Don noticed that Bill's reversed 9 looked like a lower case 'e' while the DSGB's rotated 3 was similar to an upper case 'E'. Don suggested a 7-segment display of an upper case E without the upper horizontal line (an inverted F), similar to the DSGB's modified rotated 3. See the box below for the DSGB' s modified dek and el.

	dek	el
Dwiggins	X	ε
DSA	*	#
DSGB	ᘔ or ᘔ	ε
Don		Ⱶ
Bill	∂	e
		

□

Puzzle Corner (Concluded)

Bulletin 53; Winter 1986 issue posed the following problem:

The repeating decimal .333... in base ten arithmetic represents the fraction 1/3. What fraction does it represent in base twelve arithmetic?

Solution by Jay L. Schiffman, Rowan College of New Jersey-Camden:

In base twelve, consider 0;333.... This represents the infinite geometric series 3/10; + 3/10;^{2} + 3/10;^{3} +... with first term a = 3/10; and common ratio 1/10;. Using the formula for the sum of an infinite geometric series S = a / (1 - r), we obtain
S = 3/10; / (1 - 1/10;) = (3/10;) / (#;/10;) = 3/#;.

□

PASCAL'S TRIANGLE
A Table of Duodecimal Binomial Coefficients

Jay L. Schiffman
Rowan College of New Jersey, Camden City Center

$$C(n,k) = \frac{n!}{k!(n-k)!} = \frac{n(n-1)...(n-k+1)}{k!} = C(n,n-k),\ 0! = 1$$

Note that each entry in the following table (which is often called Pascal's Triangle) is the sum of two numbers in the row above; one of these numbers is in the same column and the other is the preceding column (e.g. C(9,3) = 24 + 48 = 70).

The above important combinatorial identity can be stated as follows:

C(n,k) + C(n,k+1) = C(n+1,k+1).

Since each row in the table is symmetrical- as in a palindrome - to obtain values for k > 10, use the identity

C(n,k) = C(n,n-k).

We now construct the first two dozen rows of Pascal's Triangle in The Dozenal Base.

(Continued)

Roman Scruples

In Number Words and Number Symbols, *A Cultural History of Numbers* by Karl Menninger published by the M.I.T. Press, Cambridge, Massachusetts and London, England there is an account of Roman Duodecimal Fractions. The author demonstrates the early use of twelfths in ounces and inches. It seems our forebears commonly expressed fractions such as 5/9 as approximately 7 unciae or twelfths.

Note that 5÷9=20÷36=20÷3x1÷12=20÷3=6 2÷3 or 7 unciae.

For more accurate work they would break the unciae down into two dozen scruples and express 6 2/3 unciae as

6 x 24 + 2-3 x 24 =144 + 16 = 160 scruples.

N/K	0	1	2	3	4	5	6	7	8	9	*	#	10
1	1	1											
2	1	2	1										
3	1	3	3	1									
4	1	4	6	4	1								
5	1	5	*	*	5	1							
6	1	6	13	18	13	6	1						
7	1	7	19	2#	2#	19	7	1					
8	1	8	24	48	5*	48	24	8	1				
9	1	9	30	70	*6	*6	70	30	9	1			
*	1	*	39	*0	156	190	156	*0	39	*	1		
#	1	#	47	119	236	326	326	236	119	47	#	1	
10	1	10	56	164	353	560	650	560	353	164	56	10	1
11	1	11	66	1#*	4#7	8	#3	##0	8#3	4#7	1#*	66	11
12	1	12	77	264	6#5	11**	18*3	1#*0	18*3	11**	6#5	264	77
13	1	13	89	31#	959	18*3	2*91	3883	3883	2*91	18*3	959	31#
14	1	14	*0	3*8	1078	2640	4774	6754	7546	6754	4774	2640	1078
15	1	15	#4	488	1464	36#8	71#4	#308	1209*	1209*	#308	71#4	36#8
16	1	16	109	580	1930	42360	98#0	16500	213*6	24178	213*6	16500	*8#0
17	1	17	123	689	22#0	6890	13850	251#0	378*6	45562	45562	378*6	251#0
18	1	18	13*	7#0	2979	8#80	1*520	38*40	60*96	81248	8*#04	81248	60*96
19	1	19	156	92*	3569	#939	274*0	57360	99916	122122	150150	150150	122122
1*	1	1*	173	*84	4297	132*6	37219	82840	135076	1##*38	272272	2*02*0	272272
1#	1	1#	191	1037	515#	17581	4*503	#9*59	1#78#6	334*#2	4720**	552552	552552
20	1	20	1#0	1208	6196	20720	65*84	148360	2#5753	5307*8	7*6#*0	*04640	**4**4

DOZENAL JOTTINGS

...from members and friends...News of Dozens and Dozenalists

The New York State Math Association of Two Year Colleges holds a precalculus level math contest each semester. A free one year subscription to our *Bulletin* was given to the winners of the recent NYSMATYC contest.

A local paper reported that the *Mathletes* of Udall School, West Islip, New York scored well in a recent contest. The Society congratulated them, and sent each member and their advisors some dozenal literature. (Have you heard of any math team successes in your area? Send the information along to us, and we'll be happy to congratulate them. There may be some future dozenalists among them.)

John D. Hansen, Jr., Member 30#, a reviewer of articles submitted to our *Bulletin* from sunny California, was recently appointed Assistant Director of the *Wisdom Society*. Kudos from all of us!

It is with a great deal of sadness that the Society announces the passing of Dr. Robert E. Lovell, Member 122; after a seven year battle with cancer on May 12, 1992. He is survived by his wife Emily.

Jim Malone, during his tenure in office as Treasurer, often used his own money to pay our bills when there was a cash flow problem. In effect, Mary & Jim gave our Society interest free loans.

Recently, Jim erased our last debt to them with the words, "The Society owes us nothing." That is, the Malones have donated a substantial amount to the DSA! (Not including the interest that by right we would owe them.)

We are deeply grateful to Jim and Mary both for their generosity and for Jim's years of devoted service.

Mathematics: One of the Liberal Arts by Thomas J. Miles & Doug Nance will be published by West Publishing Co. in 1996. It contains reference to duodecimal counting and to our Society.

☐

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition--("Who needs a symbol for nothing?")--the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used.

A parallel to today seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

☐

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal-based on ten. The dozen system uses twelve as the base, which is written 10, and is called do, for dozen. The quantity one gross is written 100, and is called gro. 1000 is called mo, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, 265 represents 5 units, 6 dozen, and 2 dozen-dozen, or gross. This number would be called 2 gro 6 do 5, and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5;9'
31	694	Three ft. two in.	3;2'
96	3#2	Two ft. eight in.	2;8'
19#	1000	Eleven ft. seven in.	#;7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, which is 5 dozen and 3; so set down 53. Using this "which is" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection if you are 35 years old, dozenally you are only 2#, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12) 365
 12) 30  + 5
  12) 2  + 6
      0  + 2   Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *.

For more detailed information see Manual of the Dozen System ($1;00).

We extend an invitation to membership in our society.
Dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______________________________
LAST FIRST MIDDLE

Mailing Address (For DSA items) ______________________________

____
(See below for alternate address)

Telephone: Home ______________ Business ______________

Date & Place of Birth ______________________________

College Degrees ______________________________

Business or Profession ______________________________

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______________________________

Address ______________________________

Year & Math Class ______________________________

Instructor ______________ Dept. ______________

Other Society Memberships ______________________________

____

Alternate Address (Indicate whether home, office, school, other)

____

____

____

Signed ______________ Date ______________

My interest in duodecimals arose from ______________________________

____

____

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
%Math Department
Nassau Community College
Garden City, LI, NY 11530

DETACH HERE-- OR-- PHOTOCOPY