DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

ISSN 0046-0826

THE DUODECIMAL BULLETIN 72;

In Memorium: A Duodecimal Legend

Volume 37;
Number 1;
1994
11*2;

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00 (US) for one calendar year. Student Membership is $3.00 per year, and a Life Membership is $144.00 (US).

The Duodecimal Bulletin is an official publication of the DOZENAL SOCIETY OF AMERICA, INC., c/o Math Department, Nassau Community College, Garden City, LI, NY 11530.

BOARD OF DIRECTORS OF THE DOZENAL SOCIETY OF AMERICA

Editorial Office:
923 Spruce Street
Philadelphia, PA 19107
(215) 922-3082

THE DUODECIMAL BULLETIN

Whole Number Seven Dozen Two

Volume 37; Number 1;

11*2;

IN THIS ISSUE

FOUNDED 1944

IN MEMORIAM - FREDERICK A. NEWHALL

Jay Schiffman

It is with a great deal of sadness that the Society announces the passing of one of its most energetic and dedicated members, Frederick A. Newhall (Member number 279;) on November 27, 1993 at the age of 63; due to complications arising from a recent heart attack. Known to his friends as Fred, he epitomized the role of a leader serving as an officer with distinction in several capacities, most recently as president.

Fred joined the Society in 1983 at the Annual Meeting. Thus commenced dek years of impeccible service to the Society and its causes. Among his numerous contributions were scholarly articles published in the *Bulletin*, inspirational presentations before our Annual Meetings, and certainly not to be outdone, his exhaustive work in periodically preparing the *Dozenal Index* which serves as an invaluable tool to editors, prospective authors, and readers desiring to secure additional information on dozenal subjects of interest.

While his Society contributions in spreading the gospel "Twelve Is Best" encompassed a great deal of his time, Fred also served as a leader of young people, arranging outings with the local Eagle Scout troop in Smithtown, LI, NY where he resided with his wife, Mary Jane for more than 26; years.

I have had the pleasure of communicating with Fred since 1987 and first met him at the 1989 Annual Meeting and would aptly classify him as a "gentle giant" in the field of Duodecimals and a scholarly gentleman in the true sense. (In fact, his modular home was built with dimensions based on dozens.) The Society is indeed poorer by virtue of his loss. If one were to make an analogy, one could rate Fred's Societal contributions over the past dek years to those of such pioneer stalwarts as F. Emerson Andrews, George Terry, and Ralph Beard during the early days of the Society in the 1940's and 1950's.

Fred is survived by his wife Mary Jane, one of our active members, son Bruce who resides with his wife Elaine and children in Ellicott City, MD, and daughter Wendy and her family living in Charlton, MA outside Boston. May they be richly blessed knowing that Fred's spirit will always be smiling down on them. □

The DSA does **NOT** endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus 1/2 = 0.5 = 0;6.

DOZENAL SOCIETY OF AMERICA MINUTES OF THE ANNUAL MEETING - 11*1

Saturday, October 16, 1993
Nassau Community College
Garden City, New York 11530

I. BOARD OF DIRECTORS MEETING

1. Dr. Patricia Zirkel, Board Chair, was unable to attend the meeting because of car trouble. In her place, Fred Newhall convened the meeting at 11:15 a.m.. The following Board members were present: Alice Berridge; Jamison Handy, Jr.; Fred Newhall; Gene Zirkel; Jay Schiffman.

   Fred thanked members for coming such great distances and read an inspirational message: "A golden thread has run through the history of the world, consecutive and continuous, the work of the best men in successive ages. From point to point, it still runs, and when near, you feel it as the clear and bright and searchingly irresistible light which truth throws forth when great minds conceive it." by Walter Moxon, from "Pilocereus Senilis and Other Papers."

2. The minutes of the June 5 meeting were approved. The Nominating Committee (A.Berridge, J. Schiffman) presented this slate:

Board Chair:	Dr. Patricia Zirkel
President:	Fred Newhall
Vice President:	Jay Schiffman
Secretary:	Alice Berridge
Treasurer:	Alice Berridge

   The slate was elected unanimously.

3. Appointments were made to the following DSA Committees:

   Annual Meeting Committee: Alice Berridge

   Awards Committee: Gene Zirkel, Chair; Dr. Patricia Zirkel; Jamison Handy, Jr.

   Volunteers to these committees are welcome at any time.

4. The following appointments were made:

   Editor of The Duodecimal Bulletin: Jay Schiffman

(Continued)

Production Manager of The Bulletin: Dr. Patricia Zirkel(until February, 1994)

Parliamentarian to the Board Chair: Jamison Handy, Jr.

Reviewers of articles submitted to The Bulletin: Anthony Catania, Jamison Handy, Jr., Dr. John Impagliazzo, Kathleen McKiernan, Fred Newhall, John Hansen, Dr. Barbran Smith, Gene Zirkel.

5. Other Business of The Board: Next meeting will be October 15, 1994 at 10:30 a.m. at Nassau Community College in Building V, unless members require an elevator building, in which case the meeting will revert to B Complex.

The Board Meeting was adjourned at 11:30 a.m..

II. ANNUAL MEMBERSHIP MEETING

1. DSA President Fred Newhall gavelled the meeting to order at approximately 11:35 a.m.

In addition to the Board Members listed above, the following Society Members were present: Vera Sharp Handy, Mary Newhall.

2. Gene Zirkel moved to accept the minutes of October 24, 1992. So approved.

3. President's Report - Fred Newhall

Fred claims that he has received numerous requests for Dozenal information. Copies of Bulletins, "An Excursion in Numbers", and other material have been sent to readers. In some cases, enough copies have been forwarded for classroom utilization. Fred exhibited the three volumes of his book, which is currently available in the Nassau Community College archives as well as the Stony Brook University archives. He plans to approach other libraries. The Dozenal Clock and the Dozenal slide rule were on hand. Fred said that all Bulletin articles are indexed and computerized.

President's Appointments:

Parliamentarian to the President: Dr. Patricia Zirkel

4. Treasurer's Report - Gene Zirkel

The untimely recent illness of our treasurer Jim Malone caused Fred to appoint Gene Zirkel as acting treasurer. The Board later voted to appoint Gene as Acting Treasurer for the balance of Jim's term of office.

(Continued)

Gene presented a set of past Treasurer's Reports and a collection of pertinent financial information to bring members up-to-date. Gene faced a number of problems related to Jim's illness and a house fire that the Malones had endured. Things, however, are getting back in order. It was agreed that Alice would contact Tony Scordato for his advice on how to improve our cash flow. Jamison Handy moved acceptance of the Treasurer's report and expressed gratitude to Gene for assuming this difficult task. So approved. The members also wish to thank Jim for his tireless work as treasurer and send him our best.

5. Editor's Report - Jay Schiffman

Jay discussed the status of articles and claimed that the current arrangement is progressing well. He expects that everything will run smoothly after Pat Zirkel ceases work as Production Manager.

Members praised Jay for his excellent work.

Committee Reports

6. Annual Meeting Committee - Alice Berridge

The next Annual Meeting will transpire October 15, 1994 following the Board Meeting which will commence at 10:30a.m. and will be held in Building V, unless members need an elevator building, in which case the meeting will be held in B Complex.

7. Awards Committee - Gene Zirkel

No names had been proposed for awards this year.

8. Nominating Committee - Alice Berridge

The committee presented the slate for 1996: Dudley George, Jamison Handy, Jr., Fred Newhall, Dr. Angelo Scordato. The slate was elected unanimously.

The Nominating Committee for next year will be Alice Berridge, Fred Newhall, Jay Schiffman.

9. Other Business - Gene mentioned that the Nassau Community College Librarian, Art Freidman, has sent him a copy of the Magazine Videography, Vol. 18, #9, Sept., 1992. In the article "The Great Finger Twirl" by Mark Schubin, there is a sidebar which mentions the Society.

Gene received two letters from Russia which require translation. He believes they may be articles for The Bulletin.

(Continued)

Gene presented copies of Arthur Willock's article "The Dozenal Society of Great Britain - Outline of Principles and Aims." Whillock requests input from our members.

The meeting was adjourned at 12:35 p.m.

III. FEATURED SPEAKERS

1. Jay Schiffman, a faculty member at The Camden City Center for Rowan College of New Jersey (formerly know as Glassboro State College), presented a very interesting talk, "Duodecimal Combinatorics." He introduced the topic by outlining The Fundamental Principle of Counting and Cartesian Products as related to routing problems. He then demonstrated that the total number of Cartesian Products could dramatically increase by employing dozens. His examples included telephone, plate and license numbers, and available radio stations.

   With telephone numbers, he explained that by using decimal numerals, the current pattern, NXX NXX XXXX, with eight digits available as N and ten as X yields a total of 8 x 10 x 10 x 8 x 10 x 10 x 10 x 10 x 10 x 10 = 6,400,000,000 possible telephone numbers. In light of the fact that the symbols for ten and eleven (*; and #; respectively) can be incorporated into the numbering plan for dozens, the total increases to: * x 10 x 10 * x 10 x10 x 10 x 10 x 10 x 10 x 10 = 8,4000,000,000; or 42,998,169,000 decimally - a dramatic increase.

   Members thanked Jay for his enlightening and challenging talk.

2. Gene Zirkel, a faculty member at Nassau Community College for 22; years delivered a talk, "About Nothing." Nothing turned out to be something indeed - zero digits. Gene's talk was inspired from material from Fred Newhall's book, specifically on the topic of factorials. This material stimulated Gene to investigate the relationship of non-zero and zero digits in factorials. From his analysis, he developed the formula for the number of digits in N!, $[[\log_b N! + 1]]$, and used the result with the factors of base ten and base twelve to predict the trailing zero digits in factorials.

   Members thanked Gene for his stimulating talk and then adjourned for a luncheon.

Respectfully submitted,
Alice Berridge ☐

Remember – your gift to the DSA is tax deductible.

DUODECIMAL COMBINATORICS

Jay L. Schiffman
Camden City Center
Rowan College of New Jersey
Camden, N.J. 08102

INTRODUCTION: The science of counting succinctly defines the mathematical discipline known as combinatorics. The nature of counting problems is broadly based and encompasses such diverse branches as probability and statistics, discrete mathematics, algebra, analysis, and number theory. The purpose of this paper is two-fold: We initially desire to illustrate the use of combinatorial problems in applications entailing telephone numbers, license plates, radio stations, number theory, and functions. Secondly, the deployment of the two additional symbols * and # (dek and el) enables one to achieve greater flexibility in alleviating the major problem of not possessing a sufficient quantity of numbers to meet our everchanging demands.

One of the most essential ideas in combinatorics entails The Fundamental Principle of Counting (also known as The Multiplication Principle) which asserts that if one has two tasks r and s such that r can be performed in n_1 ways and s can be performed in n_2 ways, then the total number of ways both tasks can be accomplished is $n_1 \times n_2$. This principle can be extended to any finite number of tasks. To illustrate, if one is traveling from Philadelphia to Chicago via Cincinnati and there are five possible routes from Philadelphia to Cincinnati and three possible routes from Cincinnati to Chicago, then one has a choice of 5 x 3 = 13; routes to travel from Philadelphia to Chicago via Cincinnati. One can illustrate this in the following manner: Let the five routes from Philadelphia to Cincinnati be denoted by r_1, r_2, r_3, r_4, and r_5. Similarly, let the three routes from Cincinnati to Chicago be denoted by s_1, s_2, and s_3. We depict the result as follows:

$$\{(r_1,s_1),\ (r_1,s_2),\ (r_1,s_3),\ (r_2,s_1),\ (r_2,s_2),\ (r_2,s_3),\ (r_3,s_1),\ (r_3,s_2),\ (r_3,s_3),\ (r_4,s_1),\ (r_4,s_2),\ (r_4,s_3),\ (r_5,s_1),\ (r_5,s_2),\ (r_5,s_3)\}.$$

Interpreting this result, we have 13; ordered pairs, each one corresponding to a different route. The ordered pair (r_2,s_3) means that one takes route r_2 from Philadelphia to Cincinnati followed by route s_3 from Cincinnati to Chicago. The set of all ordered pairs that can be formed is called The Cartesian Product, where the first coordinate in an ordered pair originates from the first set while the second coordinate in an ordered pair arises from the second set. If we let $A = \{r_1, r_2, r_3, r_4, r_5\}$ and $B = \{s_1, s_2, s_3\}$, then we have formed A X B, which denotes the Cartesian Product of the sets A and B and is read A cross B. Tree Diagrams are also utilized to depict such situations.

Our applications dealing with duodecimal combinatorics now follow.

FIRST APPLICATION: (Telephone Numbers)

Let us observe that a telephone number including the area code consists of dek digits-a three digit area code, a three digit exchange, and a four digit station code. Signaling considerations invoke certain constraints on these digits. To specify the allowable format, suppose we let X denote a digit that can aassume any of the values from 0 through 9, let N denote a digit that can assume any of the values from 2 through 9, and let Y denote a digit that must be either 0 or 1. Two numbering plans are discussed in [1] (reference [1] in the appended bibliography). Some thirty years ago, the so-called old plan was employed, while the so-called current plan will ultimately be deployed throughout North America and permits the use of additional numbers to meet the demand. To summarize, the respective formats under the old plan are NYX, NNX, and XXXX. Under the current plan, the respective formats of these codes are NXX, NXX, and XXXX. (Of course, we list, in turn, the area code, exchange, and station code.) To illustrate, consider the New York City telephone number (212) 407-4427. Under the old plan, this number would not have been permitted. Observe that the second digit in the exchange is 0 which is not among the digits in the range 2-9. Under the current plan, this number is permissible and very likely has been assigned by NYNEX, the parent company of New York Telephone. Numerous area codes have been added in the past half dozen years including 908 (Central NJ.), 610 (parts of Southeastern PA.), 708 (suburban Chicago, IL.), 508 (Foxboro, MA. area), and 410 (Baltimore, MD. area). Eventually, the current plan will allow the use of area codes such as 325 and 537, not possible under the old plan which restricts the second digit of any area code to either a 0 or a 1.

Let us now calculate the number of different North American telephone numbers that are possible under both the old and the current plans respectively.

Under the old plan, we calculate the number of possible numbers that employ format NYX. One has eight choices for the first digit (any one of the digits 2-9) followed by two choices for the second digit (either 0 or 1) followed by dek choices for the third digit (any one of the digits 0-9). By the Multiplication Principle, there are 8 x 2 x * = 114; area codes with format NYX. We next determine the number of possible office codes using format NNX. One has eight choices for the first digit followed by eight choices for the second digit followed by dek choices for the the third digit. By the multiplication Principle, there are 8^2 x * = 454; exchanges with format NNX. Finally, we determine the number of station codes having format XXXX. One has dek choices for each of the four digits yielding a total of (*)^4 = 5954; station codes with format XXXX. Applying the Multiplication Principle a final time to these three results yields a total of 114; x 454; x 5954; = 246,#287,14; different numbers available in North America under the old plan.

We next calculate the number of different possible telephone numbers available under the current plan in North America. Using the format NXX for the area code, one has 8 x (*)^2 = 568; different area codes having format NXX. Next using the same format for the office code, we generate 568; diverse exchanges having format NXX. Finally, as in the old plan, one has 5954; station codes with format XXXX. A final application of the Multiplication Principle yields 568; x 568; x 5954; = 1,2*7,41#,854; different numbers available under the new plan, more than a six-fold increase. (Decimally, one has 1,024,000,000 different numbers available under the old plan and 6,400,000,000 different numbers available under the current plan.)

(Continued)

Observe that we have not allowed our symbols for TEN and ELEVEN in the duodecimal system to be incorporated into either telephone numbering plan. Our next goal hence is to demonstrate that under either plan, more numbers would be available if one resorted to the duodecimal system of numeration. The symbols * and # are incorporated into any touch-tone phone. Unfortunately these symbols are employed only for special services by local telephone companies. Let us for the sake of argument envision the flexibility achieved by the augmentation of these two symbols into the telephone number framework.

The following table will be helpful in the duodecimal base:

N: digits employed are 2-#; (Total of *)
X: digits employed are 0-#; (Total of 10;)
Y: digits employed are 0-1 (Total of 2)

If we thus argue in a manner similar to the base dek analogue and employ the Multiplication Principle, then we have under the Old Plan with Formats NYX, NNX, and XXXX respectively (* x 2 x 10;) x (* x * x 10;) x (10; x 10; x 10; x 10;) = (180;) x (840;) x (10000;) = 1,1*8,000,000; different duodecimal telephone numbers. (decimally 5,971,968,000 distinct numbers using all twelve symbols where appropriate) Under the current plan with Format NXX, NXX, and XXXX respectively, one has (* x 10;^{2})(* x 10;^{2})(10;^{4}) = (*00;) x (*00) x (10000;) = 8,400,000,000; different duodecimal telephone numbers. (decimally 42,998,169,600 distinct numbers using all twelve symbols where appropriate) The reader can thus easily note a far greater abundance of duodecimal telephone numbers available under either plan when contrasted with their decimal counterparts.

SECOND APPLICATION: License Plate Problems.

The traditional license plate problem consists of three letters followed by three digits. Let us consider the number of license plates that can be issued in both the decimal and duodecimal bases under the given conditions.

1. Determine the number of different license plates that can be formed from three digits which follow three letters if there are no restrictions placed on the letters or digits.

Solution: In base ten, one has 26^3 x 10^3 = 17,576,000 different plates. We reason that there are 26 choices for each letter and 10 choices for each digit and invoke the Multiplication Principle. In a similar vein, one has 22;^{3} x 10;^{3} = *,208,000; distinct plates duodecimally (which is decimally 30,371,828).

2. A Variation of the License Plate Problem:

Determine the number of different license plates that can be made using three letters followed by three digits if all the letters and all the digits are distinct.

(Continued)

Solution: In base ten, one has 26 x 25 x 24 x 10 x 9 x 8 = 11,232,000 distinct plates. We reason that while there are 26 available letters to fill the first slot, there are just 25 ways to fill the second slot and 24 ways to fill the third slot since we are imposing the condition that all the letters are necessarily distinct. Similar reasoning is employed when dealing with the digits. The Multiplication Principle is finally utilized to complete the solution. In the duodecimal base, one has 22; x 21; x 20; x 10; x #; x *; = 6,*90,800; plates (decimally 20,592,000).

3. A Second Variation of the License Plate Problem:

Determine the number of different license plates that can be made using three letters followed by three digits with no restrictions on the letters and the sole restriction on the digits is that the first digit cannot be zero.

Solution: In base ten, one has 26^3 x 9 x 10^2 = 15,818,400 distinct plates. (Observe that there are only 9 choices available for the first digit since 0 is not permitted.)
In the duodecimal base, one has $22;^3$ x #; x $10;^2$ = 9,3*7,400; plates (decimally 27,840,384).

THIRD APPLICATION: Radio Stations.

In North America, a radio station is designated by four call letters although the last letter is sometimes left blank. If we limit ourselves to the United States of America, the first letter is restricted to either W or K. Most radio stations east of the Mississippi begin with W while the majority west of the Mississippi begin with K. (There are, of course, exceptions to the rule as radio stations KDKA and KQV in Pittsburgh, PA.) Let us for argument sake consider only AM radio stations.

There are potentially 1 x 26^3 = 17,576 AM radio stations beginning with the letter W and an equal number beginning with the letter K, a total of 35,152 AM stations decimally. (2 x 22^3 = 18,414 duodecimally.) We note that there are 2 choices for the first call letter (W or K) and 26 (22;) choices for each of the last three call letters. This result is based on radio stations that have all four letters utilized in their identification. If one were concerned with the number of potential AM radio stations in the USA having three call letters, then observe that one has 2 choices for the initial call letter followed by 26 choices for both the second and third call letters yielding a total of 2 x 26^2 = 1,152 AM radio stations having three call letters. (Duodecimally one has 2 x 22^2 = 948 stations.) Hence one has 35,152 + 1,152 = 36,304 AM radio stations with either three of four call letters. (Duodecimally there are a total of 19,160 stations.)

If we pursue a somewhat related, yet diverse direction, we can discuss the idea of palindromic radio stations. A palindrome is a word or number (such as TOT or 323) which reads identically whether written in the natural order or the reverse order. Let us calculate how many of the potential 19160; AM radio stations in the USA are indeed palindromic.

We consider two cases corresponding to those stations having three and four call letters respectively. For the case of three call letters, there are 2 choices for the first call letter

(Continued)

(either K or W) and only one choice for the third call letter (the letter assigned as the first call letter). Meanwhile, one has 22; choices for the second call letter (any of the 22; letters from the alphabet). By the Multiplication Principle, one potentially has 2 x 22; x 1 = 44; palindromic AM stations with three call letters.

In the case of four call letters, there are 2 choices for the first call letter (either K or W) and only one choice for the fourth call letter (the letter assigned as the first call letter). Meanwhile, one has 22; choices for the second call letter (any one of the 22; letters from the alphabet) and one choice for the third call letter (the one selected as the second call letter since we are seeking palindromes). By the Multiplication Principle, one potentially has 2 x 22; x 1^2 = 44; palindromic radio stations with four call letters. We thus have a total of 2 x 44; = 88; potential palindromic radio stations on the AM dial in the USA with three or four call letters.

Some of these stations really exist! For example, WLW in Cincinnati, OH is a palindromic radio station. The five-time World Champion Cincinnati Reds plus a variety of sports and NewsTalk Programs are featured on NEWSRADIO 700.

FOURTH APPLICATION: Number Theoretic Problems.

1. How many five digit duodecimal integers are divisible by 6?

Solution: Observe that there are only two choices for the units digit (0 or 6), but a dozen choices for each of the other four digits. Thus by the Multiplication Principle, one has $10;^4$ x 2 = 20000; different five digit duodecimal integers (including those below 10000;).

2. How many duodecimal integers below 1000; are divisible by either 2 or 3?

Solution: We employ two methods of solution here.

Method 1: Let us first find the number of integers that are in turn divisible by 2, 3, and both 2 and 3.
Recall that an integer is divisible by 2 if its last digit is either 0, 2, 4, 6, 8, or *;.
An integer is divisible by 3 if its last digit is 0, 3, 6, or 9.
An integer is divisible by both 2 and 3 (and hence by 6 since 2 and 3 are co-prime) if the last digit is either 0 or 6.
One thus has six choices for the units digit and a dozen each for the dozens and gross digits respectively, yielding $10;^2$ x 6 = 600; integers divisible by 2 employing the multiplication principle.
In a similar manner, one has four choices for the units digit and a dozen each for the dozens and gross digits respectively, yielding $10;^2$ x 4 = 400; integers divisible by 3.
Finally there are two choices for the units digit and a dozen each for the dozens and gross digits respectively, yielding $10;^2$ x 2 = 200; integers divisible by 6 deploying our familiar principle.
Utilizing the fact that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$, where $n(A \cup B)$, $n(A)$, $n(B)$, and $n(A \cap B)$ connote the respective number of elements in the sets $A \cup B$ (A union B), A, B, and

(Continued)

A∩B (A intersection B), we arrive at our desired result. There are 600; + 400; -200; = 800; duodecimal integers ≤ 1000; that are divisible by either 2 or 3.

In order to present an alternative and more elegant solution, we first define ⌊x⌋ = the greatest integer in x, which corresponds to the largest integer ≤ x, where x constitutes a real number. To cite some examples, ⌊0;6⌋ = 0, ⌊3⌋ = 3, ⌊-2;4⌋ = -3, ⌊-3;2⌋ = -4, and ⌊-3⌋ = -3.

A brief problem solution achieves our goal of furnishing an elegant alternative proof of the above divisibility problem.

Let n and d be positive integers. How many positive integers not exceeding n are divisible by d?

Solution: The positive integers divisible by d are all the integers of the form dk where k is a positive integer. Hence, the number of positive integers divisible by d that do not exceed n equals the number of integers k with 0 < dk ≤ n or with 0 < k ≤ n/d. Therefore, one has ⌊n/d⌋ positive integers not exceeding n that are divisible by d.

We now present a second solution to our original problem using the inclusion-exclusion principle with the greatest integer function.

One has ⌊1000;/2⌋ positive integers not exceeding 1000; that are divisible by 2, ⌊1000;/3⌋ positive integers not exceeding 1000; that are divisible by 3, and ⌊1000;/(2 x3)⌋ = ⌊1000;/6⌋ positive integers not exceeding 1000; that are divisible by both 2 and 3. Hence the number of positive integers not exceeding 1000; that are divisible by either 2 or 3 is given by ⌊1000;/2⌋ + ⌊1000;/3⌋ - ⌊1000;/6⌋ = 600; + 400; - 200; = *00; - 200; = 800;.

3. How many duodecimal integers not exceeding 1000; are divisible by either 2, 3, or #;?

Solution: We initially employ the fact that n(A∪B∪C) = n(A) + n(B) + n(C) - n(A∩B) - n(A∩C) - n(B∩C) + n(A∩B∩C).
Now the number of positive integers not exceeding 1000; that are divisible by either 2, 3, or #;, is given by ⌊1000;/2⌋ + ⌊1000;/3⌋ + ⌊1000;/#;⌋ - ⌊1000;/(2 x 3)⌋ - ⌊1000;/(2 x #;)⌋ - ⌊1000;/(3 x #;)⌋ + ⌊1000;/(2 x 3 x #;)⌋ = 600; + 400; + 111; - 200; - 66; -44; + 22; = 845;.

We conclude the article with a fifth application dealing with an entity known as a function. Functions permeate the mathematical landscape in numerous branches including algebra and analysis.

A function is a rule of correspondence between two sets X and Y which assigns to each element in X (called the domain) one and only one element in Y (called the co-domain). Moreover, we denote by f: X→Y the function f taking points from set X into set Y. In

(Continued)

addition, a function f: X→Y is classified as one-to-one if different elements in X are transformed under f to different elements in Y. In other words, if f is one-to-one, then it is not possible for two different domain elements to be "mapped" to the same co-domain element.

In our first example that follows, we count the number of functions from X into Y, while in our second example, we seek the number of one-to-one functions from X into Y. In each case, let n(X) = c and n(Y) = d.

1. We claim that the number of distinct functions from X into Y is d^c and argue as follows: A function corresponds to a choice of one of the d elements in the co-domain for each of the c elements in the domain. Hence by The Fundamental Principle of Counting, one has d x d x ... x d = d^c functions from a set with c elements to one with d elements.

2. We claim the number of distinct one-to-one functions from X into Y is d x (d-1) x (d-2) x ... x (d-c+1) and argue as follows: If c > d, observe the impossibility of producing any one-to-one functions from a set having c elements to a set having d elements. Hence we restrict ourselves to the case where c ≤ d. Let the domain elements be denoted by x_1, x_2, x_3, ...,x_c. For the initial domain element, x_1, one has d possible elements in the co-domain it could be assigned. Select one of these d elements. The second element in the domain, x_2, can be assigned any one of the remaining d-1 elements in the co-domain (save the one assigned to the initial element in the domain, x_1, by virtue of the one-to-one character of f). Choose one such element. Similarly, the third domain element, x_3, can be assigned any of the remaining d-2 elements in the co-domain. Select one of them. Proceeding, the c th element in the domain, x_c, can be assigned to any of the remaining d-(c-1) = d-c+1 elements in the co-domain. Choose one of these. Appealing to The Fundamental Principle of Counting, one has d x (d-1) x (d-2) x ... x (d-c+1) possible one-to-one functions from a set with c elements to a set with d elements. (It should be noted that the case corresponding to c = d will admit d! = d x (d-1) x (d-2) x ... x (d-d+1) = d x (d-1) x (d-2) x ... x 1 possible one-to-one functions from X into Y.)
Hence the number of functions from X into Y if n(X) = 3 and n(Y) = 4 is 4^3 = 54;, while the number of one-to-one functions from X into Y under the same circumstances is 4 x (4-1) x (4-2) = 4 x 3 x 2 = 20;.

REFERENCE

[1] Kenneth H. Rosen--Discrete Mathematics and its Applications, Random House, New York, 1988.

□

SIGNIFICANT DIGITS & FACTORIALS

Gene Zirkel
Nassau Community College
Garden City, LI, NY

INTRODUCTION

What is a factorial? Factorial 6, written 6! is 6·5·4·3·2·1 = 500; In general, n! = n·(n-1)·(n-2) ··· 3·2·1

Two things got me thinking about factorials:

1st President Fred Newhall's work with factorials (which are used often in math, for example in combinations and permutations in probability and statistics) which were discussed at the Board of Director's meeting in the summer of 1993.

2nd My new TI85 graphing calculator which has a factorial key in its probability menu. I began thinking of the problem that factorials quickly become very large — too large for a finite machine. For example the TI85 displays only dek significant digits, and, as Fred had shown, 12! had more than dek decimal digits, so that we could only express 11! accurately in base dek using dek significant digits. For n > 11, 11! could not be expressed accurately on this calculator

Of course, the last 2 digits in 11! were zero, so that we could use scientific notation to express the trailing zeros as an exponent of dek. Using this idea we could express 13! since its 11 digits end in 3 trailing zeros. However, even this won't help us with n! for n > 13.

What Fred's work showed was that base twelve provided many, many more trailing zeros than base dek did, and that was what started me thinking.

If we express our factorials in base twelve, we find that if we use scientific notation and dek digits, we can express the do-six digits of 18! {20!} accurately because it has 8 trailing zeros.

(As an aside let me mention, a second advantage to using dozenals to express the lengthy numbers produced by factorials, is the well known fact that numbers in general use less digits in base twelve than in base dek. And the larger the number, the greater this advantage becomes.)

Thus 13! requires a baker's dozen of digits in base dek, but only a dozen digits in base do.

(Continued)

The number of digits in N! can easily be found as follows:

(1) If N is not a power of the base B the number of digits in N! is

$$[\log_B(N!) + 1]$$

where [d] is the largest integer d, (i.e., d rounded down). (This integer can easily be computed on a modern hand held calculator using $\log_B(N!) = \ln(N!)/\ln(B)$.)

(2) If N is a power of the base B the number of digits in N! is the exponent of B.

However it is the trailing zeros, that really are the important factor (pun intended), and base do produces the zeros in factorials precisely because of its *factorability* — an advantage that we dozenalists have been touting for years!

The rest of this paper investigates this advantage, and is concerned with predicting how many trailing zeros there are in N!

SOME BACKGROUND INFORMATION

If $S < L$, then $N/S > N/L$, and $[N/S] \geq [N/L]$.

In what follows, *all* division will be integer division, and hence n/d will be presumed to mean [n/d] from now on.

From the above it follows that

$$N/S + N/S^2 + N/S^3 + N/S^4 + \ldots \geq N/L + N/L^2 + N/L^3 + N/L^4 + \ldots$$

or

$$\sum_{q=1}^{\infty} N/S^q \geq \sum_{q=1}^{\infty} N/L^q$$

In particular, if p is a prime, the number of factors of p in n! is

$$\sum_{i=1}^{\infty} N/p^i$$

which is a *finite* sum.

EXAMPLE: Count the number of factors of 2 in #!

1	2	3	4	5	6	7	8	9	*	#		
	1		1		1		1		1		→	5
			1				1				→	2
							1				→	1
	1	+	2	+	1	+	3	+	1		=	8

(Continued)

But

$$\sum_{i=1}^{\infty} N/p^i = \sum_{i=1}^{\infty} \#/2^i = \#/2^1 + \#/2^2 + \#/2^3 + \#/2^4 + \dots$$

$$= 5 + 2 + 1 + 0 + 0 + 0 + \dots$$
$$= 8$$

We use the symbol {N,p}' to indicate the number of factors of p contained in N!. Thus, in this example, we have

{#,2}' = 8

EXAMPLE: Count the number of factors of 3 in *!

Answer: {*,3}' = */3 + */9 + */27 + ...
= 3 + 1 + 0 + 0 + 0 + ... = 4

Note that the above formula only works for primes.

{*,6}' */6 + */36 = 1 + 0 = 1. However, the correct answer is four since *! = $6^4 \cdot 1754$ and since 1754 ends in 4, it is not divisible by 3. (See "The Role of Congruence to Prove Duodecimal Divisibility Tests", by Jay Schiffman, this *Bulletin*, Number 70; pages 11 & 12;.)

THE NUMBER OF ZEROS

In base dek, the number of trailing zeros in N! is the minimum of {N,2}' and {N,5}'.

But since

$$\sum_{i=1}^{\infty} N/2^i \geq \sum_{i=1}^{\infty} N/5^i$$

{N,5}' ≥ {N,2}', and the answer is N! has {N,5}' zeros in base dek

In base do, the number of trailing zeros in N! is the minimum of {N,3}' and {N,4}'. However, we note that 4 is not a prime.

Clearly the number of factors of 4 in any number is ½ the number of factors of 2, since $4 = 2^2$.

EXAMPLE: Consider 8!

(Continued)

In base dek, {8,5}' = 8/5 + 8/25 + ... = 1 + 0 + 0 + ... = 1. Thus 8! {40320.} has 1 trailing zero in the awkward base dek.

In the more convenient base do, we find the minimum of {8,3}' and {8,4}'.

{8,3}' = 8/3 + 8/9 + ... = 2, and

{8,4}' = ½{8,2}' = ½(8/2 + 8/4 + 8/8) = ½(4 + 2+ 1) = 3.

The minimum of these results, 2 and 3, is 2, hence 8! (or 1#400;) has 2 trailing zeros.

Recall that the number of digits required in base dek is

$[\log_*(8!) + 1] = [4.6 + 1] = 5$

In base do, the number of digits is

$$[\log_{10}(8!) + 1] = \left[\left[\frac{\ln(8!)}{\ln(10)} + 1\right]\right] = [4.3 + 1] = 5$$

Therefore we need 5 - 1 or 4 leading digits to express 8! accurately in the inefficient base dek, but only 5 - 2 or 3 leading digits in base do.

EXAMPLE: Consider 14!

In base *, {14,5}' = 14/5 + 14/25 + ... = 3 terminal zeros, and

$[\log_*$ 14! + 1] = 12. Thus we need 12 - 3 or # decimal digits for accuracy.

In base do, {14,3}' = 14/3 + 14/9 + 14/23 + .. = 5 + 1 + 0 = 6,
and {14,4}' = ½ {14,2}' = ½ (14/2 + 14/4 + 14/8 + 14/14+ ...) =
½ (8 + 4 + 2 + 1 + 0) = 7.

The minimum of 6 and 7 is 6 terminal zeros.

Further, $[\log_{10}$ 14! + 1] = 11 Thus we need only 11 - 6 or 7 dozenal digits.

ONE LAST UNSOLVED QUESTION

Can YOU find a formula for the minimum of {N,3}' and {N,4}' = ½{N,2}'? I ran a FORTRAN program to investigate this. My results follow. At this point I do not see any pattern. Do You?

(Continued)

N	{N,3}'	{N,4}'		
1	0	0		
2	0	0		
3	1	0		4 is less
4	1	1		
5	1	1		
6	2	2		
7	2	2		
8	2	3	3 is less	
9	4	3		4 is less
*	4	4		
#	4	4		
10	5	5		
11	5	5		
12	5	5		
13	6	5		4 is less
14	6	7	3 is less	
15	6	7	3 is less	
16	8	8		
17	8	8		
18	8	9	3 is less	
19	9	9		
1*	9	9		
1#	9	9		
20	*	#	3 is less	
21	*	#	3 is less	
22	*	#	3 is less	
23	11	#		4 is less
24	11	10		4 is less
25	11	10		4 is less
26	12	11		4 is less

N	{N,3}'	{N,4}'		
27	12	11		4 is less
28	12	13	3 is less	
29	13	13		
2*	13	14	3 is less	
2#	13	14	3 is less	
30	15	15		
31	15	15		
32	15	15		
33	16	15		4 is less
34	16	17	3 is less	
35	16	17	3 is less	
36	17	17		
37	17	17		
38	17	18		3 is less
39	19	18		4 is less
3*	19	19		
3#	19	19		
40	1*	1#	3 is less	
41	1*	1#	3 is less	
42	1*	1#	3 is less	
43	1#	1#		
44	1#	20	3 is less	
45	1#	20	3 is less	
46	22	21		4 is less
47	22	21		4 is less
48	22	22		
49	23	22		4 is less
4*	23	23		
4#	23	23		
50	24	24		

(Continued)

ELECTION RESULTS

The Board of Directors is pleased to announce the election of Jay Schiffman as president of the DSA, and of Pat Zirkel to the Class of 1996. Both of these elections are to fill out the unexpired terms of Fred Newhall. Our Board is now once again composed of three classes each of which consists of four directors.

decimals					DOZENALS				
n	n! *in base dek*	number of significant digits			N	N!	number of significant digits		
		digits	non zero	zero			digits	non zero	zero
1	1	1	1	0	1	1	1	1	0
2	2	1	1	0	2	2	1	1	0
3	6	1	1	0	3	6	1	1	0
4	24	2	2	0	4	20	2	1	1
5	120	3	2	1	5	*0	2	1	1
6	720	3	2	1	6	500	3	1	2
7	5040	4	3	1	7	2#00	4	2	2
8	40320	5	4	1	8	1#400	5	3	2
9	362880	6	5	1	9	156000	6	3	3
*	3828800	7	5	2	*	1270000	7	3	4
#	39916800	8	6	2	#	11450000	8	4	4
10	479001600	9	7	2	10	114500000	9	4	5
11	6227020800	*	8	2	11	1259500000	*	5	5
12	87178291200	#	9	2	12	14*8#*00000	#	6	5
13	13076743680 E 02	11	*	3	13	19152960000 E 01	10	7	5
14	20922789888 E 03	12	#	3	14	241*#880000 E 02	11	7	6
15 ?	35568742809 E 04	13	10?	3	15	33*86734000 E 03	12	8	6
..	....	.....	....	....	..	....	......	....	...
1*					1*	3627596972* E 09	18	#	9
1# ??	25852016738 E 10	1#	17??	4	1# ?	68#041404*5 E 09	19?	10	9

? loss of 1 significant digit
?? loss of 8 significant digits

THINK 12 . . .twelve . . .10; . . .do . . . .

EDITOR, PRESIDENT, BOARD MEMBER, INSPIRATION

Gene Zirkel

When we first met Henry Clarence Churchman, his courtly ways moved my wife to describe him as the perfect "Southern Gentleman". Actually, Henry was from the midwest, but his cheerful politeness endeared him to all of us who knew him.

Member number 72; — there are only one and a half dozen two-digit members still on our mailing list — Henry joined our Society in 1952.

A stalwart of the DSA, Henry was elected vice president at Alamogordo, New Mexico in 1962, and he held that position until 1971. In 1970 he took on the additional responsibility of editing our *Bulletin*, and he served in that capacity until 1978. Along the way he guided the DSA during his three terms as president beginning in 1971.

In 1976, while still editor, he again took on the responsibilities of vice president, a position he held until illness forced him to retire in 1981.

In 1980, our Society honored him, bestowing upon him our annual award. It has never gone to a more deserving honoree.

Always an active member of the DSA and an ardent proponent of duodecimals, he wrote many articles under both his own name and several pseudonyms. We discovered this latter fact upon going thru his papers.

In President Fred Newhall's comprehensive index, Henry has one of the longer entries: a dozen and a half lines of references. In addition John Jarndyce has a line and a half containing five additional articles and Egbert Pardiggle also has a line and a half with four more. (This latter alias indicates the sense of humor Henry brought to everything he did.)

In 1981, he was too ill to continue his work, and my wife Pat, my son George who was dek years old, and I traveled to Iowa to collect our Society's archives and Henry's papers. His son, John, graciously helped us pack dozens of cartons which we shipped to Nassau Community College.

Henry was an active lawyer, involved in many things. For example, among his papers we found a church bulletin from the 1970s(!) which he had edited, crossing out all the male references, changing them to non-sexist terms. Henry was always a man ahead of his times.

We miss him as a friend and an inspiration to dozenalists everywhere. □

TWELVES AND LCM'S

Monte J. Zerger
Adams State College
Alamosa, CO. 81102

Let LCM_n = LCM {1, 2, 3, ... , n} represent the least common multiple of the first n positive integers. Then

LCM_1 = LCM {1} = 1

LCM_2 = LCM {1, 2} = 2

LCM_3 = LCM {1, 2, 3} = 6

LCM_4 = LCM {1, 2, 3, 4} = 10;

.

.

.

LCM_n = LCM {1, 2, 3, ... , n}

First note that LCM_1 * LCM_2 * LCM_3 = LCM_4 =10;.

Now consider $(LCM_{10;})$/10; = 1406;. This is precisely the LCM of all the entries in the 10;th row of Pascal's Triangle. Is this unusual? Yes; for it has been demonstrated that there are only two other values of n for which (LCM_n)/n is also the LCM of the nth row of Pascal's Triangle. One of these is a factor of 10; and the other is a multiple of 10;.

(LCM_3)/3 = 2 = LCM { C(2,0), C(2,1), C(2,2) }

$(LCM_{10;})$/10; = 1406; = LCM { C(#,0), C(#,1), ... , C(#,#) }

$(LCM_{20;})$/20; = 62868806; = LCM { C(1#,0), C(1#,1), ... , C(1#,1#) }

As a bonus, note that in all three of these exclusive cases, (LCM_n)/n is also equal to the product of all primes less than n. Observe the following:

(LCM_3)/3 = 2

$(LCM_{10;})$/10; = 1406; = (2)(3)(5)(7)(#)

$(LCM_{20;})$/20; = 62868806; = (2)(3)(5)(7)(#)(11)(15)(17)(1#)

<u>REFERENCE</u>

[1] Joe Roberts, <u>Lure of the Integers</u>, The Mathematical Association of America, 1992, P. 148. □

JOTTINGS

...from members and friends...News of Dozens and Dozenalists...

IN APPRECIATION

Thanks to all who sent in dues PLUS the many generous contributions. It is only your kindness that keeps us going.

Thanks also for the many kind words, greetings and news items you sent in along with your checks, words such as:

We feel the loss in the passing of Dozenal stalwarts like Hammond, Churchman, and Newhall. Incidentally we have been married 64 years, 4 children, 11 grandchildren, 20 great - grandchildren. (Congrats! - Ed.)

- Charlie and Miriam Bagley

Holiday greetings from Ruby and Arthur Whillock of the DSGB usually arrive on a card illustrating the twelve days of Christmas starting with the partridge in a pear tree and proceeding thru the el other gifts. This year, the card was decorated with one dozen Christmas trees. The trees were in a three by four pattern illustrating the factorability of do. Dek of them were plain silver, but two were colorfully trimmed (representing dek and el no doubt).

Thanks for all the kind words about Don Hammond, Henry Churchman and Fred Newhall.

Our gratitude to all of you.

Our warmest (pun intended) thanks to all those who sent in a replacement check, when their first check did not clear because it was presumably destroyed when our records were burnt up. We depend upon your generosity to keep the message of dozenal counting alive and in the public eye.

Thanks especially to Mary Newhall for the donation she made in memory of our past president, Fred Newhall.

A DOZENAL WRISTWATCH

Member number 320;, Paul Rapoport of McMaster University, Canada is trying to design a base twelve watch. One half dozen years ago, Paul created the dozenal clock. (See this *Bulletin* No. 5#)

(Continued)

FIND THE NEXT TERM - II

Jean Kelly

In Shaun Freguson's article, 'Find the Next Term', in issue Number 71; of this *Bulletin*, he offers the poser: "Can you write down a sequence of n numbers such that there is only one (unambiguous) value for the (n+1)st term?". The answer is NO.

Any proposed rule, R1, for the kth term, can easily be altered to yield another rule, R2, so that the first n terms derived from both rules will be identical, but the (n+1)st terms will differ.

As an example, if the first gross of terms of a series are given as the even numbers 2, 4, 6, 8, *, 10, ... 200, one might think that the rule R1 for the kth term is 2k, and hence the next term must needs be 202.

However, rule R2 for the kth term could just as well be

$$2k + (k-1)(k-2)(k-3)...(k-100)Q/(100!)$$

Where Q can be any expression at all such as $\sqrt{\pi}$, or $k^{17;6}$

Obviously, the same technique can be used no matter how many terms of the series are offered — a gross, a great-gross, a great-great-gross, etc. Guessing at the next term of a series, is always just that, a *guess!*

(By the way, how many aptitude tests ask questions such as "Find the next term of the series 2, 4, 6, 8, ?", rewarding the mediocre students for a mundane answer such as dek, while penalizing the brighter students who can see a variety of possible correct responses?)

□

Jottings (Concluded)

Welcome new members:

337	Dr. James McElhattan,	Greece
338	Glenn Bernius,	NY
339	Michael Wilson,	MA
33*	Michael Kinyon,	IN
33#	Rafael Marino,	NY

Professor Marino agreed to serve the Society as vice president and was elected to that position, filling the vacancy left when Professor Jay Schiffman was elected to the presidency.

And a grateful welcome to our most recent *Life Member*: George Jellis, member number 316; of East Sussex, England. Thanks for the support, George.

□

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal-based on ten. The dozen system uses twelve as the base, which is written 10, and is called do, for dozen. The quantity one gross is written 100, and is called gro. 1000 is called mo, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, 265 represents 5 units, 6 dozen, and 2 dozen-dozen, or gross. This number would be called 2 gro 6 do 5, and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5;9'
31	694	Three ft. two in.	3;2'
96	3#2	Two ft. eight in.	2;8'
19#	1000	Eleven ft. seven in.	#;7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, which is 5 dozen and 3; so set down 53. Using this "which is" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection if you are 35 years old, dozenally you are only 2#, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12) 365
12) 30  + 5
12) 2   + 6
    0   + 2   Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *.

For more detailed information see Manual of the Dozen System ($1;00).

DETACH HERE -- OR PHOTOCOPY

We extend an invitation to membership in our society.
dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______________________________
LAST FIRST MIDDLE

Mailing Address (for DSA items) ______________________________

____
(See below for alternate address)

Telephone: Home ______________ Business ______________

Date & Place of Birth ______________________________

College ______________ Degrees ______________

Business or Profession ______________________________

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______________________________

Address ______________________________

Year & Math Class ______________________________

Instructor ______________ Dept. ______________

Other Society Memberships ______________________________

____

Alternate Address (indicate whether home, office, school, other)

____

____

____

Signed ______________ Date ______________

My interest in duodecimals arose from ______________________________

____

____

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530