DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

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THE DUODECIMAL BULLETIN 71;

DON HAMMOND, EDITOR

THE DOZENAL *JOURNAL*

R.I. P. - 1993

-See Page 6;

1993

ANNUAL MEETING

October 16, 1993

-See Page 7;

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11*1;

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00 (US) for one calendar year. Student Membership is $3.00 per year, and a Life Membership is $144.00 (US).

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BOARD OF DIRECTORS OF THE DOZENAL SOCIETY OF AMERICA

Editorial Office:
923 Spruce Street
Philadelphia, PA 19107
(215) 922-3082

THE DUODECIMAL BULLETIN

Whole Number Seven Dozen One

Volume 36; Number 2;

11*1;

FOUNDED 1944

IN THIS ISSUE

MINUTES OF THE DSA BOARD OF DIRECTORS MEETING

June 5, 1993
Nassau Community College

The meeting was chaired by President Fred Newhall (in the absence of Board Chair Dr. Pat Zirkel who was at Cornell University studying Medieval Latin).

Present were Fred & Mary Newhall, Editor Jay Schiffman, Gene Zirkel, and by mail proxy: Pat Zirkel and Alice Berridge.

Our thanks to Barbran Smith and Alice who did the work of setting up the meeting, but were unable to attend. (Barbran recently moved into a new apartment & Alice, who has just retired from teaching, was vacationing in Florida with her husband Edmund.)

Many topics and questions relating to our *Bulletin* were raised by our new editor. Among other items, all agreed that Jay's first issue was superb. The Board reminded Jay that he was an editor, not an author. He did not have to rewrite articles that needed redoing. Simply return them to the authors indicating what was needed to be suitable for our *Bulletin.*

Regarding our ad in the *Journal of Recreational Mathematics*, it was suggested that we include a dozenal puzzle, since the readership is puzzle oriented. Mary, Jay, and Gene seemed interested in trying to create one.

We are now sending *Bulletins* to winners of mathematics contests. Hopefully, these are likely candidates for future members. If you read of any such winners in your newspaper, send the article to us.

We now have the *Bulletin* published via a word processing program. The question was raised regarding our ability to preserve future *Bulletins* on disk. This will be looked into.

(Continued)

The DSA does **NOT** endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus 1/2 = 0.5 = 0;6.

An inquiry was made to us about the Oughtred Society, a group named for William Oughtred, the inventor of the slide rule. If any one can help us with information or an address, please contact us. Thanks.

In trying to find if there were Roman Numerals larger than M, we discovered under the listing "Numerals" in the *Encyclopedia Americana* more than two columns on the "Duodecimal System", including a reference to the DSA. The edition is copyright 1968, volume 9 pages 409-410. It was a pleasant surprise to discover the author was none other than our own Ralph Beard. (Incidentally, a bar over a Roman Numeral multiplies that number by 1000.)

Gene announced that the result of the vote to bestow Honorary Membership on Dr. Anton Glaser, author of *The History of Binary and Other Non-Decimal Numeration*, was a resounding **yea**.

The meeting then evolved into several interesting discussions especially with regard to dozenal factorials, permutations and combinations.

We then adjourned for lunch at a nearby diner.

Respectfully submitted,

Gene Zirkel,
Acting Secretary

□

THE FOLLOWING ARE AVAILABLE FROM THE SOCIETY

1. Our brochure. (Free)
2. "An Excursion in Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, October 1934. (Free)
3. *Manual of the Dozen System* by George S. Terry. ($1;00)
4. *New Numbers* by F. Emerson Andrews. ($10;00)
5. Dozenal Slide Rule, designed by Tom Linton. ($3;00)
6. Back issues of the *Duodecimal Bulletin*, as available, 1944 to present. ($4;00 each)
7. *TGM: A Coherent Dozenal Metrology*by T. Pendlebury ($1;00)
8. *Modular Counting* by P.D. Thomas ($1;00)
9. *The Modular System* by P.D. Thomas ($1;00)

BRITISH EDITOR PASSES

"Whatever may happen in the future, I shall always continue to support and advocate the supremacy of the dozenal base for arithmetic as the foundation needed for an enlightened numeracy. Please accept not only my thanks, but also my best wishes for the vitality of the DSA in the years to come. Long may you flourish!"

These were the words of Don Hammond in 1991 when the Board of Directors of the DSA bestowed the Ralph Beard Memorial Award upon him. Don had been editor of the *Dozenal Journal* for more than a half dozen years, and it was fitting that an award named for a founder of the DSA, its first editor and its long time secretary should go to Don, who was similarly an editor and a secretary in the DSGB. Don was also a Fellow of the DSA.

Don succumbed to asthma on June 4th, 1993. A stalwart of our sister organization, the Dozenal Society of Great Britain, Don was 5 and ½ dozen years old. A chief petty officer when he retired from the Royal Navy, his interests were in mechanical engineering. Later he taught both physics and math at several secondary schools.

Don became interested in dozenals on his own, before he ever heard of the DSGB or the DSA. He preferred Isaac Pitman's notation - using a rotated 2 for dek (ten) and a rotated 3 for el (eleven).

When he discovered the DSGB's existence in 1977, he joined at once. His enthusiasm marked him for rapid ascent thru the ranks until he became the obvious successor to Shaun Ferguson as General Secretary and Editor. Under Don's guidance, the *Journal* rose to a peak of excellence in both content and presentation. He left a legacy that will be difficult to maintain.

Altho I never met Don, I had the pleasure of communicating with him often by mail, and once I spoke with him on the fone. His many letters were frequently quoted in our "Jottings" column, where he was quick to point out corrections whenever they were needed. His sharp eyes, along with his eagerness and fervor will be sorely missed by dozenalists thruout the world.

I am sure Don is smiling down upon us in the company of the Twelve Apostles as they recount for him the time that they picked up a dozen baskets of scraps after feeding the multitudes.

We extend our sympathies to his widow, Mrs. Judith Hammond, whose loss is much greater than ours.

-Gene Zirkel

□

Remember – your gift to the DSA is tax deductible.

ANNOUNCEMENT OF OUR ANNUAL MEETING

Saturday, October 16, 1993
Nassau Community College
Garden City, LI, NY

10:30 AM -- Complex B

The 1993 Annual Meeting of The Dozenal Society of America will take place on Saturday, October 16, 1993 at Nassau Community College, Garden City, LI, NY, at 10:30 A.M. in Complex B.

As in the past, we will commence with The Board of Directors Meeting to be followed by The Annual Membership Meeting.

The meeting will feature a presentation by Jay L. Schiffman of The Camden City Center, who is also editor of *The Bulletin*. Jay's topic will deal with the subject of Duodecimal Combinatorics. In addition, we anticipate several presentations by other members of the Society. A late luncheon or early dinner will follow.

For a good time both educationally and socially, mark this date on your calendar. We will be extremely disappointed if you miss our Annual Meeting.

□

Boy, are we burned up!

A fire recently devastated the home of Treasurer Jim Malone. It also destroyed many of our financial records.

Several members have written us that their dues check never cleared. These checks were presumably destroyed in the fire (along with our check book and several outstanding bills).

If your dues check or donation never cleared, *please* send us a duplicate.

Thanks for your support.

CHECKING ARITHMETIC COMPUTATIONS: AN APPLICATION OF MODULAR ARITHMETIC

Jay L. Schiffman
Camden County College
Camden, NJ

A number of elementary mathematics textbooks [1] furnish some kind of check, albeit a partial one, for the correctness of computations with the four fundamental operations of arithmetic. In any number base, two favorites are "casting out base b-1" [2] and "casting out base b+1." (In the case of the decimal system, this would correspond to "casting out nines" and "casting out elevens," while in the duodecimal system, this would correspond to "casting out el's" and "casting out do one's.")

To check by casting out base b-1, find a representative for each entry in the problem. This is accomplished by adding the digits of each entry and finding a number to which the sum is congruent (mod b-1). Generally this procedure is stated "Add together all the digits in each entry, divide by b-1, and represent this entry by the remainder." The same operations are performed on the representatives of the entries as are performed on the entries. The sum of the digits in the answer to the problem (sum, difference, product, or quotient) must be congruent (mod b-1) to the answer obtained utilizing the representatives if the computation is correct. It is important to realize that if the original computation were correctly performed by arithmetic procedures, then the check by casting out base b-1 would not indicate an error. Similarly, if the mistake in the original computation happened to be a multiple of b-1, casting out base b-1 would not indicate the error.

In a similar manner, one can verify the usual rule used for checking computations by casting out base b+1. In casting out base b+1, every alternate digit commencing with the units digit is added, and the other digits are subtracted. A check by casting out base b+1 consists of finding a representative for each entry in the problem by adding and subtracting the digits as indicated above, then finding a number to which the sum is congruent (mod b+1). The same operations are performed on the representatives as on the original entries. The answer from the original entries must be congruent (mod b+1) to the answer obtained from the remainders. While casting out base b+1 will generally indicate an error in the original calculation resulting from an interchange of the digits, it will not generally indicate an error if the original computation happened to be a multiple of b+1. Thus casting out base b+1 is likewise only a partial check for our arithmetic computations.

However, if both casting out b-1 and casting out b+1 check, there is a high probability that the computation is correct. This is especially true when b-1 and b+1 are relatively prime to one another as is the case in both decimals and duodecimals.

We conclude with several illustrations in the duodecimal system.

Example 1: Perform the addition and check by casting out #'s and 11's:

(Continued)

By casting out #'s:

4*# $\equiv$ 4 + * + # = 21 $\equiv$ 3 (mod #)
+ *0# $\equiv$ * + 0 + # = 19 $\equiv$ * (mod #)
12#* 11 (mod #)

12#* $\equiv$ 1 + 2 + # + * = 20 $\equiv$ 11 (mod #)

Since 20 $\equiv$ 11(mod #), the answer 12# may be correct.

By casting out 11's:

4*# $\equiv$ # - * + 4 = 5 $\equiv$ 5 (mod 11)
+*0# $\equiv$ # - 0 + * = 19 $\equiv$ 8 (mod 11)
12#* 11 (mod 11)

12#* $\equiv$ * - # + 2 - 1 = 0 $\equiv$ 0 (mod 11)

Since 11 $\equiv$ 0(mod 11), the answer 12#* is probably correct.

Suppose on the other hand, we incorrectly added 4*# and *0# as follows:

4*#
+ *0#
12*#

We will observe that casting out #'s will not detect the error in which the last two digits in the sum were interchanged.

By casting out #'s:

4*# $\equiv$ 4 + * + # = 21 3 (mod #)
+ *0# $\equiv$ * + 0 + # = 19 * (mod #)
12*# 11 (mod #)

12*# $\equiv$ 1 + 2 + * + # = 20 $\equiv$ 11 (mod #)

Since 20 $\equiv$ 11(mod #), while the sum 12*# is incorrectly obtained, the check employing casting out #'s did not reveal the error.

Observe that the mistake in the original computation was a multiple of #; for 12#* = $(2158)_{10}$, a difference of # = 1 x #.

(Continued)

"Each one teach one."
-- Ralph Beard, Founder of the DSA

On the other hand, if we check by casting out 11's:

$$\begin{array}{r l}
4*\# & \equiv \# - * + 4 = 5 \equiv 5 \ (\text{mod } 11) \\
\underline{+\ *0\#} & \equiv \# - 0 + * = 19 \equiv \underline{8 \ (\text{mod } 11)} \\
12*\# & \qquad\qquad\qquad\qquad 11 \ (\text{mod } 11)
\end{array}$$

$$12*\# \equiv \# - * + 2 - 1 = 2 \equiv 2 \ (\text{mod } 11)$$

Since 11 $\not\equiv$ 2(mod 11), the sum 12*# is definitely incorrect.

Observe that we have a situation where casting out #'s is not sufficient to find an error.

Example 2: Find the error in the following addition problem.

$$\begin{array}{r}
100 \\
\underline{+\ 1*1} \\
290
\end{array}$$

(Clearly, the correct sum is 2*1.)

Let us check the problem by casting out #'s and by casting out 11's.

By casting out #'s:

$$\begin{array}{r l}
100 & \equiv 1 + 0 + 0 = 1 \equiv 1 \ (\text{mod } \#) \\
\underline{+1*1} & \equiv 1 + * + 1 = 10 \equiv \underline{1 \ (\text{mod } \#)} \\
290 & \qquad\qquad\qquad\qquad 2 \ (\text{mod } \#)
\end{array}$$

Now 290 $\equiv$ 2 + 9 + 0 = # $\equiv$ 0 (mod #)

Since 2 $\not\equiv$ 0 (mod #), the sum 290 is definitely incorrect.

On the other hand, by casting out 11's:

$$\begin{array}{r l}
100 & \equiv 0 - 0 + 1 = 1 \equiv 1 \ (\text{mod } 11) \\
\underline{+\ 1*1} & \equiv 1 - * + 1 = -8 \equiv \underline{5 \ (\text{mod } 11)} \\
290 & \qquad\qquad\qquad\qquad 6 \ (\text{mod } 11)
\end{array}$$

Now 290 $\equiv$ 0 - 9 + 2 = -7 $\equiv$ 6(mod 11).

Since 6 $\equiv$ 6(mod 11), the incorrect sum of 290 has not been detected by casting out 11's. We thus encounter an illustration where casting out 11's is not sufficient to find an error. Observe that the correct sum 2*1 differs from the incorrect sum 290 by 11, a multiple of 11.

Example 3: We cite an example of an incorrect computation where neither casting out #'s nor casting out 11's is sufficient to detect the error.

(Continued)

Suppose we add 149 and 254 and arrive at the incorrect sum of 1*3. (The correct sum is 3*1.)

Let us illustrate our contention as a means of justification by casting out #'s and by casting out 11's:

By casting out #'s:

$$\begin{array}{r l}
149 & \equiv 1 + 4 + 9 = 12 \equiv 3 \ (\text{mod } \#) \\
\underline{+\ 254} & \equiv 2 + 5 + 4 = \# \equiv \underline{0 \ (\text{mod } \#)} \\
1*3 & \qquad\qquad\qquad\qquad 3 \ (\text{mod } \#)
\end{array}$$

Now 1*3 $\equiv$ 1 + * + 3 = 12 3(mod #)

Since 3 $\equiv$ 3 (mod #), the incorrect sum 1*3 is not detected by casting out #'s.

By casting out 11's:

$$\begin{array}{r l}
149 & \equiv 9 - 4 + 1 = 6 \equiv 6 \ (\text{mod } 11) \\
\underline{+\ 254} & \equiv 4 - 5 + 2 = 1 \equiv \underline{1 \ (\text{mod } 11)} \\
1*3 & \qquad\qquad\qquad\qquad 7 \ (\text{mod } 11)
\end{array}$$

Now 1*3 $\equiv$ 3 - * + 1 = -6 $\equiv$ 7 (mod 11)

Although 7 $\equiv$ 7(mod 11), the incorrect sum 1*3 is not detected by casting out 11's. What is remarkable here is that the correct sum 3*1 differs from the incorrect sum 1*3 by 1#*, which is a multiple of both # and 11 (and hence is a multiple of # x 11 = ## since # and 11 are relatively prime.)

The moral of this example is to exhibit extreme caution!

Example 4: Perform the indicated subtraction and check by casting out #'s and by casting out 11's:

$$\begin{array}{r}
80* \\
\underline{-\ 1\#4} \\
616
\end{array}$$

By casting out #'s:

$$\begin{array}{r l}
80* & \equiv 8 + 0 + * = 16 \equiv 7 \ (\text{mod } \#) \\
\underline{-\ 1\#4} & \equiv 1 + \# + 4 = 14 \equiv \underline{5 \ (\text{mod } \#)} \\
616 & \qquad\qquad\qquad\qquad 2 \ (\text{mod } \#)
\end{array}$$

Now 616 $\equiv$ 6 + 1 + 6 = 11 $\equiv$ 2 (mod #)

Since 2 $\equiv$ 2 (mod #), the difference may be correct.

(Continued)

By casting out 11's:

```
 80*   ■ * - 0 + 8 =16  ■   5 (mod 11)
-1#4   ■ 4 - # + 1 =-6  ■   7 (mod 11)
 616                       -2 (mod 11)

 616   ■ 6 - 1 + 6 = #    ■# (mod 11)
```

Since # ■ -2 (mod 11), the difference is probably correct.

Example 5: Perform the indicated multiplication and check by casting out #'s and by casting out 11's:

```
 831
x173
```

By casting out #'s:

```
  831   ■ 8 + 3 + 1 = 10  ■  1 (mod #)
 x173   ■ 1 + 7 + 3 = #   ■  0 (mod #)
 2093                        0 (mod #)
 4997
 831
112#43  ■ 1 + 1 + 2 + # + 4 + 3 = 1* ■ 0 (mod #)
```

Since 0 ■ 0 (mod #), the product 112#43 may be correct.

By casting out 11's:

```
  831   ■ 1 - 3 + 8 = 6   ■ 6 (mod 11)
 x173   ■ 3 - 7 + 1 =-3   ■ * (mod 11)
 2093                       50 (mod 11)
 4997
 831
112#43  ■ 3 - 4 + # - 2 + 1 - 1 = 8   ■ 8 (mod 11)
```

Since 50 ■ 8 (mod 11), the product 112#43 is probably correct.

REFERENCES

[1] Ruric E. Wheeler--Modern Mathematics--an Elementary Approach, Brooks/Cole, Belmont, CA 1967

[2] Jean Kelly--Casting Out 'Base Minus One'--The Duodecimal Bulletin, 5*; , Volume 31;, Number 2;, Summer 1988

Acknowledgment: The author would like to thank the former editor Dr. Pat Zirkel and the reviewers for their useful suggestions and comments. □

THE FIRST 30; POWERS OF TWO:

Brian M. Dean

1;	2;
2;	4;
3;	8;
4;	14;
5;	28;
6;	54;
7;	*8;
8;	194;
9;	368;
*;	714;
#;	1228;
10;	2454;
11;	48*8;
12;	9594;
13;	16#68;
14;	31#14;
15;	63*28;
16;	107854;
17;	2134*8;
18;	426994;
19;	851768;
1*;	14*3314;
1#;	2986628;
20;	5751054;
21;	#2*20*8;
22;	1*584191;
23;	38#48368;
24;	75*94714;
25;	12#969228;
26;	25#716454;
27;	4##2308*8;
28;	9#*461594;
29;	17#8902#68;
2*;	33#5605#14;
2#;	67*#00#*28;
30;	1139*01#854;

□

FIND THE NEXT TERM

Shaun Ferguson
Dozenal Society of Great Britain

"Find the next number in the sequence..." is a favorite problem of newspaper "Puzzle Corners" and numerically-minded Quiz Shows. This activity ranges from nice simple patterns such as 2,4,6,8,... to quite complicated structures. Implied in the request to find the next number is the idea there is a next number, and that it is unique. Are there sequences that admit of more than one answer - that are in fact 'ambiguous'? If so - how many (or how few) terms of the sequence do we need to make it unambiguous?

Given the pattern 2,4,6,8,... most people would assume the next number is ten; obviously jumping around from base to base we could produce different patterns depending on how <u>ten</u> is written: 11 in base nine, * in base eleven or twelve, 10 in base ten and so on. So in what follows, no jumping around is allowed and, as you might expect, all numbers are in base twelve.

Since two terms could lead anywhere, we might assume that 3 terms were the minimum: 1,2,3,... or 2,4,6,... or 1,3,5,... But even the innocent-looking sequence 1,2,3,... is wide open: is 1,2,3,... produced by adding 1 each time? Or is the third term (as in the Fibonacci sequence) the sum of the previous two terms? Does the pattern go 1,2,3,4,5,6,... or 1,2,3,5,8,11,...? So should we assume we need at least four terms? ... What sparked all this was the fact that the pattern of the first three primes (2,3,5,...) <u>could</u> imply the next items were meant to be primes - but did not need to do so. In fact, there is quite a choice of sequences ...

(A) 2,3,5,7,#,11,15,17,... (primes)

since 2 x 3 - 1 = 5, we could have:

(B) 2,3,5,12,59,... (12 = 3 x 5 - 1, 59 = 5 x 12 - 1, etc.)

(C) 2,3,5,9,15,... (nth term is 2^{n-1} + 1)

or there could be a pattern of increasing addends:
2 + <u>1</u> = 3, 3 + <u>2</u> = 5, whence

(D) 2,3,5,8,10,15,...

and if you note that the first four terms of pattern (D) are the same as those of the Fibonacci sequence

(E) 2,3,5,8,11,19,...
maybe we need at least five terms....

(Continued)

Poser for our readers: Can you write down a sequence of n numbers such that there is only one (unambiguous) value for the (n+1)st term? Just the numbers, not a rule stated in algebraic or set-theoretic terms... No, you can't have 2,4,6,8..., attractive though it be; the next number <u>might</u> be **ten**, but it could equally well be # or 10;... Another valid sequence is (a) 2,4,6,8,10,1*,... and another (b) 2,4,6,8,#,15,... - can you deduce the rule for constructing them?

Take nothing for granted! Here are two more to puzzle out:

(1) 1,2,4,8,14,<u>26</u>,<u>44</u>,... (not <u>28</u>, <u>54</u>,...)

and

(2) 1,2,4,8,14,28,<u>5*</u>,...

Can you find out what I've done and give the next term? See the next page for a hint.

□

A CRYPTARITHM

Solve the crytarithm in base twelve (of course), and then read the message below. Note *, #, and ; are used for ten, eleven, and the fraction point respectively.

```
1            OEMA;IIUD
2      MDU)DOZEN
3          MDU
4           LDE
5           COZ
6           OOZN
7           OOZA
8              O I I I
9                HUO
*                 AHI
#                 ALD
10                   C
```

Message:

```
_ _ _ _ _   _ _ _ _ _ _ _ _,  _ _ _ _ _   _ _ _ _ _ _ _ _ _ _
6 3 0 4 5   3 4 9 0 2 6 *     # 4 * * 1   3 5 1 3 4 9 0 2 6 *
```

-GZ

A NEW APPROACH TO SYMBOLS

George P. Jelliss
East Sussex, UK

A large obstacle to the adoption of the dozenal system, it seems to me, is the choice of symbols for the two extra digits.

The DSGB use of rotated 2 and 3 is impractical because the symbols are simply not available on most typewriters (unless you go to all the trouble of leaving spaces and filling in the gaps by typing it upside down, or having the typewriter specially doctored). These signs are not even available on word processors, which have a wide range of alternative symbols, though I suppose they could be specially programmed with a lot of expenditure of effort, which most people are not going to feel is worth all the bother.

The DSA symbols * and # are readily available by single key-strokes, but are already heavily used for other purposes that could conflict with their use as numerals.

The best solution I can find on my system (a personal computer with "WordPerfect") is to use the Greek letters delta (δ), the initial letter of the Greek word for ten (deka), and epsilon (ε), similar to the inverted 3 or to E for eleven. The next letter of the Greek alphabet, zeta (ζ), as it happens, is also suitable as a symbol for the dozen. The delta and epsilon are available as ASCII characters (which appear on the computer screen) [by holding down the Alt key and pressing the digits 235 or 238 on the number-pad]. The zeta, however, is only available from the Greek font, [accessed by pressing the Control and v keys at the same time and typing 8, 13 and pressing the Enter key] but appears on the screen only as a blob.

The symbols go quite well with the digits in the "Courier" typewriter style:

0 1 2 3 4 5 6 7 8 9 δ ε ζ

□

HINT for "FIND THE NEXT TERM" on page 13

The 2,4,6,8... patterns in the last paragraph are created by the formulae:

(a) $$f(n) = 2n + \frac{(n-4)(n-3)(n-2)(n-1)}{10}$$

and

(b) $$f(n) = 2n + \frac{(n-4)(n-3)(n-2)(n-1)}{20}$$

A similar idea works for (1) and (2) at the end. [I think that (1) is connected with the number of segments in a circle cut by n intersecting lines.]

By the way, in (2) 5 x 12 - 1 = 59 is true in any base b, where b ≥ 10.

□

THE ORIGINS OF THE ASTERISK AND THE OCTOTHORPE

Gene Zirkel
Nassau Community College
Garden City, LI, NY

Which symbols to use has always been a bone of contention among advocates of dozenal counting. Some favor using the current 0 through 9 plus two additional symbols for ten and eleven. Others prefer a completely new set of digits. Even among those who agree on one or the other of the above ideas, there is a great deal of disagreement concerning which symbols should be used.

I have always been of the school that advocates only adding two new symbols, and I really don't have strong feelings as to what they may be. I think that we gain more converts at this time by keeping the familiar digits from 0 to 9. I also feel that when dozenals are finally accepted, society will decide on what symbols it then wants, not the DSA.

In the interim, I can use any two symbols that we agree upon, as long as they do a reasonable job of signifying ten and eleven. The DSA long used the script X and E designed for us by the typographer, Dwiggins. When the telephone company introduced the dozenal push-button phone, it was decided to switch to their asterisk (*) and octothorpe (#) because it was felt that these would become familiar to many people.

The DSA does not endorse any particular symbols, but for the sake of uniformity, and to make it easier on our readers -- especially newcomers -- the editors of our Bulletin have always adhered to the following policy:

> Unless an article was about symbols, the Bulletin would use the same symbols in all articles. In earlier issues, these were the symbols designed by Dwiggins and now they are * and #.

Our British cousins in the DSGB have adopted the rotated 2 and 3 which were first suggested by Sir Isaac Pitman, the inventor of Pitman Shorthand. Some of their members have suggested that we should all use the same symbols. I do not see any advantage to the DSA making another change of symbols at this time. (I sometimes regret that we abandoned Dwiggins.) One pair of symbols is just as good as another, and further change might tend to confuse our readers and make us appear inconsistent. The rotated 2 and 3 are not on standard typewriters nor computer printers, while * and # (or X and E) are readily available.

Recently, Honorary Member number 262; Arthur Whillock of Great Britain indicated that the first use of the * and the # was by Edna E. Kramer. Following his lead, I found her book in the Nassau Community College Library:

> Edna E. Kramer, The Main Stream of Mathematics (New York: Oxford University Press, 1955 [Fourth Printing, 1963]), pages 16 and 17.

(Continued)

She uses * and # without naming them. Kramer mentions that some mathematicians have agreed that twelve would be the best base since it has the divisors 2, 3, 4, 6, which would have made fractional work easier than it is in base ten. The author, however, remains neutral on the subject.

In addition, she makes reference to the story that King Charles XII (what else?) of Sweden was about to mandate duodecimals when he died. [This story is apocryphal. See this Bulletin, Number 52; Vol. 2*, No. 3, Fall 1985, p. 17; -Ed.]

Giving credit where credit is due, perhaps we should refer to *Kramer's* * and #, rather than to the telephone company's.

□

TIME PERIODS

Jean Kelly
New York, NY

When using the dozenal clock invented by Dr. Paul Rapoport[1], time is read with four digits, for example 795;6. This indicates:

7	two-hour periods {one twelfth of a day},
9	dek-minute periods {one twelfth of two hours},
5	fifty-second periods {one twelfth of dek minutes}, and
6	periods of four and one sixth seconds {one twelfth of fifty seconds}.

This translates into 14:00 + 1:30 + 0:04:10 + 0:00:25 or 15:34:35 hours (or 3:34:35 PM in the old fashioned system).

It is felt that we need some words to describe these periods of time in everyday conversation, words such as, "I'll meet you in 2 hours and thirty minutes."

Some proposals have been:

2 hours	ten minutes	fifty seconds	4 1\6 seconds
Dours[2]	Temins[2]	Minettes[2]	Thirds[3]
Duors[4]	DekMins[5]		MicroDays[6]
Du-Hour[4]			

Thus our example above of 795;6 o'clock might be read as:

"7 dours, 9 dekMins, 5 minettes, and 6 thirds"

while "2 hours and thirty minutes" could translate into "a dour and 3 dekMins".

What do you think? Please send us your suggestions.

1 See this Bulletin, Volume 31; Number 3; pp. 10-14; Fall 1988.
2 Jay M. Anderson, this Bulletin, Volume #; Number 1; pp. 22-24; May 1955. [Note that a new Minette is just less than an old minute.]
3 Fred Newhall. [Editor's note: this is approximately a third of one dozen old seconds.]
4 Admiral Elbrow, this Bulletin, Volume 4; Number 1; p. 13.
5 Gene Zirkel
6 Anonymous

□

DUODECIMAL CROSS-NUMBER PUZZLE SOLUTION

Jay L. Schiffman
Camden City Center
Camden, NJ 08102

In Duodecimal Bulletin 70;, I presented a puzzle at the annual meeting. The puzzle is reproduced here for reference where each entry is a positive digit.

1	3	9
9	*	8
4	6	8

ACROSS:

1. These three digits in order forms a geometric progression.

2. This duodecimal integer having dek divisors is divisible by the integers 14; and 75;.

3. These three digits in order form an arithmetic progression.

DOWN:

1. A permutation of these digits generates the first three square integers. The resulting duodecimal integer is a perfect square, a perfect fourth power, and a perfect eighth power.

2. A rearrangement of these digits produces in sequence three triangular numbers.

3. This duodecimal integer has two dozen divisors.

(Continued)

ANALYSIS

The integers 1, 3, and 9 represent the initial three terms of the geometric progression 1, 3, 9, 23;, 69;,..., 3^n,... with common ratio 3 between successive terms.

We are next given that the duodecimal integer is divisible by both 14; and 75;. Since 14; and 75; are co-prime, this integer is divisible by their product 14; x 75; = 9*8;. Observe that 9*8; = 14; x 75; = 2^4 x 75; which has dek divisors. The number of divisors of an integer n, is denoted by $\tau(n)$, a multiplicative number-theoretic function for relatively prime integer pairs. Hence τ(9*8) = $\tau(2^4$ x 75;) = $\tau(2^4)$ x (75;) = (4+1) x (1+1) = 5 x 2 = *;. (Note that $\tau(p) = 1 + 1 = 2$ if p is prime and $\tau(p^r) = r + 1$ if p is prime.)

The integers 4, 6, and 8 represent three consecutive positive even integers and form an arithmetic progression with common difference 2 between successive terms.

The first three square integers are 1, 4, and 9. The numeral 149 is a permutation of the digits 194. In the duodecimal base, 194; = $2^8 = 4^4$ = $(14;)^2$ and is hence a perfect eighth power, a perfect fourth power, and a perfect square. It should also be noted that the the square integers are 1, 4, 9, 14;, 21;, 30;, 41;,..., n^2,....

The triangular numbers are 1, 3, 6, *;, 13;, 19;, 24;,.... A rearrangement of the digits 3, *;, and 6 is 3, 6, and *; which represent the second, third, and fourth triangular numbers respectively.

The duodecimal integer 988; is resolved into prime factorization as follows: 988; = 2^3 x 5^2 x 7. Using our number-theoretic function τ (since 2, 5, and 7 are co-prime in pairs, the multiplicative character of τ yields τ(988;) = $\tau(2^3$ x 5^2 x 7) = $\tau(2^3)$ x $\tau(5^2)$ x $\tau(7)$ = (3+1) x (2+1) x (1+1) = 4 x 3 x 2 = 20;. Hence 988; has two dozen divisors.

□

REVERSED AND EQUAL

An unsolved problem

The number 32 in base five represents the same number as the reversed numeral 23 does in base seven, (i.e., 15;).

(In fact, in any odd base b, b > 3,
32 in base b equals 23 in base 3(b-1)/2 + 1.)

Can you find a numeral in base twelve that equals its reversal in the cumbersome base dek?

-GZ

PROVING IDENTITIES BY A CHANGE OF BASE

Gene Zirkel
Nassau Community College
Garden City, LI, NY

Certain identities become apparent when we write them in an appropriate base. For example, to show that an equation such as

$$1 + 5 + 5^2 + 5^3 + \ldots + 5^{n-1} = \frac{5^n - 1}{5 - 1}$$

is an identity, we rewrite it in base five notation:

$$1 + 10 + 100 + 1000 + \ldots = \frac{10^n - 1}{4} \qquad [\#1]$$

This follows at once from the fact that in base five

$$444\ldots44 = 10^n - 1$$

or

$$111\ldots11 = \frac{10^n - 1}{4}$$

which is exactly equation [#1] above. ☐ QED

In general, if we let d = b - 1, the largest digit in base b, then

$$111\ldots11 = \frac{10^n - 1}{d} \text{ in base b}$$

Hence $1 + b + b^2 + b^3 + \ldots + b^{n-1} = \frac{b^n - 1}{b - 1}$. ☐ QED

A *Second Example*

We can also use a change of base to establish identities such as

$$\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \ldots = \frac{1}{2} \qquad [\#2]$$

Simply rewrite this series in base three, obtaining:

$$.1 + .01 + .001 + \ldots = .111\ldots$$

(Continued)

DOZENAL JOTTINGS

...from members and friends...News of Dozens and Dozenalists...

As we go to press, we are informed that **HENRY CHURCHMAN** passed away in Council Bluffs, Iowa on July 4, 1993. Henry is a former Editor of this Bulletin, and a long-time member and leader of the DSA. A full Obituary will be included in the next issue. We extend our sympathies to his son, JOHN CHURCHMAN, and to the rest of his family...

Welcome to *New Member*:

336; **TEGAN CHESLACK-POSTAVA** Ridgewood, NJ

☐

Do you have an idea to share with our members? Why not submit an article to the *Bulletin*?

PROVING IDENTITIES BY A CHANGE OF BASE

(Continued from page 1)*

We recall that in base three, .222... = 1

and thus $.111\ldots = \frac{1}{2}$ which is precisely equation [#2] above. ☐ QED

In order to generalize this type of series, let d = b - 1. We know

that .ddd... = 1 in base b.

Thus $d \times .111\ldots = 1$

or $.111\ldots = \frac{1}{d}$

and hence we have $\frac{1}{b} + \frac{1}{b^2} + \frac{1}{b^3} + \ldots = \frac{1}{b-1}$ ☐ QED

☐

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition--("Who needs a symbol for nothing?")--the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used.

A parallel to today seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

□

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal-based on ten. The dozen system uses twelve as the base, which is written 10, and is called do, for dozen. The quantity one gross is written 100, and is called gro. 1000 is called mo, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, 265 represents 5 units, 6 dozen, and 2 dozen-dozen, or gross. This number would be called 2 gro 6 do 5, and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5;9'
31	694	Three ft. two in.	3;2'
96	3#2	Two ft. eight in.	2;8'
19#	1000	Eleven ft. seven in.	#;7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, which is 5 dozen and 3; so set down 53. Using this "which is" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection if you are 35 years old, dozenally you are only 2#, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

12) 365
12) 30 + 5
12) 2 + 6
0 + 2 Answer: 265

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or*.

For more detailed information see Manual of the Dozen System ($1;00).

DETACH HERE -- OR PHOTOCOPY

We extend an invitation to membership in our society.
dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ____________________
LAST FIRST MIDDLE

Mailing Address (for DSA items) ____________________

____
(See below for alternate address)

Telephone: Home ____________ Business ____________

Date & Place of Birth ____________________

College ____________ Degrees ____________

Business or Profession ____________________

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ____________________

Address ____________________

Year & Math Class ____________________

Instructor ____________ Dept. ____________

Other Society Memberships ____________________

____

Alternate Address (indicate whether home, office, school, other)

____

____

____

Signed ____________ Date ____________

My interest in duodecimals arose from ____________________

____

____

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530