DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

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THE DUODECIMAL BULLETIN 70;

Gene Zirkel (center) enjoys an English tea with Arthur and Ruby Whillock--
(See *Minutes of the Annual Meeting of 1992)*

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	11*1;

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00 (US) for one calendar year. Student Membership is $3.00 per year, and a Life Membership is $144.00 (US).

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BOARD OF DIRECTORS OF THE DOZENAL SOCIETY OF AMERICA

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THE DUODECIMAL BULLETIN

Whole Number Seven Dozen

Volume 36; Number 1;

11*1;

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IN THIS ISSUE

FROM THE EDITOR

With the advent of this issue, a change in the editorship has transpired. Dr. Patricia McCormick Zirkel, who has served so ably in this capacity since 1981, has relinquished the duties due to her faculty position which is extremely demanding. As your new editor, I will endeavor to continue to produce a bulletin of high journalistic quality and sound dozenal content. As in the past, expository articles dealing with various aspects of number bases, puzzles, information concerning our annual and board meetings, and dozenal jottings will encompass these pages.

We owe a debt of gratitude to Pat for her excellent tenure as editor of *The Bulletin* which in no small measure is responsible for the Society's revival.

I submitted one of the first articles to *The Bulletin* in 1981 which appeared in the winter 1982 issue. In the interim, I have contributed additional articles, puzzles, and jottings. I joined the society in 1987 and have participated in society affairs since 1989. I have found Pat extremely delightful to work with and it is my sincere hope that we progress as well over the next dozen years.

Best Wishes Twelvefold,

Jay L. Schiffman

Editor

> The DSA does **NOT** endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.
>
> When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.
>
> Thus 1/2 = 0.5 = 0;6.

DOZENAL SOCIETY OF AMERICA
MINUTES OF THE ANNUAL MEETING—11*0

Saturday, October 24, 1992
Nassau Community College
Garden City, LI, NY. 11530

I. BOARD OF DIRECTORS MEETING

1. Gene Zirkel, Board Chair, opened the meeting at 10:45 a.m.. The following Board Members were present: Alice Berridge; Jamison Handy, Jr.; James Malone; Fred Newhall; Gene Zirkel; Jay Schiffman. All of us missed Pat Zirkel who was unable to attend the meeting because of bronchitis.

2. The minutes of the June 6 meeting were approved. The Nominating Committee (A. Berridge, J. Malone, J. Schiffman) presented this slate:

Board Chair:	Dr. Patricia McCormick Zirkel
President:	Fred Newhall
Vice President:	Jay Schiffman
Secretary:	Alice Berridge
Treasurer:	James Malone

   The slate was elected unanimously.

3. Appointments were made to the following DSA Committees:

   Annual Meeting Committee: Dr. Barbran Smith, Chair, Alice Berridge.

   Awards Committee: Gene Zirkel, Chair; Dr. Patricia Zirkel.

   Volunteers to these committees are welcome at any time.

4. The following appointments were made:

   Editor of The Duodecimal Bulletin: Jay Schiffman.
   Production Manager of The Bulletin: Dr. Patricia Zirkel.
   Parliamentarian to the Board Chair: Jamison Handy, Jr.
   Reviewers of articles submitted to The Bulletin: Anthony Catania, Jamison Handy, Jr., Dr. John Impagliazzo, Kathleen McKiernan, Fred Newhall, Dr. Barbran Smith, Gene Zirkel.

5. Other Business of the Board: Next meeting will be June 5, 1993 at 10:30 a.m. at Nassau Community College in Complex B.

   Gene Zirkel and the Society were featured in the "LIFESTYLES" section of This Week (a weekly Long Island publication) on October 3, 1992. The article

(Continued)

"Easier by the Dozen" by Valerie Kellogg generated good publicity. Gene received several letters concerning the article. Gene was able to obtain a clean xerox copy of Dozens vs Tens, the 1866 text by Thomas Leech, F.C.S. which was featured in Fred Newhall's article in volume 6*; Number 1 and part of Jottings in Number 2. Gene and his family met with Arthur Whillock on a trip to England last summer. Whillock of the DSGB said that he will present the original model of the Dozenal slide rule to DSA. Gene and he spent an afternoon in his library in England perusing Whillock's archives. Whillock is hoping to interest a library in England preserving his large collection of materials pertinent to dozens. Gene said that DSA would preserve these archives if necessary. John D. Hansen, Jr., author of "Dozenal Music Revisited" in Volume 6*; Number 2, sent a message to members included with a copy of an article in a San Diego paper related to Relativity Theory, a special interest of Hansen's.

Gene showed us a full-page DSA advertisement which appeared in a recent issue of The Journal of Recreational Mathematics. We have a reciprocal advertising arrangement with the Journal. Gene was a recent call-in participant on a New York City radio station (WOR): "The Gene Burns' Talk Show". Gene responded to remarks about the metric system and talked about dozens, DSA, and Nassau Community College.

The Board Meeting was adjourned at 11:15 a.m.

II. ANNUAL MEMBERSHIP MEETING

1. DSA President Fred Newhall gavelled the meeting to order at approximately 11:15 a.m.

   In addition to the Board Members listed above, the following Society Members were present: Vera Sharp Handy, Mary Malone, Mary Newhall, Barbran Smith.

2. Gene Zirkel moved to accept the minutes of October 19,1991. So approved.

3. President's Report- Fred Newhall.

   Fred has been working very hard on a regular index for the Bulletin and a larger, 60 page index which will include items from the Great Britain Journal as well. This larger index should be available soon and can be requested from Fred. Fred said he is impressed with the thousands of names from all over the world who have worked to advance the cause of dozens. The indices are separated into three sections: titles of articles and books, names of people, and subjects. This is almost all on disk and is very painstaking work. We are very grateful for his efforts. Fred presented each of those present with a copy of a page of the index which features each of us. Fred's index will be available as printed from his PC, and is to be enclosed in a folder.

(Continued)

   Fred mentioned that there were about 60 inquiries about DSA last year.

   President's Appointments: Parliamentarian to the President: Dr. Patricia Zirkel

4. Treasurer's Report- Anthony Catania

   Barbran Smith presented copies of October 16, 1992 report to compare with Tony's report of this date. Members were concerned that our net dollar value has dropped $1,657.29. It was noted that expenses are increasing, our Certificate of Deposit decreased in value, and our net stock growth for the year was only $10.00 more than last year. Members agreed that the decision to change from three to two issues of the Bulletin is sensible in terms of the economic stalemate. The dollar value of DSA is currently $15,241.43.

   Jamison Handy moved acceptance of the Treasurer's report with the recommendation of the Treasurer. So approved.

5. Editor's Report- Jay Schiffman

   Jay has begun work on the next issue. He said he was impressed with the variety and volume of articles that he received from Pat Zirkel. He says there are several long articles but he is interested in short as well as long articles. Jamison mentioned that the filler "Think About It!!!" was especially intriguing and ought to be reprinted.

Committee Reports

6. Annual Meeting Committee- Dr. Barbran Smith. Members thanked Barbran for her efforts in arranging this meeting.

   The next Annual Meeting will be on October 16, 1993 following the Board meeting which will begin at 10:30 a.m. and will be held in B cluster at Nassau Community College.

7. Awards Committee- Gene Zirkel

   Tony Catania was unable to attend this meeting. As a result, Gene arranged to present a plaque and a bottle of champagne to Tony at a recent faculty meeting at Nassau Community College in recognition of Tony's service to DSA.

   Gene asked for names of persons who might be honored by the society.

(Continued)

8. Nominating Committee- Alice Berridge

The committee presented the slate for the Class of 1995: Alice Berridge, Dr. John Impagliazzo, Robert R.McPherson, Gene Zirkel. The slate was elected unanimously.

The Nominating Committee for next year will be Alice Berridge, James Malone, Jay Schiffman.

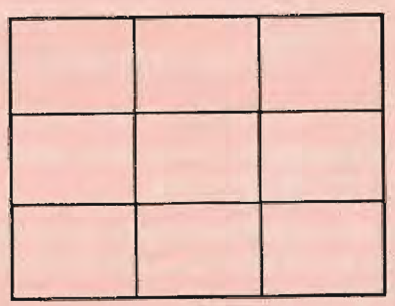
9. Other Business

Jamison and Vera Handy spoke of their experiences in New Zealand with member Bruce Moon. The meeting was adjourned at 12:00 noon.

III. FEATURED SPEAKER

Jay Schiffman, a faculty member at Camden County College Extension Center (NJ) presented a very interesting talk,"A Duodecimal Cross Number Puzzle." He said his interest had been piqued while working with a colleague on crossword puzzles. He presented members with the following puzzle:

(Each entry is a positive digit.)



ACROSS:

1. These three digits in order form a geometric progression.
2. This duodecimal integer having dek divisors is divisible by the integers 14; and 75;.
3. These three digits in order form an arithmetic progression.

(Continued)

DOWN:

1. A permutation of these digits generates the first three square integers. The resulting duodecimal integer is a perfect square, a perfect fourth power, and a perfect eighth power.
2. A rearrangement of these digits produces in sequence three triangular numbers.
3. This duodecimal integer has two dozen divisors.

The group had a lot of fun solving this puzzle. Jay guided us through the analysis of the clues.

Readers may enjoy referring to Jay's recent article related to this topic which appeared in Volume 6*; Number 2. Jay also discussed "Duodecimal Divisibility Tests for any Integer n by each of the integers from one to one gross." He provided seven pages of analysis and helped the group work out several of the examples.

Gene mentioned that he is aware that high stake investment brokers and government agencies use a secret code which relies on prime factors of the multiple of two huge prime numbers. It seems that computers can decode the problems but require 24 hours to do the job. He mentioned that when information needs to be available, this process works fast and maintains secrecy.

Members thanked Jay for both of his intriguing and instructive talks.

Members met together for a pleasant lunch after the meeting.

Respectfully submitted,

Alice Berridge □

MEMBERS

Your 11*1 (1993) dues are due as of January 1st. Please forward your check for one dozen dollars to Treasurer Jim Malone. Student dues are $3 and LIFE Memberships are one gross dollars. As you know our continued work depends upon the tax deductible dues and gifts of our members.

THANKS to those of you who have already paid and/or sent gifts.

THE ROLE OF CONGRUENCES TO PROVE DUODECIMAL DIVISIBILITY TESTS

Jay L. Schiffman
Camden County College
Camden, N.J. 08102

INTRODUCTION:

It is often useful to quickly determine when one integer is divisible by another without resorting to tedious long division. To cite an example in the decimal base, it is well known that an integer is divisible by 3 if the number formed by the sum of the digits (no matter how many digits the integer possesses) is divisible by 3, and conversely, if the number formed by the sum of the digits is divisible by 3, then the integer is divisible by 3.

The purpose of this article is to utilize congruences to furnish duodecimal divisibility for an integer by each of the integers from 1 to 14;. Throughout the article, the set Z will denote the set of all integers. We initiate our discussion with the definition of congruence.

DEFINITION 1: For a,b, n ε Z with n ≥ 2, we write a ≡ b (mod n) read "a is congruent to b, modulo n," to denote that a - b = k * n. (The difference between a and b is a multiple of n) for some k ε Z. Otherwise we write a ≢ b (mod n).

The number is called modulus of the congruence. The term modulus is derived from the Latin meaning "little measure."

To illustrate our definition, 5 ≡ 17; (mod 7)
(since 5 - 17; = - 12; = - 2 * 7) while 5 ≢ 17; (mod 3)
(since 5 - 17; = - 12; which is not an integral multiple of 3.)

The following theorem furnishes one with the properties of congruences which are essential for our work. The proof of Theorem 1 can be found in any standard number theory textbook.

THEOREM 1: For a,b,c,n Z with n ≥≥ 2:

(1) . a ≡ a (mod n)

(2) . If a ≡ b (mod n), then b ≡ a (mod n).

(3) . If a ≡ b (mod n) and b ≡ c (mod n), then a ≡ c (mod n).

(4) . If a ≡ b (mod n) and k ε N, then ka ≡ kb (mod n) where N = 1, 2, 3, . . . , the set of positive intergers.

(5) . If a ≡ b (mod n) and c ≡ d (mod n), then a + c ≡ b + d (mod n).

(Continued)

(6) . If a ≡ b (mod n) and c = d (mod n), then a * c ≡ b * d (mod n).

(7) . If a ≡ b (mod n) and k ε N, then $a^k \equiv b^k$ (mod n) where N = 1, 2, 3, . . ., the set of positive integers.

(8) . Let a ≡ b (mod n). Then for all $c_0, c_1, \ldots, c_k$ ε Z, the following congruence is valid:
$c_0 + c_1 * a + c_2 * a^2 + \ldots + c_k * a^k \equiv c_0 + c_1 * b + c_2 * b^2 + \ldots + c_k * b^k$ (mod n).

We present an illustration for each Part of Theorem 1 in the dozenal base.

(1) . 5 ≡ 5 (mod 10;) since 5 - 5 = 0 = 0 * 10;.

(2) . 10; ≡ 5 (mod 7) since 10; - 5 = 7 = 1 * 7. Now 5 ≡ 10; (mod 7) since 5 - 10; = -7 = 1 * -7.

(3) . 15; ≡ 5 (mod 3) since 15; - 5 = 10; = 4 * 3.
Also 5 ≡ 2 (mod 3) since 5 - 2 = 3 = 1 * 3.
Jointly these two congruences imply 15; ≡ 2 (mod 3) since 15; - 2 = 13; = 5 * 3.

(4) . 23; ≡ 15; (mod 5) since 23; - 15; = *; = 2 * 5. Now 5 * 23; ≡ 5 * 15; (mod 5) or # 3; ≡ 71; (mod 5) since #3; - 71; = 42; = *; * 5.

(5) . Observe that 23; ≡ 11; (mod 7) since 23;-11; = 12; = 2 * 7. Also 30; ≡ 25; (mod 7) since 30; - 25; = 7 = 1 * 7. Now 23; + 30; ≡ 11; + 25; (mod 7) since 53; ≡ 36; (mod 7) as 53; - 36; = 19; = 3 * 7.

(Continued)

(6) . 15; ≡ 7 (mod 5) and 10; ≡ -8 (mod 5). Observe that 15; ≡ 7 (mod 5) since 15; -7 = *; = 2 * 5. In addition, 10; ≡ -8 (mod 5); for 10; -(-8) = 10; + 8 = 18; = 4 * 5. Now 15; * 10; ≡ 7(-8) (mod 5) since 150; ≡ -48; (mod 5) as 150; -(-48;) = 150; + 48; = 198; = 44; * 5.

(7) . 10; ≡ -3 (mod 5); for 10; - (-3) = 10; + 3 = 13; = 3 * 5. Now 10; * 3 ≡ (-3)3 (mod 5) since 1000; ≡ -23; (mod 5) as 1000; - (-23;) = 1000; + 23; = 1023; = 253; * 5.

(8) . 10; ≡ 4 (mod 8) since 10; -4 = 8 = 1 * 8. Now 6 + 4 * 10; + #; * $(10;)^2$ = 6 + 4 * 4 + #; * $(4)^2$ (mod 8). Observe that 6 + 40; + #; * 100; ≡ 6 + 14; + #; * 14; (mod 8). 46; + # 00; ≡ 6 + 14; + 128; (mod 8). # 46; ≡ 146; (mod 8) since #46; - 146; = *00; = 130; * 8.

Before presenting our next theorem, some machinery is required.

DEFINITION 2: the greatest common divisor of two integers is the largest integer which is a divisor of both. Two integers are said to be relatively prime or co-prime if their greatest common divisor is one. We denote the greatest common divisor of a and b by (a,b).

To illustrate, the greatest common divisor of 13; and 18; (decimally 15 and 20 respectively) is 5 since 5 is the largest counting integer which is a divisor of both 13; and 18;. On the other hand, the greatest common divisor of 8 and #; (decimally 8 and 11 respectively) is 1 since 1 is the largest counting integer which is a divisor of both 8 and #;. We state the next theorem without proof. The reader is invited to consult Reference (2) in the bibliography at the conclusion of this article for a formal proof of Theorem 2.

THEOREM 2: If d = (a,b), then there exist m,n Z such that d = m * a + n * b.

From Theorem 2, we obtain the following useful corollary:

Corollary 1: If (a,b) = 1, then there exist m, n ε Z such that m * a + n * b = 1.

Based on the Preceding Corollary, we derive Theorem 3 which provides the final touch needed for verifying the divisibility properties desired in the dozenal base. In the theorem, the notation a / b will denote that a divides b, where a, b ε Z.

THEOREM 3: Suppose (a,b) = 1, a / c and b / c. Then (a * b) / c.
PROOF: Given (a,b) = 1, Corollary 1 guarantees us the existence of integers m and n satisfying the equation m * a + n * b = 1. Multiplying both sides of this equation by c yields mac + nbc = c. (1)

Since a / c, we can find an integer s such that c = a * s. (2)

Since b / c, we can find an integer t such that c = b * t. (3)

Substituting (2) and (3) in (1) yields mabt + nabs = c.

(Continued)

Now clearly ab / mabt and ab / nabs so that ab / (mabt + nabs) or ab / c as required.

It should be noted that since a, b, m ,n, s, t ε Z so does mabt + nabs = c utilizing the closure Properties of Z under addition and multiplication.

We are now in position to achieve our main goal. We state the dozenal divisibility tests for the integers 1 - 14;.

THEOREM 4: (The Duodecimal Divisibility Tests for an integer n by each of the integers 1 - 14;.)

(1) . All integers n are divisible by 1.

(2) . An integer n is divisible by 2 if the last digit, a_0 is even; that is a_0 is either 0, 2, 4, 6, 8, or *; .

(3) . An integer n is divisible by 3 if the last digit, a_0, is either 0, 3, 6, or 9.

(4) . An integer n is divisible by 4 if the last digit, a_0, is either 0, 4, or 8.

(5) . An integer n = $...a_4\ a_3\ a_2\ a_1\ a_0$ is divisible by 5 if $a_0 + 2a_1 - a_2 - 2a_3 + a_4 + ...$ is divisible by 5.

(6) . An integer n is divisible by 6 if it is divisible by both 2 and 3; that is, the last digit, a_0, is either 0 or 6.

(7) . An integer n is divisible by 7 if $a_0 - 2a_1 - 3a_2 - a_3 + 2a_4 + 3a_5 + a_6 - ...$ is divisible by 7.

(8) . An integer n is divisible by 8 if $a_0 + 4a_1$ is divisible by 8.

(9) . An integer n is divisible by 9 if $a_0 + 3a_1$ is divisible by 9.

(10) . An integer n is divisible by *; if n is divisible by both 2 and 5.

(11) . An integer n is divisible by #; if the digital sum $a_0 + a_1 + a_2 + ... + a_k$ is divisible by #;.

(12) . An integer n is divisible by 10; if it is divisible by both 3 and 4; that is the last digit, a_0, is 0.

(13) . An integer n is divisible by 11; if the alternating digital sum $a_0 - a_1 + a_2 - a_3$ +.. is divisible by 11;.

(14) . An integer n is divisible by 12; if n is divisible by both 2 and 7.

(15) . An integer n is divisible by 13; if n is divisible by both 3 and 5.

(Continued)

(16). An integer n is divisible by 14; if $a_0 - 4a_1$ is divisible by 14;.

Before proving these divisiblity tests, if should be noted that a duodecimal integer $a_k a_{k-1} \ldots a_1 a_0$; in expanded notation can be written as $a_k * 10;^k + a_{k-1} * 10;^{k-1} + \ldots + a_1 \cdot 10; + a_0$. In addition, $1 \equiv 1 \pmod{n}$ where $n \in N$ and $n \geq 2$.

PROOF OF THEOREM 4:

(1). n is divisible by 1 since $n = 1 * n$ for all $n \in Z$.

(2). To determine when n is divisible by 2, observe that $10; \equiv 0 \pmod{2}$. Hence $a_k * 10;^k \equiv 0 \pmod{2}$ for all $k \geq 1$. (Theorem 1 (7)). so that $n = a_0 + a_1 * 10; + a_2 * 10;^2 + \ldots + a_k * 10;^k \equiv a_0 \pmod{2}$. That is, if n is divisible by 2, then the last digit, a_0, is even.

(3). To determine when n is divisible by 3, note that $10; \equiv 0 \pmod{3}$ so that $10;^k \equiv 0 \pmod{3}$ for each $k \in Z$, $k \geq 1$. Hence $n = a_0 + a_1 * 10; + a_2 * 10;^2 + \ldots + a_k * 10;^k \equiv a_0 \pmod{3}$ which occurs if $a_0 = 0, 3, 6$, or 9. Observe that $10;^k \equiv 0 \pmod{3}$ for each $k \in Z$, $k \geq 1$ implies that $a_k * 10;^k \equiv 0 \pmod{3}$.

(4). To determine when n is divisible by 4, note that $10; \equiv 0 \pmod{4}$ so that $10;^k \equiv 0 \pmod{4}$ for each $k \in Z$, $k \geq 1$. Hence $n = a_0 + a_1 * 10; + a_2 * 10;^2 + \ldots + a_k * 10;^k \equiv a_0 \pmod{4}$ which occurs if $a_0 = 0, 4$, or 8.

(5). To determine when n is divisible by 5, observe that $10; \equiv 2 \pmod{5}$ so that $10;^2 \equiv 2 * 2 = 4 \equiv -1 \pmod{5}$ (Theorem 1 (7)). Also $10;^3 = 10; * 10;^2 \equiv 2 * (-1) = -2 \pmod{5}$, $10;^4 \equiv 1 \pmod{5}$ and the process of these multipliers repeats.

(Continued)

DOZENAL INDEX

President Fred Newhall has completed his DOZENAL INDEX AND presented copies to the DSA and the DSGB. Don Hammond, Secretary of the DSGB writes:

> I thank and congratulate you for the magnificent work you have done in compiling the DOZENAL INDEX. This is a fundamental reference needed by the dozenal movement worldwide and all of us will be indebted to you for undertakling such a massive and painstaking task. One can only guess at the time it has taken; but it has been time well-spent, to judge by the result which, with its usefully-separated categories and the foolproof reference code given, makes it a pleasure to use. Well done!

Note that $a_1 * 10; \equiv 2a_1 \pmod{5}$, $a_2 * 10;^2 \equiv a_2 \pmod{5}$, $a_3 * 10;_3 \equiv 2a_3 \pmod{5}$, and $a_4 * 10;^3 \equiv a_4 \pmod{5}$. (Theorem 1 (7)). Hence $n = a_0 + a_1 * 10; + a_2 * 10;^2 + a_3 * 10;^3 + a_4 * 10;^4 + \ldots + a_k * 10;^k \equiv a_0 + 2a_1 - a_2 - 2a_3 + a_4 + \ldots \pmod{5}$. (Theorem 1(7)).

(6). To determine when n is divisible by 6, note that if 2/n and 3/n, since (2,3) = 1, theorem 3 guarantees (2 * 3)/n or 6/n. (Since 2/n if the last digit, a_0 is either 0, 2, 4, 6, 8, or *; and 3/n if a_0 is either 0, 3, 6, or 9, then taking the intersection of these, we observe that 6/n if a_0 is either 0 or 6.)

(7). To determine when n is divisible by 7, note that $10; \equiv -2 \pmod{7}$ so that $10;^2 \equiv (-2)^2 \equiv -3 \pmod{7}$, $10;^3 \equiv 6 \equiv -1 \pmod{7}$, $10;^4 \equiv 9 \equiv 2 \pmod{7}$, $10;^5 \equiv 3 \pmod{7}$, $10;^6 \equiv 1 \pmod{7}$ and the process of these multipliers repeats. In addition, $a_1 * 10; \equiv -2a_1 \pmod{7}$, $a_2 * 10;^2 \equiv -3a_2 \pmod{7}$, $a_3 * 10;^3$ $-a_3 \pmod{7}$, $a_4 * 10;^4$ $2a_4 \pmod{7}$, $a_5 * 10;^5 \equiv 3a_5 \pmod{7}$, $a_6 * 10;^6 \equiv a_6 \pmod{7}$ etc. so that $n = a_0 + a_1 * 10; + a_2 * 10;^2 + a_3 * 10;^3 + a_4 * 10;^4 + a_5 * 10;^5 + a_6 * 10;^6 + \ldots + a_k * 10;^k \equiv a_0 - 2a_1 - 3a_2 - a_3 + 2a_4 + 3a_5 + a_6 - \ldots \pmod{7}$. (One should observe that since $10;^2 \equiv -3 \pmod{7}$ and $10;^3 \equiv -1 \pmod{7}$, then $10;^5 = 10;^2 * 10;^3 \equiv -3 * -1 = 3 \pmod{7}$.)

(8). To determine when n is divisible by 8, observe that $10; \equiv 4 \pmod{8}$ so that $10;^2 \equiv 4^2 = 14; \equiv 0 \pmod{8}$. Also note that $10;^k \equiv 0 \pmod{8}$ for all $k \in Z$ with $k \geq 2$. In addition, we have $a_1 * 10; \equiv 4a_1 \pmod{8}$ and $a_k * 10;^k \equiv 0 \pmod{8}$ for each $k \geq 2$. Hence $n = a_0 + a_1 * 10; + a_2 * 10;^2 + \ldots + a_k * 10;^k \equiv a_0 + 4a_1 \pmod{8}$.

(9). To determine when n is divisible by 9, observe that $10; \equiv 3 \pmod{9}$ so that $10;^2 \equiv 3^2 = 9 \equiv 0 \pmod{9}$. Also note that $10;^k \equiv 0 \pmod{9}$ for all $k \in Z$ with $k \geq 2$. Hence $n = a_0 + a_1 * 10; \equiv a_2 * 10;^2 + \ldots + a_k * 10;^k \equiv a_0 + 3a_1 \pmod{9}$.

(10). To determine when n is divisible by *; note that if 2/n and 5/n, then since (2,5) = 1, Theorem 3 assures us that (2 * 5)/n or *; /n.

(11). To determine when n is divisible by #;, note that is $10; \equiv 1 \pmod{\#;}$ and $10;^k \equiv 1 \pmod{\#;}$ for each $k \geq 1$, $k \in Z$ and $a_k * 10;^k \equiv a_k \pmod{\#;}$ for each $k \geq 1$. Hence $n = a_0 + a_1 * 10; + a_2 * 10;^2 + \ldots + a_k * 10;^k \equiv a_0 + a_1 + \ldots + a_k \pmod{\#;}$.

(12). To determine when n is divisible by 10;, note that if 3/n and 4/n, since (3,4) = 1, Theorem 3 guarantees that (3 * 4) /n or 10;/n. (since 3/n if the last digit, a_0, is either 0, 3, 6, or 9 and 4/n if the last digit, a_0, is either 0, 4, or 8, then taking the intersection of these, we observe that 10; /n if $a_0 = 0$.)

(Continued)

Do you have an idea to share with our members? Why not submit an article to the *Bulletin*?

(13). To determine when n is divisible by 11; note that $10; \equiv -1 \pmod{11;}$ and $10;^2 \equiv (-1)^2 = 1 \equiv 1 \pmod{11;}$. Also $10;^3 \equiv -1 \pmod{11;}$ and $10;^4 \equiv 1 \pmod{11;}$. In general, $10;^k \equiv 1 \pmod{11;}$ if k is even and $10;^k \equiv -1 \pmod{11;}$ if k is odd. Now, $a_k * 10;^k \equiv a_k \pmod{11;}$ if k is even and $a_k * 10;^k \equiv -a_k \pmod{11;}$ is odd. Hence $n = a_0 + a_1 * 10; + a_2 * 10;^2 + a_3 * 10;^3 \ldots + a_k * 10;^k \equiv a_0 - a_1 + a_2 - a_3 + \ldots \pmod{11}$.

(14). To determine when n is divisible by 12;, note that 2/n and 7/n, then since (2,7) = 1, Theorem 3 yields (2 * 7) / n or 12;/n.

(15). To determine when n is divisible by 13;, note that if 3/n and 5/n, then since (3,5) = 1, invoke Theorem 3 to obtain (3 * 5)/n or 13; /n.

(16). To determine when n is divisible by 14;, note that $10; \equiv -4 \pmod{14;}$ so that $10;^2 \equiv (-4)^2 = 14; \equiv 0 \pmod{14;}$ and $10;^k \equiv 0 \pmod{14;}$ for each $k \varepsilon Z$ with $k \geq 2$. Also $a_1 * 10; \equiv -4 a_1 \pmod{14;}$ and $a_k * 10;^k \equiv 0 \pmod{14;}$ for each $k \geq 2$. Hence $n = a_0' + a_1 * 10; + a_2 * 10;^2 + \ldots + a_k * 10;^k \equiv a_0 - 4a_1 \pmod{14;}$.

We conclude by illustrating Theorem 4 with an example.

EXAMPLE: The duodecimal integer 28100; is divisible by each of the integers 1-14; save 11;. Clearly 28100; is divisible by 1. 28100; is divisible by the integers 2, 3, 4, 6, 8, 9, and 10; since the last digit is 0. To show that 28100; is divisible by 5, note that $a_0 + 2a_1 - a_2 - 2a_3 + a_4 = 0 + 2(0) -3 (1) -8 + 2 = -13; \equiv 0 \pmod 5$ (-13; -0 = -13 = -3 * 5).

(Continued)

DON'T KEEP THIS MAGAZINE

Do you discard your copies of the *Bulletin* after you have read them? Or do they gather dust on a shelf in your attic? Why not pass them along to your local library, or to a school library. Perhaps some nearby math teacher would appreciate a copy. You can also just leave them in a dentist's office or other waiting area.

Help spread the word!

(If you ever need a back copy, we'd be glad to help.)

To show that 28100; is divisible by 7, note that $a_0 - 2a_1 - 3a_2 - a_3 + 2a_4 = 0 -(0) - 3(1) -8 + 2(2) = -7 \equiv 0 \pmod 7$. (-7 - 0 = - 7 = 1 * 7). Now 28100; is divisible by 13; since it is divisible by both 3 and 5. Similarly 28100; is divisible by *; since it is divisible by both 2 and 5. 28100; is divisible is #; since $a_0 + a_1 + a_2 + a_3 + a_4 = 0 + 0 - 1 + 8 + 2 = #; \equiv 0$ (mod #;). (#; -0 = #; = 1 * #;). 28100 is divisible by 14; since $a_0 - 4a_1 = 0 -4(0) = 0 -0 \equiv 0 \pmod{14;}$ (0 -0 = 0 = 0 * 14;). On the other hand, 28100; is not divisible by 11; since $a_0 - a_1 + a_2 - a_3 + a_4 = 0 -0 +1 -8 + 2 = -5$ 0 (mod 11;) (since $-5 \not\equiv -0 = -5$ is not an integral multiple of 11;.) It is interesting to note that 28100; = $2^4 * 3^2 * 5 * 7 *$ #;, and decimally is equal to 55,440.

BIBLIOGRAPHY

(1). I.A. Barnett - Elements of Number Theory, Prindle, Weber, and Schmidt, Boston, MA., 1969.

(2). R. Courant and H. Robbins, What is Mathematics?, Oxford University Press, New York, 1941.

(3). J.E. Maxfield and M.W. Maxfield, Discovering Number Theory, W.B. Saunders Company, Philadelphia, PA.,1972.

(4). K. H. Rosen, Discrete Mathematics and its Applications, Random House, New York, 1980.

(5). S.K. Stein, Mathematics - The Man Made Universe, W.H Freeman and Company, San Francisco, CA., 1958.

□

PANDA STAMP

As announced last December, among the 126 new stamps issued this year, would be five wild-animal stamps, including our mascot, the six-fingered giant panda. Animal stamps have long been among America's most popular stamp subjects.

Color posters advertising the new stamps were displayed in the lobbies of our forty thousand post offices for weeks, and special cancellations including outlines of the stamps designs appeared on our nation's mail.

Painted by Robert Giusti of New Milford, Connecticut, who created last years 29-cent wild duck stamps, the new stamps were issued by the US Postal Service in booklets of 20 stamps on October 1, 1992. They were not printed by the Bureau of Engraving and Printing, but by the private printers, Stamp Venturers of Fairfax, VA on the gravure presses of J W Fergusson & Sons of Richmond, VA.

WHICH NUMBER BASE IS BEST?

Ed Nu

We have been so thoroughly entrenched in school with base ten arithmetic, that few of us ever question the fact that there are other bases in mathematics. Most of us have been taught that there is a binary 2 base used by computers and electronics, and that multiples of it, 4, 8, and 16 (hexadecimal base) also appear in computer work. But we should question whether the traditional base ten is really the most efficient number base.

A base has to be a smaller number. Some civilization have used twenty (they counted on fingers and toes) and the Babylonians used 60 as a base, but in arithmetic you have to be able to count backward as easily as forward. Try saying the alphabet backward quickly and you'll stumble on w, v, u, or some of the other letters. Sixteen is about the upper limit for a practical number base. The lower limit should be about eight since anything below that requires too many digits to express a number. For instance, in base two our number 16 would have to be written 10000; very awkward!

Also let's assume that the best base has to be an even number. Then each base will be divisible by 2 and by half:

8 is divisible by 2 and 4
10 is divisible by 2 and 5
12 is divisible by 2 and 6
14 is divisible by 2 and 7
16 is divisible by 2 and 8, etc.

That's fine for two, but how about 3 and 4? The only small even base divisible by 3 is 12; also by 4. Sixteen is divisible by 4 but not by 3. Nine and fifteen are divisible by three but not by two.

How much better than 10 is 12? Since 12 is divisible by two more low numbers than 10, you could argue that 12 is 3 times more efficient than 10! Ten is only divisible by 2, but twelve is divisible by 2, 3, and 4.

To be fairer, the higher the number base, the more chance that it will have more even-divisors or factors. So let's make a fraction of the base and its factors:

8 has factors 2 and 4 or a total of 2 so 2/8 = 1/4
10 has factors 2 and 5 or a total of 2 so 2/10 = 1/5.
12 has factors 2,3,4,and 6 or a total of 4 so 4/12 = 1/3.
14 has factors 2 and 7 or a total of 2 so that 2/14 = 1/7.
16 has factors 2,4,and 8 or a total of 3 so that we have 3/16.

So the fraction for 12 (1/3) is much larger than the fraction for 10 (1/5). Even base 8 would be better than base 10 since 1/4 is larger than 1/5! Percentagewise, comparing 1/3 to 1/5, base 12 is 67% more efficient than our old traditional base 10!

□

KAPREKER'S PROCESS GENERALIZED

Charles Ashbacher
Cedar Rapids, IA 52402

Kaprekar's Process in base ten is defined as follows:

1. Take any three digit number where the digits are not all equal. Sort the digits.
2. Use the digits from highest to lowest to form one number and the digits from lowest to highest to form another.
3. Subtract the smallest from the largest. If the result is the same as the original, then terminate. Otherwise use the result as the input into step 1.

For example, with the initial number 123:

321 - 123 = 198 981 - 189 = 792 972 - 279 = 693

963 - 369 = 594 954 - 459 = 495 954 - 459 = 495

495 is the terminal number for each three digit number that is not constructed from a single digit.

If we perform the process in base twelve, we always obtain the terminal number 5#6, or (b/2 - 1) (b - 1) (b/2).

Carrying out the subtraction after the sorting:

b - 1	b/2	b/2 - 1
b/2 - 1	b/2	b - 1
b/2 - 1	b - 1	b/2

and we can see that if the base is even, the number (b/2 - 1) (b - 1) (b/2) will return the same number when the process is applied.

Examining all three digit numbers in base 8, we see that the process terminates with 374 for all appropriate three digit numbers.

□

THE VALUE OF PI IS

3.184809	493#91	866457	3*6211	##1515	51*057
29290*	7809*4	927421	40*60*	55256*	0661*0 ...

-Brian M. Dean

EDUCATION NEEDED

Gene Zirkel
Nassau Community College
Garden City, LI, NY

Gene Burns hosts a daily three-hour talk show on WOR radio in NYC. On Monday 14 September his topic *Should we change to the metric system?* caught my attention. Driving home, I tuned in and listened to the pros and cons. I hurried home and dialed his number only to find that the line was busy. Thanks to my redial button, I patiently sat and listened while I pushed redial continuously. After thirty minutes, I was connected and informed that I would be the first caller following the news. An additional 55 minutes transpired along with two news breaks as we came close to the conclusion of the show. At the last minute I was put on - the last caller of the day.

I was able to comment on a number of the silly things that I had heard, such as the metric system was "more precise" than our current English system.

I was able to explain that it was not "tens" that made the system seem easier but the use of the base. I spoke briefly about computers with base two or sixteen (the binary and hexadecimal bases respectively), and that the symbol 'one zero' could represent any number.

I only had a short time, but I was able to mention factorability, and the DSA along with our address, and an offer to listeners to write for our literature.

I brought an historic view to the discussion by mentioning Lagrange and his committee who considered whether to change our measurements to tens or our counting to dozens. I used my favorite analogy - that it was like the problem of - your foot is too large for your shoe. You can either cut off your toes or get a larger shoe. Lagrange opted for changing the natural measurements with their halves, thirds, and quarters instead of the artificial counting based upon the biological accident of having ten fingers.

Further, I noted that of all the countries that have gone metric, not one ever did so voluntarily. In every instance, laws were passed outlawing the sale of butter by the pound, rugs by the yard, etc. It would appear that if the metric system is as convenient as its advocates maintain, then somewhere, just once, ordinary people would have seen the truth and would have been willingly converted.

As our host noted, to replace all the highway signs with metric signs would cost "hundreds of millions of dollars." To the many people who repeated our need to switch to metric because of world trade and economic reasons, he replied - Fine, let manufacturers switch. "But I don't need to drive in kilometers per hour [in order for you] to manufacture in metric."

One caller, identified as Fred from Plainview, stated that 10 or 15 years ago the Encyclopedia Britannica had a six page essay on the advantages of keeping our current

(Continued)

system, but it was pulled from later editions. If anyone can verify this, it would be extremely interesting. Gene Burns pointed out that it was about then that there was a big push for the USA to go metric.

Fred also said that most of the world lives in the temperate zones, and as a consequence our Fahrenheit temperature usually ranges from about 0° (cold) to about 100° (hot). On the other hand, those who use the awkward Celsius scale usually range from about 18° to about 38°! Interesting.

In conclusion, I believe that the more than one hour it took for me to be connected is indicative of the vast interest in this subject, both pro and con. In addition, I heard a great deal of misconceptions, a sign that our task of educating the public is as essential as ever. □

Edgar Allan Poe's <u>Eldorado</u> Revisited

Gaily bedight,
A gallant knight,
In sunshine and in shadin',
Had journeyed long.
Singing a song,
In search of el do dozen.

But he grew old-
This knight so bold-
And o'er his heart a shadin',
Fell as he found
No spot of ground
That looked like el do dozen.

And, as his strength
Failed him at length,
He met a pilgrim shadin'-
"Shading", said he,
"Where can it be-
This land of el do dozen?"

"Over the Countin'
Out by tens,
Down the Valley of the Weighin',
Ride, boldly ride,"
The shade replied,-
"If you seek for el do dozen!"

--Henry Churchman

DOZENAL JOTTINGS

...from members and friends...News of Dozens and Dozenalists...

President **FRED NEWHALL**'s holiday greeting card listed '123456789*_' and asked What Number is Missing? The Answer , of course is No-El!

JOHN CHURCHMAN reports that his dad is 8 dozen and 1 years old. **HENRY**, former Editor of this *Bulletin*, is still mentally alert and enjoys reading of the Society's activities.

IAN PATTEN of Alaska voices concern over our 'government, now fully committed to metrication, despite overwhelming public opposition'.

Speaking of our government, former Member of our Board of Directors, **ANTON GLASER**, wrote to the Federal Reserve Board re an error in the Truth in Savings Act. Anton pointed out where they had confused 6.17% with 6.17 (which is 617%!). The response from John P. LaWare, Member of the Board of Governors of the Federal Reserve System, states:

> While I certainly understand your point that a decimal is equivalent to a percentage, ... We've tried to communicate as clearly as possible to the wide range of individuals ... Our belief is that the best way to do this is through the form of expression used.

Thus, the men we pay though our taxes feel that 'the best way' to communicate to the wide range of us poor benighted individuals is via deliberate error! What hope is there for us to convince such people of the awkwardness of a decimal metric system?

JOHN HANSEN writes on the subject of "Leaping into a Dozenal Future:"

> Here is my proposal for reckoning leap years in the dozenal calendar of the future. It consists of the following rules (all numbers are dozenal unless otherwise stated):
>
> 1. Every year has 265 days.
>
> 2. EXCEPT years which end in 0, 4, or 8, which have 266 days.
>
> 3. EXCEPT years which end in 60 or are divisible by 800, which have 265 days.
>
> Note: Years divisible by 800 end in e000, e800, or o400, where 'e' is any even digit and 'o' is any odd digit.
>
> My scheme produces an average calendar year of 265;2**6 years, which is the same as the length as the astronomical, i.e. tropical, year

(Continued)

> to an accuracy of four decimal places. Using vulgar fractions, this value is equal to 265 + 1/4 - 1/*8 days.
>
> For comparison, the current decimal scheme is only accurate to three decimal places, producing an average calendar year of decimal 365.2425 days, which is too large by decimal .0003. (This can best be corrected by omitting a leap day in the years divisible by decimal 3200).
>
> (P.S.: References are to <u>TGM</u> by Tom Pendlebury, copyright 1985, pages 45 and 58.)

DON HAMMOND of the DSGB having reached the age of 55; writes that he is now retired and will be spending sometime working on his motorbike. Ad multos annos, Don.

Welcome to new <u>Life Member</u> **ERICH KOTHE** of Illinois. And to new members:

324;	**JAMES G. STEIGERWALD**	New York, NY
325;	**JOHN STEIGERWALD**	Califon, NJ
326;	**B. GERVER** - North Shore High School	Glen Head, NY
327;	**GEORGE P. JELLIS**	England
328;	**VITA B. ALAIMO**	Ridge, NY
329;	**JACOB CORBIN**	San Francisco, CA
32*;	**GUNEY MENTES**	Canada
32#;	**GEORGE FAULKNER III**	Long Island, NY
330;	**BILL LAURITZEN**	Los Angeles, CA
331;	**DERRICK SMITH**	Berkeley, CA

In addition, **GEORGE S. CUNNINGHAM**, Member number 107; and DSA Fellow, enrolled four new student members. George writes:

> Here are four student memberships. All four plan to teach next year and are now in my "Math for Teachers" course:

332;	**IRENE SAMSON**	Bangor, ME
333;	**JACKIE GAMMON**	Orono, ME
334;	**VALERIE BURNETT**	Carmel, ME
335;	**AMLE GELLEN**	Old Town, ME

□

THE FIRST 30; FACTORIALS ARE:

Brian M. Dean

1;	1;
2;	2;
3;	6;
4;	20;
5;	*0;
6	;500;
7;	2#00;
8;	1#400;
9;	156000;
*;	1270000;
#;	11450000;
10;	114500000;
11;	1259500000;
12;	14*8#*00000;
13;	191529600000;
14;	241*#88000000;
15;	33*86734000000;
16;	4#*09*#00000000;
17;	7*8#383500000000;
18;	111***19840000000;
19;	1#040#91#700000000;
1*;	3627596972*00000000;
1#;	68#041404*5200000000;
20;	115*0828098*400000000;
21;	2411#51698356400000000;
22;	50*629148##5#8800000000;
23;	#4#8026182**546000000000;
24;	2277285*3#2944660000000000;
25;	54445661#61876#8600000000;
26;	114*#1*34*9436#533000000000;
27;	2*78299697*212#674900000000000;
28;	7845#61618#173*978800000000000;
29;	1920447*1*967518862*0000000000;
2*;	4##905229470906*818040000000000;
2#;	126#343161447248118*4#8000000000;
30;	3789*0946411972035272#00000000000;

□

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition--("Who needs a symbol for nothing?")--the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used.

A parallel to today seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

□

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro. 1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3#2*	Two ft. eight in.	*2;8'*
19#	*1000*	Eleven ft. seven in.	*#;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this *"which is"* step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2#*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
12 ) 30   + 5
12 ) 2    + 6
     0    + 2   Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *.

For more detailed information see *Manual of the Dozen System* ($1;00).

DETACH HERE — OR PHOTOCOPY

We extend an invitation to membership in our society.
dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______ LAST ______ FIRST ______ MIDDLE ______

Mailing Address (for DSA items) ______

______ (See below for alternate address)

Telephone: Home ______ Business ______

Date & Place of Birth ______

College ______ Degrees ______

Business or Profession ______

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______

Address ______

Year & Math Class ______

Instructor ______ Dept. ______

Other Society Memberships ______

______

Alternate Address (indicate whether home, office, school, other)

______

______

______

Signed ______ Date ______

My interest in duodecimals arose from ______

______

______

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530