DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

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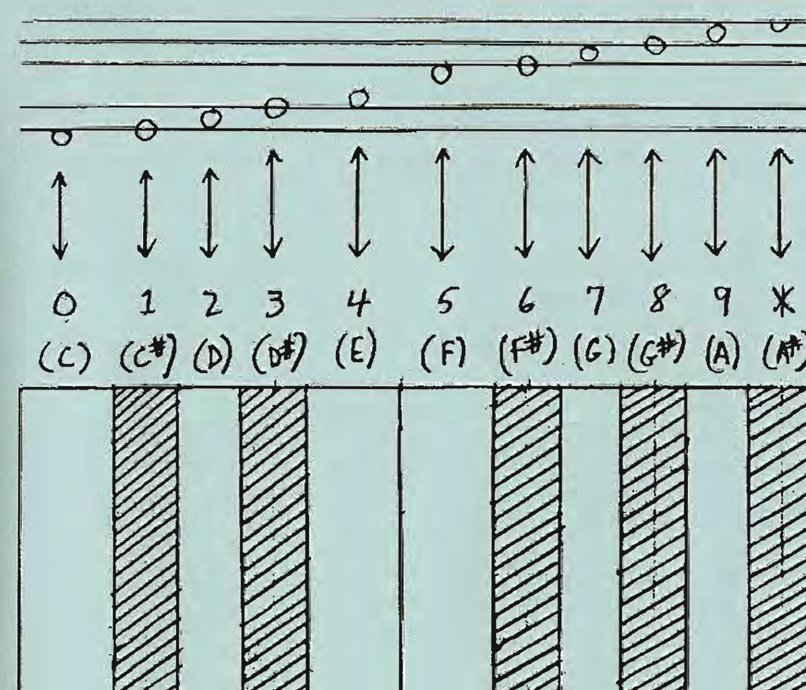
THE DUODECIMAL BULLETIN 6#;



See **DOZENAL MUSIC REVISITED**, page 14

Volume 35;
Number 2;
1992
11*0;

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00 (US) for one calendar year. Student Membership is $3.00 per year, and a Life Membership is $144.00 (US).

The Duodecimal Bulletin is an official publication of the DOZENAL SOCIETY OF AMERICA, INC., c/o Math Department, Nassau Community College, Garden City, LI, NY 11530.

BOARD OF DIRECTORS OF THE DOZENAL SOCIETY OF AMERICA

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THE DUODECIMAL BULLETIN

Whole Number Six Dozen El

Volume 35; Number 2;

11*0;

IN THIS ISSUE

FOUNDED 1944

DOZENAL SOCIETY OF AMERICA BOARD OF DIRECTORS MEETING

Saturday, June 6, 1992
Nassau Community College
Garden City, LI, NY

The Meeting was called to order at Noon by Board Chair Gene Zirkel.

Those present were:

Alice Berridge
Jamison Handy, Jr.
James Malone
Fred Newhall
Mary Newhall
Jay Schiffman
Gene Zirkel
Patricia Zirkel

The following business of the Board was addressed:

1. James Malone was unanimously elected to the Board of Directors to fill a vacancy in the previously elected slate of the Class of 1994. Jim also agreed to re-assume the duties of DSA Treasurer as soon as an orderly turnover of records from the current Treasurer, Anthony Catania, can be arranged.

2. The date of the next Annual Meeting of the DSA has been changed to October 24, 1992. In addition, the meeting is now scheduled for 10:30 AM, and will be held in an elevator-accessible building.

3. With regard to the DSA Bulletin:

   Since it was previously decided that there would in future be two yearly issues of the Bulletin, the per issue cost of back Bulletins will now be $6.00.

   Jay Schiffman has agreed to assume the duties of Editor, commencing with the January/February issue of the Bulletin. Current Editor Patricia Zirkel will remain briefly as Production Editor, so that an orderly transfer of duties may be accomplished. (See related article, this issue.)

After some brief discussion of miscellaneous dozenal concerns, the meeting was adjourned at 1:40 PM.

The Board Members and guests present then shared a luncheon at a local restaurant.

□

CONVERSION OF NUMBERS TO BASE DEK

Jean Kelly
New York, NY

There are two common methods to convert a number from one base into its base dek equivalent.

(I) The most common (but not the easiest) is to expand the number in powers of the original base.

Thus dozenal 37269 is written as

$$3(12^4) + 7(12^3) + 2(12^2) + 6(12^1) + 9(12^0) =$$

$$3(20736) + 7(1728) + 2(144) + 6(12) + 9(1) =$$

$$62208 + 12096 + 288 + 72 + 9 = 74673 \text{ in base dek}$$

(II) If we think of the above as evaluating a polynomial in base b:

$$P(b) = 3b^4 + 7b^3 + 2b^2 + 6b^1 + 9b^0$$

we are led to the alternative approach, *nested factors*:

$$P(b) = b(b[b\{3b + 7\} + 2] + 6) + 9$$

(Continued)

The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus 1/2 = 0.5 = 0;6.

DYHEXAL NUMBERS: HOW THEY FACILITATE ARITHMETIC[1]

Gerard Robert Brost
Department of Psychology
University of Florida
Gainesville, FL

This article describes a new system of notation that facilitates mathematical operations in base twelve. Some advantages of the base twelve system are briefly summarized, followed by a description of the dyhexal system of notation and its advantages over the conventional Hindu-Arabic system. It is argued that the dyhexal system provides a superior system of notation as well as a useful educational device for teaching number concepts.

1. Introduction

The greatest mathematical development of the millennium was probably the conversion to Hindu-Arabic notation from Roman numerals. The discovery of the zero and the use of the place system allowed complex operations to be performed with relative ease. It had the equivalent effect of boosting the mathematical aptitude of the average person to that of genius. Average persons could routinely perform mathematical operations that had stumped the most brilliant mathematicians of the Roman era.

The dyhexal notation proposed here is a refinement of the place system that facilitates computations even more. It is a base twelve system of arithmetic made by combining two sub-bases of six. Hence the name 'dyhexal' meaning 'double sixes'.

The base twelve system of counting is alternately known as the 'dozenal' or 'duodecimal' system. Because the dyhexal system is a variant of the dozenal system, I shall briefly review the advantages common to all base twelve systems before introducing the new dyhexal notation.

2. Some advantages of base twelve mathematics

The advantages of the dozenal system are numerous and well documented [1,2,3,4]. The superiority of the dozenal base derives from the remarkable property of the number twelve to be factored by every number less than half itself except the number five. This results

(Continued)

[1]Reprinted with permission from: International Journal of Mathematical Education in Science and Technology, 1989, Vol. 20, No. 2, 249-53.

in calculations producing a high frequency of terminal zeros, which in turn facilitates factoring and makes multiplication and long division easier [1]. Fractions are converted into uncials (base twelve decimals) with a small number of uncial places and a low frequency of non-terminating uncials [1,2,3]. This simplifies interpolation [3], multiplication, division, addition and subtraction because there are less digits.

The factorability of the number twelve has many practical uses that have made base twelve systems indispensable in certain applications. Base twelve is preferred by the packaging industry because twelve items can be packed in so many convenient ways [2,4]. Similarly, the clock is divided into twelve hours to facilitate dividing up the day. It is further subdivided into five dozen minutes containing five dozen seconds to interface dozenal measures of time with the prevalent base ten system of counting.

The standard system of angular measure is also semi-dozenal. The remarkable properties of the 30:60:90 triangle dictate that the number of degrees in a right angle should be a multiple of three, because ninety degrees is three times thirty. Since there are four right angles in an entire revolution, the total number of degrees in a circle should be a multiple of both three and four, or twelve. Our present system divides the circle into thirty dozen degrees to make our geometry compatible with base ten. However, if a pure dozenal system were used, such alteration would be unnecessary and geometric operations would be simplified. If a pure dozenal system were also used to measure time, then the longitude separating two cities would equal the time difference between them [3].

The factorability of the number twelve has another important implication: the arithmetic properties of numbers in the natural sequence repeat themselves in cycles of twelve. For this reason the last digit in a dozenal number indicates important arithmetic properties of the whole number. Dyhexal notation uses this feature to make arithmetic easier.

(Continued)

COMING SOON -- TWO BULLETINS PER YEAR

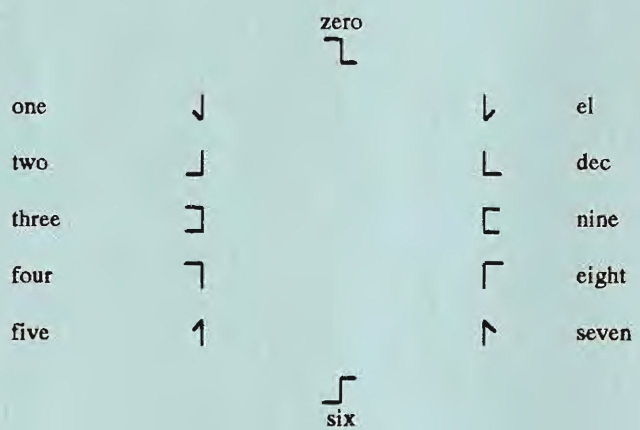
We are sorry, but in order to continue to both meet DSA expenses and the schedules of busy volunteer Editors, the Duodecimal Bulletin will, in future, be published twice yearly.

This change will ensure that the Bulletins you continue to receive will retain the quality to which you have become accustomed.

3. Dyhexal notation

The figure shows the dyhexal symbols. Dozenal systems contain two extra numbers for the tenth and eleventh numbers, traditionally called dec and el. The number twelve is called do (pronounced doe), short for 'dozen'. The numbers following do are called do-one, do-two, do-three, etc. A dozen dozen is one gro, short for gross. A dozen gross is one mo. The number ↲ ┘] ┐ (1234) is pronounced one mo, two gro, three do-four. The nomenclature is defined more extensively elsewhere [2].

The dyhexal symbols are easy to memorize because of the patterns they follow. Notice that all twelve numbers are represented by four basic symbols (┐└, ↲ , ┘ ,]) in different orientations. They are all composed of one vertical line with one or two protruding horizontal or angular lines called features. Identification is aided because when the numbers are arranged in order they form a symmetric pattern about the number six, with the features pointing away from the six. Numbers less than six point left. Numbers greater than six point right. Numbers close to six have their features in the high position, while numbers distant from six have features in the low position.



Dyhexal symbols for base twelve notation.

All numbers that are evenly divisible by two but not three (i.e. two, four, eight and dec) have one horizontal feature in one of four positions (┘ , ┐ , ┌, └). If the feature is in either of the high positions, then the number is also divisible by four. If the feature is low, then it is not.

(Continued)

> The Babylonians of Mesopotamia established the time measures of five dozen seconds to one minute and five dozen minutes to one hour.

All numbers that are evenly divisible by three but not two (i.e. three and nine) have two horizontal features on a single side of the vertical ( ] & [ ).

Numbers that are evenly divisible by both two and three (i.e. six and zero) have two horizontal features on opposite sides of the vertical (┘┌ & ┐└). If the left feature is high, the number is also divisible by four. If the left feature is low, it is not.

The remaining numbers (one, five, seven and el) are not evenly divisible by either two or three.They are signified by an angular feature in one of four different positions (↲ , ↰ , ↱, ↳).

This notation conspicuously groups numbers according to their arithmetic properties. This is particularly useful in a base twelve system, because multi-digit numbers retain the properties of their terminal digit. Arithmetic is easier, because one can determine the properties on the whole number by simple inspection of the final digit.

The following rules help make arithmetic easier in the dyhexal system:

(1) All numbers ending in ┐└ (zero) are evenly divisible by two, three, four, six and do.
(2) All numbers ending in ┘┌ (six) are evenly divisible by two, three and six but never by four, eight or do.
(3) All numbers ending in] or [(three or nine) are evenly divisible by three but never by an even number.
(4) All numbers ending in ┘ , ┐ , ┌ or └ (two, four, eight or dec) are evenly divisible by two but never by three, six, nine or do.
(5) All numbers ending with ┐ or ┌ (four or eight) are evenly divisible by two and four but never by three, six, nine or do.
(6) All numbers ending with ┘ or └(two or dec) can be evenly divided by two but not three, four, six, nine or do.
(7) All numbers ending in ↲ , ↰ , ↱ or ↳ (one, five, seven or el) are indivisible by all numbers other than one or itself, with the possible exceptions of other numbers ending with one of these four symbols.
(8) If the last two digits are evenly divisible by eight, so is the whole number.
(9) If the last two digits are evenly divisible by nine, so is the whole number.
(10) All prime numbers greater than three end in ↲ , ↰ , ↱ or ↳ (one, five, seven or el).
(11) If a number ends in ┘ , └ or ┘┌ (two, dec or six) it is not an exponential of any whole number.

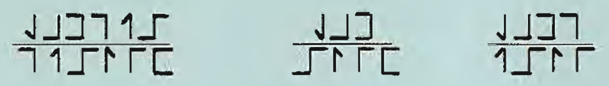
All these shortcuts may seem too numerous to remember at first. However, with a little familiarity they soon seem so obvious that they become second nature. As an illustration, consider the last two principles that are shared by the base ten system as follows.

(Continued)

(12) All even numbers are evenly divisible by two.
(13) Odd numbers can be divided evenly only by other odd numbers.

Although these last two rules may seem trivially obvious to a veteran of the base ten system, they are in fact very useful. Similarly, the other eleven rules soon become equally automatic with experience.

With a little practice one can quickly appreciate the simplicity of dyhexal notation. Otherwise obscure relationships are often readily apparent in dyhexal. For instance, one can usually detect fractions that are not in lowest terms by simple inspection of the terminal digits. Consider, for example, the following problem:
Which of the following fractions are not in lowest terms?



We can solve this problem without even knowing what the fractions are. Inspection of the final digits reveals that the first fraction is not in lowest terms. The numerator is divisible by three because it ends in [symbol] (rule 2). The denominator is also divisible by three because it ends in [symbol] (rule 3). The second fraction is not in lowest terms either. The numerator and denominator are both divisible by three (rule 3). Likewise, both terms in the last fraction are evenly divisible by four (rule 5). Therefore, none of the above fractions are in lowest terms.

Dyhexal notation also simplifies the identification of prime numbers. Consider the following example.

Which of the following numbers are prime?

We can eliminate the first three numbers immediately. None of them is prime because none end in the symbols [symbol], [symbol], [symbol] or [symbol] (rule 10). The fourth number is prime if it cannot be divided by any number greater than one that ends in any of these symbols. We start with the smallest of these numbers [symbol] (five), and attempt division. Division reveals that it is not divisible by five. We proceed to [symbol] and [symbol] (seven and el), the next highest numbers in the group. Division fails here also. However, it is evenly divisible by [symbol] (do-one). Therefore, none of the above numbers is prime.

(Continued)

Similarly, the above rules also facilitate multiplication, division, factoring, and other operations. Addition and subtraction are likewise easier, because complementary pairs are always mirror images of each other (with the sole exception of [symbol] + [symbol] = [symbol]).

With so many advantages of the dyhexal system, one may wonder about the possibility of adopting dyhexal as a standard notation. Such a change would probably be quite lengthy. In the meantime dyhexal notation provides a useful instructional device for teaching an understanding of the recurrent arithmetic properties of numbers, and for familiarizing students with base twelve mathematics. Base twelve is easier to learn in dyhexal, because one does not mistake base twelve numbers for base ten.

An alternate system of notation for base twelve was previously introduced by Camp [5]. However, Camp was apparently unaware of the mathematical advantages that an organized system of notation can offer. He proposed his system merely as an easier and more aesthetically pleasing way to write numbers.

The present system differs from Camp's in many ways. The symbols are coordinated with the mathematical properties of the numbers. A new category is created that recognizes the special properties of compound multiples of both two and three. A symbol for zero is introduced. The new symbols are simpler and easier to write. They are somewhat more aesthetically pleasing as well. Their aesthetic appeal accrues from their ability to capture formally the natural mathematical rhythm of numbers.

References

[1] Andrews, F.E., 1935, New Numbers (New York: Harcourt, Brace).
[2] Duodecimal Society of America, Inc., 1960, Manual of the Dozen System (Garden City, NY: Dozenal Society of America).
[3] Terry, G.S., 1938, Duodecimal Arithmetic (New York: Harcourt, Brace).
[4] Parkhurst, H.M., 1956, The Duodecimal Bulletin, Vol. 10, No. 2, p. 34;.
[5] Camp, K., 1946, The Duodecimal Bulletin, Vol. 2, No. 1, p. 16;.

□

DSA ANNUAL MEETING

Saturday, October 24, 1992 10:30 AM

Nassau Community College Garden City, LI, NY

This is a change from the date previously published in the Minutes of the 1991 Annual Meeting -- **Call for Information: (516) 669-0273**

MATH FORUM

Betsy Colwell
Foster, RI

[Editor's note: The following column was an assignment submitted by a student in a course entitled "Development of the Number System" at The Community College of Rhode Island. The assignment was to write a column for an imaginary magazine *Math is Fun.*]

Counting from 1 to 10 is child's play, but it formed the basis of a number system so popular that it is the only number system to be used universally. It's only been in use for 300 years or so, but that's quite an accomplishment when you consider the number of languages spoken and the changes and developments in them over the past 300 years.

There's a group of people though who maintain that our number system leaves a lot to be desired. They'd opt for the duodecimal system which uses base 12. We have 10 fingers and 10 toes, but when you think about it, we also have lots of things that come in twelves. There are 12 doughnuts or 12 eggs packed to a box, 12 hours on the clock, 12 months in our year, 12 dozen to a gross. You get the idea. We're already working with twelves when we measure, --12 inches to a foot. There are a lot more compelling reasons to use a base 12 system though.

Twelve can be divided by twice as many factors as 10, making it much easier to work with in many respects. Division is faster, fractions more sensible and percentages more accurate. Factoring is so simple that long division could be nearly obsolete. More fractions in the 12 system can be expressed as whole numbers and are more accurate since a two place decimal in base 10 is figured to the closest 100th part; in base 12 it's figured to the closest 144th part. And percentages are much easier to work with since base 12 has half as many repeating numbers. Now we can easily express 1/3 as 40%, 2/3 as 80%. Even 1/16 comes out evenly at 9%, all because 144 has 14 factors as compared to 100's 8 factors.

What would conversion to the base 12 system mean? There would be a few changes in the numbers themselves. The Duodecimal Society has worked it all out. Numbers would look a little different,

(Continued)

Why not give some of our literature to a friend? Brochures, *Excursions* and *Bulletins* are available.

1	2	3	4	5	6	7	8	9	X	E	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

11	12	13	14	15	16	17	18	19	1X	1E	20
do one	do two	do three	do four	do five	do six	do seven	do eight	do nine	do dek	do el	two do

and sound different too. Twelve as we know it would cease to exist.

There would be new addition and multiplication tables to learn, a new number language, new prime numbers, new mathematics discoveries and a lot of new jokes about the conversion process. A simple addition problem could look like these:

203	39EX
688	XE02
88E	12900

Of course, all this would never happen overnight. Remember that as a group, we were very reluctant to give the metric system a chance. Remember too though, that it took 500 years or so for the Hindu-Arabic number system to be widely accepted and used in Europe. So if conversion to a base 12 system took a couple of generations, that wouldn't be so bad. It sure gives a whole new meaning to the term "The New Math," doesn't it?? □

THE FOLLOWING ARE AVAILABLE FROM THE SOCIETY

1. Our brochure. (Free)
2. "An Excursion in Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, October 1934. (Free)
3. *Manual of the Dozen System* by George S. Terry. ($1;00)
4. *New Numbers* by F. Emerson Andrews. ($10;00)
5. Dozenal Slide Rule, designed by Tom Linton. ($3;00)
6. Back issues of the *Duodecimal Bulletin*, as available, 1944 to present. ($6;00 each)
7. *TGM: A Coherent Dozenal Metrology*by T. Pendlebury ($1;00)
8. *Modular Counting* by P.D. Thomas ($1;00)
9. *The Modular System* by P.D. Thomas ($1;00)

(Continued)

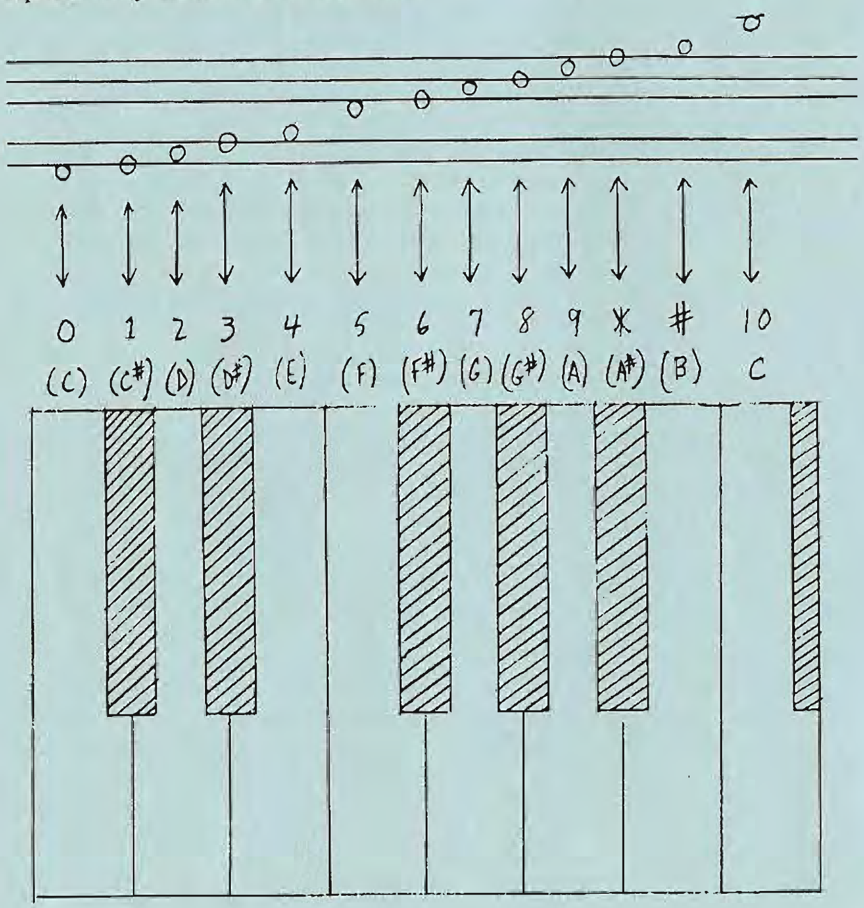
DOZENAL MUSIC REVISITED

John D. Hansen, Jr.
Vista, CA

I just finished reading Tom Pendlebury's article "Music a la Dozen" from ten years ago. (The Dozenal Journal, No. 1, Winter 1981.)

His article inspired me to come up with a musical notation of my own. Unlike his system, my system does not aspire to be an all-purpose musical notation. But it is a good tablature for piano and similar keyboards.

The white keys are represented by notes in the white spaces, and the black keys are represented by the notes on the black lines.



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DOZENAL JOTTINGS

...from members and friends...
News of Dozens and Dozenalists...

The April 1992 issue of *The Mathematics Teacher* ran an ad from MMEI Corp. which is oversimplified and hence not true.

The ad starts with large print: The Metric System is Coming! It then goes on to state that at the end of September 1992, the Federal government must use metric units. (Public Law 100-418). It then goes on to talk about "the" metric system.

The truth is:

1. The many exceptions contained within the lengthy - several pages long - law will allow many people to wisely save money and avoid the costly switch to metric units.

2. Any reference to "the metric system" is misleading. There are as many metric systems as their are countries that use them. No two countries agree 100%. Cf., any issue of the *American Metric Journal* for confirmation of this.

It behooves a journal intended for educators to present the truth, even when they may not like it. Running such ads is misleading to thousands of teachers, and then to millions of students! . . .

KEN STILWELL of Northeast Missouri State University has a letter on page 322 of the April 1992 issue of the *Mathematics Teacher*. He complains about the use of the oxymoron, a "decimal in base three".

Right on the mark, Ken! Math is an exact science, and much of the trouble people have with it stems from inaccurate language. We should avoid such abuses. . .

New member **UNDERWOOD DUDLEY**'s book, *A Budget of Trisections*, received a highly favorable review in the Winter 1992 issue of The Mathematical Intelligencer, pages 73-77. . .

Member **BILL CROSBY** called. He had lost touch with the Society and was pleased to find us again. A search of our records indicated that Private William S. Crosby joined the DSA in the 1940s! Bill is member number 15; and we are glad to have him back. In the course of our conversation he let slip that he had worked at Douglas College, that he was in his 80th decade and that he was calling from a hospital pay phone where he was staying for 'nothing serious'. We wish him a quick recuperation. . .

(Continued)

CONGRATULATIONS! To **JAY SCHIFFMAN** on his election to the DSA *Board of Directors* and to the office of *Vice President*. Jay spoke on the topic "Duodecimals are Better" at the Metro New York section of the MAA in May, 1992. . .

CONGRATULATIONS! To **ALICE BERRIDGE** on her election to the office of *Secretary* of the DSA and to our *Nominating Committee*. . .

JOHN CHURCHMAN (Council Bluffs, IA) wrote concerning his father, **HENRY CHURCHMAN**, a former Editor of this Bulletin. Henry was 96 at the end of last year and still enjoys receiving The Duodecimal Bulletin. He especially enjoyed Bulletin number 69; because it contained an article which he originally wrote in 1975. . .

The publication Radio Electronics ran a contest to find "any professional organization in any field that has book or publication resources of possible interest to hardware hackers." **JOHN D. HANSEN, JR** (CA) sent them full information concerning the DSA, and we look forward to hearing from their readers.

JOHN also called and, among other things, mentioned that he wouldn't be surprised to see Russia revert to our old, convenient, dozenal measuring system. His reasoning is that the communists were *gung ho* on the awkward decimal metric systems and that the new regimes are fervently trying to distance themselves from the old USSR. It certainly would be good news if John's idea became reality. . .

ARTHUR WHILLOCK writes from England concerning Bulletin number 6*; "Fred's point on carbon 12 being essential for life is more pertinent than he knows. (See "Minutes," p. 6) The life substance called protein is composed of hydrogen, oxygen, nitrogen and carbon whose valencies comprise the dozenal factors 1, 2, 3, and 4, thus making it possible to assemble the wide range of forms to build complex structures." Regarding the "Oldest Book on Dozens," Arthur is going to try to obtain a copy, since it was printed in England originally.

On another matter, **ARTHUR** allows as to how the symbols * and # may yet come into their own internationally. He refers to an article in New Scientist dated December 14 (1991). . .

CHARLES ASHBACHER writes from Cedar Rapids, IA concerning references to the DSA in books. (See "Minutes," Bulletin 6*; p. 5) To the book mentioned in the last issue he adds The Penguin Dictionary of Curious and Interesting Numbers by David Wells (Penguin: 1986), see p. 86; and The Nature of Mathematics, Sixth Edition, by Karl J. Smith (Brooks/Cole Publishing: 1991), see p. 223. . .

(Continued)

DON HAMMOND writes from England: "Now, I can't let this pass! In his article on Thomas Leech's book [last issue], **FRED NEWHALL** notes that Leech advocated setting 0 degrees Fahrenheit at the freezing point of water, and said he had not seen the idea before. However, if Fred looks at DOZENAL REVIEW No. 32, or DOZENAL JOURNAL No. 1, he will find my article: "Did Fahrenheit Have It Right?" in which I described my proposed Rational Fahrenheit Scale. In this, the ice point is set at 0 degrees F. The scale is then numbered in dozenal notation, which gives the round numbers 130 degrees F for boiling point and -350 degrees F for absolute zero. The absolute (Rankine) scale in this notation then gives 0 degrees R for absolute zero, 350 degrees R for the ice point and 480 degrees R for the boiling point. What could be better? I didn't know that Leech had thought of it -- perhaps it's a matter of great minds . . .?

Several members have recently made sizable donations to the DSA: **RICHARD TRELFA**, a Life Member, number 159; . . . **HENRY CHURCHMAN**, a Life Member and Fellow, number 72; also a former President and Vice President of the Society . . . **DR. ROBERT LOVELL**, Member number 122; . . . and **"SKIP" SCIFRES**, a Life Member and Fellow, number 11; and a former Treasurer of the Society. . .

Welcome to new member:

323;	**ROBERT HIGH**	New York, NY

□

THINK ABOUT IT!!!

Hindu-Arabic numerals will never replace Roman numerals.

The automobile will never replace the horse.

This country needs a half-dozen computers.

USSR: We'll bury you.

Germany will never be re-united.

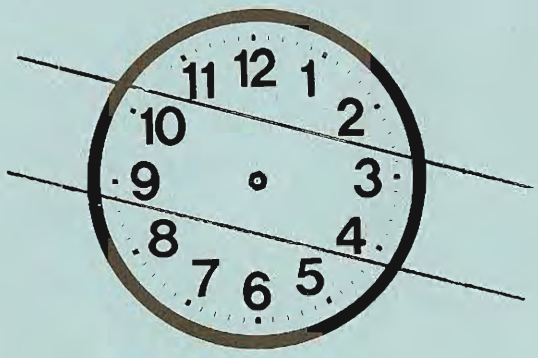
AND -- *Duodecimals will never replace tens. . .*

DSA SEAL PUZZLE SOLUTIONS

Charles Ashbacher
Cedar Rapids, IA

Part A:Divide the face of a clock into three parts with two straight lines so that the sums of the numbers in the three parts are equal.

Solution:



(Continued)

DON'T THROW THIS *BULLETIN* AWAY --

Give it to a friend or

Leave it in your dentist's office.

Part B:If you use the Society's Seal (found on the front cover of this Bulletin) can you divide it into three such parts having equal sums? Can you divide it into two parts with equal sums?

□

IN MEMORIAM -- ISAAC ASIMOV

Isaac Asimov, member number 293; died on April 6, 1992.

The author of more than 3 gross books, several of which contained positive references to the advantages of duodecimal counting, Asimov accepted honorary membership in the DSA more than one half dozen years ago. At that time he added the terse caveat, "as long as it is understood that I don't have to do anything." His comments about the advantages of twelve base counting were read by thousands of people.

On April 29, 1992 the "Business Day" section of the NY Times mentioned the following: "With a precision people can seldom duplicate, often under conditions humans connot endure, industrial robots are slowly making reality of the science-fiction servants that the author Isaac Asimov began describing in the early 1950's." The article began: "With the death of Isaac Asimov this month, the robotics industry lost its patron saint..."(D1).

□

ANOTHER SOLUTION TO A BASE TWELVE CROSS NUMBER PUZZLE

Jay L. Schiffman
Camden County College
Camden City Center
Camden, NJ 08102

In Bulletin 68;, Summer 1991, Charles Ashbacher of Hiawatha, IA. proposed an intriguing puzzle which was solved in Bulletin 69; (p. 19;). What follows is an alternate solution.

A BASE TWELVE CROSS NUMBER PUZZLE
(Each entry is a non-negative digit)

	1	2	3
1	5	0	9
2	1	5	6
3	9	*	9

ACROSS:

1. A perfect square and a perfect cube.
2. The three most widely used bases are divided by two.
3. Palindromic in two common bases.

DOWN:

1. A permutation of the digits form an arithmetic sequence.
2. The dozenal sum of these digits occurs as part of another solution in this puzzle.
3. Easily seen to be divisible by 3 in base ten and base twelve.

(Continued)

ANALYSIS

Across:

(1) 509; is simultaneously a perfect square and a perfect cube; for 509; = 9^3 = 23;^{2}. (Decimally, we are saying that $729 = 9^3 = 27^2$.)

(2) The most widely utilized bases are the binary (scale of 2), decimal (scale of *;), and the duodecimal (scale of 10;). When each of these three bases is divided by two, one obtains 1, 5, and 6 respectively.

(3) The duodecimal numeral 9*9; = $(858)_{13}$. (Decimally, 1,425 is palindromic in two bases as 9*9; and $(858)_{13}$. These three digit numerals read the same in either the natural order or the reverse order.)

Down:

(1) The three digit duodecimal numeral 519; when its digits are rearranged, gives rise to six distinct permutations: 159;, 195;, 519;, 591;, 915;, and 951;. The first and last of these form arithmetic sequences with respective common differences ±4.

(2) The dozenal sum of the digits 0, 5, and *; is 13; which coincides with the dozenal sum of the digits 5, 1, and 9, the solution to 1 DOWN. Moreover, the set of digits 0, 5, and *; forms an arithmetic sequence with common difference 5.

(3) In base ten, a positive integer is divisible by 3 if the sum of the digits of the number (positive integer) is divisible by 3. Moreover, in base twelve a positive integer is divisible by 3 if the integer terminates in one of the digits 0, 3, 6, 9. If we treat 969 as a decimal numeral, then 969 is divisible by 3 by observing that 9 + 6 + 9 = 24 decimally which is divisible by 3. If we treat 969; as a duodecimal numeral, then 969; is divisible by 3 since 969; terminates in the digit 9. What is indeed intriguing is 969; = 1,377 decimally, which is divisible by 3. (1 + 3 + 7 + 7 = 18 decimally, divisible by 3.) On the other hand, 969 as a decimal numeral = 689; which is divisible by 3; for 689; terminates in the digit 9. 969 is also palindromic!

□

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition--("Who needs a symbol for nothing?")--the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used.

A parallel to today seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

□

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents 5 units, *6* dozen, and 2 dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3#2*	Two ft. eight in.	*2;8'*
19#	*1000*	Eleven ft. seven in.	*#;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down 53. Using this *"which is"* step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2#*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
  12 ) 30 + 5
    12 ) 2 + 6
         0 + 2   Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *.

For more detailed information see *Manual of the Dozen System* ($1;00).

We extend an invitation to membership in our society.
dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______________________________
LAST FIRST MIDDLE

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Signed ______________ Date ______________

My interest in duodecimals arose from ______________________________

____

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Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

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