ISSN 0046-0826

THE DUODECIMAL BULLETIN 69;

SEE -- Schedule of the DSA Annual Meeting for 1991 - p. 4;

Dr. Barbran Smith, James Malone, Gene Zirkel and Fred Newhall conduct the business of the 1990 DSA Annual Meeting.

The 1991 DSA Annual Meeting will be held
October 19, 1991 at 1 p.m.
Nassau Community College
Garden City, LI, NY

Volume 34;
Number 3;
1991
119#;

DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00 (US) for one calendar year. Student Membership is $3.00 per year, and a Life Membership is $144.00 (US).

The Duodecimal Bulletin is an official publication of the DOZENAL SOCIETY OF AMERICA, INC., c/o Math Department, Nassau Community College, Garden City, LI, NY 11530.

BOARD OF DIRECTORS OF THE DOZENAL SOCIETY OF AMERICA

Editorial Office:

6 Brancatelli Court
West Islip, NY 11795
(516) 669-0273

THE DUODECIMAL BULLETIN

Whole Number Six Dozen Nine

Volume 34; Number 3;

119#;

FOUNDED 1944

IN THIS ISSUE

DOZENAL SOCIETY OF AMERICA
SCHEDULE OF THE ANNUAL MEETING -- 119#;

Saturday, October 19, 1991
Nassau Community College
Garden City, LI, NY 11530

1 P.M. Administrative Tower, Twelfth Floor

All business at the Annual Meeting is conducted by outgoing officers.

I BOARD OF DIRECTORS MEETING -- Tentative Agenda

1. Call to order, Gene Zirkel, Chair
2. Report of the Nominating Committee (L. Aufiero, J. Malone, J.Schiffman) and proposal of a slate of DSA Officers:

Board Chair	
President	**TO**
Vice President	**BE**
Secretary	**ANNOUNCED**
Treasurer	

3. Election of said Officers. The new Officers will be installed later in the day.

(Continued)

The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus 1/2 = 0.5 = 0;6.

4. Appointments to the following DSA Committees:

   Annual Meeting
   Awards
   Finance

5. Other appointments:

   a) Parliamentarian (to the Board Chair)

6. Other business of the Board
7. Adjournment

II ANNUAL MEMBERSHIP MEETING -- Tentative Agenda

1. Call to order, F. Newhall, President
2. Minutes of the 1989 Annual Meeting, L. Aufiero
3. President's Report, F. Newhall

   Appointments:
   a) Parliamentarian (to the President)
   b) Editor of the Duodecimal Bulletin

(Continued)

THE FOLLOWING ARE AVAILABLE FROM THE SOCIETY

1. Our brochure. (Free)
2. "An Excursion in Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, October 1934. (Free)
3. *Manual of the Dozen System* by George S. Terry. ($1;00)
4. *New Numbers* by F. Emerson Andrews. ($10;00)
5. Dozenal Slide Rule, designed by Tom Linton. ($3;00)
6. Back issues of the *Duodecimal Bulletin*, as available, 1944 to present. ($4;00 each)
7. *TGM: A Coherent Dozenal Metrology* by T. Pendlebury ($1;00)
8. *Modular Counting* by P.D. Thomas ($1;00)
9. *The Modular System* by P.D. Thomas ($1;00)

4. Treasurer's Report, A. Catania

Tony Catania (left) discusses his Treasurer's Report at the 1990 DSA Annual Meeting. Next to Tony is Jamison Handy.

5. Reports of other Officers and individuals, as called for.

   a) Editor's Report

      1) Appointment of Reviewers of Articles for the Bulletin

6. Committee Reports:

   Annual Meeting - B. Smith, Chair; L. Aufiero, A. Catania, A. Razziano

   Awards - G. Zirkel, Chair; J. Impagliazzo, J. Malone, A. Scordato, P. Zirkel

   Finance - A. Scordato, Chair; L. Aufiero, A. Catania, D. George, J. Malone, A. Razziano, P. Zirkel

   Reports of other Committees, as called for.

(Continued)

7. Report of the Nominating Committee (L. Aufiero, J. Malone, J. Schiffman)

   a) Nomination and election to the Board of the Class of 1993.

   SLATE
   TO
   BE
   ANNOUNCED

   b) Election of a Nominating Committee for the period 1991-1992.

8. New business of the Membership.

III SPEAKERS **TO BE ANNOUNCED**

IV DINNER AND EVENING ENTERTAINMENT

TO BE DECIDED BY ATTENDEES

Gene and Pat Zirkel during the DSA "night out" following the Annual Meeting in October 1990.

□

THE REMARKABLE PRIME NUMBERS

Jay Schiffman
Camden County College
New Jersey

Introduction

In a recent issue of The Dozenal Journal [1], Shaun Ferguson presented a variety of intriguing ideas on prime numbers as expressed in various number bases, as well as raising a number of thought-provoking problems. The purpose of this article is to extend some of his initial researches and resolve some of his mind bogglers.

Our first interesting idea with prime numbers concerns the notion of palindromic primes. Recall that an English word such as "dad" or "tot" which reads the same whether read in the natural order (from left to right) or in reverse order is called a Palindrome. People from walks of life as diverse as the English professor to the sports announcer are often fascinated as to how a word or name would read backwards. For instance, Hall of Fame Cubs broadcaster Harry Caray on WGN radio and television often fills in the dead air time between pitches playing such word games. I do not recall how many palindromes he arrived at. Similarly numbers such as 131 and 14641 (in any base) read the same backwards as well as forwards and hence are called palindromic numbers. We provide an enumeration of the palindromic primes in bases ten and twelve which fall below 1000 in these bases.

Base ten: 11, 101, 131, 151, 181, 191, 313, 353, 373, 383, 727, 757, 787, 797, 919, 929

Base twelve: 11, 111, 131, 141;, 171;, 181, 1#1, 535, 545, 565, 575, 585, 5#5, 727, 737, 747, 767, 797, #1#, #2#, #6#

A second idea of excitement for number theory buffs is the notion of a prime reversal. A prime reversal of a two-digit number is the number formed with the digits interchanged. Of course, both the original two-digit number and the number formed by interchanging the digits must both be prime.

We list the two-digit prime reversals in bases ten and twelve for reference.

Base ten: 13 and 31, 17 and 71, 37 and 73, 79 and 97.

(Continued)

Base twelve: 15 and 51, 57 and 75, 5# and #5.

Let us note that 15; = (1 x 12) + (5 x 1) = 17 decimally (prime)

51; = (5 x 12) + (1 x 1) = 61 decimally (prime)

Notice the scarcity of prime reversals in bases ten and twelve. In any base with an even number of characters, no prime reversal pair can contain an even digit, which vastly limits the number of potential candidates. (In any base with an even number of characters, any multiple digit number ending in an even digit is divisible by 2 and hence cannot be a prime.)

A third useful idea concerns cyclic primes. Consider the decimal integer 197. 197 is prime in base ten, and if we take a cyclic arrangement of its digits, we obtain the integers 971 and 719. What is rather fascinating is that 971 and 719 are likewise prime in this base. Let us formulate a list of three-digit integers in the decimal and duodecimal bases where all so-called cyclic permutations of the digits of a prime likewise produces a prime.

Base ten:
113, 131, and 311
197, 971, and 719
199, 991, and 919
337, 373, and 733

Base twelve:
117, 171, and 711
11#, 1#1, and #11
175, 751, and 517
1#7, #71, and 71#

A fourth interesting notion concerns primes which are at minimal distance from one another. Such primes (known as twin primes) differ by two and are necessarily odd. (2 is the only even prime in any base). Although Euclid proved over 2,000 decimal years ago that there are infinitely many primes, it remains unknown whether or not there are infinitely many twin prime pairs. A list of the twin primes below 1000 in bases ten and twelve follows.

Base Ten:

(3,5), (5,7), (11,13), (17,19), (29,31), (41,43), (59,61), (71,73), (101,103), (107,109), (137,139), (149,151), (179,181), (191,193), (197,199), (227,229), (239,241), (269,271), (281,283), (311,313), (347,349), (419,421), (431,433), (461,463), (521,523), (569,571), (599,601), (617,619), (641,643), (659,661), (809,811), (821, 823), (827,829), (857,859), (881,883).

(Continued)

Base Twelve:

(3,5), (5,7), (#,11), (15,17), (25,27), (35,37), (4#,51), (5#,61),
(85,87), (8#,91), (#5,#7), (105,107), (12#,131), (13#,141),
(145,147), (16#,171), (17#,181), (1*5,1*7), (1#5,1#7), (23#,221),
(24#,251), (2*#,2#1), (2##,301), (325,327), (375,377), (3#5,3#7),
(41#,421), (435,437), (455,457), (46#,471), (575,577), (585,587),
(58#,591), (5#5,5#7), (615,617), (70#,711), (735,737), (745,747),
(76#,771), (7##,801), (865,867) (905,907), (91#,921), (9*#,9#1),
(*0#,*11), (*35,*37), (*3#,*41), (#1#,#21), (#2#,#31), (#6#,#71),
(#95,#97), (##5,##7).

Our final activity in this article is to resolve three mind bogglers involving primes. Two of these problems were posed by Shaun Ferguson in his paper and a third is a variation of his problem.

Problem 1:

1193 is prime in base ten. This is prime even when 'circulated'. (A cyclic permutation of the digits always produces a prime): 1931, 9311, and 3119. Can one find a similar example in the dozenal base?

Solution:

Indeed one can find two examples as follows:

First Solution:

157#;, 57#1;, 7#15;, and #157; are all prime. Note that 157#; =(1x10;^{3}) + (5x10;^{2}) + (7x10;) + (#;x1) = 1,728 + 720 + 84 + 11 = 2,543 decimally (prime).

57#1; = (5x10;^{3}) + (7x10;^{2}) + (1x10;) + (1x1) = 8,460 + 1,008 + 132 + 1 = 9,781 decimally (prime).

7#15; = (7x10;^{3}) + (#x10;^{2}) + (1x10;) + (5x1) = 12,096 + 1,584 + 12 + 5 = 13,697 decimally (prime).

#157; = (#x10;^{3}) + (1x10;^{2}) + (5x10;) + (7x1) = 19,008 + 144 + 60 + 7 = 19,219 decimally (prime).

To check that 2,543 is prime, test all primes $\leq \sqrt{2543}$; that is test all primes $\leq$ 47. Namely, test the primes 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47. None of these primes is a factor of 2,543, therefore 2,543 is a prime. This 'square root test' is useful for determining whether or not an integer is prime. Similar remarks hold for the other decimal integers claimed to be prime.

(Continued)

Second Solution:

555#;, 55#5;, 5#55;, and #555; are all prime. Note that 555#; = (5x10;^{3}) + (5x10;^{2}) + (5x10;) + (#;x1) = 8,640 + 720 + 60 + 11 = 9,431 decimally (prime).

55#5; = (5x10;^{3}) + (5x10;^{2}) + (#;x10;) + (5x1) = 8,640 + 1,584 + 60 + 5 = 9,497 decimally (prime).

5#55; = (5x10;^{3}) + (#;x10;^{2}) + (5x10;) + (5x1) = 8,640 + 1,584 + 60 + 5 = 10,289 decimally (prime).

#555; = (#;x10;^{3}) + (5x10;^{2}) + (5x10;) + (5x1) = 19,008 + 720 + 60 + 5 = 19,793 decimally (prime).

The reader is invited to check that the decimal integers 9,431, 9,497, 10,289, and 19,793 are indeed prime.

Problem 2:

Can any one find a 'superprime' (in any base) where, the prime being abcd, a, ab, abc, and abcd are all prime?

Solution:

In base twelve, 27#1; is such a prime.

Note that 2 is prime.

27; is prime since 27; = (2x10;) + (7x1) = 24 + 7 = 31 decimally.

27#; is prime since 27#; = (2x10;^{2}) + (7x10;) + (#;x1) = 288 + 84 + 11 = 383 decimally.

Finally 27#1; is prime since 27#1; = (2x10;^{3}) + (7x10;^{2}) + (#;x10;) + (1x1) = 3,456 + 1,008 + 132 + 1 = 4,597 decimally.

To show that 4,597 is prime, test all primes $\leq \sqrt{4597}$ (all primes $\leq$ 67). It can be shown that none of these primes divides 4,597 so that 4,597 is prime.

Problem 3:

Is it possible to find a prime where any combination of the digits (1, 2, 3, or all 4 at a time) (in order) is always prime in a given base?

(Continued)

Solution:

There are C(4,1) + C(4,2) + C(4,3) + C(4,4) = 4 + 6 + 4 + 1 = 15 = 2^4 - 1 ways decimally in which to accomplish this task.

C(4,1) means the number of ways of selecting 4 objects taken one at a time, namely 4 ways. There are C(4,2) or 6 ways of choosing 4 objects taken two at a time. Similarly, there are C(4,3) or 4 ways of choosing 4 objects taken three at a time. Finally, there are C(4,4) or only one way of choosing 4 objects taken all four at a time.

To see this, consider a four digit number (in any base) labelled abcd.

Choosing one digit from the four can be done in four ways:

a, b, c, or d

Choosing two digits from the four can be done in six ways:

ab, ac, ad, bc, bd, or cd

Choosing three digits from the four can be done in four ways:

abc, abd, acd, or bcd

Choosing four digits from the four can be done in only one way:

abcd

Adding, we have a total of 15 ways decimally.

For those familiar with elementary set theory, this problem is a model of the problem asking one to determine the number of non-empty subsets for a set having 4 elements, also 15 decimally.

To solve our problem, consider the duodecimal integer 35#7;
The following is valid:

3 is prime.
5 is prime.
is prime.
7 is prime.

35; = (3x10;) + (5x1) = 41 decimally, prime.
3#; = (3x10;) + (#;x1) = 47 decimally, prime.
37; = (3x10;) + (7x1) = 43 decimally, prime.
5#; = (5x10;) + (#;x1) = 71 decimally, prime.
57; = (5x10;) + (7x1) = 67 decimally, prime.
#7; = (#;x10;) + (7x1) = 139 decimally, prime.

35#; = (3x$10;^2$) + (5x10;) + (#;x1) = 503 decimally (prime).
357; = (3x$10;^2$) + (5x10;) + (7x1) = 499 decimally (prime).
3#7; = (3x$10;^2$) + (#;x10;) + (7x1) = 571 decimally (prime).
5#7; = (5x$10;^2$) + (#;x10;) + (7x1) = 859 decimally (prime).

Finally,

35#7; = (3x$10;^3$) + (5x$10;^2$) + (#;x10;) + (7x1) = 6,043 decimally.

Now 6,043 is indeed prime. Note that $(73)^2$ = 5,329 while $(79)^2$ = 6,241. Test all primes $\leq$ 73. It can be shown that none of these primes divides 6,043 so that 6,043 is indeed prime, completing our problem solution.

BIBLIOGRAPHY

1. Ferguson, Shaun. "Palindromic Primes and Primes - Some other Ideas". The Dozenal Journal, No. 6, Spring 1988.

2. Hammond, Donald. "Base Twelve and The Prime Numbers." The Dozenal Journal, No. 5, Spring 1987.

3. Pettofrezzo, A.J. and D.R. Brykit. Elements of Number Theory. Prentice Hall, Englewood Cliffs, NJ, 1970.

□

A BASE PUZZLE

In every base, b, there is always a three digit number which is equal to the sum of its digits multiplied by (b+1). In base two the number is 110 since

(1+1+0) x (11) = 10 x 11 = 110.

Can you find the three digit numbers with this property for all the bases from three through do-one?

-Adapted from a question in the Mathematics Teacher.

□

GENERAL SHERMAN ON TWELVES

Sent in by: *H.K. Baumeister*
Adams Run, SC

A cavalry regiment is now composed of twelve companies, usually divided into six squadrons of two companies each, or better subdivided into three battalions of four companies each. This is an excellent form, easily admitting of subdivision as well as union into larger masses.

Nevertheless, the regimental organization for artillery has always been maintained in this country for classification and promotion. Twelve companies compose a regiment, and, though probably no colonel ever commanded his full regiment in the form of twelve batteries, yet in peace they occupy our heavy sea-coast forts or act as infantry; then the regimental organization is both necessary and convenient.

But the infantry composes the great mass of all armies, and the true form of the regiment or unit has been the subject of infinite discussion; and, as I have stated, during the civil war the regiment was a single battalion of ten companies. In olden times the regiment was composed of eight battalion companies and two flank companies. The first and tenth companies were armed with rifles, and were styled and used as "skirmishers;" but during the war they were never used exclusively for that special purpose, and in fact no distinction existed between them and the other eight companies.

The ten-company organization is awkward in practice, and I am satisfied that the infantry regiment should have the same identical organization as exists for the cavalry and artillery, viz., twelve companies, so as to be susceptible of division into three battalions of four companies each.

Taken from pages 383 & 384 of the second volume of Memoirs of Gen. W.T. Sherman, Written by Himself, 4th edition, Charles L. Webster & Co., New York, 1892; Chapter 25 entitled "Conclusion - Military Lessons of The War".

☐

Do you have an idea to share with our members? Why not submit an article to the *Bulletin*?

THE METRONS ARE COMING!

Henry C. Churchman
Council Bluffs, IA

The thoughts which follow are excerpted from an article written for publication in 1975.

There is little new about metronic dimensions. Each is exactly related to our foot, inch or line. Still in powers of 12. As American as apple pie.

Base-twelve measurements have been in use around this earth, and in the zodiac of the heavens, well over 2000 years. The early Romans put together a Latin word "uncia" (meaning one-twelfth), which by Englishmen was pronounced, perhaps from Roman occupation days, as "inch" or one-twelfth foot. The civilizations thriving on the banks of the Euphrates delighted in the factors 2, 3, 4, and 6 parts of a dozen, over 3000 years ago.

The French meter, however, dates from the 1600s when Fr. Gabriel Mouton, then vicar of St. Paul's Cathedral at Lyons, France, urged a system of measurements based on ten and plus or minus powers of ten alone, with no variations whatever. In principle perfect, faultless, but the system applies to base-twelve measurements better than to base-ten.

The metric system was further developed by Prof. Antoine L. Lavoisier at the request of the French Academy of Sciences in the 1790s, and adopted by the government while the French people were in a revolutionary mood, with strong desires to change not only their form of government but all everyday habits such as church feast days, King Louis' calendar, his hours in a day, his foot---to be forever rid of the monarchy and all other reminders of hunger and poverty.

The common people, nevertheless, even as today, did not rush to follow their leaders; so, about 1840, as is customary in these matters, the French metric system was forced down their throats by new statutory requirements to abandon the former units entirely. Tax money was spent lavishly.

(Continued)

The next DSA Annual Meeting will be held on Saturday, October 19, 1991 at 1 P.M. on the campus of Nassau Community College in Garden City, LI, NY. We hope to see YOU there!

If you were born before about 1950, you probably have been taught nothing of the French base-ten metric system in grade school, and only a smattering of it in high school, although our Congress allowed the French metric system for the people of the United States more than a century ago (1866).

We perhaps should encourage the teaching of the base-ten metric system in every school room. Otherwise the American people will not know the good points and, even more important, the bad parts of the French metric system. Those unfavorable or unworkable parts, without much argument, make its permanent use impossible in America. These weaknesses, however, could be removed by Americans, who, so familiar with the advantages of base-twelve divisions of measurement units, can impose metronic changes upon the French decimetric system here in these United States. A Frenchman, M. Jean Essig, suggested most of these needed improvements. Read his Douze! Notre Dix Futur (Dunod, Paris, 1955).

All metronic unit lengths today are defined, not in terms of meters, which are always subject to change, but more exactly and precisely by the number of wavelengths of orange-red krypton 86 light, in vacuum, which they represent. No metronic unit differs proportionately from the present commonly employed Canadian units of length by more than 33 of 100,000 equal parts, when each is described in wavelengths of kr 86 light. U.S. licensed survey crews are not more precise in their measurements.

A table of comparisons of some American metronic units with French metric or English measurement units follows:

(Continued)

DON'T THROW THIS *BULLETIN* AWAY --

Give it to a friend or

Leave it in your dentist's office.

A TABLE OF COMPARISONS

Units of Length	Symbol	Approximate Relationships
Metron** (153,792 Kr. 86)	met**	44 lines, 3-2/3"
dozenal meter*	md*	44 in., 3-2/3 ft
dozenal decameter*	damd*	44 ft, 2-2/3 rds
dozenal hectometer*	hmd*	528 feet, .1 mi.
dozenal kilometer*	kmd*	1.2 mile
navinaut*** (air travel)	nav**	1.2 mile in 50 seconds; 86.4 mph
Area		
dozenal hectare*	had*	6.4a, 1% Section
camp** (1 gross had)	cp**	1.2 mile squared
congressional township****	Twp	36 Sec, 25 camps
Volume		
dozenal liter* (1^3 met)	ld*	four-fifth liter
half jon, jean, or juan**	1/2 j**	13.5 fl. oz., 2/5 liter
quarter jon, jean, or juan**	1/4 j**	6.75 fl. oz., 1/5 liter
Weight		
kal** (calc, rock) (kgd*)	k**	1.75 lb., 4/5 kg
dokal** (dozen k)	dk**	21 lbs, 10 kg
rekal** (gross k)	rk**	250 lbs, 115 kg
mikal** (greatgross k)	mk**	3000 lbs, 1 1/2 tons

Temperature

144°H	=	212°F	=	100°C	=	boiling point water
36°H	=	104°F	=	40°C	=	feverish temperature
0°H	=	68°F	=	20°C	=	Hawaii mean temp.**
-36°H	=	32°F	=	0°C	=	freezing temperature
-72°H	=	-4°F	=	-20°C	=	double freezing
-528°H	=	-460°F	=	-273°C	=	absolute zero

* suggested by M. Jean Essig, Paris, France.
** suggested by Henry Churchman, Council Bluffs, IA
*** suggested by Charles S. Bagley, Alamogordo, NM
**** Act of Congress, Northwest Ordinance of 1787.

(Continued)

NUMBER BASES AND SCIENTIFIC NOTATION: A Classroom Example

Gene Zirkel
Nassau Community College
Garden City, LI, NY

Many introductory math courses include the topics of number bases and scientific notation. What follows is a real life example using these concepts.

One of my better students wrote a simple computer program which subtracted a 10% discount from a given cost and then added a 7¼% tax.

When given a cost of $35, the program correctly printed the following output:

Cost	$35.00
Discount	-3.50
Subtotal	31.50
Tax	+2.28
Total	$33.78

However when given a cost of $10.35, the output she obtained was:

(Continued)

The Metrons Are Coming *(Concluded)*

Addenda: One-fourth metron equals 11 lines or 11/12 inch. One-fourth md is the exact equal of 11 inches, or of 1 "nufut," in describing plane altitude. The dozenal hectometer should exactly match 0.1 Canadian mile. A dozen hmd equal 1 "aeromile," or 1 kmd. The kmd would, as suggested by Essig, ultimately replace the nautical and statute miles and the kilometer, on water, on land, and in the air. The navinaut is a natural successor to the knot but can also replace miles per hour and kilometers per hour, as knowledge about it spreads. Fifty-four miles per hour equal 5/8 Navinaut. Five navinauts equal 432 mph.

In reporting acres of growing crops, the United States Department of Agriculture might record area by dozenal hectares, 100 in each square mile of farm lands, or by Sections and percent of section, say 123.45 sections, with which U.S. farmers are familiar. □

Cost	$10.35
Discount	-1.03
Subtotal	9.31
Tax	+0.68
Total	$9.99

Note the results of the subtraction which appear on the third line!

The way that I make use of this in class is as follows:

First I show the correct example given above. Then I ask the students to calculate similar results for a cost of $10.35. Most students produce one of the following:

Cost	$10.35	Cost	$10.35
Discount	-1.04	Discount	-1.03
Subtotal	9.31	Subtotal	9.32
Tax	+0.67	Tax	+0.68
Total	$ 9.98	Total	$10.00

Students who use a calculator may produce

Cost	$10.35
Discount	-1.035
Subtotal	9.315
Tax	+0.6753 . . .
Total	$9.9903 . . .

Which they round off to

Cost	$10.35
Discount	-1.04
Subtotal	9.31
Tax	+0.68
Total	$9.99

Of course, they have to "fudge" to do this by rounding 1.035 *up* while at the same time they round 9.315 *down*.

After discussing these various correct possibilities, I show them the computer's erroneous output given above, and ask them to explain what went wrong. The discussion generally raises the question:

(Continued)

Why doesn't the computer round 1.035 up to 1.04 as is customary?

Realizing that computers work in binary (which can be equivalently expressed in hexadecimal) notation, we change the numbers to base sixteen (or base two, if you prefer).
Now 1.03_{ten} = the repeating fraction $1.08F5C28F5C28F5C2..._{sixteen}$
and real numbers are stored in the computer's memory in a scientific notation format with a base sixteen exponent. Hence the above is written as

0.108F5C28F5C28F5C2... Exponent(01)

A typical computer uses 32 binary digits (or bits) which is equivalent to 8 hexadecimal digits (or hits) to store a real number. Two hits are used to store the exponent and 6 hits are used to store the mantissa. Thus only the 6 hits 108F5C are put into the computer memory.

When the students express the number 0.108F5C Exponent(01) as $1.08F5C_{sixteen}$ and then convert it back to base ten they obtain 1.03125_{ten}, and they then understand why the machine rounds *down* to 1.03.

The final idea is to convey to them the difference between the contents of the computer's memory and the output that they see.

Thus memory contains,	while the output reads
\$10.35	\$10.35
-1.035	- 1.03
9.315	9.31
+0.675335	+0.68
\$9.990335	\$9.99

(In computer classes, I also discuss two more topics: (1) The difference between algorithms that are mathematically correct and those that yield correct results when used in a program. (2) Now that we understand the problem, how can we have the computer produce correct output.)

All in all, this is a very satisfying lesson, interrelating several diverse ideas: number bases - including conversions to and from base ten, rounding off, scientific notation, computer memory versus computer output, using algorithms in computer programs, and methods of debugging the above type of errors.

Try it. You'll like it, and so will your students. □

Remember -- your gift to the DSA is tax deductible.

IN A PERFECT WORLD -- Solution

1; Twelve Commandments, not 10
2; Twelve seasons, not 4
3; The House of the Twelve Gables, not 7
4&
5; Friday the Twelfth: Part Twelve, not 13th & VII
6; The Twelve Hills of Rome, not 7
7; One Thousand seven hundred and twenty eight Arabian Nights, not 1001
8; The Wonderful Twelve Hoss Shay, not 1
9; The Twelve Wonders of the Ancient World, not 7
*; Twelve Seas, not 7
#; The Twelve Lost Tribes, not 10
11; Twelve Gentlemen of Verona, not 2
12; Twelve Years Before the Mast, not 2
13; Twelve Comedies & Twelve Interludes by Cervantes, not 8
14; The Mighty Twelve Russian composers, not 5
15; The Civil War Battle of the Twelve Forks, not 5
16; Twelve Characters in Search of an Author, not 6
17; The Twelve Sisters by Chekov, not 3
18; Valley of the Twelve Peaks in Canada, not 10
19; I Led Twelve Lives, not 3
1*; Twelve River National Forest in CA, not 6
1#; Goldilocks & the Twelve Bears, not 3
20; The Twelve Musketeers, not 3

□

BASE TWELVE CROSSNUMBER PUZZLE -- Solution

1	5	0	9
2	1	5	6
3	3	*	3

[Another solution has been received from Jay Schiffman, and will be printed in the next issue. -Ed.]

Charles Ashbacher □

TWELVE MONTHS IN A YEAR

Jean Kelly
New York, NY

As each month passes, one-twelfth (0;1) of the year goes by. A simple way to indicate the current month and year would be to make use of the fact that there are twelve months in the year. Thus we could write:

11*0;0	(1992;0)	for January 11*0;
11*0;1	(1992;1)	for February 11*0;
11*0;2	(1992;2)	for March 11*0;
11*0;3	(1992;3)	for April 11*0;
11*0;4	(1992;4)	for May 11*0;
11*0;5	(1992;5)	for June 11*0;
11*0;6	(1992;6)	for July 11*0;
11*0;7	(1992;7)	for August 11*0;
11*0;8	(1992;8)	for September 11*0;
11*0;9	(1992;9)	for October 11*0;
11*0;*	(1992;*)	for November 11*0;
11*0;#	(1992;#)	for December 11*0;

□

BOOK REVIEW

Jean Kelly
New York, NY

The Mote in God's Eye by Larry Niven & Jerry Pournelle, PocketBooks, NY 1974.

An engrossing tale of the first encounter between humans and aliens. Of course the aliens have six fingers on each hand -- actually four fingers and two opposing thumbs. This detail is not beaten to death, but subtly woven into the story in a half dozen places.

One of the first communications that the aliens send is three, one eight four eleven, which are the starting digits of Pi in base twelve.

In approximating time the aliens naturally refer to half a gross of years rather than to half a century as decimal counting humans would.

For an enjoyable read get a copy of this science fiction narrative. □

DOZENAL JOTTINGS

...from members and friends...News of Dozens and Dozenalists...

Welcome to new members:

317; AL STANGER — St. Louis, MO

318; HAROLD M. STEIN — Hauppauge, NY

Harold's interest in duodecimals arose from meeting Fred Newhall through Scouting activities and from reading science fiction. He has travelled to Europe and Israel on a scholarship.

319; DR. JOHN A. SCHUMAKER — Rockford, IL

John heard about the Society through seeing our advertisement in the Journal of Recreational Mathematics..."I had not heard of or from the Duodecimal Society for many years and had thought it to be defunct!" □

THE SHORT AND THE LONG OF IT

The following are some further, random thoughts on measurement. The reader is directed to an article in the last issue of this Bulletin: "History Professor Speaks to DSA Members."

Evidence of our ancestors' wisdom in using dozens exists in reference to the English *short* hundred (of dek^2 items) and the *long* hundred or dek dozen. Similarly they used the *short* thousand versus the *long* thousand of a dozen *short* hundreds.

The Norwegians referred to the *great* hundred, and in Iceland before the year 1000 they distinguished between the *ten*-hundred and the *twelve*-hundred. The French sometimes referred to the *short* hundred as the *ecclesiastical* hundred.

Note also, that a tun is equal to a *great* thousand liters.

So if you order one hundred items, don't get short changed!

For more information on these ideas see Number Words and Number Symbols, *A Cultural History of Numbers* by Karl Menninger published by the M.I.T. Press, Cambridge, Massachusetts and London, England.

-Gene Zirkel □

JAPAN COMES BACK!

Gene Zirkel

According to a report in July/August Unit of the *American metric Journal*, which was headlined "Millions in Japan Join UK & Aussies", Japan has re-legalized the old units. This reverting to customary measures follows dozens of years of trying to force the Japanese version of the metric system upon its citizens.

An aside in the article mentions that US importers of Japanese cars weren't pleased with the Japanese variation of metric screw threads, so they invented a third "modified metric system" of threads. (The 1975 metric act gives the Secretary of Commerce the power to "interpret and modify the SI metric system".)

A second article worth reading in the same July/August Unit is entitled "Metric Inching Into Oblivion". It details the failure of the attempts to convert the USA to the awkward decimal metric system. Among the current headlines it quotes are:

94% OF USA STILL CUSTOMARY, and

IT'S ALL OVER, THE FAT LADY HAS SUNG.

One cannot help but wonder: Where would we be today if all the time and money spent trying to foist uncomfortable tens on people had been channeled into promoting a factorable system based on dozens? Lagrange[1] - hang your head in shame!

[1]After the French Revolution, Joseph Lagrange was instrumental in leading the committee considering whether to change measures to tens or counting to twelves in the wrong direction. He mistakenly thought that people would accept the tens, but history has proved him wrong. Not one country has ever voluntary accepted the impoverished decimal metric system. □

DON'T KEEP THIS MAGAZINE

Do you discard your copies of the *Bulletin* after you have read them? Or do they gather dust on a shelf in your attic? Why not pass them along to your local library, or to a school library. Perhaps some nearby math teacher would appreciate a copy. You can also just leave them in a dentist's office or other waiting area.

Help spread the word!

(If you ever need a back copy, we'd be glad to help.)

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition-("Who needs a symbol for nothing?")-the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accomodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used, and as early as 1585, Simon Stevin stated that the duodecimal base was to be preferred to the base ten.

The parallel seems tenable. The notation of the dozen base better accomodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses)* to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTORy** because it has **NOT ENOUGH FACTORS.**

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if y6u think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited. □

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro. 1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3#2*	Two ft. eight in.	*2;8'*
19#	*1000*	Eleven ft. seven in.	*#;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down 53. Using this *"which is"* step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2#*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
12 ) 30 + 5
12 )  2 + 6
      0 + 2   Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *.

For more detailed information see *Manual of the Dozen System* ($1;00).

We extend an invitation to membership in our society.
dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______________________________
LAST FIRST MIDDLE

Mailing Address (for DSA items) ______________________________

____
(See below for alternate address)

Telephone: Home ______________ Business ______________

Date & Place of Birth ______________________________

College ______________ Degrees ______________

Business or Profession ______________________________

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______________________________

Address ______________________________

Year & Math Class ______________________________

Instructor ______________ Dept. ______________

Other Society Memberships ______________________________

____

Alternate Address (indicate whether home, office, school, other)

____

____

____

Signed ______________ Date ______________

My interest in duodecimals arose from ______________________________

____

____

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

DETACH HERE -- OR PHOTOCOPY