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A BASE TWELVE

CROSS NUMBER

PUZZLE

--See page 1#

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THE DOZENAL SOCIETY OF AMERICA

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THE DUODECIMAL BULLETIN

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IN THIS ISSUE

SOLUTION TO "REMEMBERING PI"

(Winter 1991 -- p. 1#)

In the last issue of the Bulletin we asked readers to provide us with a mnemonic for remembering the duodecimal value of Pi, which was given as:

3;184 809 493 #91 866 457 3*6 212...

a total of 21; digits.

Our first solution comes from Charles Ashbacher, Hiawatha, IA whose suggestion contains 15; digits. Please note that since it is impossible to have a word of length zero, Charles simply placed the word "zero" in that position.

Gee. A computer that displays zero answering time, beguiling all individuals, beginning a stampede toward owning many.

Our second solution contains 14; digits and comes from new member Monte J. Zerger, Adams State College, Alamosa, CO. Monte uses the period between the two sentences to signify the zero in the expansion.

Ten, I maintain, isn't suitable. Examining real carefully the mathematics, discloses a superior number, twelve.

Monte also writes:

I found it pleasing that the first three digits in the base twelve representation of Pi are the first three digits in the base ten representation of 1/Pi.

$Pi = (3;18)_{12}$

$1/Pi = (.318)_{10}$

End

The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus 1/2 = 0.5 = 0;6.

WHAT WOULD BE A GOOD BASE?

Cedric Smith
University College
London, England

[Reprinted from Colson News, Vol. 4, No. 2, pp. 45-46, July MCMLXXXIX; with permission.]

We are accustomed to write numbers in the scale of ten, or the decimal or denary scale. That is, when we write the year as 1989, the last figure, 9 represents the units, the next figure on the left, 8, the tens, the next figure on the left, 9, the hundreds (tens times tens), and so on. This regular progression makes arithmetic easier; thus the addition table shows that 2 + 3 = 5, whether we are referring to units, tens, hundreds or thousands. That is, 20 + 30 = 50, 200 + 300 = 500. But that is not true of measurements of time. 20 + 20 + 20 + 20 seconds make 1 minute 20 seconds, 20 + 20 + 20 + 20 minutes make 1 hour 20 minutes, 20 + 20 + 20 + 20 hours make 3 days and 8 hours. But, while the progression should be regular for convenience, there is no necessary reason why we should count by tens, hundreds (tens of tens), thousands, and so on. We could equally well count by units, dozens, gross (dozens of dozens), dozens of dozens of dozens, and so on, in which case a number such as 231 would mean

2 gross + 3 dozens + 1 (= 3 hundred + 2 tens + 5)

It seems very clear that this counting by tens originates from counting on the ten fingers. Some languages count differently, especially those rather isolated from Western culture. Even in Western nations there are traces of different ways of counting, especially by twenties, presumably due to counting on both fingers and toes. Thus, in French, French 80 is "four twenties" ("quatre-vingts" but "octante" in Belgium). In Danish, one form of 60 is "tresindstyve", from "tre" = 3 and "tyve" = 20. We may ask for a dozen or 2 dozen eggs in a shop, rather than 10 or 20.

However, the world needs a simple system, which shall be easy to operate, and uniform in all nations, with the result that virtually everybody uses the decimal scale of numbering (though some languages tend to retain their own symbols for the digits 0, 1, 2, ... side by side with the international ones) and the International System (SI) of measures. The irregularities are measures of time (day - hour - minute - second), of angle (complete revolution - right angle - degree - minute - second), and the remains of the traditional systems of measurement used side by side with international ones in some Anglophone countries (mile - furlong - yard - foot - inch, etc.) But even these are now legally defined in terms of the international units.

An obvious question is, "Have we done well in having ten as our basis of enumeration? Could we have done much better with another base?" One could go further, and suggest that it might be worth while to consider changing to another base, if that would greatly simplify life. That is not a nonsensical suggestion: almost all computers use the base 2, or the "binary scale", rather than the customary decimal scale, so as far as computers are concerned, such a change of base has already been made. Such a change in everyday life is hardly thinkable at present, in view of the enormous disruption

(Continued)

which would be involved in introducing a systems of units completely incompatible with the present ones. (What the distant future might bring is less predictable. Circumstances could be different.) However, that does not prevent one looking into the question of the relative advantages of different scales, and, indeed, there are even now situations in which the use of a different base is possible and advantageous. For example, the rules for winning quite a lot of games depend on the use of the scale of 2.

The relative advantages of different bases depend very much on the sort of calculations one wants to do. Suppose that one wants to find the factors of a number. In the usual denary scale, one knows that a number is divisible by 2 if the last figure is even, and it is divisible by 5 if the last digit is 0 or 5. One can go further, and say that it is divisible by 4 if the number formed by the last 2 figures is exactly divisible by 4, for example, 1325244 is exactly divisible by 4. To find whether a number is divisible by 3 is more trouble; the rule is that a number is divisible by 3 if the total of the figures is divisible by 3, and it is divisible by 9 if the total is divisible by 9. Thus 1 + 3 + 2 + 5 + 2 + 4 + 4 = 21, which is divisible by 3 but not by 9, so 1325244 divides exactly by 3 but not by 9. One can construct rules for divisibility by other numbers, such as by 7, but most of such rules tend to be rather complicated. If, instead of using the scale of ten, we used that of twelve, matters would be easier. A number would then be divisible by 2 when the last figure is divisible by 2, and similarly for 3, 4, and 6. However, tests for divisibility such as these are more matters for amusement than of practical use these days. They may well have been useful in the past, when one could have to add fractions such as 1/2 + 3/5 + 5/6 + 1/4, and so had to reduce them to a common denominator. But how often does one now have to deal with fractions? Fractions can be useful in certain technical contexts, such as some algebraic calculations, but are rarely needed in everyday life, apart from halves and quarter, which can anyway be written as .5 and .25, etc.

(Continued)

The Society regrets to announce the death of member number 1*3, James A Forster, on October 28, 1990.

Mr. Forster had been a member of the DSA for over two dozen years, and had made use of the base twelve circular slide rule during a long teaching career.

We extend our deepest sympathies to his family.

Before looking into the question further, one needs to think what sorts of calculations need to be done at all often at present. The situation has changed dramatically in the last 50 years. In 1940 calculating machines would do the four elementary operations of addition, subtraction, multiplication and division, but rather slowly, and they were expensive. Anything beyond that was bothersome. Tables gave the values of square roots, logarithms, antilogs, sines, cosines, etc., conveniently to about 4 figures, but any greater accuracy involved the slow process of looking them up in substantial books of tables. Nowadays a scientific pocket calculator has a very modest price, and can perform all these calculations and many more (quite often to an accuracy of something like a dozen figures), and do them virtually instantaneously. We can classify the typical sorts of calculations which are now regularly performed somewhat as follows:

(1) Financial calculations (e.g., bank accounts). These involve doing very simple operations, such as adding, subtracting, classifying, recording, sorting large masses of data, best done by a large computer network.

(2) Technical and scientific calculations. These possibly involve doing rather complicated mathematical operations on a medium sized body of data. Usually, a moderate sized computer will cope adequately, although there are occasions on which extremely long calculations are performed, which are only possible when large versatile computers are available.

(3) Simple technical and mathematical calculations, arising from time to time in an investigation, which can be adequately performed on a pocket calculator or personal computer. With the increasingly modest prices and ready availability of calculators and simple computers, these will rarely be done by hand.

(4) Small, off-the-cuff calculations which can best be done by hand, or which occur when no calculator is readily available. In practice, these will almost always amount to some additions, subtraction and multiplications, as when one buys goods in a shop, or measures a room for decorating, or wants to know how much a prospective purchase will cost, or something similar.

(Continued)

PRIME ENDINGS

Mathematicians know that every prime number greater than 3 is of the form 6k ± 1, where k is a natural number. In base twelve this means that all primes end in either 5, 7, # or 1.

Hence we know that

1) Numbers greater than 3 which end in the other eight digits (0, 2, 3, 4, 6, 8, 9, *) are factorable, and

2) When searching for prime numbers, we need only consider numbers which end in one of the four digits: 5, 7, # or 1.

Thus, in practice, the question of what would be a good scale resolves itself into 2 rather distinct questions: what would be a good scale for a computer, and what would be good for a human calculator?

Computers are not bound by tradition. Some of the first computers worked in the scale of ten, as we do. But it was soon realized that that was inefficient. It requires quite a deal of effort for a human to translate a number from one scale to another, so that changing the scale in an ordinary calculation is unproductive. But, on the other hand, a computer can change the scale very quickly, so that it can be designed to operate in any scale which gives the best performance. In practice, this almost always is taken to be the scale of 2, or binary scale. A positive binary number is simply a series of 1's and 0's. Thus 1 represents 1, 10 represents 2, 100 represents 2 x 2 = 4, 1000 binary = 2 x 2 x 2 = 8, and so on, giving a sequence of powers of 2. Any other positive number can be expressed as the sum of powers of 2, as 5 = 4 + 1, written 101, ten = 8 + 2, written 1010, 15 = 8 + 4 + 2 + 1, or 1111, and so on. This is convenient, since there is an enormous possible choice of "presence" and "absence" alternatives which can be used to denote 1 and 0. A black mark on a piece of paper can mean "1", and the absence of such a mark "0". A magnetic tape can be magnetized in one direction to signify "1", the opposite direction "0".

That means that a binary number can be recorded or transmitted in a number of different ways. The elements representing "1" and "0" are called "bits" (short for "binary digits"). The addition and multiplication tables are extremely simple:

0 + 0 = 0;   0 + 1 = 1 + 0 = 1;   1 + 1 = 10.

0 x 0 = 0 x 1 = 1 x 0 = 0;   1 x 1 = 1.

But the great snag is that the binary system, in this form, only represents positive numbers. The usual device to allow it to represent negative numbers is to let the left most digit take the values q1 (= -1) and 0, instead of 1 and 0. Thus, if we are dealing with 5 digit numbers, 00000 = 0, 00001 = 1, 00100 = 4, 00101 = 5 exactly as before, but q10000 = -16 (=q24), so that q10110 means -16 + 4 + 2 = - 10 (= q10). Any number between -16 and +15 can then be represented.

A possible rival to the scale of 2 is the scale of 3, or ternary scale. This has several advantages:

(1)There are only 3 digits, 1, 0, q1 = -1

(2)Any whole number is represented in one and only one way, as 2 = 1q1 (=3 - 1), 7 = 1q11 (=3 x 3 - 3 + 1), -2 = q2 = q11, etc.

(3)To reverse a number, one just reverses each digit = 7 = 1q11, -7 = q11q1.

(4)The addition and multiplication tables are very simple:

(Continued)

Addition of 0 does not change a number; also

q1 + q1 = q11
q1 + 1 = 1 + q1 = 0
1 + 1 = 1q1

Out of the 9 possible additions of pairs of digits, only 2 (q1 + q1 and 1 + 1) produce a carry.

Multiplication by 0 always gives 0, and

q1 x q1 = 1 x 1 = 1
q1 x 1 = 1 x q1 = q1

No one of these multiplications produces a carry.

(Continued)

IN A PERFECT WORLD. . .

we would have had "Snow White and the Twelve Dwarfs", not seven. Can you restore the correct numbers in the examples that follow?

1; Twelve Commandments, not ________
2; Twelve seasons, not ________
3; The House of the Twelve Gables, not ________
4 & 5; Friday the Twelfth: Part Twelve, not ________ & ________
6; The Twelve Hills of Rome, not ________
7; One Thousand seven hundred and twenty eight Arabian Nights, not ________
8; The Wonderful Twelve Hoss Shay, not ________
9; The Twelve Wonders of the Ancient World, not ________
*; Twelve Seas, not ________
#; The Twelve, Lost Tribes not ________
11; Twelve Gentlemen of Verona, not ________
12; Twelve Years Before the Mast, not ________
13; Twelve Comedies & Twelve Interludes by Cervantes, not ________
14; The Mighty Twelve Russian composers, not ________
15; The Civil War Battle of the Twelve Forks, not ________
16; Twelve Characters in Search of an Author, not ________
17; The Twelve Sisters by Chekov, not ________
18; Valley of the Twelve Peaks in Canada, not ________
19; I Led Twelve Lives, not ________
1*; Twelve River National Forest in CA, not ________
1#; Goldilocks & the Twelve Bears, not ________
20; The Twelve Musketeers, not ________

ANSWERS -- NEXT ISSUE

However, the use of such a scale of 3 requires a supply of "trits", i.e. 3 states in which an object can exist. Thus a magnetic tape could be magnetized in one direction or the other, or left unmagnetized. An electric current could flow in one direction along a wire, or the other, or there could be no current. A dot could be colored green, red, or left white. But it is rather more difficult to think of suitable trits than to think of suitable bits, and the idea has so far not found great favor.

But one could, of course, use a pair of bits to represent a trit. 10 could mean 1, 01 could mean q1 = -1, and 11 could mean 0. This would make the representation a little longer than using the binary scale; to represent a number in this way, with pairs of bits representing each trit, would require about 26 (=3q4) percent more space. (With the price of computer memories falling dramatically, this may no longer be so important.)

But it has a compensating advantage, in that there is a fourth combination, 00, which can be taken to mean "not known", or "uncertain". Think of the statement, "the population of England is 53 million. (Never mind, if that is not strictly correct; this is an illustration.) We would naturally take this to be stated to the nearest million, i.e., that the population lies between 52 500 000 and 53 000 000. The zeros here do not really mean zero. They are just put in to fill the spaces, and represent figures whose values we are not worrying about. Similarly, if we say that the population is 50 million, we would again take that to mean to the nearest million, unless the contrary was stated. If we write that as 50 000 000, the first zero is a real exact zero, but the others just represent figures whose value is unknown or ignored.

There is nothing in the ordinary way of writing 50 000 000 to show which 0's are real zeros and which are indeterminate or unknown figures. That is, 50 000 000 might just as well be the value to the nearest hundred thousand, instead of to the nearest million. There is nothing in the way the number is written to indicate which is meant. Thus there is a real need for a figure which can mean "unknown", rather than each figure expressing a precise value.

The second question is rather different. If we could go back in history, and choose which base of notation we liked best for calculations, which would we choose? In discussing this, we will suppose in any case that we are using two-way numbers, since

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IT IS A DOZENAL UNIVERSE -- Official!!

Arthur Whillock

Recent studies in nuclear physics have shown that all matter in the Universe is composed of twelve fundamental particles only, arranged into three families, with four in each, which are bonded together with twelve forces. An article describing this significant fact is being prepared for the DSGB Journal. DSA members should refer to the Scientific American for December 1989: "The Lost Generation," p. 15. British readers can more conveniently see New Scientist for 28th October 1989: "Cosmology Limits the Number of Neutrinos," p. 30.

they make arithmetic quite noticeably easier than using the traditional one-way numbers. There is also a personal element in this. Professor A. C. Aitken, FRS, of Edinburgh, used to like calculating in the scale of sixty. But that gives addition and multiplication tables which are much too big for ordinary mortals to learn conveniently. Possibly sixteen is about the largest base which one could reasonably consider. And any small base, say, less than 6, will tend to make even reasonably large numbers so long that they would strain the memory. Remember that, for example, 1989 or 20q1q1 becomes 11111000101 in the binary scale.

There seems little question that, for elegance, twelve is easily the best scale. We have already mentioned that in that scale we can immediately determine whether a number is divisible by 2, 3, 4 or 6 by looking only at the last digit. We can have analogs of "decimal fractions" in the scale of twelve, but we can not call them decimals. They are usually called "radix fractions". Thus, ;6 in the scale of twelve would mean six twelfths, or 1/2, ;4 would mean 4 twelfths, or 1/3, and ;16 = (twelve + 6)/ (twelve x twelve) = one eighth.

One advantage of the scale of twelve is that so many fractions, like these, which occur fairly naturally, can be expressed as simple short radix fractions, whereas, in our ordinary decimal scale, 1/2 = .5, 1/4 = .25, 1/5 = .2, but 1/3 and 1/7 and 1/9 and many other fractions become recurring decimals. Also, in the multiplication table, various combinations such as 2 x 6 = 10, 3 x 4 = 10, give simple answers, whereas the only one such in decimals is 2 x 5 = 10 (ten). Further comments, showing the elegance of the dozenal scale, are to be found in Colson News, Vol. 3, issue 2.

However, what we are looking for is not elegance, but practical usefulness. One may very well hope that the two will go hand in hand, but it does not necessarily follow. We have seen that in practice by far the greatest number of calculations not done on a calculator or computer are additions (or subtractions, which amount to much the same with two-way numbers). The next most common operation is multiplication. Other operations, such as finding factors of numbers, or turning fractions into radix fractions, may be useful on rare occasions, but they form a very small part of everyday life. The operations of division and square roots are unquestionably useful in many contexts, but by far the easiest way of performing them is to use a pocket calculator.

What is wanted, then, is some measure of the difficulty of calculation, by which we can compare the merits of the different bases. And, because addition and multiplica-

(Continued)

Do you have an idea to share with our members? Why not submit an article to the *Bulletin*?

tion are easily the two most common operations performed in practice, we will be particularly interested in how comparatively difficult they are. Now when we say that the multiplication 2 x 5 = 10 (base ten) is simple, the feature which would justify that is that there is only 1 nonzero digit, 1, in the product, whereas in a product like 3 x 5 = 15, there are 2 nonzero digits.

One possible measure of difficulty which springs to mind is the average number of nonzero digits in the entries in the addition and multiplication tables. For example, in the addition table for the scale of 2 there are 4 entries, with sums 0, 1, 1, 10 (=2), giving 3 nonzero digits, or an average of 3/4 = (decimally) .75 per entry. In the scale of 3 there are 9 (=1q1) entries in the table, and 8 nonzero digits giving an average number 8/9 = .89 (=1.q1q1) nonzero digits per entry. From that point of view, the binary scale is simpler than the ternary. As regards multiplication, the binary table gives 1 nonzero digit in 4 entries, or an average of .25, while the ternary scale has 4 out of 9, or 4/9 = .44. Again, binary does better than ternary. But 2 and 3 are too small to be practical scales. Suitable candidates for a good base of notation could be 6, 8, 10, 12 and 16. One would expect that 12, being the most elegant, would turn out to be the simplest scale. The actual values are shown in Table 4.1.1q4.1.

Table 4.1.1q4.1 Average no. of nonzero digits

Base	In addition table	In multiplication table
06 = 1q4	1.01	0.83 = 1.q23
08 = 1q2	1.07 = 1.1q3	1.02
10	1.10	1.18 = 1.2q2
12	1.13	1.20
16 = 2q4	1.16 = 1.2q4	1.40

These results are somewhat unexpected. As regards addition, 10 is a bit simpler than 12 or 16, though not quite as simple as 8. As regards multiplication, where one would have expected 12 to score most heavily, it actually comes out as slightly worse than 10. So from this particular point of view, 10 is not really such a bad choice, though 8 would have been better.

However, in this calculation we have neglected another important factor, namely the length of the numbers to be added. The larger the base, the shorter the number when written down, and hence the smaller amount of work to be done in adding or multiplying it. This is more important in multiplication, since in multiplying 2 numbers, say, x and y, the number of multiplications to be done is the product of the number of digits in x and the number of digits in y. For long numbers, containing many digits, this means that in base B the number of operations to be performed in addition is (log 10/ log B) times the number required with base 10. We will call this the "length factor". For multiplication, the length factor is the square of the length factor for addition. Thus, with base 12, we find that, in this sense, an addition only requires about 93 per cent as much work as in base 10, and a multiplication only 86 per cent as much work. The results are set out in Table 4.1.1q4.2.

(Continued)

Table 4.1.1q4.2 Length Factors

Base	For addition	For multiplication
06 = 1q4	1.29 = 1.3q1	1.65 = 2.q3q5
08 = 1.q2	1.11	1.23
10	1	1
12	0.93 = 1.q13	0.86 = 1.q14
16 = 2q4	0.83 = 1.q23	0.69 = 1.q3q1

These length factors are more than large enough to offset the difficulty factors in the previous table. So, on multiplying the two together, it seems that the larger the base (in our table) the smaller the difficulty, and 12 is, after all, better than 10.

It would seem that the best measure of the difficulty of calculation, if we are to measure it in terms of the expected number of nonzero digits would be the product of the factors in the two Tables, 4.1.1q4.1 and 4.1.1q4.2, as Table 4.1.1q4.3.

Table 4.1.1q4.3 Combined measure of difficulty

Base	For addition	For multiplication
06 = 1q4	1.30	1.37 = 1.4q3
08 = 1q2	1.19 = 1.2q1	1.25
10	1.10	1.18 = 1.2q2
12	1.05	1.04
16 = 2q4	0.96 = 1.0q4	0.96 = 1.0q4

The general conclusion suggested by this table is that the larger the base the easier the calculation, though there is no very dramatic difference between different bases among those in this table.

However, this is still not the end of the story. We have already remarked that there is also a psychological factor, which may vary from person to person. Prof. Aitken could calculate in the scale of 60. But that means that (even if had he used two-way numbers) he would have had to learn effectively 1860 (=2q1q40) entries in the addition table, and 435 entries in the multiplication table. Not many people would have the ability to do that. The larger the base, the larger the number of entries in the addition and multiplication tables to be learnt, and recalled when needed. The effort needed to do this will vary from person to person, so it is not easy to put forward a suitable measure of the difficulty that presents. In brief, though disappointingly, it doesn't seem possible to give a really objective measure of comparison of the difficulties of calculating in the different bases.

End

BASE 32 BACKWARDS!

Andrew Hodges
From ALAN TURING: The Enigma (Simon and Schuster).

By 1949, Alan Turing had lost interest in doing certain kinds of basic computer work...

"The 'fussy little detail' of binary to decimal conversion, for instance, he now found not worth bothering about. He himself found it simple to work directly in the base-32 arithmetic in which the machine could be regarded as working, and expected other people to do the same.

"To use base-32 arithmetic it was necessary to find 32 symbols for the 32 different 'digits'. Here he took over the system already used by the engineers, in which they labelled the five-bit combinations according to the Baudot teleprinter code. Thus the 'twenty-two'digit, corresponding to the sequence 10110 of binary digits, would be written as 'P', the letter that the sequence 10110 encoded for an ordinary teleprinter. To work in this system meant memorising the Baudot code and the multiplication table as expressed in it -- something he, but few others, found easy.

"The ostensible reason for sticking to this hideously primitive form of coding, which entailed so much work for the user, was that the cathode ray tube storage made it possible -- indeed necessary -- to check the contents of the store by 'peeping', as Alan called it, at a monitor tube. He insisted that what one saw as spots on the tube had to correspond digit by digit to the program that had been written out. To maintain this principle of correspondence it was actually necessary to write out the base-32 numbers backwards, with the least significant digit first. This was for technical electronic engineering reasons, the same as those which obliged cathode ray tubes always to scan from left to right. Another awkwardness arose on account of the five-bit combinations which did not correspond to a letter the the alphabet on the Baudot code. . . Geoff Tootill had already introduced extra symbols for these, the zero of the base-32 notation being represented by a stroke '/'. The result was that pages of programs were covered with strokes."

"Each one teach one."
-- Ralph Beard, Founder of the DSA

A PROPOSAL

Charles S. Bagley
Alamogordo, NM

[Editor's note: *Due to the untimely death of Ralph Beard many years ago, a note from our former Board Chair, Charles Bagley, was misplaced. The note included the following proposal. Our apologies to him for the long publication delay!*]

I suggest that the following numbering system is more rhythmic than DEK EL DO.

I use the ampersand (&) for ten and a modified greek rho (ρ) for eleven, and call ten, eleven and twelve deci, alif and tan respectively. Counting is as follows:

0	zero	10	tan			100 candred
1	one	11	monotan			
2	two	12	duotan	20	twenta	1,000 tan candred or
3	three	13	thirtan	30	thirta	one dozend
4	four	14	fortan	40	forta	
5	five	15	fiftan	50	fifta	10,000 tan dozend
6	six	16	sixtan	60	sixta	
7	seven	17	seventan	70	seventa	100,000 candred dozend
8	eight	18	eightan	80	eighta	
9	nine	19	ninetan	90	nineta	1,000,000 zillion
&	deci	1&	decitan	&0	decita	
ρ	alif	1ρ	aliftan	ρ0	alifta	1,000,000,000 one dozend zillion

The rule is: in the teens change 'een' to 'tan'. In the ty's change 'y' to 'a'. Use candred for a gross (hundred), dozend for a great gross (thousand), and zillion for a dozend^2 (million).

The ordinal numbers follow the decimal system in common use. For example: monotanth, duotanth, twentah, thirtafirst, thirtasecond, decitath, aliftath, candreth, dozendth, zillionth.

The year 119ρ (1991) would be one dozend one candred ninetalif.
104& is one dozend fortadec (with the c pronounced as s). A mile of 3080 (5280) feet is three dozend eighta feet. The $200,000,000,000 deficit would fall in the candred zillion class.

What do you think? We would like to hear your reactions to this proposed numbering system.

End

The next DSA Annual Meeting will be Saturday, October 19, 1991 at 1 P.M.

VARIATIONS ON 153_{DEK}

153_{DEK} (Number One)

Brian Dean
Ohio

There is a text from the desert fathers called 'On Prayer: One Hundred and Fifty-Three Texts' by St. Evagrios the Solitary.

Now $153_{DEK} = 109_{twelve}$, and 109; is an interesting number. It is a triangular number, i.e. it is a number such that it is a member of t(n) = 1+2+3+...+n, and in fact 109 = t(15).

It is also a hexagonal number which means that it is a member of h(n) = 1+5+9+11+...+n (109 = h(29)).

It is also the sum of 84 (which is a square number), 24 (which is triangular (24 = t(7)) and 21 (which in base * is 25). This means that 25. is a spherical/circular number -- one that is a square number whose square root gets reproduced as the last digit (in base *, 25 = 5 x 5, also 36 = 6 x 6) although in the dozenal system 21 would not be circular/ spherical.

* * *

153_{DEK} (Number Two)

S. Ferguson
Dozenal Society of Great Britain

While working on patterns involved with the "153" problem (where $1^3+3^3+5^3 = 153_{dek}$) I happened to note that for 7^3 in base dek there is a pattern

$$7^3 = 343, \text{ and } (3+4)^3 = 343.$$

Was this an isolated case? or could there be similar examples in base twelve or some other base?

Up till now I have the following:

$(1+2)^3 = 123$ in base 4

$(3+4)^3 = 343$ in base *

$(3+1)^4 = 314$ in base 9

and $(1+4)^4 = 144$ in base 1#

(Continued)

Since the unit digit of the number written in a given base is the same as the power of the bracketed expression, we are in fact looking at numbers which fit the congruence

$$x^n = n \text{ (mod base).}$$

I suspected (a) that the above four might well be the only solutions with three digits and (b) that there aren't any solutions with four or more digits, but further investigation led me to discover

$(9+0+8)^4 = 9084$ in base 19

and $(4+3+1+5)^4 = 43154$ in base 9

* * *

153_{DEK} (Number Three)

Gene Zirkel
New York

In response to Shaun Ferguson's contribution "153_{dek} - Number two," note that

$2^4 = 24_{six}$

$2^6 = 26_{25;}$

$2^8 = 28_{(*4;)}$

etc.

In fact,

$2^{2n} = 2(2^{2n-1} - n) + 2n$ is an identity as is

$k^{kn} = k(k^{kn-1} - n) + kn$
where the base $(k^{kn-1} - n) >$
the 2 digits k and kn.

This latter yields examples such as

$3^3 = 33_8$, $3^6 = 36_{(181;)}$ and $4^4 = 44_{(53;)}$

End

HISTORY PROFESSOR SPEAKS TO DSA MEMBERS

Gene Zirkel

On Tuesday, 12; May 119#, on very short notice, history Professor Jens Ulff-Müller from Copenhagen conducted a brief but interesting seminar at Nassau Community College on the history of counting and measuring. He was returning home to Denmark from the 26th Annual Congress on Medieval Studies held at Western Michigan University in Kalamazoo, where he spoke and also organized two sessions of a half dozen speakers. We were indeed fortunate to secure his presence as he spoke about a variety of topics relating to medieval counting and measurement.

It was very interesting to learn of the difficulties historians and linguists had in deciphering such phrases as 'a year had three hundred and five days'. The problem is not that our ancestors used shorter years, but rather they used longer 'hundreds'.

According to Professor Ulff-Müller, many people used a hundred containing dek dozen units. We refer to this today as the *long hundred*, and differentiate it from the *narrow hundred* of only eight dozen and four units. (You may recall that the long ton of 2400 - rather than 2000 - pounds is still in use today.)

In the middle ages, most numbers were written out, and algorithms for operations were not easy to come by, division being an especially vexing problem. This led people to desire that the number of partitions in a given unit of measurement be highly factorable and hence - the long hundred of dek dozen units. It divides evenly by 2, 3, 4, 5, 6, 8, *, 10, 13, 18, 20, 26, 34, 50, and *0.

Plato's perfect world may have led academicians to prefer the regularity of either dek times dek or do times do as the number of subdivisions of a measurement, but the common people did things for convenience and the result was the long hundred.

This preference for convenience over standardization may be the reason we had so many hybrid combinations of partitions of our units of measurement. (These were to be found on the back cover of our black and white notebooks in grammar school.) A preference for convenience may also explain the present resistance to being forced to adopt the awkward decimal metric system. People everywhere seem to demand halves and then thirds and/or quarters in their measurements in order to avoid fractions of units as much as possible.

(Continued)

We tend to think of things being codified and universal. However many measurements were regional and things were written differently in different localities. Thus we find Roman Numerals not quite as standard as we were taught in elementary school. For example:

> IV is not the only four - IIII was also used. Two hundred appears as II hundred as well as CC. But sometimes CC stands <u>for</u> two long hundreds! V^{XX} is found as denoting 5 times 20 and VI is used for six thousand.

The reader who is not aware of these variations would have difficulty in attempting to decipher the meaning of some passages in medieval texts.

To some extent, it appears that the popular culture preferred the long hundred while the narrow hundred prevailed in sacral use.

Professor Ulff-Müller is due to return to the States in November. We hope to be able to announce a date when he will speak to us again, and we trust that you will have the opportunity to hear him.

Don't be surprised if you read that

> 100 fish = 6 score fish,
> 3 (units) of fish = 100, and
> 1 (unit) = 40.

Just pity the poor historian who is trying to comprehend a text in which the symbol 100 has two different interpretations in the same document.

End

DOZENAL JOTTINGS

. . . from members and friends . . .News of Dozens and Dozenalists . . .

JOHN D. HANSEN, JR. (30#; Vista, CA) writes: "I find myself more optimistic about dozenal's chances of eventual adoption. Until recently I felt we had a better chance of getting mankind to grow twelve fingers than to adopt base twelve.. But when I think of all the improbable things that have happened on the world scene in the past three years, I begin to think that anything is possible." John also reports that his robotics professor has read our "Manual of the Dozen System" and is interested in other material from the Society . . .

Member **JAY L. SCHIFFMAN** (Jersey City State College, NJ) recently presented a paper "Congruences and Divisibility Tests" at the 49th Annual Meeting of the Metropolitan New York Section of the Mathematical Association of America . . .

An announcement about **PETER D. THOMAS'S** work in modular counting and arithmetic appeared in the March-April 1991 issue of the American Metric Journal. (Peter received the DSA Ralph Beard Memorial Award posthumously in 1990, having passed away early in that year.). . .

Member **PAUL SCHUMACHER** (Cherry Hill, NJ) recently had an article published in Topical Time: "The Presidential Election of 1856" . .

Bulletin Editor **DR. PATRICIA ZIRKEL** (St. John's University, NY) recently presented two papers on medieval topics: one at the Twelfth Medieval Forum, Plymouth State College (USNH), Plymouth, NH; and a second at the 26th International Congress on Medieval Studies, Western Michigan University, Kalamazoo, MI . . .

Thanks to **HENRY C. CHURCHMAN** (Council Bluffs, IA) for his recent gift to the DSA, which was forwarded to us by his son **JOHN P. CHURCHMAN** . . .

Thanks also to **KAY McKIERNAN** for her recent gift to the DSA in memory of her husband **DR. ELLIS R. VON ESCHEN**. Kay wrote to express appreciation for the tribute to Ellis which appeared in the last issue of the Bulletin. She said: "Ellis was very fond of the group and enjoyed the times he spent with us." Dr. Von Eschen had twice presented papers at DSA Annual Meetings . . .

New member **MONTE J. ZERGER** (Adams State College, Alamosa, CO) writes:

> I notice that the symbols for dek and el are apparently from two of the twelve push buttons on the telephone.
>
> (1) To obtain Directory Assistance for numbers outside their area code, callers must dial 1 + area code + 555-1212. Curiously, the number 5551212 ends with a repetition of 12, and is divisible by 12.

(Continued)

> (2) How appropriate then that BELL ends in a repetition of the 12th letter of the alphabet. And what about HELLO's successive L's?
>
> (3) Counting from either side of the alphabet to the 12th letter we find L and O. These letters exactly balance in TELEPHONE as well.

```
ABCDEFGHI JK  L   M  N O PQRSTUVWXYZ
          TE  L   EP H O NE
```

. . .

BRUCE MOON, DSA Fellow, writes from Diamond Harbour, NZ:

> Thank you for my copy of Bulletin 67; which I enjoyed reading. I am glad to know that the Society is going from strength to strength. I noticed too the paragraph apologising for your rather amusing error in confusing New Zealand and Australia -- apologies accepted of course.
>
> I have come to the conclusion that you folk down under are really rather droll, for you go on to say that **JAMISON** and **VERA HANDY** visited us this past summer. Now that really is quite comical because their visit was in the depths of winter. While her son John went skiing, Jamie, Vera and John's friend Corinne drove with me across the South Island from Diamond Harbour (east coast) to Greymouth (west coast). During the trip across the Southern Alps we had magnificent views(...)
>
> Dating of Bulletin 67; as "Winter 1991" is also rather quaint since we haven't had winter yet this year! . . .

Welcome to New Members:

310;	**JOANNE ALICE YOUNG**	Crosslanes, WV
311;	**NADER RAFATY MALAKY**	University of Tabriz Malekan, IRAN
312;	**ROBERT S. HARRIS**	Knoxville, TN

> Bob writes: "If it's good enough for eggs, it's good enough for me!" (See "Eggsactly a Dozen," James Malone; Whole Number 42; (1981), p. #). He is also interested in sexagesimal counting and would like to hear from other aficionados. . .

313;	**MONTE JAMES ZERGER**	Adams State College Alamosa, CO

(Continued)

SOLUTION TO "A CRYPTARITHM"

(Winter 1991 -- p. 1#)

The puzzle:

Replace each letter by a different digit to make this sum correct. Hint: the digit * is not used.

AAA
BBB
+CCC
DEFG

According to Charles Ashbacher, Hiawatha, IA, there are 6 equivalent solutions. The values of A, B and C can be assigned any permutation of the numbers 6, 8 and 9, leading to a sum of 210# in all cases.

End

WHY CHANGE, INDEED?

"At this juncture, there is not one nation on record that uses the SI exclusively. Some countries nationalize metrics, others use terms and units outside the system and or old traditional European metric."
-from the *American Metric Journal*
volume XIX, page 3, UNIT 3, May/June 1991
(emphasis added)

Whenever someone raises the issue of changing to the metric system (SI), I ask them, "Which metric system?'.
-GZ

Dozenal Jottings *(Concluded)*

314; JOHN BARTON — Port Perry High School, Port Perry, Ontario, CANADA

315; DALE S. MILNE — Boulder, CO

Dale writes that one of the issues that spurred his interest in duodecimals was a study of (American) Indian languages. "A few had base 4, 5, 8, and many the mixed 5-20 system." [We would like to hear more about this! -ED.]

316; GEORGE PETER JELLISS — St.Leonard's on Sea, East Sussex, ENGLAND

George is also interested in six-based systems.

End

A BASE TWELVE CROSSNUMBER PUZZLE

Charles Ashbacher

(Each entry is a non-negative digit.)

	1	2	3
1			
2			
3			

ACROSS:

1. A perfect square and a perfect cube.
2. The three most widely used bases are divided by two.
3. Palindromic in two common bases.

DOWN:

1. A permutation of the digits form an arithmetic sequence.
2. The dozenal sum of these digits occurs as part of another solution in this puzzle.
3. Easily seen to be divisible by 3 in base ten and base twelve.

SOLUTION IN NEXT ISSUE!

"In numerology, the number 12 has always represented completeness, as in the 12 months of the year, the 12 signs of the zodiac, the 12 hours of the day, the 12 gods of Olympus, the 12 labors of Hercules, the 12 tribes of Israel, the 12 Apostles of Jesus, the 12 days of Christmas and so on. Since 13 exceeds 12 by only one, the number lies just beyond completeness and, hence, is restless to the point of being evil."

from the Smithsonian, 2/87

DUODECIMAL PRIME PALINDROMES

Charles W. Trigg

An integer is palindromic if it is the same when read forward or backward. That is, a palindrome is identical with its reverse. Every palindrome with an even number of digits is divisible by 11. Thus 11 is the only prime palindrome with an even number of digits.

Of the 110 three-digit palindromes, 18 are prime. They are:

111	181	565	727	797
131	1#1	575	737	#1#
141	535	585	747	#2#
171	545	5#5	767	#6#

Included above are four pairs and two trios of consecutive palindromes. Note that 565 and 575 are consecutive primes, while 727, 737, 747 and 767 are alternate primes.

_______*End*

HEADLINES AND EXCERPTS

from a recent issue of the American Metric Journal (Volume XVIII, Jan/Feb 1990)

'No case has been made for metric' 1990

Britain Still Dragging Its Feet Over Metric System
(Feet? - Ed.)

Conversion would hurt the U.S. economy (from the Government Accounting Office Report to the Congress)

There is no need to convert to the metric system (from the GAO Report)

European countries do NOT all use the same metric. They even mix in the inch/pound system and always have.
(Emphasis added - Ed.)
COSTS IN THE BILLIONS
Conversion would be enormously expensive

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3#2*	Two ft. eight in.	*2;8'*
19#	*1000*	Eleven ft. seven in.	*#;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this *"which is"* step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2#*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
12 ) 30  + 5
12 ) 2   + 6
     0   + 2   Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *.

For more detailed information see *Manual of the Dozen System* ($1;00).

DETACH HERE -- OR PHOTOCOPY

We extend an invitation to membership in our society.
dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______________________________
LAST FIRST MIDDLE

Mailing Address (for DSA items) ______________________________

____
(See below for alternate address)

Telephone: Home ______________ Business ______________

Date & Place of Birth ______________________________

College ______________ Degrees ______________

Business or Profession ______________________________

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______________________________

Address ______________________________

Year & Math Class ______________________________

Instructor ______________ Dept. ______________

Other Society Memberships ______________________________

____

Alternate Address (indicate whether home, office, school, other)

____

____

____

Signed ______________ Date ______________

My interest in duodecimals arose from ______________________________

____

____

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530