

# THE DUODECIMAL BULLETIN 67;



*President Fred Newhall and his wife Mary enjoyed the evening's entertainment following the 1990 DSA Annual Meeting. See page 4;*



DOZENAL SOCIETY OF AMERICA  
c/o Math Department  
Nassau Community College  
Garden City, LI, NY 11530



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# THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are \$12.00 (US) for one calendar year. Student Membership is \$3.00 per year, and a Life Membership is \$144.00 (US).

*The Duodecimal Bulletin* is an official publication of the DOZENAL SOCIETY OF AMERICA, INC., c/o Math Department, Nassau Community College, Garden City, LI, NY 11530.

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# THE DUODECIMAL BULLETIN

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FOUNDED

1944

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## DOZENAL SOCIETY OF AMERICA

## MINUTES OF THE ANNUAL MEETING — 119\*;

Saturday, October 13, 1990  
Nassau Community College  
Garden City, LI, NY 11530

## I BOARD OF DIRECTORS MEETING

1. James Malone, Board Chair, opened the meeting at 2:40 p.m.

The following Board Members were present:

Alice Berridge	Fred Newhall
Anthony Catania	Gene Zirkel
Jamison Handy, Jr.	Dr. Patricia Zirkel
James Malone	

2. The Nominating Committee (J. Malone, L. Aufiero, A. Catania) proposed this slate of DSA Officers:

Board Chair	Gene Zirkel
President	Fred Newhall
Vice President	Alice Berridge
Secretary	Larry Aufiero
Treasurer	Anthony Catania

3. The slate was elected unanimously.
4. Appointments were made to the following DSA Committees:

Annual Meeting Committee:

Barbran Smith, Chair  
Larry Aufiero  
Anthony Catania  
Anthony Razziano

Awards Committee:

Gene Zirkel, Chair  
Dr. John Impagliazzo  
James Malone  
Dr. Angelo Scordato  
Dr. Patricia Zirkel

Finance Committee:

Dr. Angelo Scordato, Chair  
Larry Aufiero  
Anthony Catania  
Dudley George  
James Malone  
Anthony Razziano  
Dr. Patricia Zirkel

Volunteers to these Committees are welcome at any time.

5. The following other appointments were made:

Editor of The Duodecimal Bulletin:

Dr. Patricia Zirkel

Parliamentarian:

Dr. Patricia Zirkel

Reviewers of articles for the Bulletin:

Anthony Catania  
Jamison Handy, Jr.  
Dr. John Impagliazzo  
Kathleen McKiernan  
Fred Newhall  
Barbran Smith  
Gene Zirkel

6. Other Business of the Board:

The Society regrets to announce the recent death of Ellis R. Von Eschen, husband of long-time member Kathleen McKiernan. (See article, this issue.)

The Board Meeting was adjourned at 2:55 p.m.

(Continued)

Have you written a letter to some local newspaper or to a newsletter extolling the advantages of Base Twelve? Remember to mention that free literature is available from the Society.



## II ANNUAL MEMBERSHIP MEETING

1. DSA President Fred Newhall gavelled the meeting to order at approximately 3:00 p.m.

In addition to the Board members listed above, the following Society members were present:

Vera Sharp Handy  
Mary Newhall  
Jay Schiffman

2. Gene Zirkel moved to accept the minutes of October 14, 1989. So approved.
3. President's Report - Fred Newhall

Fred said that there is ample evidence that interest in dozens is growing and that awareness of the DSA is spreading.

Fred also spoke about the ample correspondence of the Society during the past year -- there have been many inquiries resulting from publicity notices, and we have also received more than eight dozen pieces of correspondence from school children. Whenever someone requests information, they receive a packet of materials, followed in a week or two by the current Bulletin.

4. Treasurer's Report - Anthony Catania

Tony submitted copies of the 10/15/88, the 10/14/89 and the current 10/13/90 reports. Total assets are down from 1989, but up from 1988. The increase in 1989 was due to a large influx of dues from new life members. He mentioned that there is a decline in total dues, that AMJ had been inadvertently paid twice, and that a new line item for binding should be created.

He also expressed concern about the decrease in value of the Certificate of Deposit. Since our expenses are paid in large part by interest from this CD, the decline in principal is serious. He pointed out that the dividends from stocks remain stable in spite of the drop in stock value.

Members questioned whether costs should be reduced, dues increased, or the Certificate be further decreased; whether stocks should be sold or converted to CD's; and whether the number of issues of the Bulletin should be reduced. It was agreed that the number of issues of the Bulletin should not be reduced, since this would be contrary to the fundamental goals of the Society.

Pat suggested that the Finance Committee should meet to discuss the following:

(Continued)

1. Since the shares held by the DSA pay a lower dividend than CD interest, should all or part of the stocks be sold and the funds deposited in a CD?
2. Would raising the dues be appropriate and by how much?
3. The publishing of a budget by the Finance Committee for the Society.

After discussion, Fred moved that dues be raised to \$15. Jay suggested that the motion be amended to ask for a voluntary contribution of \$3. Motion withdrawn.

Jamison moved that the Society express a need for more funds, saying that it should be explained to members that it had been considered that dues should be raised to \$15, but that it was decided to continue our tradition of \$12 dues. He suggested that members be asked for a voluntary contribution of \$3 or more. Motion approved.

Tony agreed to attach a list of paid members to his next Treasurer's Report. Tony was thanked for his excellent report.

5. Reports of other Officers and Individuals

Vice-President's Report - Gene Zirkel

Gene drew attention to Fred's Dozenal watch, which Fred had obtained from Image Watch Company. Its face is in the style of the DSA logo.

Miscellaneous comments: The DSA has been invited by Prairie View A&M College (Texas) to the inauguration of their fifth president. Shaun Ferguson (DSGB) informed us that Scientific American and New Scientist have both published evidence that the world is based on twelve elements. Gene mentioned that in a current math text used in colleges, our terms "dek" and "el" are used as part of a number base discussion. Gene suggested that an adjustment to our publicity flyer/brochure may be in order. It was also suggested that the ongoing article "Why Change?" (in the Bulletin) be promoted more extensively.

Editor's Report - Dr. Patricia Zirkel

Fred praised and thanked Pat for her continuing work as Editor.

Pat said that there have been three issues published this year, that printing costs are up slightly, and that there is a big backlog of longer articles. Brief articles (i.e., up to 5 typed, double-spaced pages), articles aimed at teachers, and elementary articles are needed. Articles about number bases in general and about computers would be very welcome.

(Continued)



Pat also announced that, dependent on the status of her new faculty position at St. John's University, she may not be able to continue as Editor after May 1991. Pat and Gene then outlined the nature of the job in the hope of encouraging another person to step forward as Editor. It was also suggested that faculty at Nassau Community College with expertise in this area should be contacted.

#### 6. Committee Reports

##### Annual Meeting Committee - Dr. Barbran Smith

Barbran explained that this meeting marks the first time we have met for just one day.

The Board had convened in March for a very successful meeting, and it was agreed that the Board again meet in March, 1991, at the Hofstra Club. The next DSA Annual Meeting will be Saturday, October 19, 1991. Since members present felt that the assigned time at the current meeting was a bit brief, it was decided that the 1991 meeting would begin one hour earlier, at 1 p.m.

Barbran and the Committee were thanked for their efforts.

##### Awards Committee - Gene Zirkel

Gene announced that the Ralph Beard Memorial Award (formerly the DSA Annual Award) had been presented to Peter D. Thomas of Australia for his work on modular counting and measurement. Mr. Thomas has since passed away.

Jamison moved that, at the request of Board member Robert McPherson of Florida, Honorary Membership be granted posthumously to Parry Moon for his work in dozens. So approved.

#### 7. Nominating Committee - James Malone

The following slate was proposed for the Board of Directors, Class of 1993:

Dudley George  
Jamison Handy, Jr.  
Fred Newhall  
Dr. Barbran Smith

The slate was elected unanimously.

(Continued)

The following members were elected to the Nominating Committee:

Larry Aufiero  
James Malone  
Jay Schiffman

#### 8. New Business

Tony Catania (Treasurer) discussed a change in our method of disbursing funds.

The meeting was adjourned at 4:00 p.m.

Respectfully submitted,

Alice Berridge  
(for Larry Aufiero,  
Secretary)

### III FEATURED SPEAKER

Jay Schiffman was the featured speaker for the latter part of the afternoon. Jay's topic was "A Survey of Modular Systems with Applications to Checking Arithmetic Computations," and related (mod 10;) to clock and calendar problems. A four-hour clock was used to work out the multiplication with (mod 4), which Jay related both to the four seasons of the year, and to coterminal angles. The presentation concluded with interesting examples for casting out #'s and 11; using modular arithmetic.

Jay also pointed out that this subject would fit neatly into the curriculum of two-year colleges. Members agreed that Jay's presentation was very interesting and stimulating.



Jay Schiffman at the 1990 DSA Annual Meeting

(Continued)



IV EVENING ENTERTAINMENT

DSA members were joined by spouses and friends for supper and an evening of entertainment at a local Comedy Club.



Alice Berridge, shown with her husband Edmund, after being presented with the coveted red lobster award for creative theatre ticket buying.

End

Use decimal metric? Oh well,

It has one advantage to sell.

The fractional point moves;

But it really behooves

Us to count with both \* and with #.

A SURVEY OF FRACTIONS AND DUODECIMALS IN THE BASE TWELVE SYSTEM OF NUMERATION

Jay Schiffman  
 Jersey City State College  
 Jersey City, NJ

In a previous article of mine dealing with Fundamental Operations in the Duodecimal System (*The Duodecimal Bulletin* 5#; Volume 31; Number 3; Fall 1988), a treatment of operations with whole numbers was presented. The purpose of this article is to survey how one performs computations with fractions in base twelve. A number of examples will be furnished to promote understanding.

To initiate our discussion, let us recall the decimal system. One has in succession immediately to the right of the fraction point the place values tenths, hundredths, thousandths, ten thousandths, etc, or  $10^{-1}$ ,  $10^{-2}$ ,  $10^{-3}$ ,  $10^{-4}$ , ... where  $10^{-n} = 1/10^n$  for any counting integer n.

Thus 0.49573 represents in expanded notation  $(4 \times 10^{-1}) + (9 \times 10^{-2}) + (5 \times 10^{-3}) + (7 \times 10^{-4}) + (3 \times 10^{-5})$ , or equivalently,  $4/10 + 9/100 + 5/1,000 + 7/10,000 + 3/100,000 = 49,573/100,000$ .

In a similar manner duodecimal fractions have the successive place values immediately to the right of the fraction point corresponding to twelfths, one hundred forty-fourths, one thousand seven hundred twenty-eighths, etc., or  $12^{-1}$ ,  $12^{-2}$ ,  $12^{-3}$ , ... where  $12^{-n} = 1/12^n$  for some counting integer n. (Note that we have been employing decimal notation here.)

To illustrate,  $0; *16_{\text{TWELVE}}$  represents in expanded notation  $(* \times 10;^{-1}) + (1 \times 10;^{-2}) + (6 \times 10;^{-3})$ , or in equivalent decimal notation,

$$10/12 + 1/144 + 6/1,728 = 1,458/1728 = \frac{1,458 + 54}{1,728 + 54}$$

$$= \frac{27}{32} \text{ in simplest form.}$$

We next demonstrate how one converts rational numbers in base ten to base twelve fractionals. In order to obtain the base twelve expansion of a fraction such as  $27/32_{\text{TEN}}$  we can proceed as follows, multiplying the fraction by twelve, and expressing the result as a mixed number. Thus  $12(27/32) = 10 \frac{1}{8}$ , with the computations performed in base ten.

(Continued)



We refer to the integer portion of the result as the product and the fractional portion as the remainder. Repeat the above process using the remainder obtained in the previous step. The products thus obtained are the digits of the base twelve expansion. Thus with all the computations performed in the decimal system,

$$\begin{aligned} 12(27/32) &= 8 \frac{1}{8} = 10 \frac{1}{8}, & \text{Product} &= 10, & \text{Remainder} &= 1/8. \\ 12(1/8) &= 12/8 = 1 \frac{1}{2}, & \text{Product} &= 1, & \text{Remainder} &= 1/2. \\ 12(1/2) &= 6, & \text{Product} &= 6, & \text{Remainder} &= 0. \end{aligned}$$

Inasmuch as the remainder is zero, the process terminates and our solution is complete.

Hence  $27/32_{\text{TEN}} = 0; *16$ , as we obtained in the preceding paragraph.

A nice shortcut to the above procedure can be performed in the following manner.

$27/32_{\text{TEN}}$  can be expressed in base ten as 0.84375.

$$\begin{array}{r} 0.84375 \\ 32 \overline{) 27.00000} \\ \underline{256} \\ 140 \\ \underline{128} \\ 120 \\ \underline{96} \\ 240 \\ \underline{224} \\ 160 \\ \underline{160} \end{array}$$

Repeated multiplication of the fractional part by twelve yields the whole numbers ten, 1, and 6.

	.84375
	x 12
10	.125
	x 12
1	.5
	x 12
6	.0

Thus  $27/32_{\text{TEN}} = 0.87345_{\text{TEN}} = 0; *16_{\text{TWELVE}}$

(Continued)

As a second example, we express the base ten fraction  $3/5$  in base twelve with the computations performed in the decimal system.

$$\begin{aligned} 12(3/5) &= 36/5 = 7 \frac{1}{5}, & \text{Product} &= 7, & \text{Remainder} &= 1/5. \\ 12(1/5) &= 12/5 = 2 \frac{2}{5}, & \text{Product} &= 2, & \text{Remainder} &= 2/5. \\ 12(2/5) &= 24/5 = 4 \frac{4}{5}, & \text{Product} &= 4, & \text{Remainder} &= 4/5. \\ 12(4/5) &= 48/5 = 9 \frac{3}{5}, & \text{Product} &= 9, & \text{Remainder} &= 3/5, \end{aligned}$$

and  $3/5$  is the number we started with.

If we continue in the above manner, the digits 7, 2, 4 and 9 are repeated infinitely in this pattern. Thus

$$3/5_{\text{TEN}} = 0; \overline{72497249} \dots = 0; \overline{7249}$$

so that  $3/5$  is an infinitely repeating decimal in base twelve.

If one employs the shortcut given above, then the following is obtained:

$$3/5 = 0.6.$$

	.6	
	x 12	
7	.2	
	x 12	
2	.4	
	x 12	
4	.8	
	x 12	
9	.6	
	x 12	
7	.2	repeating the cycle 7249 7249 etc.

Hence  $3/5 = 0.6 = 0; \overline{7249}$ .

As a final example, consider the infinitely repeating duodecimal fraction  $0;333\dots$

$$\text{Now } 0;333\dots = \frac{3}{12} + \frac{3}{12^2} + \frac{3}{12^3} + \dots = \frac{3}{12} + \frac{3}{144} + \frac{3}{1728} + \dots$$

(Written in base ten.)

(Continued)



This is a geometric progression. (A geometric progression is a progression in which each term after the first is multiplied by the same constant term to obtain the next term. Such a constant term is called the common ratio.) It is known that the sum of an infinite geometric progression is given by the formula

$$S = \frac{a}{1 - r}$$

where S represents the sum; a, denotes the first term and r,  $|r| < 1$ , connotes the common ratio.

In our example  $a = 3/12$  and  $r = 1/12$ .

$$\text{Thus } S = \frac{3/12}{1 - 1/12} = \frac{3/12}{11/12} = 3/11_{\text{TEN}} = 3/\#_{\text{TWELVE}}$$

For additional information on geometric progressions, please see [1] in the bibliography at the end of this article.

We conclude our discussion by illustrating four fundamental operations of arithmetic with fractions in base twelve.

Example 1: Add:

$$\begin{array}{r} ;235 \\ ;180 \\ +1;4*\# \\ \hline 1;8*4 \end{array}$$

Observe that  $5 + \# = 1$  dozen and 4. Put the 4 in the right-most column and carry the 1 over to the preceding column.  $(1+3+8+*) = 1$  dozen and \*. Bring down the \* and carry over the 1 to the preceding column.  $(1+2+1+4) = 8$ . Record the 8 and bring down the 1 in the left-most column. Line up the fractional point.

To check, we could express each addend in base ten, add in base ten and convert the sum to base twelve.

$$;235 = (2 \times 10;^{-1}) + (3 \times 10;^{-2}) + (5 \times 10;^{-3}) =$$

$$\frac{2}{12} + \frac{3}{144} + \frac{5}{1,728} = \frac{329}{1,728}$$

$$;180 = (1 \times 10;^{-1}) + (8 \times 10;^{-2}) =$$

$$\frac{1}{12} + \frac{8}{144} = \frac{240}{1,728}$$

(Continued)

$$1;4*\# = 1 + (4 \times 10;^{-1}) + (* \times 10;^{-2}) + (\# \times 10;^{-3}) =$$

$$1 + \frac{4}{12} + \frac{10}{144} + \frac{11}{1,728} = \frac{2,435}{1,728} \quad \text{AND}$$

$$\frac{329}{1,728} + \frac{240}{1,728} + \frac{2,435}{1,728} = \frac{3,004}{1,728} = 1 \frac{1,276}{1,728} \quad \text{AND}$$

$$1;8*4 = 1 + \frac{8}{12} + \frac{10}{12^2} + \frac{4}{12^3} = 1 + \frac{1,276}{1,728}$$

Example 2: Subtract:

$$\begin{array}{r} 3;475 \\ -2;369 \\ \hline 1;108 \end{array}$$

Since 9 is greater than 5, we must borrow one dozen from the preceding column. This yields a base ten sum of  $12 + 5 = 17$ . Now subtracting 9 from 17 gives a difference of 8. Complete the problem in the standard manner. The 7 in the preceding column becomes a 6.  $6-6=0$ ,  $4-3=1$ , and  $3-2=1$ . Line up the fractional point. One can check the problem by addition.

Example 3: Multiply:

$$\begin{array}{r} 12;9 \\ \times 4;68 \\ \hline 9*0 \\ 746 \\ \hline 4\#0 \\ \hline 57;240 \end{array}$$

The multiplication algorithm you learned in base ten can be used in duodecimal multiplication. Multiplying:  $9 \times 8 = 6$  dozen and zero. Record the 0 in the right-most column and carry the 6 over to the preceding column.  $[(2 \times 8) + 6] = 1$  dozen and \*. Record the \* and carry the 1 over to the preceding column.  $[(1 \times 8) + 1] = 9$ . Thus we have our first partial product from  $129 \times 8$ :

$$\begin{array}{r} 129 \\ \times 8 \\ \hline 9*0 \end{array}$$

(Continued)



Next  $(6 \times 9) = 4$  dozen and 6. Record the 6 and carry the 4 in the preceding column.  
 $[(6 \times 2) + 4] = 1$  dozen and 4. Record the 4 and carry the 1 in the preceding column.  
 $[(6 \times 1) + 1] = 7$ . We now have the following accomplished:

$$\begin{array}{r} 129 \\ \times \quad 68 \\ \hline 9*0 \\ 746 \\ \hline \end{array}$$

Finally  $(4 \times 9) = 3$  dozen and 0. Record the 0 and carry the 3 to the next column. Now  $[(4 \times 2) + 3] = \#$ . Now to complete:  $(4 \times 1) = 4$ .

We thus have

$$\begin{array}{r} 12;9 \\ \times \quad 4;68 \\ \hline 9*0 \\ 7 \ 46 \\ \hline 4\# \ 0 \\ \hline 57;240 \end{array}$$

Adding, we record the 0. Also  $(*+6) = 1$  dozen and 4. Record the 4 and carry the 1 over to the preceding column.  $(1+9+4) = 1$  dozen and 2. Record the 2 and carry the 1 over to the preceding column.  $(1+7+\#) = 1$  dozen and 7. Record the 7 and carry the 1 over to the preceding column.  $(1+4) = 5$ . Finally count the number of fractional places. There are three. Hence our answer is 57;240 in base twelve.

Example 4:  $0;7 \overline{)0;235}$

Using base twelve multiplication (forgetting the fraction points for the moment), the largest multiple of the divisor 7 in base twelve is 3; for  $(7 \times 3)_{\text{TEN}} = 1$  dozen and 9.

$$\begin{array}{r} 3 \\ 7 \overline{) 235} \\ \hline 19 \\ \hline 65 \end{array}$$

Now 7 goes into 65 exactly # times.

(Continued)

Hence

$$\begin{array}{r} 3\# \\ 7 \overline{) 235} \\ \hline 19 \\ \hline 65 \\ \hline 65 \\ \hline \end{array}$$

Now use the fact that to make the divisor a whole number, we must move the fraction point one place to the right. We do the same with the dividend. (Note that we are multiplying the divisor and dividend by twelve.)

$$0;7 \overline{)0;2*35} = 7 \overline{)2;3*5}$$

$$\begin{array}{r} 3\# \\ 7 \overline{) 2;3*5} \\ \hline 19 \\ \hline 65 \\ \hline 65 \\ \hline \end{array}$$

One can check by multiplication.

#### REFERENCES

1. I.A. Barnett, Elements of Number Theory, Prindle, Weber, and Schmidt, Boston, MA, 1969.
2. J.E. Maxfield and M.W. Maxfield, Discovering Number Theory, W.B. Saunders Company, Philadelphia, PA, 1972.

Acknowledgment: The author would like to thank Professor Gene Zirkel and the reviewers for their useful suggestions and comments.

\_\_\_\_\_End

### You Can Count On Them

Gene Zirkel

Instead of counting on your ten fingers, try using the dozen phalanges (digital bones) on the four fingers of your right hand. Using your right thumb as a pointer, and starting at the top of your pinky you can easily count to a dozen on one hand.

If you use your left hand in a similar manner to keep track of the dozens, you can count up to one gross on your fingers. Thus if your left thumb is on the top phalanx of your index finger and your right thumb indicates the middle phalanx of your ring finger you are pointing to dek dozen and five.



## WHY I JOINED THE DOZENAL SOCIETY

*Brian Dean  
Kent, OH*

Recently I read an article in a number theory book about the Dozenal Society and the idea of base twelve being better than base ten. I was very surprised when I saw the article because I had had the same idea myself since high school (six or seven years ago). I didn't read the article closely enough to see why you people think so, but here's what I came up with.

My theory is that a number base is easy if the proportion of the number of divisors matches in high proportion to the base. So base twelve would be pretty good because you have six divisors (1,2,3,4,6,12) and  $6/12 = .5$ . Whereas in base ten you have four divisors and  $4/10 = .4$  which is less than .5 for base twelve.

I can't formally prove this, but intuitively, if a number is a divisor of the base or a multiple of a divisor of the base, then it follows that if you divide something by that number, you will end up with a terminating fraction. Otherwise you won't. Also in the multiplication tables the numbers that are divisors or multiples of divisors will have more easily recognizable patterns than numbers that aren't so.

Also, dividing the number of divisors by the base makes sense because if you took base sixty, it has twelve divisors (1,2,3,4,5,6,10,12,15,20,30,60) and  $12/60 = .2$ . This indicates that there are a lot more numbers that don't divide evenly into 60 than ones that do.

Of all of the bases that could possibly exist, base twelve is the best if you use my rule. I am convinced (and was convinced even long before I ever saw the article) that base twelve would be a better base to work with for multiplying, dividing, and therefore probably better for other things as well.

If any of you have some information that I don't have, or if any of you have any formal proofs of anything I stated in this letter, please feel free to contact me at

P.O. Box 3241  
Kent, OH 44240

I can at the very least lend moral support to your cause.

(As an afterthought — is "do" the best name for twelve that we can come up with? Does anyone have a better suggestion?)

## IN MEMORIAM —

### DR. ELLIS R. VON ESCHEN

The Society regrets to announce the death of Dr. Ellis R. Von Eschen, husband of long time DSA member Kathleen McKiernan, on August 25, 1990. At the time of his death, Dr. Von Eschen was a Professor of Mathematics and Computer Science at Suffolk County Community College on Long Island in New York.

Dr. Von Eschen received his Mathematics Degrees from Willamette University, OR; Stanford University, CA; The Courant Institute, and New York University. He also did graduate study at the Universities of Göttingen and Berlin, Germany. He was also recently awarded a Master's Degree in Computer Science from the University of Oregon.

Twice Dr. Von Eschen spoke at DSA Annual Meetings. In 1986 his presentation dealt with methods of progressing from an additive to a multiplicative magic square, and with patterns in multiplicative magic squares. In 1987 he spoke on divisibility tests — decimal, terminal and recursive.



*Dr. Ellis R. Von Eschen is shown as he took questions following his presentation at the 1986 DSA Annual Meeting.*



## DOZENAL JOTTINGS

... from members and friends ... News of Dozens and Dozenalists ...

JAY SCHIFFMAN is now teaching at Camden County College Extension Center (NJ) where he has proposed a mini course in other numeration systems. Good luck, Jay! ...

Thanks to DICK TRELFA the Society now has Membership Cards. Yours will be in the mail in response to your 1991 dues payment ...

H.K. BAUMEISTER (SC) sent us some material from the memoirs of General W.T. Sherman, which says in part: "A cavalry regiment is now composed of twelve companies, usually divided into six squadrons, of two companies each, or better subdivided into three battalions of four companies each. This is an excellent form, easily admitting of subdivision as well as union into larger masses." The Baumeisters recently visited the site of an old family home "...the Kilpatrick plantation belonging to my great great grandfather which was burned the night after General Kilpatrick, General Sherman's cavalry officer, stopped in for dinner on the march ..."

We received the following in response to CEDRIC SMITH's last entry in these pages:

"Dear Jean and Pat:

Eye just had two right too you, and say how much eye in joyed the article on homonyms. Recently, I modified a program dealing with them.

Even you may be sir prized at the number in hour language. Their could even bee a miss steak hear, in my let her.

Eye no four shore, there is an era sum wear hear, butt I'm knot shore wear. Keep up the good work!" JOHN RYAN (NCC) ...

BRUCE MOON wrote to correct our spelling of his home in Diamond Harbour and to remind us that he resides in New Zealand, which is as far removed from Australia (our mistake) as the distance from Vancouver, BC to Tijuana, Mexico! With all apologies, we stand corrected. He was visited this past summer by JAMISON HANDY and his wife VERA ...

JOHN CHURCHMAN (Council Bluffs, IA) wrote to send best wishes to all in 1991. He also reports that his father HENRY CHURCHMAN, past Editor of this Bulletin, remains in reasonably good health ...

DSA President FRED NEWHALL (Smithtown, NY) spoke at his local library on January 30th about all the materials he has written — many of which, of course, are dozenal ...

Several communications have been received from DSGB members:

SHAUN FERGUSON hopes to be able to attend our Annual Meeting at some time in the future. He also wants to know whether we have "managed to persuade all current dictionaries to accept the word 'dozenal' yet? (More work for the PROs of the Societies...)" ...

Thanks to ARTHUR WHILLOCK we now have a copy of the original "Methods of Conversion" (decimal to dozenal and vice versa) which originally accompanied the fourth in a series of tests administered to applicants for DSA membership. Arthur also requested a new supply of copies of the "Manual of the Dozen System" since "Those you sent me before have all gone to good homes." Good work, Arthur! ...

Quite a while ago (Bulletin 63; p. 21;) DAVID FAIRCHILD asked several questions pertaining to a dozenal slide rule. DSGB member DON HAMMOND wrote to say that there "is a fold-out copy of all the scales of Tom Pendlebury's Dozenal Slide Rule at the back of each TGM booklet, from which a rule may be assembled." (Copies of TGM are available from the DSA.) ...

Welcome to new members:

309; RANDALL JAMES ROGES  
30\*; SR. MONICA ANN ZORE

A student member from Iowa City, IA  
A teacher from Marian College  
in Indianapolis, IN

30#; JOHN D. HANSEN, JR.

A computer programmer from Vista, CA

John heard about the DSA from his third year algebra teacher at Notre Dame High School in Niles, IL. "This was in 1963 - 64. I tracked you down with the help of a library reference that listed academic, professional, and scholarly organizations." John expressed an interest in metrology. "My interest in this subject comes from a friendship with someone who designed a coherent (much better than metric!) measurement system for base ten. (Of course, his system suffers from the clumsy base.) John also wrote to remind DSA members of some approaching anniversaries:

1992 (= 11\*0;) 160; years since the start of the American Revolution. (1776 = 1040;)

2009 (= 11#5;) 100; years since the end of the Civil War. (1865 = 10#5;)

Also: October 30, 2044 = \*/26/1224; which is day 300,000; of the Common Era (AD).

Four years later is binary year 2K, or 1000 0000<sub>two</sub>. One year after that will be the binary bi-millennium.

About ten years after day 300,000; will be day 750,000<sub>ten</sub>, on June 4, 2054.

End



## DIGIT REVERSAL SOLUTION

Charles Ashbacher

On page 11 of volume 33; number 3;, you have a digit reversal puzzle.

Working through possibilities, up to base 100, I found the solutions:

- 14 divides 41 in base 11
- 15 divides 51 in bases 7 and 19
- 16 divides 61 in base 29
- 17 divides 71 in bases 9, 17 and 41
- 18 divides 81 in bases 13 and 55
- 19 divides 91 in bases 11, 31 and 71
- 1\* divides \*1 in bases 23 and 89
- 1# divides #1 in bases 13, 19, 29 and 49.

Thanks to Jay Schiffman, who also sent a solution to this puzzle.

\_\_\_\_\_ End

**THE FOLLOWING ARE AVAILABLE  
FROM THE SOCIETY**

1. Our brochure. (Free)
2. "An Excursion in Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, October 1934. (Free)
3. *Manual of the Dozen System* by George S. Terry. (\$1;00)
4. *New Numbers* by F. Emerson Andrews. (\$10;00)
5. *Douze: Notre Dix Futur* by Jean Essig. In French. (\$10;00)
6. Dozenal Slide Rule, designed by Tom Linton. (\$3;00)
7. Back issues of the *Duodecimal Bulletin*, as available, 1944 to present. (\$4;00 each)
8. *TGM: A Coherent Dozenal Metrology* by T. Pendlebury (\$1;00)
9. *Modular Counting* by P.D. Thomas (\$1;00)
10. *The Modular System* by P.D. Thomas (\$1;00)

## REMEMBERING PI

Jean Kelly  
New York

Many people use mnemonics to help them recall things. Over the centuries a favorite among mathematicians has been to create sentences to recall the value of Pi or 3.141 592 653 58 . . . We even have examples in Latin from the Middle Ages. Recently the Mathematics Teacher printed some student creations. A few of these were:

Sir, I need a large microwave to simmer, broil and roast.

Joe, I have a radio somewhere in Peggy's black and white suitcase.

Now I have a green Chevrolet to travel daily all about Maryland.

Each word in the sentence has as many letters as the corresponding digit in Pi. I especially like those which have a period where the fraction point belongs such as:

Wow. I need a white telephone on . . .

Can any of our readers provide us with an example of a mnemonic for the duodecimal value of Pi:

3;184 809 493 #91 866 457 3\*6 212 . . .

\_\_\_\_\_ End

**A CRYPTARITHM - - A PUZZLE**

Replace each letter by a different digit to make this sum correct. Hint: the digit \* is not used.

AAA  
BBB  
+CCC  
DEFG

Send us your solutions to or your extensions of this problem. Perhaps you know of other problems which are related to dozenals or to number bases in general.



## MORE ABOUT BINARY CODED DIGITS

Donald Hammond  
DSGB  
Hampshire, England

The "Binary Coded Digits" which were the subject of an article in Bulletin number 66; deserve careful consideration. The idea is quite impressive, and I will continue to explore its possibilities (as, no doubt, will many others!). It is difficult to think of a better alternative seven-segment display set, but there remains the question of pencil-and-paper. The great strength of the idea is that one gets unique digits by the sheer mathematics of the thing, and early trials to develop hand-written digits from these hold promise.

It is noteworthy that - once again - six turns out to be the opposite of nine (is there some deep earlier reason for this in human history?), and that ten/dek/decim is opposite to five (which latter distinctly resembles our usual 5). The symbol for two is very like the modern Arabic two. Four and eight are also opposites. These symbols can also be made with the typewriter using \_ and /.

⌋	┘	⌌	└	┘	┘	┘	┘	┘	┘	┘	—
1	2	3	4	5	6	7	8	9	*	‡	0

We will try to get all members of the DSGB informed of this interesting development in numeration!

End

### UNTITLED

Since we parted yester eve,  
I do love you, love, believe,  
Twelve times dearer, twelve hours longer,  
One dream deeper, one night stronger,  
One Sun surer, this muc more  
Than I loved you, love, before.

-Meredith

## A DREAM YOU CAN COUNT ON

Someday you'll be able to say that logic and ease have finally WON. People have stopped counting by awkward deks. The new base is do.

Not just counting, but weights and measures TOO. All is conveniently in dozens.

Scolding, THREAping, and chiding people didn't accomplish this change. Just plain common sense.

No one will have to struggle with an infinite decimal FOR one- third any longer.

"'FIVE' been obstructing true progress, I'm sorry"; the realists will confess.

The insane holds out against the better way. He rants. He raves. He SICS his dog on us. But eventually, of course, he is defeated.

"'S HEAVEN" is how people will be describing the newly adopted system.

Those who realize that their resistance has been counter- productive will gladly admit that they ATE their words.

To the few diehards around the world we'll say "No, NEIN, non, nyet, . . ."

Weighing butter, dividing portions, measuring lumber for a new DECK, will all be easier now.

Crostics and puzzles that used the letters "a" through "j", will now employ "k" and "L" also.

Though it may cost a bit to convert, in the long run we know that we will save a lot of DOUGH.

### DON'T KEEP THIS MAGAZINE

Do you discard your copies of the *Bulletin* after you have read them? Or do they gather dust on a shelf in your attic? Why not pass them along to your local library, or to a school library. Perhaps some nearby math teacher would appreciate a copy. You can also just leave them in a dentist's office or other waiting area.

*Help spread the word!*



## COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, 265 represents 5 units, 6 dozen, and 2 dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5;9'
31	694	Three ft. two in.	3;2'
96	3#2	Two ft. eight in.	2;8'
19#	1000	Eleven ft. seven in.	#;7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down 53. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only 2#, which  $12 \overline{) 365}$  is two dozen and eleven. For larger numbers,  $12 \overline{) 30} + 5$  keep dividing by 12, and the successive remainders are the desired dozenal numbers.  $12 \overline{) 2} + 6$   
 $0 + 2$  Answer: 265

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12<sup>2</sup> (or 144) times the third figure, plus 12<sup>3</sup> (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by #, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or #.

For more detailed information see *Manual of the Dozen System* (\$1.00).

We extend an invitation to membership in our society.

Dues are only \$12 (US) per calendar year; the only requirement is a constructive interest.

## Application for Admission to the Dozenal Society of America

Name \_\_\_\_\_  
LAST FIRST MIDDLE

Mailing Address (for DSA items) \_\_\_\_\_

(See below for alternate address)

Telephone: Home \_\_\_\_\_ Business \_\_\_\_\_

Date & Place of Birth \_\_\_\_\_

College \_\_\_\_\_ Degrees \_\_\_\_\_

Business or Profession \_\_\_\_\_

Employer (Optional) \_\_\_\_\_

Annual Dues ..... \$12.00 (US)

Student (Enter data below) ..... \$3.00 (US)

Life ..... \$144.00 (US)

School \_\_\_\_\_

Address \_\_\_\_\_

Year & Math Class \_\_\_\_\_

Instructor \_\_\_\_\_ Dept. \_\_\_\_\_

Other Society Memberships \_\_\_\_\_

Alternate Address (indicate whether home, office, school, other)

Signed \_\_\_\_\_ Date \_\_\_\_\_

My interest in duodecimals arose from \_\_\_\_\_

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America  
 c/o Math Department  
 Nassau Community College  
 Garden City, LI, NY 11530

DETACH HERE -- OR PHOTOCOPY