DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

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THE DUODECIMAL BULLETIN 65;

ROCKED
THE
\+ CLOCK
AROUND

See ROCK-IT!
page 7

RALPH BEARD
MEMORIAL AWARD
to
Peter D. Thomas
(Australia)

See page 21;

Volume 33;
Number 2;
Summer 1990
199*;

THE DUODECIMAL BULLETIN

Whole Number Six Dozen Five

Volume 33; Number 2;

Summer 119;*

FOUNDED
1944

IN THIS ISSUE

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DOZENAL SOCIETY OF AMERICA

BOARD OF DIRECTORS MEETING

Friday, March 9, 1990 (119*)
Hofstra Club, Hofstra University
Garden City, LI, NY

I CALL TO ORDER

Board Chair James Malone gaveled the meeting to order at 4:45 p.m.

The following Board Members and Members were present:

Board:	Member:
Larry Aufiero	Edmund Berridge
Alice Berridge	Mary Malone
Anthony Catania	Mary Newhall
Dr. John Impagliazzo	Dr. Barbran Smith
Fred Newhall	
Dr. Angelo Scordato	
Gene Zirkel	

Continued . . .

The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve. Thus 1/2 = 0.5 = 0;6.

II BUSINESS OF THE SOCIETY

1. Vice President Gene Zirkel reported receiving letters from various people who had expressed interest in the Dozenal Society and its activities. Among other correspondence, Gene received a letter from Jay Schiffman, who said he would attend our next Annual Meeting and would again speak at that meeting. [The DSA Annual Meeting will be held October 13, 1990; details to be announced in the next issue.]

   Gene announced that the 1990 Ralph Beard Memorial Award had been sent to Peter D. Thomas in Australia. [See related story, this issue.]

   Gene also talked about and encouraged the use of the duodecimal clock in some of Nassau Community College's math classes.

2. President Fred Newhall has discovered someone living in Iran who was interested in the goals and activities of our group. Fred also has been in contact with Board Member Robert R. McPherson at the University of Florida.

   Fred visited the Goudreau Museum in New Hyde Park (LI), which is devoted to mathematics. He will continue to keep in contact with them for the Society.

3. At the request of Board Member Robert R. McPherson (FL), a motion was made to name Parry Moon an Honorary Member of the Dozenal Society in recognition of his dozenal writings. The motion was passed with the stipulation that Mr. Moon is still living.

4. Charles Trigg, who frequently had contributed to the Bulletin, recently passed away and was posthumously named an Honorary Member of the Society. Tony Scordato reported that he had received a letter from Mrs. Trigg in which she expressed her gratitude, saying: "I wish to thank you and the members of your Society for your thoughtfulness."

Continued . . .

5. Treasurer Anthony Catania reported that the Society was in a good financial position. The Certificates of Deposit have been rolled over, and the balance in checking is sufficient for the Society's needs.

6. Following a suggestion from Gene Zirkel that the balance of time be used to discuss anything of interest to the Society, Fred Newhall expressed his admiration for the quality of our *Bulletin*. Those present all agreed, and passed congratulations on to Editor Pat Zirkel who was absent due to illness.

7. It has been decided that the next Annual Meeting of the Society will be held on Saturday, October 13, at 2:00 p.m. First, the business portion of the day will be conducted, followed at approximately 4:00 p.m. by the talks and presentations. The social activities (dinner, or dinner and theatre) will then follow.

Continued . . .

John Impagliazzo, Tony Catania, Larry Aufiero, Alice and Edmund Berridge, at dinner following a productive Board of Directors Meeting, held March 9, 1990 at the Hofstra Club.

8. Fred Newhall has not yet completed his referenced Index of our *Bulletins* together with the DSGB *Journals*. This currently consists of over 60 typewritten pages. It was agreed that we attempt to place this information into a computerized database file. Tony Scordato said he would meet with Kathy Gutleber regarding the possibility of designing a database containing Fred's indexed material.

The meeting was adjourned at 5:55 p.m. The participants then met for a delicious dinner. The accommodations for both meeting and dinner were excellent. Thanks to John Impagliazzo for recommending the Hofstra Club and to Barbran Smith for making the arrangements.

Respectfully submitted,

Larry Aufiero

End

ROCK - IT -- *A Puzzle*

Here is an alphanumeric puzzle sent to us by member Charles Ashbacher:

```
  ROCKED
     THE
+  CLOCK
  AROUND
```

Replace the twelve letters with the digits from 0 to # so that the addition is correct. There is more than one solution possible, so as an added challenge try to find the solution which maximizes the value of AROUND.

End

MUSIC, SCALES AND DOZENS

Part II -- Just Intonation and the Chromatic (Dozenal) Scale

Dr. John Impagliazzo
Hofstra University
Garden City, LI, NY

Part I of this Article -- "Mathematical Considerations and Pythagorean Scales" -- appeared in issue number 64; Winter 1990.

THE SCALE OF JUST INTONATION

Considering the harmonic of a vibrating string, the octave (first harmonic), the fifth (third harmonic) and the third (fifth harmonic) are most prominant while at the same time, the most concordant. These have frequency ratios of 1/1 for the tonic, 2/1 for the octave, 3/2 for the fifth, and 5/4 for the third. Hence, the most natural concordant harmony would be a combination of these three tones. Thus, a tonic chord as

C-E-G-C

would have the frequency ratios of

1/1, 5/4, 3/2, 2/1

respectively.

If the same scheme is started with the fifth, its ratios become

(1/1)(3/2), (5/4)(3/2), (3/2)(3/2), (2/1)(3/2)

or

3/2 , 15/8 , 9/4 , 3/1

This represents the dominant chord G-B-D-G. If the scheme is started with the fourth, which is the inverted fifth with ratio 1/(3/2) = 2/3, the ratios are

Continued . . .

(1/1)(2/3), (5/4)(2/3), (3/2)(2/3), (2/1)(2/3)

or

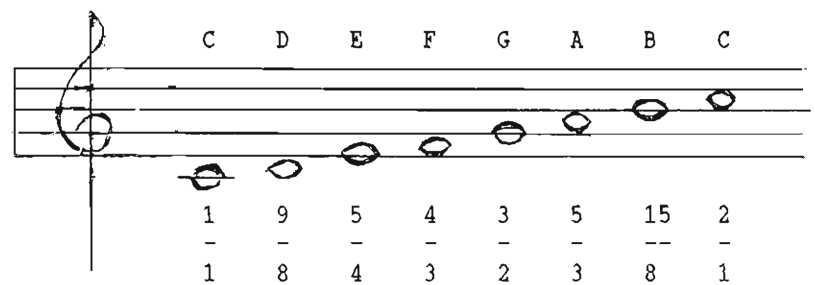
2/3 , 5/6 , 1/1 , 4/3

This represents the subdominant chord F-A-C-F. Summarizing the previous discussion, the three sequences of tones are

C - E - G - C	:	1/1	5/4	3/2	2/1
G - B - D - G	:	3/2	15/8	9/4	3/1
F - A - C - F	:	2/3	5/6	1/1	4/3

When these ratios are adjusted so that they fall within the range of one octave and in ascending order, they form the scale of Just Intonation [Coxeter, p. 318]. This is shown in Figure 3.

Figure 3



James Jeans wrote:

> It is found to be a quite general law that two tones sound well together when the ratio of their frequencies can be expressed by the use of small numbers, and the smaller the numbers the better is the consonance [Jeans, p.154].

The Scale of Just Intonation is considered "purer" than the Pythagorean Scale because it is composed of ratios of numbers which are composed of smaller numbers.

Continued . . .

DEFECTS IN TONAL SCALES

In the Pythagorean Scale the ratio between successive whole notes is 9/8 = 1.12500 (e.g., 243/128 divided by 27/16) while the ratio between successive semitones is 256/243 = 1.05350 (e.g., 4/3 divided by 81/64) [Jeans, p.167]. Two successive semitones do not equal a whole tone. This is a cause for musical distortion especially when modulating from one tonic key to another.

In the Scale of Just Intonation, semitone ratios such as F to E and C to B have constant ratios equal to 16/15. However, the whole tone ratios are not constant. For example, the ratio E to D results in the ratio 5/4 to 9/8 which equals 10/9. The ratio B to A, however, has the ratio 15/8 to 5/3 which equals 9/8. Clearly, $9/8 \neq 10/9$. Music played in a key which is tuned to Just Intonation would sound optimal. A change in key, however, would create a definite tonal distortion.

THE CHROMATIC SCALE, OR SCALES AND DOZENS

Suppose successive ratios of 3/2 are taken from a fundamental tonic of ratio 1. Then new notes are generated (those with sharps) to form what is called the Chromatic Scale which contains one dozen notes. If the tonic is the note C, then

$(3/2)^{-1}$	corresponds to	F		
$(3/2)^{0}$	corresponds to	C		
$(3/2)^{1}$	corresponds to	G		
$(3/2)^{2}$	corresponds to	D		
$(3/2)^{3}$	corresponds to	A		
$(3/2)^{4}$	corresponds to	E		
$(3/2)^{5}$	corresponds to	B		
$(3/2)^{6}$	corresponds to	F#		
$(3/2)^{7}$	corresponds to	C#		
$(3/2)^{8}$	corresponds to	G#		
$(3/2)^{9}$	corresponds to	D#		
$(3/2)^{10}$	corresponds to	A#		
$(3/2)^{11}$	corresponds to	E#	=	F
$(3/2)^{12}$	corresponds to	B#	=	C

Continued . . .

These values when normalized within a single octave produce what is commonly known in music theory as a circle of fifths. However, the value

$$(3/2)^{12} = 129.74634$$

while the value

$$(2/1)^{7} = 128.$$

Herein lies the discrepancy. A dozen factors of 3/2 should have encompassed seven octaves. It did not. Over seven octaves there exists a ratio of 129.74634/128 = 1.01364 (often called the "comma of Pythagoras") rather than the ideal 1.

In order to compromise the aforementioned discrepancies, a scale based on a dozen semitones each possessing a ratio of $2^{1/12} = 1.05946$ was constructed. In this case, the ratio of any two adjacent notes on the scale would equal a constant. As a case in point, the construction of a fifth from a given tonic would be $2^{7/12} = 1.49831$, very close but less than the ideal of 3/2 = 1.5. The third, which is four semitones from the tonic is $2^{4/12} = 1.25992$. This differs from both the Pythagorean third of 81/64 = 1.26563 and the Just Intonation third of 5/4 = 1.25000.

The scale based on a dozen semitones of ratio $2^{1/12}$ is called the Well-Tempered Scale. A table of ratios of the dozen notes is shown:

C	:	2^{0}	=	1.00000	C#	:	$2^{1/12}$	=	1.05946
D	:	$2^{2/12}$	=	1.12246	D#	:	$2^{3/12}$	=	1.18921
E	:	$2^{4/12}$	=	1.25992	F	:	$2^{5/12}$	=	1.33484
F#	:	$2^{6/12}$	=	1.41421	G	:	$2^{7/12}$	=	1.49831
G#	:	$2^{8/12}$	=	1.58740	A	:	$2^{9/12}$	=	1.68179
A#	:	$2^{10/12}$	=	1.78180	B	:	$2^{11/12}$	=	1.88775
C	:	$2^{12/12}$	=	2.00000					

Continued . . .

Note that with the Well-Tempered, dozenal scale the tones C#, D#, F#, G# and A# could be replaced by Db, Eb, Gb, Ab and Bb respectively. In the Pythagorean or the Scale of Just Intonation this is impossible.

The Well-Tempered scale has its defects also. Its greatest defect is that it is not always harmonious. It is always "almost in tune." A vocalist or instrumentalist such as a violinist will sing or play toward notes of Just Intonation. A piano is almost always tuned to Well-Tempered balance. Singing or playing an instrument together with a piano sometimes presents problems, since it will always sound a "little" out of tune. The greatest asset of the Well-Tempered scale is that the tonality will always be the same irrespective of the key in which it is played.

The following compares values of the ratios to the note C in the Well-Tempered, Just Intonation and Pythagorean scales. The corresponding frequencies are based on note A tuned at a frequency of 440 Hertz.

Note	Well-Tempered	Frequency	Just Inton.	Freq.	Pythag.	Freq.
C	1.00000	261.6258	1.00000	264	1.00000	260.7407
D	1.12246	293.6645	1.12500	297	1.12500	293.3333
E	1.25992	329.6276	1.25000	330	1.26562	330.0000
F	1.33484	349.2286	1.33333	352	1.33333	347.6543
G	1.49831	391.9966	1.50000	396	1.50000	391.1111
A	1.68179	440.0000	1.66667	440	1.68750	440.0000
B	1.88775	493.8842	1.87500	495	1.89843	495.0000
C	2.00000	523.2517	2.00000	528	2.00000	521.4814

The discrepancies shown present definite problems when tuning fixed-tone instruments such as the harp or piano.

Continued . . .

CONCLUSION

Without a doubt there is a correlation between mathematics and music which is supported from general observation, from historical considerations and from a philosophical viewpoint. From a physical point of view there are other aspects such as acoustical environments, vibrating membranes, air columns and harmonic analysis. The feeling of an affinity is more than coincidental. Music is very much mathematical and mathematics is very related to music as shown from the construction of scale, the very basis of music.

The mathematician J. Sylvester, in a footnote to his memoirs on "Algebraical researches containing a disquisition on Newton's rule for the discovery of imaginary roots," stated the relationship very nicely. He wrote:

> May not Music be described, as the Mathematic of Sense, Mathematic as the Music of reason? the soul of each the same! Thus the musician feels Mathematic, the mathematician thinks Music, - to receive its consummation from the other when the human intelligence, elevated to the perfect type, shall shine forth glorified in some future Mozart - instinctly foreshadowed in the genius and labors of a Helmholtz! [Archibald, p.2]

How true this is. How true.

Continued . . .

SHAME ON YOU!

In Bulletin Number 64; the solution to the Farmyard puzzle given by Charles Ashbacher states that the number of chickens was 30! Shouldn't that be 26; chickens?

Jean Kelly

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End

HOW MANY FRUITS ON THE TREE OF LIFE?

B.A.M. Moon
Diamond Harbor,
New Zealand

Since times of the greatest antiquity, twelve-based systems have found a place in all kinds of human activities, not least in religion and the law. Robert Graves gives a long list, including the Twelve Knights of the Round Table, the Twelve Companions of Odysseus, The Twelve Shepherds of Romulus and the Twelve Sons of Jacob.[1]

It seems also that a witches' coven had twelve followers assembled under the leader. On the other hand, the Mongolian hordes of Genghis Khan were organized decimally!

Turning to history, a Roman consul was accompanied by twelve lictors and a dictator by two dozen. There were twelve tables of Roman Law. One jurist, Danish-born O.T.J. Alpers commented: "The instinct of the English people in deciding upon twelve as the number of triers was as sound in that as in most other things . . . A British jury of twelve is more intelligent than any member of it."[2]

The most remarkable testimony of all may be that of the Biblical book, the Revelation of John, wherein the number twelve occurs frequently. To cite only one example, in the vision of the heavenly Jerusalem (whose walls have twelve gates):

> "In the midst of the street of it, and on either side of the river, was there the Tree of Life, which bore twelve manner of fruits, and yielded her fruit every month; and the leaves of the tree were for the healing of nations."[3]

[1] The White Goddess (New York: Noonday Press), 201.

[2] Cheerful Yesterdays (London: J. Murray & Sons, 1928), 239, 224.

[3] Revelation 22.2, Holy Bible, Authorized Version.

End

SECOND ORDER DUODECIMAL DIGITAL ROOT BRACELETS

Charles W. Trigg

The digital root of an integer is obtained by adding the digits, adding the digits of the sum, and continuing the process until a single digit, the digital root, remains. This is equivalent to casting out elevens[1] except that # is kept as a final digit rather than a zero. Thus 754678;--> 31;--> 4, the digital root of 754678.

A bracelet is one period of a simple periodic series, considered as a closed sequence, with terms equally spaced around a circle. Hence, a bracelet may be regenerated by starting at any arbitrary point on the circle and applying its generating law. For example, a second order digital root bracelet is formed by repeated application of

$$d_{n+2} = \text{digital root of } (d_n + d_{n+1}).$$

For example, if we start with the arbitrary digits 3 and 7 we obtain

3 + 7 = * (for the third digit)

7 + * = 15, 1 + 5 = 6 (fourth digit)

* + 6 = 14, 1 + 4 = 5

6 + 5 = #

5 + # = 14, 1 + 4 = 5

+ 5 = 14, 1 + 4 = 5

5 + 5 = *

5 + * = 13, 1 + 3 = 4

* + 4 = 12, 1 + 2 = 3

4 + 3 = 7

Continued . . .

ROOT BRACELETS, Continued

and 3 + 7 are the two digits that we started with. They generate the bracelet: 3 7 * 6 5 # 5 5 * 4 / 3 7 . . . This is listed as bracelet E in Table 1.

TABLE 1: SECOND ORDER DUODECIMAL DIGITAL ROOT BRACELETS

A:	1	1	2	3	5	8	2	*	1	#	1	1
B:	2	2	4	6	*	5	4	9	2	#	2	2
C:	3	3	6	9	4	2	6	8	3	#	3	3
D:	4	4	8	1	9	*	8	7	4	#	4	4
E:	5	5	*	4	3	7	*	6	5	#	5	5
F:	6	6	1	7	8	4	1	5	6	#	6	6
G:	7	7	3	*	2	1	3	4	7	#	7	7
H:	8	8	5	2	7	9	5	3	8	#	8	8
I:	9	9	7	5	1	6	7	2	9	#	9	9
J:	*	*	9	8	6	3	9	1	*	#	*	*
K:	1	8	9	6	4	*	3	2	5	7	1	8
L:	1	4	5	9	3	1	4					
M:	2	8	*	7	6	2	8					
N:	#	#	#	#								

Continued . . .

[1] "Casting Out 'Base Minus One'", this *Bulletin* Volume 31; Number 2; pages 4-9; "Casting Out Σ's", this *Bulletin* Volume 16; number 1; pages 21-23;.

There are el dek-digit bracelets (A to K), two five-digit bracelets (L and M), and a single one-digit bracelet (N), the result of starting with ##. Together, these bracelets include all of the (#)(#) = *1; ordered zero-free dyads. Straight line representatives of the 12; bracelets are shown in Table 1.

When each digit, d, of a digital root bracelet is replaced by the digital root of the product, dm, the bracelet has been multiplied by m. For example, if we multiply Bracelet D by 4, we obtain bracelet E:

4 D = 4(4 4 8 1 9 * 8 7 # #)

= 5 5 * 4 3 7 * 6 5 # = E.

Multiplication of any member of the set, A, B, ..., J, by a positive digit <# will produce another member of the set. As shown in the box in Table 2, all members of the set can be derived from any individual member by multiplication.

Continued . . .

DSA
ANNUAL MEETING
Saturday
13 October 1990

2 p.m.-Business Meeting

4 p.m.-Speakers

TABLE 2: BRACELET MULTIPLES

bracelet \ m	1	2	3	4	5	6	7	8	9	*	#
A	A	B	C	D	E	F	G	H	I	J	N
B	B	D	F	H	J	A	C	E	G	I	N
C	C	F	I	A	D	G	J	B	E	H	N
D	D	H	A	E	I	B	F	J	C	G	N
E	E	J	D	I	C	H	B	G	A	F	N
F	F	A	G	B	H	C	I	D	J	E	N
G	G	C	J	F	B	I	E	A	H	D	N
H	H	E	B	J	G	D	A	I	F	C	N
I	I	G	E	C	A	J	H	F	D	B	N
J	J	I	H	G	F	E	D	C	B	A	N
K	K	K	K	K	K	K	K	K	K	K	N
L	L	M	L	L	L	M	M	M	L	M	N
M	M	L	M	M	M	L	L	L	M	L	N
N	N	N	N	N	N	N	N	N	N	N	N

Note that the box entries are symmetrical with reference to the box center.

Multiplication of any bracelet by # produces a repetitious string of #'s. Otherwise multiplication of Bracelet K by a positive digit <# merely rotates the bracelet. Multiplication of a five-digit bracelet (L or M) by such a digit produces another five-digit bracelet.

End

DOZENAL JOTTINGS

. . .from members and friends . . .News of Dozens and Dozenalists. . .

Member **CHARLES ASHBACHER** of Mount Mercy College, Iowa, had his solution to a difficult geometry problem published in Mathematics and Computer Education, Volume 2 dozen, Number 1, Winter 1990, pages 91-92. Congratulations! . . .

DR. TONY SCORDATO (NCC) recently had our Ralph Beard Memorial Award engraved for presentation to **PETER D. THOMAS** of Australia. While he was at the engraver's (who had created previous awards for us) he interested the owner and his daughter in the subject of Dozenals. They asked for literature and Tony left some with them. Would you have had literature with you? Why not keep some of our brochures at home, at work, and in your car -- you never know when opportunity may knock! . . .

New member **ROBERT BRODA** sent us the book Astrology of Inner Space, which re-explains the twelve houses of the zodiac. It is a welcome addition to our library . . .

The Society recently heard from **ANETTA TRIGG**, widow of **CHARLES W. TRIGG**, who was a frequent contributor to our *Bulletin*. She wrote:

> You may be interested to know that, according to a 12/29/89 letter from the Mathematical Association of America, a room in the new Dolciani Mathematical Center at the MAA headquarters in Washington, D.C. has been designated "The Charles W. Trigg Room." He had been a member of the Board of Governors. I hope you will see it some time when you are in Washington.
>
> I'd like you to know that my husband thought it was fun to work in the duodecimal system and enjoyed his association with you. . . .

Continued . . .

DOZENAL JOTTINGS, Continued

DUDLEY GEORGE (CA) wrote to say that he and his wife **KAY** are well and hope to attend the Annual Meeting in October. Dudley was most interested to hear about the Australian **MODULAR CONVERSION BUREAU.** (See "Dozens in Australia," in the last issue of the *Bulletin.*) . . . **JAMISON** and **VERA HANDY** also sent greetings . . .

Member **NEELA LAKSHMANAN** (PA) wrote early in the year to wish all a happy 119*. She had just returned from a holiday visit to India . . .

Continued . . .

MORE PI

$$\text{BASE } \frac{1}{26*,680;^{3}}$$

Gregory and David Chudnovsky recently calculated *pi* to 242,793,32#; places! They used a Cray 2 super computer, writing their program in FORTRAN. They think of the series they used in the calculation as an expression of a number in base 26*,680^{-3}. The coefficients are all integers, although they are not all less than the base. Checking the results as they proceeded gave them a probability of an error at any step less than 10;$^{-1*4}$.

-From Focus, the Newsletter of the Mathematical Association of America (vol. 9, no. 5, pp. 1-4, Oct. 89)

End

DOZENAL JOTTINGS, Continued

New member **IAN PATTEN** (Alaska) has kept us abreast of his book-writing activity. He is currently working on two publications -- The first deals with the integration of customary/imperial measurement with metrics by means of a 40" meter; the second with duodecimal counting and measurement. His 40" meter concept is intriguing. He writes:

> The first step I took was to spend a winter researching material on the 40" meter concept. I was aware we could then have a 300 mm foot, 25mm inch and a 900 mm yard involving the vital 3 factor, and also that since the liter and quart were very close in size we could then have a 4" cubed liter/quart, with a gallon 4 times that dimension. This intrigued me to find out how much further we could go, so I worked through volume and weight. To my surprise, things worked out more favorably than I expected, and I then realised the only real problem to its adoption would be political.

Continued . . .

STRANGE DIGITS

WordPerfect's newest version uses some strange 'digits' in its menus. When a menu has more than dek changes, past versions would follow the digits 0 thru 9 with capital letters: A, B, C, etc.

The recently released version 5.1 uses / \ : * ? + and = to correspond to the numbers dek through do-four!

End

DOZENAL JOTTINGS, Continued

His second book attempts to bring the modular/duodecimal systems "to life in an exciting manner, sticking to our decimal symbols and naming (symbols) as much as possible so the reader will have familiar guidelines." He will market his books along with those of **PETER D. THOMAS** of the Australian Modular Conversion Bureau.

IAN is also looking for help in revising his work and wonders if anyone in the Society might be interested? We would be glad to put any interested parties in contact with him . . .

Continued . . .

"A BASIC SOLUTION" SOLUTION

Jean Kelly has found another solution to the problem posed in our Fall 1989 (1199;) issue (page 16; "A Basic Solution").

The problem:	Move one digit to make the expression true:
	101 - 102 = 1
The latest solution:	110 - 102 = 1 (base three)

Jean also notes that both 101 - 02 = 11 and 0101 - 12 = 1 would be valid in base two if we allow 2 as a digit.

End

DOZENAL JOTTINGS, Continued

CHARLES BAGLEY (NM) wrote saying: "...**MIRIAM** and I passed our 60th wedding anniversary last summer, so you can see that we are getting along in years. But dozenally we have only been married 50 years and I am only 73 and Miriam is 72. Hoopla! who can beat that?" Congratulations to Charles and Miriam on all counts! He also wrote on another occasion:

> It pleases me to know that qualified, worthy people are still carrying on the traditions and objectives of the Dozenal Society. I will never regret my association with it and the companionship of men like Terry, Beard, Humphrey, Andrews, Linton, Churchman, Handy and others. . .

CHARLEY enclosed a copy of an old letter from **GEORGE TERRY**, dated March 18, 1963, in which George mentions the then current slump in Society activities, but expresses at the same time a faith in the future of duodecimal counting and of the Society itself. It seems that this faith was well-placed . . .

BRUCE MOON (Australia) wrote to express his continuing interest in dozens. He would also like to see more use made of the practical advantages of dozenal systems. . .

DONALD HAMMOND of the DSGB was named an Honorary Member of the DSA at the last Annual Meeting. He writes:

> It was with great pleasure that I received my Certificate of Honorary Membership of the DSA. Please convey my thanks to your Board of Directors, who were also kind enough to offer encouraging comments on my work in this field. Such recognition is very much appreciated... Whatever changes may occur in the future, be assured that I shall continue to work at this most fascinating and important subject.

Continued on page 22; . . .

RALPH BEARD MEMORIAL AWARD -- 1990 (119*)

PETER D. THOMAS (AUSTRALIA)

After learning about the Australian Modular Conversion Bureau, and the work of its founder, Peter D. Thomas, the Dozenal Society of America formally presented its Ralph Beard Memorial Award to Mr. Thomas this past March. (See "Dozens in Australia," *Duodecimal Bulletin*, Whole Number 64; Vol. 33; No. 1; Winter 1990.)

Peter D. Thomas wrote two books advocating base twelve numeration and measurement (which he called "Modular Counting"). The books, titled Modular Counting and The Modular System, also opposed the Australian movement to enforce decimal metrication. Both are now available through the DSA at a cost of $3.00 and 5.00, respectively.

When it was learned earlier this year that Mr. Thomas was terminally ill, the decision was made by the DSA Board of Directors to present the award immediately. Mr. Thomas was informed by mail, and was able to enjoy this recognition of his efforts. Unfortunately, the engraved presentation plaque arrived in Australia the day following his death. His widow, Jean Thomas, wrote to us to say:

> "I feel so sad to have to tell you that the day after Peter died your very handsome plaque arrived -- the Ralph Beard Memorial Award. I do wish he could have seen it, but at least he had received your letter beforehand and I know your acknowledgment of his work meant a lot to him and I thank you on his behalf. The plaque has a prominent place in our living room."

It is hoped that Mr. Thomas's colleagues, David Caldbeck and Ian B. Patten (Alaska) will be able to carry on with his important work.

End

DOZENAL JOTTINGS, Continued from page 20;

FRED NEWHALL has been preparing a referenced Index of both DSA and DSGB material. He recently sent a copy to the DSGB and **DON HAMMOND** has responded:

> The Dozenal Index you have compiled, with evidently a considerable amount of both work and time, is quite splendid. It seems to have missed nothing; I really had no idea that there was so much material written on the subject. I have a collection of <u>most</u> of the past publications of DSGB and DSA, but your Index brings home to one the real extent of dozenal literature.

RICHARD T. TRELFA (VT) wrote recently encouraging the dissemination and use of DSA Membership Cards. He also said:

> I believe that the *Bulletin* should, now and then, dwell on the philosophy of numbers. I believe that Tolstoy said something along the line that we <u>all</u> learn the rote of numbers (1 horse, 2 rabbits, 3 dogs, etc.) before our intellects have developed to a point where we can understand the meaning of numbers. By the time our intellects have developed to the point where we <u>could</u> understand the meaning of numbers, the rote of numbers is so <u>firmly</u> fixed in our minds that we probably <u>never</u> truly understand the meaning of numbers. I know that I only occasionally get a glimpse of that meaning, but believe that more of us would understand <u>way</u> more if we had learned our numbers in a system based on some other unit than 5 X 2!
>
> Should we explore the dozenal system as a more efficient vehicle for quantitative logic? Would a dozenal system increase/decrease or have no effect on the efficiency of estimation of a population from samples? . . .

Continued . . .

DOZENAL JOTTINGS, Continued

ROBERT R. MCPHERSON (FL) has noticed that the symbol for dek in the DSA logo seems to be inverted, and hence, appears top-heavy. Has anyone else noticed this? Should it be corrected? . . .

Welcome to new member:

307; **ROBERT A. BRODA** St. Paul, MN

End

THE FOLLOWING ARE AVAILABLE FROM THE SOCIETY

1. Our brochure. (Free)
2. "An Excursion in Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, October 1934. (Free)
3. *Manual of the Dozen System* by George S. Terry. ($1;00)
4. *New Numbers* by F. Emerson Andrews. ($10;00)
5. *Douze: Notre Dix Futur* by Jean Essig. In French. ($10;00)
6. Dozenal Slide Rule, designed by Tom Linton. ($3;00)
7. Back issues of the *Duodecimal Bulletin*, as available, 1944 to present. ($4;00 each)

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition—("Who needs a symbol for nothing?")—the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accomodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used, and as early as 1585, Simon Stevin stated that the duodecimal base was to be preferred to the base ten.

The parallel seems tenable. The notation of the dozen base better accomodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS.**

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

COUNTING IN DOZENS

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*	#	*10*
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10,* and is called *do,* for dozen. The quantity *one gross* is written *100,* and is called *gro. 1000* is called *mo,* representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5,* and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3#2*	Two ft. eight in.	*2;8'*
19#	*1000*	Eleven ft. seven in.	*#;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53.* Using this *"which is"* step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2#,* which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

12) 365
12) 30 + *5*
12) 2 + *6*
0 + *2* Answer: *265*

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *.

For more detailed information see *Manual of the Dozen System* ($1;00).

DETACH HERE -- OR PHOTOCOPY

We extend an invitation to membership in our society.
dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______________________________
LAST FIRST MIDDLE

Mailing Address (for DSA items) ______________________________

____
(See below for alternate address)

Telephone: Home ______________ Business ______________

Date & Place of Birth ______________________________

College ______________ Degrees ______________

Business or Profession ______________________________

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______________________________

Address ______________________________

Year & Math Class ______________________________

Instructor ______________ Dept. ______________

Other Society Memberships ______________________________

____

Alternate Address (indicate whether home, office, school, other)

____

____

____

Signed ______________ Date ______________

My interest in duodecimals arose from ______________________________

____

____

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530