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THE DUODECIMAL BULLETIN 5*;

(L-R) Edith Andrews (Mrs. F. Emerson), NCC President Sean Fanelli, DSA President Gene Zirkel, and DSA Board Chair Fred Newhall officially open the F. Emerson Andrews Memorial Collection at the Nassau Community College Library.

LIBRARY COLLECTION OF DOZENAL MATERIALS
OFFICIALLY DEDICATED
See Pages 14 - 15;

Volume 31;
Number 2;
Summer 1988
1198;

DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00 (US) for one calendar year. Student membership is $3.00 per year, and a Life membership is $144.00 (US).

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THE DUODECIMAL BULLETIN

Whole Number Five Dozen Dek

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IN THIS ISSUE

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CASTING OUT 'BASE MINUS ONE'

Jean Kelly
New York, NY

Introduction

Many people learned to check their arithmetic calculations in grade school by 'casting out nines'. We wish to investigate this process, and to generalize it to any base. In particular, one can check dozenal arithmetic by 'casting out elevens'[1].

Some Examples

To check the base ten addition, 28 + 11 = 39, we find the remainders when 28, 11, and 39 are divided by 9. These are 1, 2, and 3 respectively. We then check the addition of these remainders, 1 + 2 = 3.

It is known that if the addition of these remainders is NOT correct, then the original addition is wrong also. (Note, the converse is NOT true. That is, if the calculation with

Continued...

[1] Wm. Bruce Knapp discussed "Casting Out Els" in this Bulletin, volume 16; number 1; pp 21-23;.

> The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Whatever symbols are used, the numbers commonly called "ten", "eleven" and "twelve" are pronounced "dek", "el" and "do" in the duodecimal system.
>
> When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve. Thus 1/2 = 0.5 = 0;6.

the remainders is correct, the original calculation may or may not be correct. Thus casting out nines is useful for detecting many, but not all, errors.)

A Short Cut

To find the remainder when a number is divided by 9 we can simply add the digits of the original number. If this sum is not a single digit repeat the process. Thus to find the remainder when 2,734 is divided by 9, we simply add, obtaining

$$S = 2 + 7 + 3 + 4 = 16,$$

and then repeating the process we add again obtaining

$$S' = 1 + 6 = 7.$$

Thus the remainder when 2,734 (and also when 16) is divided by 9 will be 7.

Notation

Mathematicians write the statement, 'the remainder when 2,734 is divided by 9 is 7', as

$$2{,}734 \equiv 7 \pmod{9}.$$

This is read '2,734 is congruent to 7 (modulo 9)', and is often abbreviated as

$$2{,}734 \equiv 7 \text{ (mod 9) or even as } 2{,}734 \equiv 7$$

when the modulus is known.

Definition

We define $a \equiv b$ (mod m) to mean that there exists an integer t, such that $a = b + tm$. Thus we also have

$$2{,}734 \equiv 16 \text{ and } 16 \equiv 7 \pmod{9}.$$

Continued...

Theorem I - Digit Sums

The above idea about adding digits to obtain the remainder is true for any base. That is, if X is a number in base B, and S is the sum of the digits of X, then X and S have the same remainder when divided by B-1. That is

$$X \equiv S \pmod{B-1}.$$

Proof

Let $X = d_n B^n + d_{n-1} B^{n-1} + \cdots + d_2 B^2 + d_1 B + d_0$.

Let $S = d_n + d_{n-1} + \cdots + d_2 + d_1 + d_0$.

Let $m = B - 1$.

Then $X = d_n (m + 1)^n + \cdots + d_1 (m + 1) + d_0$.

Expanding the powers of $(m + 1)$ in the above, and then factoring out an m from most of the terms we obtain

$X = mt + (d_n + \cdots + d_1 + d_0)$.

$X = mt + S$, which by our above definition means that

$X \equiv S \pmod{m}$

Q.E.D.

Examples

Let X = the dozenal number 346;. Then S = 3 + 4 + 6 = 11;. Since 11; is not a single digit number, we add again, obtaining

$$S' = 1 + 1 = 2.$$

Thus 346; $\equiv$ 11; $\equiv$ 2 (mod #)

Continued...

Let X = the base three numeral 1221. Then S = 1 + 2 + 2 + 1 = 20, and S' = 2 + 0 = 2. In fact all even non-zero ternary numbers will eventually yield a digit sum of 2, and all odd ternary numbers yield a digit sum of 1.

Theorem II - Casting Out Base Minus One

If $\circ$ represents either addition, subtraction or multiplication, and $X \circ Y = Z$ in base B, then

$$x \circ y \equiv z \pmod{B-1},$$

where $X \equiv x$, $Y \equiv y$, and $Z \equiv z \pmod{B-1}$.

Proof

Let $m = B-1$

Case 1. Addition or subtraction.

If $X \pm Y = Z$ then by the above definition we have

$(x + rm) \pm (y + sm) = z + tm$

$x \pm y = z + (t - r \mp s)m$, that is

$x \pm y \equiv z \pmod{m}$.

Case 2. Multiplication.

If $XY = Z$ then

$(x + rm)(y + sm) = z + tm$

$xy = z + (t - rsm - ry - sx)m$, or

$xy \equiv z \pmod{m}$.

Q.E.D

Continued...

In order to check the division, X/Y = Z, we can instead check the multiplication, X = YZ using case 2 above. Thus we know that if X/Y = Z is correct then x ≡ yz (mod B-1) must also be correct.

Examples

The multiplication 46; x 2 = 92; is wrong in base twelve since the sum of the digits

4 + 6 = *

2 = 2, and

9 + 2 = #

but * x 2 = 18; which is congruent to 1 + 8 = 9 and not #.

Again, 5*# x 87; = 42,885; may be correct since

(5 + # + *) x (8 + 7) =

22; x 13; ≡

(2 + 2) x (1 + 3) =

4 x 4 =

14; ≡

1 + 4 = 5.

And 4 + 2 + 8 + 8 + 5 = 23; ≡ 2 + 3 = 5 also.

[Note, in all of the above, if B-1 = m divides evenly into X the remainder should be zero, but the repeated sum of the digits will yield m instead of zero. For example in base dek, 9 divides 18 with a remainder of zero, but 1 + 8 = 9.

Continued...

This is not a problem, since the check works either way because m ≡ 0 (mod m). For example the base dek addition

18 + 24 = 42

can be checked either by

(a) the digit sums;

(1 + 8) + (2 + 4) = 9 + 6 = 15 ≡ 1 + 5 = 6, and

4 + 2 = 6 also, or by

(b) the remainders found by dividing by 9:

0 + 6 = 6.]

End

HAVE SOME FUN??!!

An ordinary base ten abacus usually has 2 beads above the vertical bar and 5 beads below. The Japanese Soroban has one bead above and 4 beads below. Hence an Abacus with 5 beads can be used as a duodecimal Soroban.

Figuring this out is only half the fun -- using it could be the other half!

*

THE EGYPTIANS -- A Comment

Arthur Whillock
Underhill
Moulsford
Oxon, England

The debate on Egyptian counting is interesting, but I am afraid that evidence for the dozen is very slim. For measurement there was the Royal or administrative cubit which had a centismal basis of (presumably) a hundred wheat grains, but mainly divided into 28 digits and four spans of seven digits which had a religious significance. There was also a profane or engineers cubit of a fore-arm length, which is marked on some specimens of the Royal cubit, but which clearly preceeded it since the heiroglyph for cubit is a sketch of a fore-arm. This practical unit was 24 digits long, and its half became the Attic foot of Greece.

Land measurement was decimal: a *Setat* was 10,000 square royal cubits. Multiplied by ten this was called a "thousand of land". But, according to Herodotus, a grant to members of the warrior class was <u>twelve</u> setats (about 8 acres).

This, with the divisions of day and night, also the seasons, is all that I have encountered.

Merits of the dozen were better appreciated on the north side of the Mediterranean via the Babylonians and Indo-Aryans. We had a radio lecture on the *Odyssey* in which the 3 - 12 based structure was mentioned. To add to Brost's list: 6 men from each of Odysseus' 12 ships were killed at Ismarus, and they were 9 days drifting past Cape Malea. Three men went ashore to explore the land of the Lotus Eaters.

You can also see the *Duodecimal Review* Number 24 for a list of the twelve in history. There are lots more.

End

#

ANTIQUITY REVISITED -- A REPLY

David Singmaster
Department of Computing and Mathematics
Polytechnic of the South Bank
London, SE1 0AA, England

I was pleased to see that you published my letter in The *Duodecimal Bulletin* 59; and I think the adjacent article by Brost is an interesting contrast. There are several points where I think Mr. Brost's facts are wrong and I think that he has made a number of speculative assertions which are not substantiated by what is known about Egyptian mathematics and culture.

As he says, there is a lack of papyri, but there are monumental inscriptions which help us to understand Egyptian metrology, although the earliest written materials only go back to about -3100, just two centuries before the Great Pyramid. In Richard J. Gillings, *Mathematics in the Time of the Pharaohs*, chapter 20, pp. 207-213, there is a summary of Egyptian weights and measures. Among these, there is one occurrence of 12: 1 *deben* of gold is worth 12 *shaty*. A *deben* is a weight and a *shaty* is a unit of value, not corresponding to any particular coin. The most basic units, of length, follow this pattern: 1 *hayt* = 100 cubits, 1 cubit = 7 hands (or palms), 1 hand = 4 fingers (or digits). Other conversion factors are generally 10 and 100 with 20, 30 and 320 also occurring. Hence I don't think one should speak of a 'dozenal metrology'.

The phrase 'long before the pyramids were ever built' is unreasonable since written knowledge only goes back to about 200 years earlier in -3100. The reference to -4241 is extremely dubious. The Egyptians did not use a 365¼ day calendar, but one with 365 days in every year. "This calendar is, indeed, the only intelligent calendar which ever existed in human history." [O. Neugebauer, *The Exact Sciences in Antiquity*, 2nd. Ed. (1957, 1962) p.80.] Because of the lack of leap years, this calendar gets about one day out of step with the seasons in every four years, hence it

Continued. . .

gets one year out of step in about 1460 (= 4 x 365) years, when it appears to be back in phase. The Roman scholar Censorinus, in his *De Die Natali*, of 238, asserted that the Egyptian calendar was in synchronization with the seasons in 139. Consequently, it has been asserted that the Egyptian calendar was started in 139 - 1460 n, where n is the number of complete cycles. Taking n = 3 gives -4241. However, this ignores several points:

The year is not exactly 365¼ days long.

There wasn't a year 0. Hence the date for n = 3 is better as -4228. But many scholars feel that this extrapolation can only be stretched back by n = 2 cycles, giving an origin of about -2773, though this seems too late. [See: C.B. Boyer. *A History of Mathematics*. Wiley, 1968, p. 12.] In any case, this extrapolation is over a rather long period and the datum of Censorinus may not be too reliable. Further, other calendar systems often were back-dated from the date of their creation to some significant historic era. For example, the Christian era and the division into BC and AD were set up in the 6th century. So the zero point of the calendar may not correspond to the date of creation of the calendar.

Finally, the archeological evidence does not indicate a degree of civilization in c-4000 which could have accurately recorded the passage of so many years prior to the development of writing - indeed it is quite hard to determine dates for the first 2000 years of writing!

I have just discovered an additional fact. The *Encyclopedia Britannica* (1971) article on calendars says that the later Egyptians did have leap years. If this went on for 500 years before Censorinus, then the zero point of the Egyptian calendar would move back by about 500 years. This would make n = 2 more reasonable.

Continued . . .

Mr. Brost is quite right that the Egyptians did have twelve months in the year. Each of these had 30 days and there were 5 extra days at the end. This was unusual for early civilizations since they generally used lunar months (as the word indicates), which were 28 or 29 days long, with about 12 3/8 months per year. I don't know how far back the 30 day month goes - the Britannica says the word for month involves the symbol for the moon, but that there is no evidence of a lunar calendar. So the 30 day month and the 12 month year must go back to c-3100.

The Egyptians divided the year into three four-month periods: inundation or sowing, coming-forth or growing, and summer or harvest. [Gillings, p. 235. Neugebauer, pp. 81-82.] The Britannica calls them: inundation, winter or sowing, and summer or harvest. I don't see any indication that they divided the year into four seasons and Neugebauer and the Britannica refer to the above as the Egyptian seasons.

The Egyptian divisions of the night and the day into 12 hours have their origins in the 36 '*decans*', corresponding to 10 day periods, and in the decimal division of the daylight with two extra hours for dawn and dusk, as discussed in my previous letter. Though the decans were originally based on a 360 day year, they did account for the extra 5 days by later refinements. The decans first appear during the Middle Kingdom (c-2133 to -1786), according to Neugebauer (p. 82), or the Old Kingdom (c-2613 to c-2181), according to the Britannica article on Egypt.

Neugebauer says the development of the 12 hours of the night dates from about -1800. The oldest surviving sundial, mentioned by Brost, seems to be about -1500. Neugebauer mentions a cenotaph of Seti I (c-1300) which shows a sundial and gives directions for its use, indicating the 10 hour division of the day. The assertion that the introduction of the first clock 'could easily have predated the pyramids' is clearly only speculation. The dates given above indicate

Continued . . .

that neither the 12 hour night nor the 12 hour day were in use before about -1800, though I don't know what is known about measurements of the night or the day prior to this. The assertion that 'The Egyptians had some way to measure the day at the time they measured the year, or they could not have obtained such an accurate result' is again speculation, based on incorrect facts and would appear to be at variance with what is known. Firstly, I presume the 'accurate result' refers to the length of the year. Secondly, as seen above, the Egyptians did not have any very accurate result - they used 365 days per year for nearly 3000 years. Thirdly, their method of measuring the night was based on the division of the year into decans, so they didn't have independent measures for the night and the year. Fourthly, their hours varied with the time of year and the day hours and the night hours were not generally equal on a given date.

The assertion at the top of p. 17 is combining several speculations and a definite error into a possibility which can only be described as exceedingly unlikely, even ignoring the 12 hour assertion. We don't know anything about the Egyptian calendar prior to c-3100.

Neither the Egyptians nor the Babylonians actually measured angles - instead they measured some trigonometric function of the angle, such as the ratio of the height to the base, i.e. the tangent. For example, in the Rhind Papyrus, problems 56 - 59 deal with the sloping angle of a pyramid. If we set the base of the pyramid as B and the height as H, then the *seked* of the pyramid is given by 7B/2H, where the 7 converts the ratio into palms per cubit. If θ is the sloping angle of the pyramid face, then the seked = 7 cot θ. The famous Babylonian tablet Plimpton 322 gives $\sec^2 A$ in some right triangles. The idea of an angular unit arises much later. Hypsicles (-2C) divided the day (or the zodiac) into 360 parts, undoubtedly based on the Babylonian base 60 system. Hipparchus, the father of trigonometry (c-150), popularised the use of these 360 parts as angles in general circles.

Continued . . .

There are many speculations as to the origin of the Babylonian base 60 system and most books on the history of mathematics mention several of them. [Boyer, p.28. Phillip S. Jones, "Angular measure" - Capsule 94 in the 31st Yearbook of the NCTM: *Historical Topics for the Mathematics Classroom*, 1969, pp 364-368.] Neugebauer [pp. 17-20] points out that the Babylonians used several other number systems for dealing with various measurements while the sexagesimal system was the system for mathematical work. Hence the choice of 60 as base may simply derive from some measurement conversion - Neugebauer suggests a monetary ratio - though the advantages of base 60 probably served to confirm it as the standard base.

Brost gives one of the many speculations on the origin of base 60, relating it to the approximately 360 day year. Though interesting, it somewhat mixes ideas from several millenia apart. The base 60 system occurs in the earliest Babylonian material, c-3000, long before any extensive astronomical calculations and definitely long before the division of the circle into 360 degrees. The idea that the Babylonians divided the circle into hexads does not have any historical basis that I can locate.

None of the histories of mathematics seem to discuss the early Babylonian calendar, but the *Encyclopedia Britannica* has some information. Basically, the Babylonians used a lunar calendar. Very early, they used the fact that there is a lunar eclipse about every 6 months and based a 'year' of only six months on these eclipses. Such peculiar years may account for the unusual ages of the Biblical patriarchs! Somewhat later, they used a lunar year of 12 months of 29½ days, making 354 days in a year, like the modern Islamic calendar. When the months drifted out of phase with the year, an extra month would be inserted. Later (c-500, according to Neugebauer), this was systematised with the Metonic cycle of 235 months in 19 years. An Assyrian calendar of about -19C had 360 day years, but added 15 days every third year. Hence I cannot see what Mr. Brost's

Continued on page 1. . .*

DEDICATION -- THE F. EMERSON ANDREWS MEMORIAL COLLECTION

March 19, 1988
17; March 1198;

Library
Nassau Community College
Garden City, LI, NY

At the 1987 DSA Annual Meeting, the Society passed a resolution to the effect that the Dozenal collection of materials and books which are housed in the Nassau Community College Library be dedicated to the memory of DSA Founder F. Emerson Andrews. Many of these materials were in fact written by Mr. Andrews.

On March 19, 1988 the materials were dedicated as "The F. Emerson Andrews Memorial Collection", and a plaque attesting to this and to the fact that these materials were donated to the Nassau Community College Library by the DSA was unveiled. Many members of the Society were present, along with Edith Andrews (Mrs. F. Emerson), and other members of her family.

Edith Andrews (seated), with her sons and daughter-in-law immediately behind her, and surrounded by DSA members and friends.

Continued . . .

DEDICATION, Continued

Following the dedication, Mrs. Andrews said: "The assembling of Emerson's mathematical writings constitute an accomplishment worthy of its own tribute to (the Society) . . . . It is good to know that there is a memorial collection, safely housed in a college library. Emerson would be pleased, and proud, as are our sons and I."

DSA President Gene Zirkel (right) with Nassau Community College President Sean Fanelli. The Dedication Plaque, showing the Seal of the DSA, is shown mounted on the door of the room in which the Collection is housed.

The Society is grateful to Dr. Angelo Scordato who was responsible for the organization of the event. Tony is the Chair of the Awards Committee which first proposed the naming of the Library's Dozenal collection. He was also in charge of arranging for the obtaining and mounting of the dedication plaque.

End

DUODECIMAL TEST (IV)

This is the final test in our series, the subject being the conversion of numbers from the decimal to the duodecimal radix, and the reverse. First study the conversion rules given on the next page and on the pages directly following the test. Then perform the following exercises, giving in full the methods by which the answers were reached.

1. Convert the following to dozenals

   (a) by Rule 1, and

   (b) by Rule 2:

   1,920   18,671   324,810

2. Convert the results of Exercise 1 to decimals

   (a) by Rule 3, and

   (b) by Rule 4.

3. Convert the following to common fractions of the dozen system:

   $\frac{13}{44}$ $\frac{189}{670}$ $\frac{99}{100}$

4. Put them back as they were.

5. Use Rule 5 for the following:

   .37   .144   .6336   .916,667

Continued on page 18;. . .

CONVERSION RULES FOR TEST IV

The sheet of conversion rules accompanying test four has been lost. If any of our older members have one, we would greatly appreciate a copy. Apparently it contained eight rules such as the following for converting whole numbers, fractions, and mixed numbers from base dek to base twelve and vice versa.

1. To convert 365. to dozens, divide by twelve and keep the remainders.

```
12) 365
 12) 30   5
      2   6
```

hence 365. = 265;

2. Another way to convert 365 to dozens is to multiply on the left by dek and add the next digit to the right.

```
     365
x     *
     26
+     6
     30
x     *
    260
+     5
    265; = 365.
```

3. To convert 265; to base dek, raise to powers of twelve.

265; = $2(12^2) + 6(12) + 5$ = 365.

Continued on Next Page. . .

DUODECIMAL TEST (IV), Continued from Page 16;

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6. **Use** Rule 6 for the following:

   ;*97,24*   ;372,497   ;055,555

7. Convert 18.62 by Rule 7, and reconvert it by Rule 8.

8. Convert the following by Rule 7, and back by Rule 8.

   3937   393.7   39.37   3.937   .3937

9. Add the following, giving the answer in both systems. (Note the alternation of decimal and duodecimal numbers.)

```
 23,540.
1*#,987;
422,604.
329,*81;
 67,148.
#24,*15;
```

End

CONVERSION RULES, Continued from Page 17;

---

4. Another way to convert 265; to base dek is to divide by dek and keep the remainders.

```
*) 265
 *) 30   5
     3   6
```

hence 265; = 365.

Continued . . .

CONVERSION RULES, Continued

---

5. To convert 0.125 to duodecimals, multiply by twelve and keep the integers to the left of the decimal point.

```
0 .125
  x 12
1 .500
  x 12
6 .000
```

hence 0.125 = 0;16

6. To convert 0;16 to decimals, multiply by dek and keep the integers to the left.

```
     0;16
     x  *
1     ;30
     x  *
2     ;60
     x  *
5     ;00
```

hence 0;16 = 0.125

7. To convert 25.125 to dozens, first convert 25125. to dozens using rule 1 above.

```
12) 25125
 12) 2093    9
  12) 174    5
   12) 14    6
         1   2
```

and then divide 12 659;000 by dek three times

```
*) 12659;000
 *) 1554;600
  *) 18#;300
     21;160
```

hence 25.125 = 21;160

Continued on Next Page. . .

ANTIQUITY, Continued from Page 13;

assertion that the Babylonians reckoned 360 days to a year is based on. Further, the Metonic cycle and its less systematic predecessors were trying to solve the practical problem of keeping the months (whose names often indicated the activity to be carried out in the month) in phase with the year, despite the turbulent history of Mesopotamia which caused numerous restarts of calendars, while the Egyptians, with their 365 day calendar and relative peace, made no attempts at keeping things in phase until very late on.

In Mr. Brost's penultimate and summarising paragraph, the first sentence is factually correct, but the succeeding sentences are speculative (he says 'probably' and 'most likely'). The evidence described above demonstrates that both assertions may be unlikely, though the pyramid builders probably had some form of sun clock. As a scientist, I hesitate to say that something definitely is impossible, unless there is a formal proof, but unless Mr. Brost can provide more information than I have been able to find, I think that any historian would agree that his speculations are just that.

End

CONVERSION RULES, Continued

8. To convert 21;16 to decimals, first convert 2116 to decimals by rule 4 above.

```
*) 2116
 *) 261     8
  *) 30     1
      3     6
```

and then divide 3618.00 by twelve twice

```
12) 3618.00
 12) 301.50
      25.125
```

hence 21;16 = 25.125

End

DOZENAL JOTTINGS

. . .from members and friends . . .News of Dozens and Dozenalists. . .

ARTHUR WHILLOCK writes from England: "A great honour to be elected Fellow! (At the 1987 DSA Annual Meeting -- see the *Bulletin*, number 59;) The testimonial will be framed for hanging alongside my collection of abacuses (abaci??). It is a non-sexist rank I take it. One of our nursing grades is 'sister', whether male or female". . .

Also from England **CEDRIC SMITH**, commenting on a Christmas card, said that the so-called "Prayer of St. Francis" (Lord, make me an instrument of your peace, etc.) is probably attributed incorrectly to the saint. He goes on: "It seems rather on a par with the legend that when an apple struck Newton on the head, he suddenly thought 'Of course, gravitation!', just as Archimedes shouted 'Eureka!' and jumped out of his bath. Malcolm Smithers, one of the Editors of *Colson News*, says that the idea of the apple hitting Newton on the head is a later improvement of the legend. But some close friend of Newton's originally said that it was watching apples fall from trees which suggested the idea to Newton." Cedric also reported that a friend had suggested to him that in the year 12321 his age would be 4 times a triangle and X of a square. He comments: "That is a very elegant result, but I fear I will not live long enough to see it realized!". . .

Several people wrote to us in the process of sending Dues for 1988: **PAUL SCHUMACHER** (NJ); **AL LOPEZ** (ME) enclosed an extra donation (Thanks!); **NEELA LAKSHMANAN** (PA) reports that she has been on the faculty of the University of Scranton as an Assistant Professor since last September (Congratulations!); and **CHARLES F. MARSCHNER** (FL) also sent us a donation (Many thanks!). He tells us he is now 75 years of age, and has believed in the Dozen system since he was 17, due to a good math prof! . . .

Continued . . .

DOZENAL JOTTINGS, Continued from Page 1#;

JOHN CHURCHMAN writes from Iowa regarding his father, **HENRY CHURCHMAN** (a past Editor of this *Bulletin*). "Dad continues to do well in spite of surgery this summer. He recuperated well and sends best wishes to all."

We heard from **KENNETH C. McCOLLOCH**, a weather observer with Environment Canada, working in "The Garden Spot of the Arctic" -- Eureka, North West Territories, Canada. He has been doing this work for over 30 years. "Most of this time I have been living in the Arctic, and have had almost no contacct with any members of the Dozenal Society, except by mail. The only member I met was Mr. Humphreys, whom I met in Wilmette, IL a long time ago"...

Continued . . .

A GROSS ERROR???

---

6 = ½ dozen

12 = 2 dozen

Multiplying the above we obtain:

6 x 24 = ½ x 2 = 1 dozen, OR

144 = 1 dozen!

--Rochelle Meyer
Nassau Community College

DOZENAL JOTTINGS, Continued

EDITH ANDREWS (Mrs. F. Emerson Andrews) wrote to us with some information we had requested, and commented on her first use of a computer: "I am not converted! I could have written the whole thing by hand in five minutes...This house (her son's) has four computers and no good, old-fashioned typewriter". (Many would agree with her, I'm sure.)

The Society is saddened to learn of the deaths of two members within the past year: **A. ADLER HIRSCH**, number 231; of Shreveport, LA, passed away in July of 1987. Also, **SAM SESSKIN**, number 266; passed away in December. We extend our sympathy to their families and colleagues.

Continued on Page 24;. . .

THE DSA ANNUAL MEETING

IS WHERE IT'S ALL HAPPENING!

WE'D <u>LOVE</u> TO SEE YOU THERE

October 14 and 15, 1988
(12; and 13; October, 1198;)

WATCH FOR THESE IN THE NEXT ISSUE OF THE *BULLETIN*!

Whole Number 5#, September, 1988

SCHEDULE, 1988 ANNUAL MEETING
October 14 and 15, 1988
Nassau Community College
Garden City, LI, NY

FUNDAMENTAL OPERATIONS IN THE DUODECIMAL SYSTEM
Jay Schiffman
Jersey City State College
Jersey City, NJ

SQUARE INTEGERS, N^2
Charles W. Trigg
San Diego, CA

THE DOZENAL CLOCK
Paul Rapoport
McMaster University
Hamilton, Ontario, Canada

. . .With additional debate on the issue of Dozenal metrology in antiquity, as we receive it. THIS LAST IS UP TO YOU!

The following are available from the Society

1. Our brochure (free)
2. "An Excursion in Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, Oct. 1934. (Free.)
3. *Manual of the Dozen System* by George S. Terry ($1;00)
4. *New Numbers* by F. Emerson Andrews ($10;00)
5. *Douze: Notre Dix Futur* by Jean Essig, in French ($10;00)
6. Dozenal Slide rule, designed by Tom Linton ($3;00)
7. Back issues of the *Duodecimal Bulletin* (as available) 1944 to present ($4;00 each)

DOZENAL JOTTINGS, Continued from Page 22;

Welcome to new members:

Number 2#3 **VERA SHARP HANDY**, wife of **JAMISON HANDY**, of Pacific Palisades, CA, who joined following the last DSA Annual Meeting...

2#4 **SANI USMAN BUNZA**, a lecturer in Mathematics at the College of Education in Sokoto, Nigeria...

2#5 **VICTOR J. BLIDEN**, a student from Aurora, CO. Both Sani and Victor joined in response to materials received from the Society...

Two others joined in response to an advertisement which we had placed in the *Journal of Recreational Mathematics*:

2#6 **JAVIER PEREZ**, a student at Austin Community College in Austin, TX...and,

2#7 **FRED A. MILLER**, from Elkins, WV...

End

DON'T KEEP THIS MAGAZINE

Do you discard your copies of the *Bulletin* after you have read them? Or do they gather dust on a shelf in your attic? Why not pass them along to your local library, or to a school library. Perhaps some nearby math teacher would appreciate a copy. You can also just leave them in a dentist's office or other waiting area.

Help spread the word!

(If you ever need a back copy, we'd be glad to help.)

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition—("Who needs a symbol for nothing?")—the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accomodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used, and as early as 1585, Simon Stevin stated that the duodecimal base was to be preferred to the base ten.

The parallel seems tenable. The notation of the dozen base better accomodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS.**

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

COUNTING IN DOZENS

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*	#	*10*
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10,* and is called *do,* for dozen. The quantity *one gross* is written *100,* and is called *gro. 1000* is called *mo,* representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents 5 units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5,* and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3#2*	Two ft. eight in.	*2;8'*
19#	*1000*	Eleven ft. seven in.	*#;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53.* Using this *"which is"* step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2#,* which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
 12 ) 30  + 5
  12 ) 2  + 6
       0  + 2   Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *.

For more detailed information see *Manual of the Dozen System* ($1;00).

DETACH HERE -- OR PHOTOCOPY

We extend an invitation to membership in our society.
dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______ LAST ______ FIRST ______ MIDDLE

Mailing Address (for DSA items) ______

______ (See below for alternate address)

Telephone: Home ______ Business ______

Date & Place of Birth ______

College ______ Degrees ______

Business or Profession ______

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______

Address ______

Year & Math Class ______

Instructor ______ Dept. ______

Other Society Memberships ______

Alternate Address (indicate whether home, office, school, other)

______

Signed ______ Date ______

My interest in duodecimals arose from ______

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530