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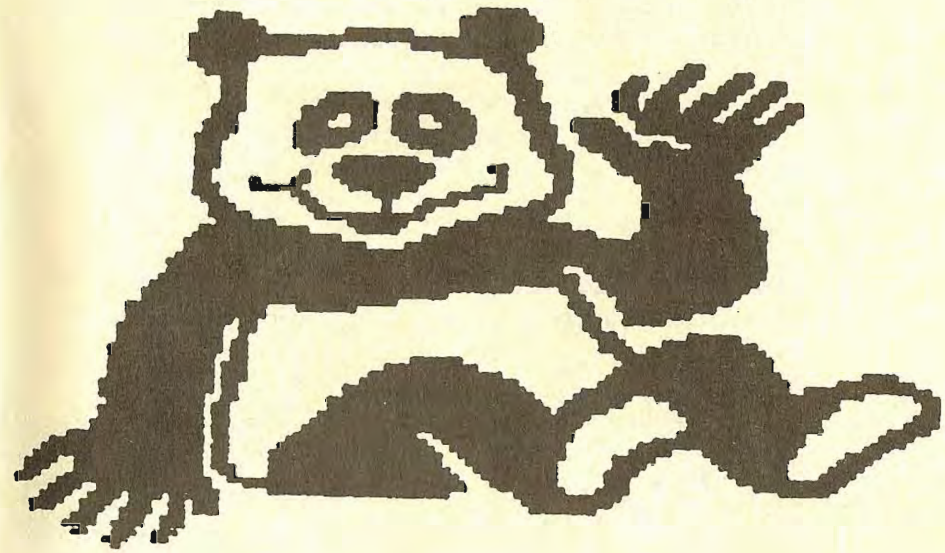

PANDA-ING TO BASE INSTINCTS
See page #;

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THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

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The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Years ago, as you can see from our seal, we used X and Ɛ. Both * and X are pronounced "dek". The symbols # and Ɛ are pronounced "el".

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus ½ = 0.5 = 0;6.

THE DUODECIMAL BULLETIN

Whole Number Five Dozen Four

Volume 2#; Number 2;

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IN THIS ISSUE

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A USEFUL APPLICATION OF TERNARY DECIMALS

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Kean College of New Jersey
Jersey City State College

INTRODUCTION:

The ternary scale (expansion in base 3 utilizing only the symbols 0, 1, and 2) plays a useful role in mathematics. One of the entities in which amateur and professional mathematicians alike take great delight is the Cantor Ternary Set K. This set owes its appeal in no small measure to its pathological behavior. (The set serves as an example illustrating exceptional rather than traditional behavior in mathematics.) While the basic goal of this article is not to present a mathematical treatise concerning the striking properties possessed by K, we nonetheless demonstrate the ternary decimal expansion and its relationship to K.

To initiate our discussion, recall the decimal system. One has in succession immediately to the right of the decimal point the place values tenths, hundredths, thousandths, ten thousandths, etc. or 10^{-1}, 10^{-2}, 10^{-3}, 10^{-4},... where $10^{-n} = 1/10^n$ for any counting integer n. Thus 0.35729 represents in expanded notation $(3\times10^{-1}) + (5\times10^{-2}) + (7\times10^{-3}) + (2\times10^{-4}) + (9\times10^{-5})$, or equivalently, 3/10 + 5/100 + 7/1,000 + 2/10,000 + 9/100,000 = 35,729/100,000. In a similar manner ternary decimals have the successive place values immediately to the right of the decimal point corresponding to thirds, ninths, twenty-sevenths, eighty-firsts, etc, or 3^{-1}, 3^{-2}, 3^{-3}, 3^{-4},... where $3^{-n} = 1/3^n$ for some counting integer n. Thus $(.2222)_3$ represents in expanded notation $(2\times3^{-1}) + (2\times3^{-2}) + (2\times3^{-3}) + (2\times3^{-4})$ or 2/3 + 2/9 + 2/27 + 2/81 = 80/81.

Our next goal is to demonstrate how one converts rational numbers in base ten to ternary decimals. In order to obtain the ternary expansion of a fraction such as 1/5, we proceed as follows. Multiply the fraction by 3, expressing the result as a mixed number. Thus 3(1/5) = 0 + 3/5.

We refer to the integer portion of the result as the <u>product</u> and the fraction as the <u>remainder</u>. Now repeat the above process using the remainder obtained in the last step. The products thus obtained are the digits of the ternary expansion. Thus

3(1/5) = 3/5, Product = 0, Remainder = 3/5

3(3/5) = 9/5 = 1 4/5, Product = 1, Remainder = 4/5

3(4/5) = 12/5 = 2 2/5, Product = 2, Remainder = 2/5

3(2/5) = 6/5 = 1 1/5, Product = 1, Remainder = 1/5.

Hence $1/5 = (.012101210121...)_3$

In expanded notation, 1/5 = (0 x 1/3) + (1 x 1/9) + (2x 1/27) +(1 x 1/81) + (0 x 1/243) +(1 x 1/729) + (2 x 1/2187) +(1 x 1/6561) +... or 1/5 = 0/3 + 1/9 + 2/27 + 1/81 + 0/243 + 1/729 + 2/2187 + 1/6561 +...

It is easily seen that the ternary expansion of a proper fraction is not necessarily unique. (To see this, note that $2/3 = (.2000...)_3$.) However, the representation can be shown to be unique in the sense that no proper fraction has a ternary expansion expressible in more than one way without utilizing the digit 1. The importance of this idea will come into play later in our discussion. We shall be concerned with those proper fractions which utilize solely the digits 0 and 2 in their ternary expansions and never use the digit 1. Such points remain in K after all other points between 0 and 1 have been deleted in some stage of the construction of this set. This fact will easily be reflected upon once the formulation of K and knowledge of ternary decimal expansions and geometric progressions become more lucid. (E.g., it will be noted that 1/5 is deleted in the second stage of construction of K, as a 1 appears as the second digit in the ternary expansion of 1/5.)

At this juncture it is useful to discuss the concept of the sum of an infinite geometric progression. Knowledge of this will aid in converting ternary decimals to base

Continued...

ten proper fractions. A geometric progression is one in which each term after the first is multiplied by the same constant term to obtain the next term. This common constant is called the common ratio of the geometric progression. To cite an example, the progression 1,3,9,27,... is geometric, and each term after the first is obtained by multiplying its predecessor by 3, the common ratio. Similarly, the geometric progression 1, 1/2, 1/4, 1/8,... has common ratio 1/2.

It is well known that if a denotes the first term of an infinite geometric progression, r connotes the common ratio, and S represents the sum, then $S = a/1-r$, if $|r| < 1$. For example, the sum of the infinite geometric progression

$2/3 + 2/9 + 2/27 + \ldots$ is given by $S = \frac{2/3}{1-1/3} = \frac{2/3}{2/3} = 1$,

where a = 2/3 and r = 1/3. ternary decimal $(.222\ldots)_3$ represents one in decimal notation. In a similar manner $(.00222\ldots)_3$ represents the fraction 1/9; since .00222... = 0/3 + 0/9 + 2/27 + 2/81 + 2/243 +... where a = 2/27 (here is the first non zero term) and r = 1/3 so that

$S = \frac{2/27}{1-1/3} = \frac{2/27}{2/3} = 2/27 \cdot 3/2 = 1/9$. One can use some ingenuity to determine the base ten representation of $(.0022002200\ldots)_3 = 0/3 + 0/9 + 2/27 + 2/81 + 0/243 + 0/729 + 2/2187 + 2/6561 + \ldots$, and examine that 2/27 + 2/81 = 6/81 + 2/81 = 8/81 and
2/2187 + 2/6561 = 6/6561 + 2/6561 = 8/6561.

Noting that the expression has alternately a pair of zeros followed by a pair of two's, one is forming the sum of the progression $8/81 + 8/(81)^2 + 8/(81)^3 + \ldots$, where a = 8/81 and r = 1/81 yielding

$$S = \frac{8/81}{1-1/81} = \frac{8/81}{80/81} = 8/81 \cdot 81/80 = 1/10$$

Thus $1/10 = (.0022002200\ldots)_3$ and is a geometric progression (in base 3, the repeating fraction 0.00220022... equals 22/10000 + 22/100000000 +...)

Our next goal is to present an elementary treatment of the Cantor Set K. For our discussion the closed unit interval [0,1] denotes all real numbers between 0 and 1 inclusive. (In general, the closed interval [a,b] denotes all real numbers between a and b inclusive, while the open interval (a,b) connotes all reals strictly lying between a and b, not including the endpoints a and b.) The length of either of these intervals is the difference between the righthand and lefthand endpoints, namely b-a. Moreover, the trisection of an interval is the subdivision of the interval into three equal parts. For example, the trisection of [0,1] consists of all points from 0 to 1/3 plus all points from 1/3 to 2/3, together with all points from 2/3 to 1. In short, we are dealing with the three closed intervals [0,1/3], [1/3,2/3], and [2/3,1].

One is now in position to describe the construction of K. Start with [0,1], and trisect it, removing (1/3, 2/3), called the open middle third. Form the remainder $K_2 = [0,1/3] \cup [2/3,1]$, where $\cup$ denotes the union of these closed intervals. Now trisect each of the closed intervals forming K_2; namely [0,1/3] and [2/3,1], deleting their respective open middle thirds (1/9,2/9) and (7/9,8/9). Form the residue $K_3 = [0,1/9] \cup [2/9,1/3] \cup [2/3,7/9] \cup [8/9,1]$. Trisect each of the above four closed intervals forming K_3, deleting their respective open middle thirds (1/27,2/27), (7/27,8/27), (19/27,20/27), (25/27,26/27) and form the remainder $K_4 = [0,1/27] \cup [2/27,3/27] \cup [6/27,7/27] \cup [8/27,1/3] \cup [2/3,19/27] \cup [20/27,21/27] \cup [24/27,25/27] \cup [26/27,1]$. Repeat the process and continue indefinitely in this manner. Form $K = \bigcap_{n=1}^{\infty} K_n$ (K is the intersection of all the K_n's and consists of those points common to every K_n.) It would appear that almost nothing is left in the Cantor Set. This is not the case, however. While K has length zero, it possesses as many points as [0,1], a number higher than the counting integers! In fact, [0,1] and K

Continued on page 19;...

DUODECIMAL TESTS *TEST NUMBER TWO*

This is the second of four tests which used to be required of all applicants to the DSA. Test #1 was printed in Bulletin #51;. Note the use of X and Ɛ .

Answers may be written or typed on this sheet. If additional sheets are used, refer to the question by numbers. Send your completed test to the Chairman, Member Qualification Committee. The next test will be sent to you upon satisfactory completion of this one. Keep one copy for your files.

1. Add the following: *84X* + *372*; *93Ɛ8* + *64X5* + *Ɛ927*.*

2. Add:

25,062	*123,456*	*4,648*	*ƐƐX,832*	*Ɛ,X98,765*
89,073	*234,567*	*4,363*	*ƐX1,514*	*4,321,0ƐX*
3Ɛ,X41	*345,678*	*3,894*	*ƐXX,956*	*9,876,543*
78,205	*456,789*	*5,117*	*ƐXX,437*	*2,108,X98*
	567,89X	*7,479*	*ƐƐX,694*	*7,654,321*
	678,9XƐ	*1,532*		*ƐX9,876*
		2,721		*5,432,10Ɛ*
		6,28X		

3. Subtract:

Ɛ,874	*43,8X1*	*ƐXX,XX9*	*123,456*	*1,111,110*
X,732	*9,742*	*ƐƐ,ƐƐƐ*	*78,9XƐ*	*987,654*

4. When the motorman started his one-man trolley-car, there were *5* passengers aboard, *3* men and *2* women. One of the women was carrying her *8* month old son. At the first stop, *18* men and *7* women got on. At the second, *1* woman and *2* men got off, while *10* women and *2* men got on, and in addition, a man and his wife with their *2* sons, one *14* and the other *4* years old. At the next stop, *15* men and *11* women got off, including the woman with the baby. Next

* Italicized numerals are duo-decimals.

TEST NUMBER TWO, Continued

stop was the end of the line. (a) How many persons were then on the car? (b) How many males, how many females? (c) How many persons rode altogether? (d) How many paid fares? (In this and the following problems, please give the process by which you reach your result.)

5. In the prairies states the roads run on section lines, an even mile apart. Mr. Tweet, a farm adviser, travels here and there, telling the farmers why they are not making any money. Starting from his headquarters at the county-seat, he drove north *47* miles, then east *32*, south *18*, west *35*, north *16*, east *1Ɛ*, south *46*, west *Ɛ*, and quit for the night. (a) How many miles did he travel? (b) How far was he from his starting point when he quit?

6. Mr. Dybwad died, leaving a wife and *4* sons, Alfred, Benjamin, Charles and David. Under his will, *1/2* of his estate went to his wife, excepting that out of her share she was to pay minor beneficiaries to the extent of *1/10* of the total estate. Al got *1/2* of his mother's entire half; Ben got half as much as Al; Charlie half as much as Ben; and Dave, poor guy, half as much as Charlie. The attorney got what was left. Dave's share *$9X6*. (a) How much did each heir get? (b) How much did the minor beneficiaries get? (c) How much did the attorney get? (d) What was the value of the estate?

7. Papa wanted to give Willie a lesson in systematic saving, so, on the first of April he said, "Now, Willie, I am dropping a penny into his box. Tomorrow I shall drop *2*, the next day *4*, and so on, doubling the number each day through the month. By the end of the month there will be a nice little pile of pennies." Assuming that Papa kept his promise, (a) How many pennies would he drop on the last day of the month? (b) How many altogether would be in the box? (c) If it had been March? (d) February?

8. Take any number of *3* digits. Double it. Double again. Add *398*. Divide by *4*. Subtract the original number plus *Ɛ5*. How much is left?

POOR MAN'S BASE CONVERSION ALGORITHMS

Jean Kelly
New York, NY

In a letter from Bill Leonhardt, 18*; regarding his HP calculator, he called the MOD (or remainder) function the "poor man's base conversion" (see Volume 2*; number 2; pages 1# to 20). This set me to thinking. We often convert from base ten to another base by repeatedly dividing by the new base and keeping the remainders. Thus to change decimal 2056 into base twelve we have:

```
12 )2056          remainder
 12 )171              4
  12 )14              3
   12 )1              2
       0              1
```

and so $2056_{ten} = 1234_{twelve}$

In addition to the MOD function, we also use the *integer* division function which is often called DIV. Thus the first division above would be equivalent to

2056 DIV 12 = 171, and
2056 MOD 12 = 4.

The following algorithm will change a decimal number, DECIM, into any other base, NUBASE, storing the digits in the first N digits of an array named DIGIT:

```
CONVERT(DECIM, NUBASE, DIGIT, N)
   K = 1
   WHILE (DECIM>0) DO
      DIGIT[K] = DECIM MOD NUBASE {Save the remainder}
      DECIM = DECIM DIV NUBASE    {Get quotient for
                                   next division}
      K = K + 1
   END WHILE
   N = K - 1                {N is the number of digits
RETURN                           in the new numeral}
```


POOR MAN'S BASE CONVERSION, Continued

The two parameters, DECIM and NUBASE, are input to the algortihm, while the contents of the array DIGIT and N are output from the algorithm.

For example, to change decimal 179 into dozenals, we would call

CONVERT(179, 12, DIGIT, N).

This would return N = 3, and

```
            -------------------------
DIGIT =     |   |   |   | 1 | 2 | 11|
            -------------------------
                          ^   ^    ^
                          |   |    |
                          3   2    1 --> elements of the
                                         array DIGIT
```

since $179_{ten} = 12\#_{twelve}$.

Continued . . .

FROM THE COVER.......

WHAT DO YOU CALL A 6-FINGERED PANDA?

After reading "Secrets of the Wild Panda" by George B. Schaller in the March 1986 issue of The National Geographic, volume 169, number 3, pp. 284 - 309, I learned that pandas have *six fingers* on each hand! I would like to propose that we make the Giant Panda the unofficial mascot of the DSA.

Of course, a mascot needs a name. Can you think of an appropriate one? Please send us your suggestions for a name for our panda.

-Gene Zirkel, President

POOR MAN'S BASE CONVERSON, Continued

In order to change a number back to base ten, we have the following algorithm:

```
CHANGE BACK(DECIM, NUBASE, DIGIT, N)
   DECIM = DIGIT[N]
   FOR K = (N-1) DOWNTO 1 DO
      DECIM = DECIM * NUBASE + DIGIT[K]
   END FOR
RETURN
```

The three parameters, NUBASE, N, and the array DIGIT, are input to this procedure, while DECIM is the output parameter.

For example if the array DIGIT contains 1, 2 and 11, as in the example above, then the call

CHANGE BACK(DECIM, 12, DIGIT, 3)

would set DECIM = $\{[(1 * 12) + 2] * 12\} + 11$
$= 1 * 12^2 + 2 * 12 + 11$
$= 179_{ten}$

Finally to change a number from any base to any other base we would use both of the above algorithms. First change the number to base ten, and then to the desired base. Thus if the array DIGIT contains the 4 digit number 1234 in base 5, and we wish to change it to base 6, we would simply invoke the two algorithms as follows:

CALL CHANGE BACK(DIGIT, 5, 4, DECIM)
CALL CONVERT(DIGIT, 6, N,DECIM).

The first call would set
DECIM = $(\{([(1 * 5) + 2] * 5) + 3\} * 5) + 4$
$= 1 * 5^3 + 2 * 5^2 + 3 * 5 + 4$
$= 194_{ten}$

The second call would change 194_{ten} into 522_{six} setting

DIGITS = | | 5 | 2 | 2 | since $194_{ten} = 522_{six}$
and storing a 3 in N since 522 contains 3 digits.

See next page . . .

SEXIST LANGUAGE

Our Constitutions and By-Laws, originally written in 1944, contain sexist language. For example, our Vice-President and Editor are referred to as "he". Most of such terminology is easy to change, but we have run into one snag -- the term "Fellow".

We have two special types of members:

Honorary Member is for a person who is not already a member of the DSA

Fellow is for a person who is presently a member.

So we need a non-sexist term, different from "honorary", to replace "Fellow".

One suggestion has been

Distinguished Member.

What do you think? Do you have a better suggestion? Please let us know. Thanks.

-Your Constitution and By-Laws Revision Committee

POOR MAN'S BASE CONVERSION, Conclusion

The two algorithms above are given generically in what is known as pseudo code. They can easily be implemented in any programming language such as BASIC, FORTRAN, Pascal, etc., as well as on either a programmable or a non- programmable calculator.

SELF-DESCRIPTIVE NUMBERS

Mark Calandra
Chappaqua, NY

A number whose expression is a complete inventory of its own digits is called, for lack of a better name, a self-descriptive number. A simple example will illustrate this esoteric quality: in base 5, the only digits are 0, 1, 2, 3 and 4. We seek a 5-digit number whose first digit gives the number of zeros that it contains, whose second digit is the number of ones in the number,... and whose fifth digit is the number of fours in the number. Each digit in the base is tallied in ascending order. If we consider, for instance, $22041_{(5)}$, we find 1 zero, 1 one, 2 twos, no threes and one four, but 11201 is not 22041. In fact, only one number in base five is self-descriptive: 21200. It provides its own description -- 2 zeros, 1 one, 2 twos, no fours and no threes.

The concept of a self-descriptive number may be a bit difficult to understand at first, but its amazing infrequency of occurrence certainly warrants consideration. At first it seems misleadingly easy to try to construct such numbers, but it is fabulously frustrating. The task is comparable to creating a sentence of the type "This sentence contains thirty-nine A's, twenty-four B's,...", in which even the most minute change creates a wave of far-reaching effects, each of which must be corrected.

I have found what may very well be the only duodecimal self-descriptive number. It is 821000001000. It provides its own complete description in terms of all twelve of the duodecimal digits: 8 zeros, 2 ones, 1 two, 1 eight and nothing of anything else.

HOW MANY POINTS ON A COMPASS?

Archeologists have discovered a sixteenth century compass which consists of a bronze disk with a central well in which a magnetic needle floated on water. *Two dozen* compass points are marked on the outer circle, not the traditional dozen and one-third.

-from *Mysteries of the Past*

NUMBER BASES ELSEWHERE *PART I*

In "Computing Large Factorials", The College Mathematics Journal, volume 16, number 5, Nov. 85, pp. 403-412, author Gerard Kiernan shows how the knowledge of number bases helped him to solve a computer problem involving significant digits.

Most computers work only with 6- or 7-digit accuracy. A typical device used to handle larger numbers is to put each digit in an array; thus the number 1,234,567,890 could be stored in in an array using ten memory locations. If you wanted to compute 2206! (2206 factorial) you would need 6,421 memory locations just for the digits of the answer not counting the locations needed for the intermediate products and 'carrys'.

But Professor Kiernan's method saves both computer time and memory space by using bases larger than ten, such as base one hundred or base one thousand. For example, to add 1,234,567,890 and 2,345,678,901 in base ten would require 3 arrays of ten memory locations each plus ten additions of the corresponding digits. However, if we express these numbers in base one thousand -- using parentheses to indicate each digit -- we are adding only 4-digit numbers,

(1)(234)(567)(890) + (2)(345)(678)(901).

Hence our arrays need only contain 4 memory locations each, and instead of ten additions with carrys we need only perform 4.

Using these ideas, Professor Kiernan shows how to compute factorials containing thousands of digits using bases such as one thousand and 100,000,000.

Do you have an idea to share with our members? Why not submit an article to the Bulletin?

HOW TO CONVERT FRACTIONS

Honorary member Igor Valevsky, number #7, writes from Brazil about an easy way to convert fractions which are less than 1.

To convert from base ten to base twelve you repeat these three steps over and over using base ten arithmetic:

1. divide by 5
2. add the quotient to the dividend
3. shift the fraction point one digit to the right, and remove the whole number which is the next digit in the answer

For example, to convert 0.125 from base ten to base twelve

	.125
divide by 5	+.025
add	.150
shift	1.50
remove the digit 1	.50
divide by 5	+.10
add	.60
shift	6.0
remove the digit 6	.0

Hence decimal .125 = dozenal ;16

To convert from dozenals to decimals we repeat the following steps using duodecimal arithmetic:

1. divide by 6
2. subtract
3. shift

For example, to convert 0;16 to base ten we have

	;16
divide by 6	-;03
subtract	;13
shift	1;3

remove the digit 1	;3
divide by 6	-;06
subtract	;26
shift	2;6
remove the digit 2	;6
divide by 6	-;1
subtract	;5
shift	5;0

Hence dozenal ;16 = decimal .125

To convert a mixed number such as 13.910 separate the whole number from the fraction, and convert them separately.

Thus 13. = 11; and we convert .910 as follows:

	.910
divide by 5	+.182
add	1.092
shift	10.92
remove the digit 10 (or *)	.92
divide by 5	+.184
add	1.104
shift	11.04
remove the digit 11 (or #)	.04
divide by 5	+.008
add	.048
shift	0.48
remove the digit 0	.48
divide by 5	+.096
add	.576
shift	5.76
remove the digit 5	.6
etc.	

Thus decimal 13.933 = dozenal 11;*#05...

BOOK REVIEW

THE SEVEN DAY CIRCLE, the History and Meaning of the Week

by Eviatar Zerubavel

The Free Press, a division of Macmillan Inc, NY, 1985

I started to read this book because of an article in a newspaper stating that it mentioned a six day week. It does. It also mentions a twelve day week used by our ancestors in ancient Southern China. But although there are many references to six day weeks, this is not an especially pro-dozenal book. What it is, however, is a fascinating look at the history and the sociology of something most of us take for granted -- the seven day week. I quickly became intrigued and absorbed in this interesting work, and I finished it the day after it arrived.

I was surprised to learn that Russia began using a six day week the day before I was born; that Africans used a six day week, and even more popularly, a week containing one-third of a dozen days. In addition, the Javanese used a week with one-half dozen days.

Unfortunately for Dozenalists, our forebears also used weeks containing from two to ten days, as well as weeks of twelve, thirteen, nineteen and twenty days duration.

The author points out that our current seven day week comes from the seven known 'planets': Saturn, Jupiter, Mars, Sun, Venus, Mercury, and Moon. If the ancients had been aware of our heliocentric model of the planetary system we might have a six day week today!

On page 100 we find both the advantages of a six day week, as well as the advantages of the six day cycle of classes used by some schools. This cycle essentially ignores both the days of the week and the disruptions that holidays otherwise would cause.

BOOK REVIEW, Continued

All in all, I found this brief book an eye-opener which made me think about something I always took for granted. I resonated with almost all of the author's well-researched conclusions, and I think that you will also. And still, I can't help but wonder about a six day week.
What if we . . . ?

PANDA - GRAM

Perfect
Arithmetic
Needs
Dozenal
Addition.

A QUESTION

John Earnest, 250; asks if there are names for digits in bases other than base do or base dek? For example, computer scientists use the letters A through F for the digits from ten through fifteen, but do they have names for these digits?

A second question arises. Is there a method for pronouncing numerals with more than one digit such as '16'? For example, in dozenals we say 'do six'. In base dek we say 'sixteen'. Is there a way to prounce '16' in hexadecimals or in octals?

Are there words for large numbers such as 'gross' or 'thousand'? Or must one always resort to such awkward phrasing as 'that is bigger than one zero zero zero' in bases other than do or dek?

Are there any generic words for the base (10 in every base) or for the base squared (100)? That is, a word which would mean 'do' in duodecimals, but 'dek' in base ten, and 2 in the binary base. Or a word that would mean 'gro' in base twelve, but 'one hundred' in base ten, and 9 in base 3?

Can anybody help John out?

DIZEBAK?

The Post Office succeeded in delivering a piece of mail to us addressed to the Dizebak Society of America. We congratulate them.

However, can you spot the simple mistake the sender made? Uf bitm wgt bit?

TERNARY DECIMALS, Continued from page 7;

contain as many points as the entire real line. The next two paragraphs address the above ideas.

Let us note that in the first stage of the construction of K, one open interval (1/3,2/3) of length 1/3 is removed, while in stage two, two open intervals (1/9,2/9) and (7/9,8/9), each of length 1/9 are deleted. In stage three, four open intervals (1/27,2/27), (7/27,8/27), (19/27,20/27), (25/27,26/27), each of length 1/27 are removed, etc. The length of the deleted open intervals is the sum of the infinite geometric progression 1/3 + 2/9 + 4/27 +... with a=1/3 and r=2/3 yielding $s = a/1-r = \frac{1/3}{1-2/3} = \frac{1/3}{1/3} = 1$. As the length of [0,1] = 1-0 = 1, the length of K = 1 - 1 = 0.

Despite the zero length of K, the number of points in K coincides with the number of points in [0,1]. To appreciate this, let us agree that each point in [0,1] has a ternary expansion of the form $\sum_{n=1}^{\infty} a_n/3^n = a_1/3^1 + a_2/3^2 + a_3/3^3 + \ldots$ where a_n is either 0 or 2. We know that each point in [0,1] has a binary expansion (expansion in the scale of 2) of the form $\sum_{n=1}^{\infty} b_n/2^n = b_1/2^1 + b_2/2^2 + b_3/2^3 + \ldots$ where each b_n is either 0 or 1 and where we assume that in certain instances not all the b_n's after a certain term are zero (i.e., the series may in certain cases be non-terminating). Establishing a one-to-one correspondence $a_n = 0$ whenever $b_n = 0$ and $a_n = 2$ whenever $b_n = 1$ verifies the claim.

Let us note that the place values in the binary decimal expansion are halves, fourths, eighths, sixteenths, etc., or 2^{-1}, 2^{-2}, 2^{-3}, 2^{-4}, etc., where $2^{-n} = 1/2^n$. Observe

Continued . . .

$1/2 = (.1000\ldots)_2$ while $2/3 = (.1010\ldots)_2$. To expand 2/3 in binary notation, consider the following set of computations:

$2(2/3) = 4/3 = 1\ 1/3$. Product = 1, Remainder = 1/3

$2(1/3) = 2/3$. Product = 0, Remainder = 2/3

$2(2/3) = 4/3 = 1\ 1/3$. Product = 1, Remainder = 1/3,

and so forth. As in the ternary expansion, continue the process and write down all the products (integer parts) obtained in succession and always multiply the remainders (fractional parts) of the mixed number by 2, expressing all improper fractions as mixed numbers.

Note that $.10101\ldots = 1/2 + 1/8 + 1/32 + \ldots$ so that $a = 1/2$ and $r = 1/4$ yielding $s = a/1-r = \frac{1/2}{1-1/4} = \frac{1/2}{3/4} = 1/2 \cdot 4/3 = 2/3$ as claimed.

While it is readily apparent that the endpoints of the closed intervals in the construction such as 1/3, 2/3, 1/9, and 2/9 remain in K after all stages, the points $1/4 = (.020202\ldots)_3$ and $3/4 = (.202020\ldots)_3$ remain even though they are not endpoints.

Obviously, the point $1/2 = (.111\ldots)_3$ is deleted in the first stage of the construction of K as the digit 1 occurs as the initial digit to the right of the decimal point in the ternary expansion. Of course, 1/2 lies in the deleted open interval (1/3,2/3). Similarly, $1/5 = (.012101210121\ldots)_3$ is deleted in the second stage of construction of K as the digit 1 is used as the second digit to the right of the decimal point in the ternary expansion. We observe that 1/5 lies in the deleted open interval (1/9,2/9). To find a point deleted in the third stage of construction of K where the digit 1 is used as the third digit to the right of the decimal point in its ternary expansion, we note that $1/25 = (.001002011\ldots)_3$ and 1/25 lies in the deleted open interval (1/27,2/27).

The reader can indeed demonstrate that each of the following points lies in K, but fail to be endpoints:

$1/12 = (.0020202\ldots)_3$

$1/36 = (.00020202\ldots)_3$

$1/108 = (.000020202\ldots)_3$

and $1/10 = (.00220022\ldots)_3$

There are a multitude (in fact, more points than the counting integers) of points in addition to the endpoints of the closed intervals in base three utilizing the digits 0 and 2 and hence belonging to K.

We conclude the article with an interesting application of K and its ternary decimal ramifications. Imagine that rain is falling on the closed unit interval (0,1) of the x-axis. How does one provide shelter from the rain? Simply trisect (0,1) and erect a tent in the form of an equilateral triangle in the central part. It protects this central part from the rain except the endpoints 1/3 and 2/3. In short, all points in (1/3,2/3) remain dry. As in the construction of K, trisect each of the two residual pieces and protect the middle part with a tent of the same form. In the third step we erect four more tents, then eight more, etc.

We have the curve now sketched:

$0 - 1/9 \wedge 2/9 - 1/3 \wedge 2/3 - 7/9 \wedge 8/9 - 1$

We observe that some, but not all, points left unprotected after a certain stage remain unprotected throughout the construction. On the other hand, points such as 1/6 are unprotected at stage 1 and then protected at stage 2. Wet points after the initial stage possess ternary expansion, having the forms

0.0... or

0.2... (the succeeding digits can be any combination of 0's, 1's, or 2's.)

Continued . . .

TERNARY DECIMALS, Continued

The dry points after the initial stage possess ternary expansion having the form

> 0.1... (the succeeding digits can be any combination of the digits 0, 1, 2 in the same manner that all points whose base ten decimals begin with the digit 1 lie between the points 1/10 and 2/10.)

After two tents of the second stage are erected, wet points are those whose ternary expansions begin with

> 0.00...
>
> 0.01...   (all succeeding digits can be any combination of 0's, 1's and 2's.)
>
> 0.20...
>
> 0.22...,

The dry points after the second stage possess ternary expansions of the forms

> 0.01...   (all succeeding digits can be any combination of 0's, 1's, and 2's.)
>
> or 0.21...

To briefly summarize, those points which remain wet correspond to those whose ternary expansions use only zeros and two's in the representation after all the tents are erected and hence belong to K. A multitude of wet points in addition to the endpoints of the intervals forming K (such as 1/4 and 3/4) will always remain unprotected. The important idea to realize is that while the endpoints of the intervals forming K represent only a countably infinite set (a set expressible in a one-to-one correspondence with the counting integers), K consists of uncountably many other points. The dry points correspond to those deleted from K after some stage. The ideas expressed in this article help to explain why K is an intricate object enjoyed by mathematicians and decimal enthusiasts on

See next page . . .

See you at the next DSA Annual Meeting
* and # October 1196; in New York!

SHOULD YOU SERVE ON A DSA COMMITTEE???

The work of the DSA is done by many volunteers who serve on Society committees or help with the *Bulletin*.

Should you want to become more involved we could use:

(1) People to review books for the *Bulletin*.

(2) People to review articles submitted to the *Bulletin*.

(3) People to write articles for our *Bulletin*.

(4) People to serve on the following committees:

- Annual Meeting
- Constitution/By-Laws
- Video
- Awards
- Financial
- Nominating

Since committees often work by mail, anyone in any geographical area can serve on any committee.

TERNARY DECIMALS, Conclusion

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all levels. The reader is invited to find additional applications of ternary decimals.

BIBLIOGRAPHY

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Acknowledgment: The author wishes to thank Professor Gene Zirkel for his useful suggestions and comments.

DOZENAL JOTTINGS

...from members and friends...News of Dozens and Dozenalists....

FRED NEWHALL has completed an inventory of the DSA Archives, located at Nassau Community College in Garden City (LI), NY. Fred generously volunteered to do the inventory at our last Annual Meeting. Thanks, Fred!...We've lost track of **JOHN MC CORMICK**, 199; of Innisfail, Alberta, Canada. Can anyone help us out with John's new address?...By this time all DSA members should

Continued . . .

THE DOZENAL TALLEY

Jerry Brost
Gainesville, FL

Almost everyone has had occasion to use the familiar decimal tally, usually when keeping score for some kind of game. In this system the number one is represented by a single hatch-mark (/). The number two has two (//), the number three has three (///), the number four has four (////), and the number five is represented by a diagonal crossmark placed over the four (~~////~~), thus making a sub-base of five. Two clusters of five make ten, producing a decimal system of counting.

This system can be easily modified to yield a dozenal tally. Simply add a second diagonal to the five to produce a cluster of six (~~~~////~~~~). Two sixes make twelve, giving us a dozenal tally.

Experiment with the dozenal tally when playing cards. Try playing up to one gross instead of one hundred. You may also want to value your poker chips in dozenal. This allows a greater variety of interesting ways to wager bets and raises.

Have fun, and try not to lose your pants!

DOZENAL JOTTINGS, Continued

have received their copy of the *Dozenal Journal*, Number 4, Spring 1196;. Note the list of publications available from the DSGB, on the inside back cover...**JAMES A. FORSTER**, 1*3; from Hopwood, PA, just retired after almost three dozen years of teaching, during which he taught our base 12 circular slide rule...**PROF. BAUMEISTER**, 140; writes and asks what * and # would look like on a seven-segment display, ¦-¦ and ¦_¦ ? He suggests _¦ and _¦ as alternates. (But, wouldn't *el* be better as ¦_ ? *-Ed.*)...Reading Hofstadter's book, *Metamagical Themas*, we ran across several references (pp. 303 ff.) to another work: *Notes on Rubik's "Magic Cube"*, by David Singmaster, member number 28*; from London...**H. W. STEELE** sent his DSA dues from his home in Mt. Maunganui, New Zealand, noting that the rate of exchange is "very unfavourable", in that $12.00 US = over $28.00 NZ. Our foreign members are indeed dedicated Dozenalists!...Member **ANTHONY H. SARNO** from Ridge, NY, has been appointed Chairman of the NY State Veterans' Affairs Commission by Governor Mario Cuomo...**CHARLES F. MARSCHNER** writes from Melbourne, Florida, recalling that

Continued...

The following are available from the Society

1. Our brochure (free)
2. "An Excursion in Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, Oct. 1934. (Free.)
3. *Manual of the Dozen System* by George S. Terry ($1;00)
4. *New Numbers* by F. Emerson Andrews ($10;00)
5. *Douze: Notre Dix Futur* by Jean Essig, in French ($10;00)
6. Dozenal Slide rule, designed by Tom Linton ($3;00)
7. Back issues of the *Duodecimal Bulletin* (as available) 1944 to present ($4;00 each)

DOZENAL JOTTINGS, Continued

he was introduced to counting in dozens in 1930 by Calvin Austin Brown, a Math Professor at Mercersburg Academy. "He made us work, but I have never forgotten him." Charles also notes from his travels that "the Europeans and others, while dedicated to metrics, still package bottled and canned goods by 12s. They want to be able to stack their cartons easily! And cartons of 20 tend to be too big and heavy. Of course clocks and navigation equipment are still based on 12 or a multiple of it."...14; DSA members and friends celebrated the Twelve Days of Christmas on January 3rd, 1196; in West Islip, NY: **GENE** and **PAT ZIRKEL, GEORGE ZIRKEL, CRISTINA RIBAUDO, JOHN** and **ANNAMARIA IMPAGLIAZZO, ALICE** and **EDMUND BERRIDGE, FRED NEWHALL, WALTER** and **ROCHELLE MEYER, JOHN** and **KAREN RYAN, TONY SCORDATO, LOU MC CORMICK,** and **HARRIET RIEDESEL.** No business was conducted at this Dozenal gathering but "a good time was had by all"...

Welcome to new members **ISAAC D. RUSSELL,** 29#; and **DONALD R. HART, JR.,** 2*0;. Isaac is the son of Henry E. Russell, formerly of Tenafly, New Jersey, member number 183;. Mr. and Mrs. Russell were friends of Mr. and Mrs. F. Emerson Andrews, and as a child, Isaac often heard of the affairs of the "Duodecimal Society". (F. Emerson Andrews was a DSA founder.) We're glad to have both Isaac and Donald in our ranks...

End

The 1986 DSA Annual Meeting will be held October 10 and 11, 1986, at Nassau Community College, Garden City (LI), NY.

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition—("Who needs a symbol for nothing?")—the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accomodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used, and as early as 1585, Simon Stevin stated that the duodecimal base was to be preferred to the base ten.

The parallel seems tenable. The notation of the dozen base better accomodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS.**

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10,* and is called *do,* for dozen. The quantity *one gross* is written *100,* and is called *gro. 1000* is called *mo,* representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5,* and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3#2*	Two ft. eight in.	*2;8'*
19#	*1000*	Eleven ft. seven in.	*#;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53.* Using this *"which is"* step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2#,* which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
  12 ) 30  + 5
    12 ) 2 + 6
         0 + 2   Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *.

For more detailed information see *Manual of the Dozen System* ($1;00).

DETACH HERE -- OR PHOTOCOPY

We extend an invitation to membership in our society.
Dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______________ LAST ______________ FIRST ______________ MIDDLE

Mailing Address (for DSA items) ______________________________

______________________________ (See below for alternate address)

Telephone: Home ______________ Business ______________

Date & Place of Birth ______________________________

College ______________ Degrees ______________

Business or Profession ______________________________

Employer (Optional) ______________________________

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______________________________

Address ______________________________

Year & Math Class ______________________________

Instructor ______________ Dept. ______________

Other Society Memberships ______________________________

____

Alternate Address (indicate whether home, office, school, other)

____

____

____

Signed ______________________________ Date ______________

My interest in duodecimals arose from ______________________________

____

____

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530