DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

ISSN 0046-0826

THE DUODECIMAL BULLETIN 52;

WHY ARE THESE PEOPLE SMILING? See page 23;.

Volume 2*;
Number 3;
Fall 1985
1195;

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00(US) for one calendar year. Student membership is $3.00 per year, and a Life membership is $144.00 (US).

The Duodecimal Bulletin is an official publication of the DOZENAL SOCIETY OF AMERICA, Inc. c/o Math Department, Nassau Community College, Garden City, LI, NY 11530.

The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Years ago, as you can see from our seal, we used X and Ɛ. Both X and * are pronounced "dek". The symbols # and Ɛ are pronounced "el".

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus ½ = 0.5 = 0;6.

The Duodecimal Bulletin

Whole Number Five Dozen Two

Volume 2*; Number 3

Fall 1195;

IN THIS ISSUE

Patricia McCormick Zirkel, *Editor*
Editorial Office:
6 Brancatelli Court
West Islip, New York 11795

DOZENAL SOCIETY OF AMERICA

Annual Meeting

Friday to Sunday
October 11 to 13, 1985
(October # to 11; 1195;)

Nassau Community College
Garden City, LI, NY 11530

Schedule

Friday Evening, October 11, 1985

A theatre party, with friends and spouses. "Two by Two, a tale of Noah's Ark with music by Richard Rodgers will delight all types of audiences. Broadhollow Theatre, Route 110, Farmingdale, LI, NY.

Following the theatre, we will gather at the home of Tony and Annette Catania for cocktails and conviviality. Tony's home is in North Seaford, approximately 12 to 15 minutes away from Nassau Community College.

Saturday, October 12, 1985

I 10 A.M. (Administrative Tower - Nassau Community College)

Business of the Society - Tentative Agenda

1. Call to order; attendance - G. Zirkel
2. Minutes of 1984 Annual Meeting - Impagliazzo
3. President's Report - G. Zirkel
4. Treasurer's Report - Malone
5. Editor's Report - P. Zirkel

SCHEDULE, 1985 ANNUAL MEETING, Continued

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6. Reports of other Officers, as called for.

Reports of Committees

7. Annual Meeting - Berridge, Catania
8. Financial - Scordato, Malone, George, Foley, P. Zirkel
9. Nominating - G. Zirkel, Foley, Berridge, Catania
   Election of new Board Members.
10. Reports of other Committees, as called for.
11. New Business, and appointment of new Committees.

II BOARD OF DIRECTORS MEETING

1. Election of Officers
2. New Business

LUNCHEON 12 - 2 P.M.

III 2 P.M. (Administrative Tower - Nassau Community College)

Speakers - Partial Listing

1. Fred Newhall *Accuracy Measurement* Fred will speak on the utilization of duodecimal symbols to rate resistors and other electronic components.

2. (Tentative) Viewing of a videotape prepared by DSA members for educational purposes. Discussion and critique.

Continued...

SCHEDULE, 1985 ANNUAL MEETING, Continued

IV Evening

As at the 1984 Annual Meeting, our banquet is planned for the top floor of the Administrative Tower, with its views of all Long Island and the New York City skyline. Spouses, guests and friends are invited. Estimated cost: $20.00 per person.

Sunday, October 13, 1985

Sightseeing and departure, at leisure.

Please let us know if you plan to attend, so that arrangements can be made for refreshments, tickets, etc. Out-of-towners who call will be directed to local hotels.

For further information, and for reservations, please call Gene Zirkel at:

(516) 222-7611/7383 (College)
(516) 669-0273 (Home)

Gene Zirkel and John Impagliazzo at the 1984 Annual Meeting.

PERFECT NUMBERS IN BASES TWO AND FOUR

Jay Schiffman
Kean College of New Jersey
Jersey City State College

INTRODUCTION:

A number which equals the sum of all its divisors smaller than the number is a rare find indeed. Only twenty-seven such numbers, called perfect numbers, are known. The purpose of this note is to explore an interesting pattern possessed by the perfect numbers in bases two and four.

In order to explore the idea of a perfect number more fully, let us provide an enumeration of the first few perfect numbers. The initial seven perfect (base ten) numbers in order of increasing magnitude are 6, 28, 496, 8 128, 33 550 336, 8 589 869 056, and 137 438 691 328. Let us note that

6 = 1+2+3 (6 is the sum of all its divisors <6).

28 = 1+2+4+7+14 (28 is the sum of all its divisors <28).

496 = 1+2+4+8+16+31+62+124+248 (496 is the sum of all its divisors <496).

The Greek Mathematician Euclid demonstrated that all even perfect numbers have the form $(2^{p-1})(2^p-1)$ where p is a prime and the second factor 2^p-1 is likewise prime. Primes of the form 2^p-1 are called Mersenne Primes after the French friar and amateur mathematician Marin Mersenne (circa 1644). One can show that the initial eight Mersenne Primes correspond to p=2,3,5,7,13,17,19, and 31 respectively. For example, if p=5, then $2^p-1=2^5-1 = 32-1=31$, a prime. (31 has no divisors other than 1 and itself). The proof of Euclid's formula concerning even perfect numbers in base ten is accessible in reference (2) in the appended bibliography.

Our basic goal is to convert perfect numbers in base ten to bases two and four. In general, let us recall how one con-

Continued...

verts from the decimal system of numeration to the base two and base four systems. The positional values in the base 2 systems are...$2^5, 2^4, 2^3, 2^2$,2,1 or...32,16,8,4,2,1.
Similarly the positional values in the base 4 system are...
$4^5, 4^4, 4^3, 4^2$,4,1 or..., 1 024,256,64,16,4,1.
These two systems of numeration contrast with the base ten system where the positional values are..., $10^5, 10^4, 10^3, 10^2$, 10,1 or..., 100 000, 10 000, 1 000, 100, 10, 1.
To cite an illustration, 45 973 in expanded decimal notation corresponds to 45 973 = $(4 \times 10^4)+(5 \times 10^3)+(9 \times 10^2)+(7 \times 10)+3$ or (4x10 000)+(5x1 000)+(9x100)+(7x10)+3.

At this juncture let us consider two methods utilized to convert 28 from a decimal number to a base two and base four number. In the first method to convert 28 from a decimal number to base two, note that $2^4=16$ is the highest power of 2 which is less than 28.
Divide 28 by 16.

$$\begin{array}{r} 1 \\ 16\overline{)\,28} \\ \underline{16} \\ 12 \end{array}$$

Therefore, there is one group of 16 in 28. Next divide the remainder, 12, by 8.

$$\begin{array}{r} 1 \\ 8\overline{)\,12} \\ \underline{8} \\ 4 \end{array}$$

There is one group of 8 in 12. Next divide the remainder, 4, by 4.

$$\begin{array}{r} 1 \\ 4\overline{)\,4} \\ \underline{4} \\ 0 \end{array}$$

There is one group of 4 in 4.
Since the remainder, 0, is less than the base, no further division is required. Accounting for all positional values in base two, there is no group of 2 in 4 and no units.
Hence $28=(11100)_2$.

To convert 28 to base four, note that $4^2=16$ is the highest power of 4 which is less than 28. Divide 28 by 16.

$$\begin{array}{r} 1 \\ 16\overline{)\,28} \\ \underline{16} \\ 12 \end{array}$$

Therefore, there is one group of 16 in 28. Next, divide the remainder, 12, by 4.

$$\begin{array}{r} 3 \\ 4\overline{)\,12} \\ \underline{12} \\ 0 \end{array}$$

There are three groups of 4 in 12.
Since the remainder, 0, is less than the base 4, no further division is necessary. Hence 28 = $(130)_4$.

There is an alternative method for changing a number in base 10 to a different base. We use this method to convert 28 to bases 4 and 2. To convert 28 to base 4, divide 28 by 4 yielding a quotient of 7 and a remainder of 0. Write the quotient below the dividend and the remainder on the right as illustrated below:

$$4\,\underline{|28}\quad \text{remainder } 0$$
$$\quad 7$$

Continue this process of division by 4.

		remainder
4	28	
4	7	0
4	1	3
	0	1

Note that since the last dividend, 1, is smaller than the divisor 4, the remainder is 1. Observe that the division continues until the quotient is zero. The answer is read from the bottom number to the top number in the remainder column. Thus 28 = $(130)_4$. Similarly, to convert 28 to base 2, divide by the base and retain the remainders until

Continued...

PERFECT NUMBERS, Continued

---

the dividend is smaller than the divisor, as illustrated below:

		remainder
2	28	
2	14	0
2	7	0
2	3	1
2	1	1
	0	1

Hence $28 = (11100)_2$.

We similarly perform the following computations to convert 496 (the third perfect number in base ten) to bases 2 and 4.

To convert 496 to base 2:

		remainder
2	496	0
2	248	0
2	124	0
2	62	0
2	31	1
2	15	1
2	7	1
2	3	1
2	1	1
	0	

To convert 496 to base 4:

		remainder
4	496	0
4	124	0
4	31	3
4	7	3
4	1	1
	0	

Hence $496 = (111110000)_2$ and $496 = (13300)_4$.

Continued on page 19 .....

Those of us who work with base two are accustomed to the word 'bit' which stands for Binary digIT. Thus 1011 contains four bits. Recently, in a talk on base sixteen, Board Member Dr. John Impagliazzo coined the word 'hit' to mean Hexadecimal digIT. For example 1A contains two hits. One wonders, then, if 7#4 contains three 'dits'?

MATHEMATICAL CONSTANTS

Mark Calandra
Chappaqua, NY

Three of the most famous mathematical constants are pi, e and the square root of two, each of which has an infinite, patternless decimal expansion. I have taken the liberty of computing their duodecimal values using double-precision computer arithmetic, which allows a sixteen-digit value to be stored and manipulated. Sixteen decimal digits contain as much information as 14.8 duodecimal digits (16 x .9266), so the results have been rounded to thirteen digits to be certain that they were accurate in their entirety. (See "Relative Sizes of Decimal and Duodecimal Numbers", 51, Vol. 2*, No. 2.)

$Pi_{(12)}$: 3; 184809493#919

$e_{(12)}$: 2; 875236069821*

square root, $two_{(12)}$: 1; 4#79170*07#76

PUZZLE CORNER

Back in volume 28; number 2; we asked for dozenal numbers which are exactly twice as large as their decimal counterparts. Igor Valevsky, number #7, of Brazil has sent us over a dozen solutions. The first is

11788; = 2(11788.)

Others are 11790, 11818, 11820, 12298, 12328, 12330, 24658, 24660, 25200, 25168, 25170, 36988, 36990, 37528, 37530, 38038, 38040, 49858, and 49860.

STRANGE BASES, PART II

Gene Zirkel
Nassau Community College
Garden City, LI, NY

Part I of this article was published in the Summer 1195; issue, whole number 51; page 6.

2.0 Introduction.

In Part I we considered the negative integers less than -1 as possible bases for a positional number system. We now look for other possibilities. Could we use fractions, algebraic irrational numbers, transcendental numbers, or imaginary numbers as bases?

2.1 Unit Fraction Bases.

First we consider unit fractions, that is fractions whose numerators are 1. But note, all that this does is to write our numbers in reverse. (We use the subscript $_F$ to indicate the unit point when writing numbers in a fractional base.) The number abc_Fde in base 1/n

$$= a(1/n)^2 + b(1/n) + c + d(1/n)^{-1} + e(1/n)^{-2}$$

$$= a(n)^{-2} + b(n)^{-1} + c + d(n) + e(n)^2$$

= edc.ba in base n.

We see that in both base n and base 1/n the unit point is placed to the RIGHT of the units digit. Thus

36; = 6_F3 in base 1/10, and 1/2 = 0;6 = 60_F in base 1/10

↑ ↑ ↑ ↑

The above says that $3(10) + 6; = 6 + 3(1/10)^{-1}$, and that 1/2 = (6;)(1/10) = 6(1/10) + 0.

STRANGE BASES II, Continued

With integral bases we often omit the unit point when it is followed by a zero. Thus 3 = 3; = 3;0. However, with a fractional base it is usually advisable to indicate the unit point even tho we could write 3 for 3_F (or 3;), and 12 for 12_F (or 2 + 1/n).

2.1.1 Digits.

For the unit fraction base, 1/n, let us use the same digits that we use for base n, namely 0,1,2, ... up to n-1.

2.1.2 Arithmetic Operations With Unit Fraction Bases.

The algorithms for addition and subtraction are the same as the ones that we learned in grade school with the exception that they are *reversed*. We now start on the LEFT instead of on the right. For example using the reciprocal of one dozen as our base:

```
  16F4
 +8F
  12F5      (or 46;1 + 8; = 52;1)
and
  163F3
 - 81F
  1*1F3     (or 33;61 - 1;8 = 31;*1)
```

Similarly the algorithm for multiplication is *reversed*, BUT the rule that the number of digits to the right of the unit point is equal to the sum of the numbers of digits to the right of the unit points in each factor is changed. The new rule is:

> THE NUMBER OF DIGITS TO THE <u>LEFT</u> OF THE UNIT POINT IS <u>ONE LESS THAN</u> THE SUM OF THE NUMBERS OF DIGITS TO THE <u>LEFT</u> OF THE UNIT POINTS IN EACH FACTOR.

For example

```
  36F1
 x 2F4
  603
 __016
  6046
```

Continued...

STRANGE BASES II, Continued

The number of digits to the left of the unit point is 2 + 1 - 1 = 2, thus the product is 60_F46 (or 16;3 x 42; = 640;6)

Continuing to work in reverse we see that 1_F2 divided by 8_F becomes

```
   613
   ____
    1F2(8F
    0 2
   01
  _8_
  04
  04
```

To determine where the unit point should be placed we proceed as follows. We move the point in the divisor to the LEFT to change the divisor to an integer, that is just behind the leftmost digit. Then we move the point in the dividend the same number of places. Finally the point goes in the quotient directly above the point in the dividend. This is essentially the same rule that we learned in grade school for placing the unit point in long division.

Thus, in the example above we did not have to move the point in the divisor so that the point is placed in the quotient above the point in the dividend, that is to the right of the 3 giving us 613_F (or21; / 8; = 3;16).

2.2 Other Fractions.

For simplicity's sake, we consider fractions which have already been reduced to lowest terms. Having already considered unit fractions, we now turn to fractions whose numerators do not equal 1. If we try a base such as 5/4, one question is: which digits shall we use?

If we were to try the digits we use for integral bases: 0, 1, ... up to but not including the base itslf, we need 0 and 1. The first few numbers that we obtain in this manner are:

0 = 0
1 = 1
10 = 1;3

STRANGE BASES II, Continued

---

11 = 2;3
100 = 1;69 (which is less than 11!)

However, if we attempt to express the integer 2 as a sum of powers of 5/4 we obtain

$$2 = (5/4)^{3} + (5/4)^{-12} + (5/4)^{-23} + \ldots$$

where the next few terms have exponents of -2#, -37, -4*, -55, -62, -69, ...

The problem of algorithms seem formidable since a simple addition such as 1 + 1 seems quite complicated. In general it seems that no matter which digits we were to choose, there would always be other simple integers which we could not conveniently express in bases which are not unit fractions. *Hence, in general, fractions which are not unit fractions do not seem to be of much use to us as the base of a positional number system.*

Obviously, If a fraction were to work as a base, so would its reciprocal, for these would be the reverse of each other just as the unit fractions were the reverse of their reciprocals which were the integers.

For an interesting application of a fractional base see "Nested Polynomials and Effecient Exponentiation Algorithms for Calculators", by Dan Kalman & Warren Page, in *The College Mathematics Journal*, Vol. 16, no. 1, January 1985, pages 57-60.

In Part III we will conclude our investigation of possibilities for bases of positional number systems by considering some irrational and imaginary numbers as bases.

EFFICIENT NUMBER SYMBOLS

Fred Newhall
Smithtown, NY

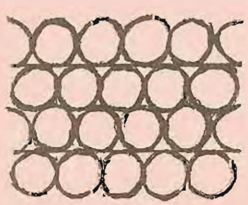
Selecting the most efficient number system, based on 12, is only half of the attempt to make numbering easier. The writing of the number symbols has to be made logical. Our arabic numbers are thousands of years old, having accidentally evolved with no engineering intent for efficiency; having no logic as to ease or speed of writing.

The first consideration in the engineering of writing is deciding on continuous vs. individual symbols. The lifting of the pen and lowering of it for each number symbol is time- and energy-consuming. Continuous writing is more efficient; for instance takes much less effort than writing separate o o o o's. Most people have to lift the pen twice to write 4, even more inefficient.

When using continuous writing, all letters should begin and end on the same base-line: there should be no symbols like , , , that end above the line. Every symbol must be attachable to any other without modification.

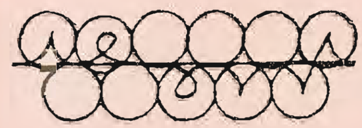
By carefully studying finger and hand movement in writing, it was determined by human engineering methods that and were the most efficient symbols. Each can be written upside down and combined with each other , . We should assume a general rule that when a line becomes tangent to the base-line, it becomes a new symbol; that is and are 4 separate symbols, whereas crossing the base-line at an angle makes this all one symbol.

Consider space to be efficiently divided into information bits which fit more closely on a page if offset:



I have shown two of the base-lines which are separated so that symbols

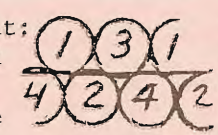
can be formed above and below each base-line the way a occupies two information bits. A bit can be filled with either a line or a circle or skipped and left blank as so:



Of course, the space bits and base-line are only imaginary guides to the actual symbols.

A computer can easily read this method of writing since the information is in two parallel lines of bits both sides of the base-line. In each bit it would read either a line, a circle, or a blank space. A computer based on a trinary number system is 50% more efficient than the conventional binary computers. Electronically a trinary computer would operate with +, -, or 0 voltage, or magnetically with North, South, or neither.

Each bit being occupied in 3 ways by either a line, circle, or blank, only 4 bits would ever be needed for one symbol, 3^4 = 81 possible symbols. The bits would be used in sequence top-to-bottom, left-to-right:



If spaces 1 & 3 are occupied with a line and if spaces 2 & 4 are left blank, then the symbol would look like this . If 1 and 3 are blank and 2 & 4 are occupied, the symbol would be upside down .

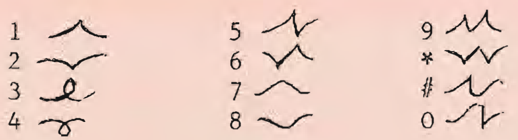
Continued...

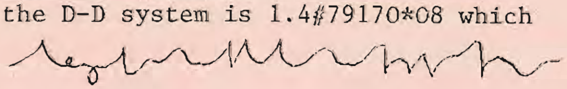
Don't throw this BULLETIN away -

- *Give it to a friend, or*
- *Leave it in your dentist's office.*

EFFICIENT NUMBER SYMBOLS, Continued

By assigning a "degree of difficulty" number to each stroke of the pen, for instance (being a curve is harder to write than a straight line / which is easier than a backhand stroke \ which is easier than a backhand curve) ; you can rate every possible combination by adding up the difficulty numbers for each stroke. The easiest symbols are then selected. This was done for an entire system of shorthand, but we need only the 12 easiest symbols for our duo-decimal numbers:



The value of $\sqrt{2}$ in the D-D system is 1.4#79170*08 which would be written - 

without lifting the pen, with easier curves, fewer points, and computer readable.

Notes: Since the roman alphabet is spelled with a small r because of its dubious origin, I spell arabic with a small a.
A complete description of assigning "degree of difficulty" numbers to the strokes of the pen is detailed in my language writings.
A shorthand system of writing and the Sound Alphabet of the sounds we use in everyday speaking, as well as a Sound Dictionary of 32,000 entries are part of my proposed international language, a lifetime project.

SIMON STEVEN AND CHARLES XII OF SWEDEN

Anton Glaser
Southampton, PA 18966

My doctoral research dealt with numeration systems (including the dozenal) and later resulted in a book History of Binary and Other Nondecimal Numeration (1971 & 1981). Before I started my research, the Dozenal Society was kind enough to provide me with an issue of its Bulletin that contained a bibliography on dozenal and related topics. I searched through all of the publications listed therein, plus others I found referenced elsewhere. At a later date I would like to share with you some particularly interesting information about dozenal advocates. Today, however, I shall deal only with two persons who have been erroneously listed as dozenal advocates, namely: Simon Stevin and Charles XII of Sweden.

Let me first deal with Charles XII (1682-1718). His science advisor was Swedberg (later enobled and called Swedenborg). In a town called Bryn Athyn (only ten minutes from my home) I found a replica of the original Swedenborg personal library containing a copy of every book that was in his original library. Swedberg had pushed base eight; his king, Charles the Great, base 64. Voltaire got the story straight when he wrote in his biography of Charles:

> Some people have wanted to pass off this prince as a good mathematician; undoubtedly he had a deeply penetrating mind; but the proof usually given of his mathematical knowledge is not very conclusive; he would change the manner of counting by ten, and he would propose sixty-four in its place, because this number contained at once a cube as well as a square, and when divided by two it would in the end reduce to unity. This idea could prove only that he loved the extraordinary and the difficult. It proved also that he had penetrated, up to a certain point, the theory of numbers, until he knew the nature and properties of these arithmetic scales.

Continued on page 21.....

BOOK REVIEW

DATA PROCESSING MATHEMATICS

by Elizabeth Bliss
Prentice-Hall, Inc., NJ

Chapter One of this textbook will be of interest to our readers. Entitled "Different Number Bases", it is a standard exposition of bases other than ten, especially those of interest to the computer scientist: binary, octal, and hexadecimal.

In a clear and straightforward manner, the author explains how to convert back and forth between base ten and other bases. She includes both whole numbers and fractions less than one. Fractions are treated in two forms: both as the quotient of two integers such as 20/32, and also in the form of .588.

The author explains binary arithmetic including addition, subtraction, multiplication, and division. She also illustrates addition and subtraction with octal and hexadecimal numbers. Exercises and examples are mostly in base two, eight, and sixteen, although base three, five and seven are also used. No mention was made of base twelve.

The remainder of the book deals with Boolean Algebra, matrices, linear algebra, logs, etc. The book is basically well done. However on page ten there is a reference to "four-place accuracy" when four-place precision is meant. (Accuracy usually refers to the number of significant digits.)

On page seven appears an unusual description of an algorithm as 'repetitive'. Also on the same page we read that the word *digit* is reserved for base ten, and the word *bit* is used for all other bases. It seems strange to refer to an octal digit as a 'bit' since the word bit is taken from the first and last letters of the words *BInary digIT*!

Finally the book contains no references nor bibliography.

All in all, it is a clear explanation of number bases that can easily be understood by any novice in the field.

*PERFECT NUMBERS, Continued from page **

Actually the conversion from base 4 to base 2 is extremely simple. One merely converts each individual digit:

0 <-> 00
1 <-> 01
2 <-> 10
3 <-> 11

Hence 130 in base 4 becomes 01 11 00 in base 2, and 13300 becomes 01 11 11 00 00, etc.

One can similarly show that $8128 = (1111111000000)_2$ and $8128 = (1333000)_4$.

In base 2 expanded notation $8128 = (1x2^{12}) + (1x2^{11}) + (1x2^{10}) + (1x2^9) + (1x2^8) + (1x2^7) + (1x2^6) + (0x2^5) + (0x2^4) + (0x2^3) + (0x2^2) + (0x2) + (0x1) = (1x4096) + (1x2048) + (1x1024) + (1x512) + (1x256) + (1x128) + (1x64) + (0x32) + (0x16) + (0x8) + (0x4) + (0x2) + (0x1)$.

Likewise, in base 4 expanded notation, $8128 = (1x4^6) + (3x4^5) + (3x4^4) + (3x4^3) + (0x4^2) + (0x4) + (0x1) = (1x4096) + (3x1024) + (3x256) + (3x64) + (0x16) + (0x4) + (0x1)$.

The fifth perfect number

$33\ 550\ 336 = (1111111111111000000000000)_2$ and

$33\ 550\ 336 = (1333333000000)_4$ as can be demonstrated by anyone who enjoys working with larger computations.

Meanwhile the sixth perfect number

$8\ 589\ 869\ 056 = (111111111111111110000000000000000)_2$

and $8\ 589\ 869\ 056 = (13333333300000000)_4$.

Finally, the seventh perfect number

$137\ 438\ 691\ 328 = (1111111111111111111000000000000000000)_2$

and $137\ 438\ 691\ 328 = (1333333333000000000)_4$.

Continued...

PERFECT NUMBERS, Continued

Let us observe that the initial perfect number $6 = (110)_2$ and $6 = (12)_4$. In base 2 all of the other perfect numbers possess an even number of zeros and an odd number of ones. All except 6 convert to a base 4 representation of a 1 followed by a string of threes and a tail of zeros. To see this, consider the following conclusions in bases two and four.

In base 2:

6 has two ones followed by one zero.

28 has three ones followed by two zeros.

496 has five ones followed by four zeros.

8 128 has seven ones followed by six zeros.

33 550 336 has thirteen ones followed by twelve zeros.

8 589 869 056 has seventeen ones followed by sixteen zeros.

and 137 438 691 328 has nineteen ones followed by eighteen zeros.

In base 4:

6 has a one followed by a two.

28 has a one followed by one three followed by one zero.

496 has a one followed by two threes followed by two zeros.

8 128 has a one followed by three threes followed by three zeros.

33 550 336 has a one followed by six threes followed by six zeros.

8 589 869 056 has a one followed by eight threes followed by eight zeros.

and 137 438 691 328 has a one followed by nine threes followed by nine zeros.

PERFECT NUMBERS, Continued

This is summarized in the table below:

Perfect Numbers	base 2 ones	base 2 zeros	base 4 ones	base 4 threes	base 4 zeros
6	2	1	-	-	-
28	3	2	1	1	1
496	5	4	1	2	2
8 128	7	6	1	3	3
33 550 336	13	12	1	6	6
8 589 869 056	17	16	1	8	8
137 438 691 328	19	18	1	9	9
⋮	⋮	⋮	⋮	⋮	⋮
$2^{p-1}(2^p-1)$	p	p-1	1	½(p-1)	½(p-1)

If one considers the formula $2^{2n}(2^{2n+1}-1)$ where n is a counting integer, we can shed light on the above results for even perfect numbers in bases two and four.

Every even perfect number with the exception of 6 has the above form where 2n+1=p for some prime p and 2n=p-1. (Recall Euclid's Formula.) One can show that in base two a number of the form 2^{2n} $(2^{2n+1}-1)$ has 2n+1 ones followed by 2n zeros while in base four a number of this form has a one followed by n threes followed by n zeros. Rather than prove this we utilize examples demonstrating the validity for certain select values of n. The second through seventh perfect numbers correspond to n = 1,2,3,6,8, and 9 respectively. For example, if n=3, then $2^{2(3)}\cdot(2^{2(3)+1}-1) = 2^6(2^{6+1}-1) = 2^6(2^7-1) = (64)(127) = 8128$, a perfect number.

Continued...

PERFECT NUMBERS, Continued

On the other hand, for some n the formula $2^{2n}(2^{2n+1}-1)$ does not yield a perfect number. If n=5, we have $2^{2(5)}(2^{2(5)+1}-1) = 2^{10}(2^{10+1}-1) = 2^{10}(2^{11}-1) = (1024)(2047) = 353,828$. Note that 2047 is not a prime; for 2 047 = 23x78 and 353 828 is larger than the sum of its proper divisors.

We conclude our discussion at this point, as the even perfect numbers become large and hence unwieldy. For example, the twenty-seventh perfect number $2^{44\ 496}(2^{44\ 497}-1)$ is so large that it would take over 25 000 digits to write it out (a more than 200-foot-wide piece of paper would be needed to write it out on an ordinary typewriter in its base ten representation). The reader is invited to pursue additional applications of the base two and base four systems of numeration.

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2. Rademacher, H. and Toeplitz, O. - The Enjoyment of Mathematics, Princeton Univ. Press, Princeton, NJ. 1957.

Acknowledgment: The author wishes to thank the reviewers for their useful suggestions and comments.

NEW REVIEWERS

With this issue we have two new reviewers. Our thanks to Tony Catania, 291; and Kay McKiernan, 284; for their efforts to make the *BULLETIN* better than ever.

SIMON STEVEN, Continued from page 17

Nevertheless numerous references to Charles as a dozenal advocate can be found. In the last analysis, they are all traceable to Levi Conant's statement included in his "Primitive Number Systems," (Smithsonian Institution Annual Report, pp 583-594, 1892): "So palpable are the advantages of 12 from this point of view that some writers have gone so far as to advocate the entire abolition of the decimal system and the substitution of a duodecimal system in this place. Charles XII, of Sweden, may be mentioned as an especially zealous advocate of this change, which he is said to have had in actual contemplation for his own dominions at the time of his death." Evidence cited above and the writings available of Swedberg, leave no doubt that Conant was mistaken.

Now for the case of Simon Stevin (1548-1620). I've acquired copies of his writings, and it is very clear to me that he was a strong advocate of DECIMALS. Base ten decimals, lest there be doubt. The notation he first advocated is not the one eventually adopted, but that is not relevant. There is not a shred of evidence that I found that he ever advocated dozenals either for whole numbers or for fractional numbers. How he ever got listed as a dozenal advocate is still a mystery to me. I wish I had been able to trace the source of the error, as I did in the case of the erroneous listing of Charles XII. But the burden of proof now lies with those who think Stevin advocated dozenals.

The following are available from the Society

1. Our brochure (free)
2. "An Excursion in Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, Oct. 1934. (Single copies free. Bulk orders available.)
3. *Manual of the Dozen System* by George S. Terry ($1;00)
4. *New Numbers* by F. Emerson Andrews ($10;00)
5. *Douze: Notre Dix Futur* by Jean Essig, in French ($10;00)
6. Dozenal Slide rule, designed by Tom Linton ($3;00)
7. Back issues of the *Duodecimal Bulletin* (as available) 1944 to present ($4;00 each)

DOZENAL JOTTINGS

News from or about the dozenal activities of members and friends....

This Fall **DR. ANTON GLASER** will be teaching (as an unpaid volunteer) for the Arthritis Foundation, instructing arthritics in self-help techniques with lots of exercises...Recently **MRS. F. EMERSON ANDREWS**, 257; sent us some material that belonged to her late husband, one of our founders, and member number 7! Among the books we received was a copy of Realm of Numbers by Isaac Asimov, honorary member number 293;. Andrews left a book mark in chapter five: "Breakage by Tens"...**JEAN KELLY** writes to point out that in our last issue on page 11; **CHARLES MARSCHNER** complains that unfortunately the inch was divided into sixteenths instead of twelfths, while on page 1# we have the usual definition of a line as one twelfth of an inch!...We have lost contact with **DR. ROBERT C. GILLES**, 97; of Germany, and with **DR. MARY C. VARNHORN**, 79; formerly of Trinity College in Washington, D.C. If anyone knows of their current addresses, please let us know. Thanks...In our last issue's list of new members, the name of **GERARD ROBERT BROST**, member number 294; was misspelled. Our apologies!...Congratulations to **KAY MC KIERNAN**, who has been elected First Vice Chairperson of the Academic Senate at Nassau Community College (NY)...**KAY** and **JOHN EARNEST** recently appeared in a faculty spoof of life at Nassau Community College. Creative and entertaining, the evening of fun was entitled "NCC Chorus Whine"...**ARTHUR WHILLOCK** writes from the UK: "Bulletin No. 51; just received is encouraging. Advise **DR. PAUL RAPOPORT** that there is an International Phonetic symbol for a voiced 'x' which is available in a

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Counting and measuring, cousin

With the base of dek just doesn'

Give us factors enough

Advantages and stuff

As when we count with a dozen

DOZENAL JOTTINGS, Continued

limited style of type -- X (upper case), γ (lower). ℓ was a medieval symbol for four...**PAUL** himself wrote to add to his remarks in Bulletin 51; concerning dozenal timekeeping. "The simplicity of the idea for an essentially dozenal metric clock has long appealed to me, and the prototype of the clock (now running) realizes what I predicted would work. Unfortunately, we cannot build a clock with 4 hands (an analog clock); but fortunately, microchip technology makes a digital clock fairly easy to create. The clock presents four digits, each running from 0 to #. The leftmost digit changes once every 1/12th of a day (2 hours on the current clock), the next digit once every 1/12th of that (10 minutes), the next digit once every 1/12th of that (50 seconds), and the rightmost digit once every 1/12th of that (4 1/6th seconds). For example, a time of 600.0 is noon, and 996.6 is 7:35:25 p.m. There are 1000.0 units in a day. A problem we face is what to use for dek and el, since the * and # are not realizable on a seven-segment display. We are planning to use a variation of the British symbols. Any suggestions on the matter would be welcome. The price of the finished clocks is likely to

Continued...

ON THE COVER

These people are smiling because they enjoyed themselves at the 1984 DSA Annual Meeting. This year's meeting is October 11 and 12 at Nassau Community College, Garden City, LI, NY. For a good time, call Gene at (516) 669-0273.

Alice Berridge and Dudley George, at the 1984 DSA Annual Meeting.

DOZENAL JOTTINGS, Continued

be $80 to 100 US, plus postage...**TOM PENDLEBURY's** T G M: A Coherent Dozenal Metrology was recently published by the DSGB. They call it "a commendable addition to the literature of base twelve mathematics", and will make it available for sale to all Society members at 75p. Because of his work in dozenals, especially his book, **TOM** has been accepted for *Honorary Membership* in the DSA. Welcome to new member number 295;...We regret to announce the recent passing of DSGB past president **SIR IAIN MONCREIFFE OF THAT ILK.** He was 55; years of age, born 9 April, 1919. Sir Iain studied Scots Law at Edinburgh University, took a PhD with a thesis on the Scots law of peerages, and was appointed QC (Scotland) in 1980. He pursued genealogical, heraldic (and, we might add, Dozenal) interests with enthusiasm...

end

READ ALL ABOUT IT!

The following headlines were gleaned from the AMERICAN METRIC JOURNAL, May/June 1985, Unit Three, Volume XIII:

1) "Canada Retreats on Shift to Metric; Will Use Dual System"

2) "SI Revolt Developing"

3) "Force Ends In Britain"

4) "AUSTRALIA REPEALS LAW"

5) "UK METRICATION BOARD WASHED UP COMPLETELY"

6) "(U.S.) STORES STICK TO STANDARD STUFF"

7) "METRIC RESISTANCE BUILDS"

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition—("Who needs a symbol for nothing?")—the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accomodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used, and as early as 1585, Simon Stevin stated that the duodecimal base was to be preferred to the base ten.

The parallel seems tenable. The notation of the dozen base better accomodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS.**

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10,* and is called *do,* for dozen. The quantity *one gross* is written *100,* and is called *gro. 1000* is called *mo,* representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5,* and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3#2*	Two ft. eight in.	*2;8'*
19#	*1000*	Eleven ft. seven in.	*#;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53.* Using this *"which is"* step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2#,* which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
12 ) 30  + 5
12 ) 2   + 6
     0   + 2    Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *.

For more detailed information see *Manual of the Dozen System* ($1;00).

We extend an invitation to membership in our society.
Dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______ LAST ______ FIRST ______ MIDDLE

Mailing Address (for DSA items) ______

______ (See below for alternate address)

Telephone: Home ______ Business ______

Date & Place of Birth ______

College ______ Degrees ______

Business or Profession ______

Employer (Optional) ______

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______

Address ______

Year & Math Class ______

Instructor ______ Dept. ______

Other Society Memberships ______

Alternate Address (indicate whether home, office, school, other)

______

Signed ______ Date ______

My interest in duodecimals arose from ______

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

DETACH HERE

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530