DOZENAL SOCIETY OF AMERICA
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530

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THE DUODECIMAL BULLETIN

51;

WHEN DOES

OCT 31 = DEC 25?

OCT / DEC

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SEE PAGE 24;

Volume 2*;
Number 2;
Summer 1985
1195;

THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are $12.00(US) for one calendar year. Student membership is $3.00 per year, and a Life membership is $144.00 (US).

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BOARD OF DIRECTORS OF THE DOZENAL SOCIETY OF AMERICA

The DSA does NOT endorse any particular symbols for the digits ten and eleven. For uniformity in publications we use the asterisk (*) for ten and the octothorpe (#) for eleven. Years ago, as you can see from our seal, we used X and Ɛ. Both X and * are pronounced "dek". The symbols # and Ɛ are pronounced "el".

When it is not clear from the context whether a numeral is a decimal or a dozenal, we use a period as a unit point for base ten and the semi-colon, or Humphrey point, as a unit point for base twelve.

Thus ½ = 0.5 = 0;6.

The Duodecimal Bulletin

Whole Number Five Dozen One

Volume 2; Number 2*

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IN THIS ISSUE

Patricia McCormick Zirkel, *Editor*
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6 Brancatelli Court
West Islip, New York 11795

IN MEMORIAM, J. HALCRO JOHNSTON

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b. *8 May 1891*
d. *26 Jan 1985*

J. Halcro Johnston, DSA member number 55; of Orkney, Scotland, UK, died on January 26, 1985, following a fall on New Year's Day in which he broke a leg. He was 79; years of age. He leaves two sons, Hugh, who continues to look after the family farm and estate in Orkney, and James, who is a consultant civil engineer (as was his father).

Mr. Johnston was born on May 8, 1891, the only son (with four sisters) of an Orkney landowner and farmer. Educated in Aberdeen and at George Watsons College, Edinburgh, he went on to read Civil Engineering at Edinburgh University, where he showed an aptitude for mathematics and won the Gold medal for his year. He trained as an engineer with Sir Robert McAlpine, and in 1914 went to India to join the Public Works Department of the British Administration. Based at Lahore in the Punjab, he designed roads and bridges for the North West Frontier area of the Himalayas.

In 1917 he served as a "sapper" (Royal Engineers) with the 36th and 45th Sikh regiments in Mesopotamia, was at the siege of Baghdad, was wounded by a shot in the hand, and honored by being "mentioned in dispatches". He survived to return to India, where he worked between the wars. There he visited some of the most remote corners of the Himalayan Mountains on foot, and is mentioned in books on travel and exploration. He saw all his sisters married after (in succession) serving him as housekeeper!

In 1934, home on leave, he met an Englishwoman, Marjorie Freeth, who was visiting Orkney. They were married in Bombay on September 19, 1935. Their first son, Hugh, was born in Lahore, the capital of the Punjab in 1936. Another son, James, was born in 1938.

In 1939 he returned to the UK to serve as a major commanding a company of engineers in France. He built hospitals which

J. HALCRO JOHNSTON, Continued

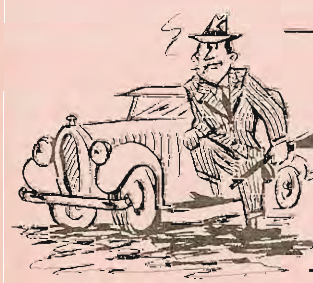
---

were over-run by the German invasion in 1940, and was lucky to escape from Normandy after the retreat from Dunkirk. He then was posted "home" to Scapa, where he was involved in the design and construction of the "Churchill Barriers" (causeways between the islands, built to protect the British Fleet from U-boat attack). They are of great benefit to Orkney today.

After a second posting to the naval dockyard at Freetown, Sierra Leone, he retired to Orkney to look after the farm (beef, dairy and sheep) and fulfill various public duties. He was a member of the Health Board, a Justice of the Peace, a President of the Boy Scout movement, and also worked for many charities. His wife pre-deceased him in 1975.

Mr. Johnston wrote The Reverse Notation which was published by Blackie and Son in 1937. The system advocated therein is also called "negative-affirmative arithmetic" and "two-way numbers". It uses both positive and negative digits to construct a system with few symbols, aimed at ease and simplicity of calculation. An article, also entitled "The Reverse Notation" was published in this BULLETIN, vol, 6, no. 2, August 1950, pp. 25-30;. He received the DSA Annual Award in 1951. He had a lifelong interest in mathematics, and was passionate both in support of duodecimals and in opposition to the metric system. Another interest was astronomy, and despite failing eyesight, he was reading a book on this subject at the time of his death.

Many thanks to Hugh Halcro Johnston, who graciously supplied us with most of the preceding biographical material.



We'll be very disappointed if you're not with us for the DSA Annual Meeting -- October 11 and 12, 1985, in New York.

STRANGE BASES *PART I*

Gene Zirkel
Nassau Community College
Garden City, LI, NY

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0.0 Introduction.

Most of the readers of this BULLETIN are accustomed to using any base. We feel comfortable doing arithmetic in dozenals, hexadecimals, binary and even in the awkward base dek. The only handicap we experience in suddenly using a new base (for example base 7) is a lack of familiarity, a lack of practice. It takes us a minute or two to get a feel for the magnitude of the answer to such questions as what is 651_{seven} divided by 13_{seven}?

```
     45
 13)651
    55
    101
    101
```

However, having broadened our outlook to see past just base dek, we still seem to limit ourselves to only positive integers greater than one as possible candidates for the base of a positional number system.

Wherever clarity demands it, we will use the customary notation to distinquish between dozenals and decimals by use of the appropriate fraction point. For duodecimals we use the semicolon (or Humphrey point), while for base dek we use the period. Thus 16; = 18.

1.0 Negative Integer Bases.

J. Halcro Johnston in his book, The Reverse Notation, Negative Digits with 12 as the Base, suggested the use of negative digits, but not a negative base. Others have worked with the negative digits -1, 0, and +1 in base 3, but still the base remained positive.

Can we use a negative integer for a base? The answer to this question is, yes. For example 211 in base -3 would have the usual meaning of

$$2(-3)^2 + 1(-3) + 1 \text{ or } 14;$$

Continued...

However the more important questions are:

1. Can every number be represented in the negative base?
2. If so, what digits do we need to do so?
3. Do there exist algorithms for the four fundamental operations of arithmetic

The answers to these questions are yes. In what follows I will stay with base -3 using the digits 0, 1, and 2 for examples, but what is said can easily be transposed to other negative bases less than -1 using the digits 0, 1, 2, ... up to 1 less than the absolute value of the chosen base.

Counting

If we begin counting by looking at the digits in their usual order:

1, 2, 10, 11, 12, 20, 21, 22, 100, 101, ...

we obtain the following strange progression

0 = 0	100 = 9
1 = 1	101 = *
2 = 2	102 = #
10 = -3	110 = 6
11 = -2	111 = 7
12 = -1	112 = 8
20 = -6	120 = 3
21 = -5	121 = 4
22 = -4	122 = 5

Each set of three numbers are consecutive increasing integers: n, n+1, and n+2. After 0, the first two are positive, then six are negative, then a dozen and a half are positive, etc. This can be summarized in the table below.

---

group of numbers with the same sign		number of integers in the group
first group,	+	2 = 3 - 1
second	-	6 = 9 - 3
third	+	16 = 23 - 9
.		.
.		.
.		.
k^{th}	$(-1)^{k-1}$	$3^k - 3^{k-1}$

Instead of looking at the integers in the above way, let us try to count in the usual ascending order

0	0	-1	12
1	1	-2	11
2	2	-3	10
3	120	-4	22
4	121	-5	21
5	122	-6	20
6	110	-7	1202
7	111	-8	1201
8	112	-9	1200
9	100	-*	1212
*	101	-#	1211
#	102	-10	1210
10	220	-11	1222
11	221	-12	1221
12	222	-13	1220
etc.			

We note that, except for zero, the integers represented by an odd number of digits are positive, and those represented by an even number of digits are negative.

1.1 Addition.

We add in base negative three in a way similar to base positve three, <u>EXCEPT THAT WE CARRY A NEGATIVE 1</u> to the next column whenever our sum contains two digits.

Continued...

STRANGE BASES (I), Continued

The addition table for base +3 follows:

+	0	1	2
0	0	1	2
1	1	2	10
2	2	10	11

For example, adding 112 and 101 we obtain:

```
  1 1 2
 +1 0 1
[  -1  ]  <-- carry
  2 0 0   <-- sum        (or 8 + * = 16;)
```

We can see how this works by analyzing an example.

```
  1   2   1   2
 +    1   1   1
[-1   0  -1     ]   <-- carry
  0   0   1   0     <-- sum    (or -* + 7 = -3)
```

We rewrite the above as follows

```
 1(-23) + 2(+9) + 1(-3) + 2
+         1(+9) + 1(-3) + 1
[-1(-23)    0      -1(-3)     ]
       ↖              ↖
        3(+9)           3
 0          0      1(-3)    0
```

When adding, if the left most digit is a negative 1, then we replace it by 12 which is its equivalent. Thus to add 21 and 21 we have 1 + 1 = 2 (that is 2 and 0 to carry). Then 2 + 2 = 11 (that is +1 and -1 to carry).

```
   21
  +21
[-10 ]  <-- carry
 -112   which becomes 1212 when we replace the leading
        -1 by its equivalent, 12 (or -5 + -5 is -*).
```

Continued on page 16; ....

RELATIVE SIZES OF DECIMAL AND DUODECIMAL NUMBERS

Mark Calandra
Chappaqua, NY

It comes as no surprise that duodecimal numbers are shorter (or the same size), generally, when compared to their decimal counterparts, but an exact value for this difference has now been calculated. The sizes of the first billion positive integers have been measured (with a shortcut formula, of course) in base ten and base twelve. Admittedly, this group is only a small subset of all possible numbers, but it includes an overwhelming majority of the numbers which the average person might need to use, so it will do for our purposes.

In a positional notation system, the number of digits in an integer directly relates to the size of the largest power of the base which it contains. In other words, numbers grow by a digit whenever a new power of the base is reached. Obviously, numbers expressed in higher bases will be shorter, because powers of the base are further apart and are reached less frequently. For our comparison, we have to ask how many powers of 12 there are in a power of 10. The answer is log 10/log 12. (The base of the logarithms is immaterial.) The result of this division is .9266, which leads to the conclusion that duodecimal numbers are, on the average, 7.34% shorter than those expressed in decimal notation. This agrees with the actual tally, which showed that in the range 1-1,000,000,000, the amount of numerical information contained in 10,000 decimal digits could be (and was) packed into 9266 duodecimal digits.

BOOK REVIEW

The Ascent of Mathematics by Raymond Coughlin and David E. Zitarelli, McGraw-Hill Book Co., 1984.

This is an excellent book about mathematicians and the creation of mathematics. Written in the spirit of Jacob Bronowski's classic, The Ascent of Man, the authors have produced a text that covers a broad spectrum of mathematics (including non-Euclidean geometry, linear programming, probability and statistics) in an elementary fashion. The novel idea of separating the historical portion of each section from the skills portion allows the reader to choose different modes of reading the book. One can skim thru all the history for a fast interesting read, stopping only at those exercises which intrigue him, or one can delve deeply into the exercises, learning more about each topic covered. An excellent flow chart in the Preface shows the chapter dependency so that one does not have to read the chapters in strict order.

Of special interest to our readers would be the section on number bases in chapter 1. The binary system is covered in detail, including base conversion and addition in base two. On page 42 of the text brief reference is made to the question: 'Which system is the best?', and under related readings for the section on the binary system, this BULLETIN is listed.

"Each one teach one."
- Ralph Beard, Founder of the DSA

WHY EUROPEANS LAG IN PRODUCTION

Charles F. Marschner
Melbourne, FL.

After retiring I became involved in the international sale of a fastening device. As an experienced engineer previously involved in producibility and production management and operations, I had always been interested in why the U.S.A. so often outpaced the Europeans. Recent dealings with Sweden on the matter of tolerancing fasteners finally gave me one clue.

The metric linear system leads design engineers into a trap. This is especially true in countries where graduate engineers are loath to get their hands dirty or learn to do shop work.

It just happens that .001" is the tolerance borderline below which ultra precision machine work begins. It is a measure that can be easily read and confirmed on a micrometer. But this .001" is, in metric 0,025mm. Now that looks like a big number when you compare 0,025 with .001. As a result the inexperienced designer tries to reduce tolerances by using a smaller metric number! This forces the production line to resort to more expensive machining techniques. For example, on a shaft with a close ± .002" (± 0,050mm) tolerance which could be lathe finished, if the designer used metric ± 0,020mm (which looks big!) he would be asking for ± .0008" and the shaft would have to be ground. Furthermore, more precise instruments than a micrometer would be necessary to verify it. Result: higher cost, lower production.

The millimeter is too large to be a precision unit of measurement and any portion of it - expressed in decimals - that is less than about 0,030 is too tiny. In general the millimeter is too small for use in non-precision work like wood working, and the centimeter is too large. The inch, which unfortunately was divided into 1/16ths, 1/32nds, etc. instead of 1/12ths, is otherwise a suitable measurement unit.

AN UNFULFILLED PROPHECY

It was widely believed, a decade ago, that the United States would complete its conversion to life under the metric system by 1985.

Road signs would be in kilometers, and motorists would be interested in liters per 100 kilometers instead of miles per gallon. Most of us would be shopping, cooking and working in metric.

The metric boosters were wrong.

There has been some (change), such as in automobile manufacturing and product packaging. But overall, the conversion seems to be barely inching - or centimetering - along. Many gas stations, for example, ARE GOING BACK TO GALLONS. (Emphasis added)

Some experts today say the nation's transition may not be completed until after the year 2000. (And if we are fortunate that transition may be to a sensible TWELVE based metric system instead of an awkward decimal system.)

-Reprinted from Newsday, Long Island, NY.
Parenthetical remarks added.

VACANCIES/AWARDS

Should you be on the Board of Directors? Do you know a member of the Society who should be considered for election to the Board? Send your name or theirs to us. Do you know of someone who deserves to be considered for the Society's Annual Award? Send your recommendations to us please.

DUODECIMAL TEST I

In the early years of our Society aspirants had to pass four tests in duodecimal arithmetic before they became full-fledged members. Thinking that some of our readers might wish to try their hands at these old tests, we reprint a version of test number one below. We will print other tests in future issues of the Bulletin.

THE DUODECIMAL SOCIETY OF AMERICA

Answers may be written or typed on these sheets. If additional sheets are used, refer to the questions by number.

Be careful to distinguish between decimal and duodecimal quantities by italicizing or underscoring the latter.

A careful study of the folder originally sent to you will enable you to answer these questions. We suggest that you also study the first *8* pages of Mr. Terry's "The Dozen System."

Two copies of the Test are enclosed. One is for your files.

Send your completed tests to the Chairman, Member Qualification Committee, at the indicated address. The next test will be sent to you as soon as this has been reviewed.

1. What does "Duodecimal" mean?

2. What does "Dozenal" mean?

3. What system of counting is in general use?

4. How many symbols are used in decimal counting? What are they?

Continued...

5. How many are used in dozenal counting?

6. What is the function of the 0, or zero?

7. What is meant by place value?

8. Has the zero the same function in dozenals as in decimals?

9. How is the zero used in fractions?

X. What is the general theory of duodecimal counting?

Ɛ. What new symbols are used, and what are they called?

10. What is *10* called, and what does it mean? (Italicized numerals are duo-decimals.)

11. Write in sequence the dozenal numbers for *1* to *100*, and under them the corresponding decimal numbers.

12. In decimal counting, what would be the number of this question?

13. What is the quantity commonly known as twelve called, duodecimally, and why?

14. What dozenal quantity corresponds to the decimal quantity 144, and what is it called? Why?

15. *10* articles can be arranged in how many equal piles of how many articles in each pile?

16. 10 articles?

17. There are 5280 feet in a mile. How many inches?

18. There are *3080* feet in a mile. How many inches?

19. Add the following:

14	*689*	*6,Ɛ95*
36	*4Ɛ3*	*5,238*

1X. Subtract the following:

689	*6,Ɛ95*
4Ɛ3	*5,238*

1Ɛ. Convert the following to dozenals: 14, 30, 84, 100.

20. Convert the following to decimals: *14, 30, 84, 100.*

I hope that you got some fun out of these questions. Next time you will get some problems in addition and subtraction, so study the table on Page *8* of The Dozen System. I shall be glad to answer any questions.

Learn to think in dozens.

A sheet of eight conversion rules which originally accompanied Test Four of this series is not in our files. If any of our long-term members has such a list, we would greatly appreciate receiving a copy. Thank you.

SOME HISTORY

"The ancient units of linear measurement were ... the digit (the width of a finger); the palm (the width of 4 fingers, i.e., 4 digits); the span (the spread between the outstretched thumb and little finger, equal to 3 palms); the cubit (distance between the elbow and the top of the middle finger, equal to 2 spans or 6 palms); ... and the fathom (the distance between outstretched arms, or 4 cubits). That makes a span equal a dozen digits, and a cubit equal two dozen digits.

- AMERICAN METRIC JOURNAL, volume XIII, MAR/APR 1985.

*STRANGE BASES (I), Continued from page **

1.2 Subtraction.

An easy way to perform subtraction is to add the negative of the number to be subtracted. Thus m-n = m + (-n).

To find the negative of a number in base -3 we first introduce the concept of (temporary) negative digits. We will let 0', 1', and 2' represent the digits -0, -1, and -2. Thus -(102) = 1'0'2'.

```
Note that -0 = 0' = +0
          -1 = 1' = 12
          -2 = 2' = 11
```

One could regularly employ negative digits in base -3, but our intention here is to make only temporary use of them, passing from the negative of an integer expressed in non-negative digits, thru that same number expressed in negative digits, and then to our result expressed in non-negative digits. (This is analogous to the way in which Gauss first used imaginary numbers to obtain real solutions to real problems.)

Example:

The negative of 102 = -102 = 1'0'2' = +1211 (or -[#] = +[#]).

In the example above we proceed as follows. Starting at the right we change 2' to 11. This gives a rightmost digit of 1 and a carry of +1 to be added to the next digit which is a zero. Since 1 + 0 is 1 we now have 1'11. Finally we change 1' to 12 obtaining 1211.

More examples:

Find the negatives of 11 and of 112021.
To find the negative of 11 we proceed as follows. -11 = 1'1'. Now starting at the right we replace 1' by 12 placing the 2 beneath 1' and carrying the 1. Repeating this we obtain

STRANGE BASES (I), Continued

```
   1'1'
 [ 1 1    ]   <-- carry for changing 1' into 12
   2 2        <-- step 1
 +__________  Now we add the carry to step 1,
              obtaining
 [-1      ]   <-- carry from adding
   0 0 2      <-- sum    (or -(-2)) = +2)
```

```
Similarly
-112021 = 1'1'2'0'2'1'
    [ 1 1 1 0 1 1   ]   <-- carry for changing
  +_____2_2_1_0_1_2__   <-- step 1
    [  -1           ]   <-- carry from adding
       1 2 0 1 1 2 2       (or -[-165] = +165;)
```

Now, returning to our algorithm for subtraction, we simply change (m-n) into (m + [-n]) and proceed as in addition.

```
Example, subtract 122 from 1221. First find
   -122 = 1'2'2'
     [ 1 1 1   ]    <-- carry from changing digits
    +____2_1_1____  <-- step 1
     [-1       ]    <-- carry from adding
       0 0 2 1      <-- sum
```

```
Next add 1221 and 21
     1221
    +__21
   [ -1  ]
     1112    (or [-12] - [5] = [-12] + [-5] = -17;)
```

We can sometimes shorten the above procedure by using temporary negative digits as follows:

```
      1221
     -__122

   becomes

     1 2 2 1
    +___1'2'2'
     1 1 0 1'  = 1112
```

Continued...

1.3 Multiplication.

Multiplication follows the usual algorithm we learned in grade school using the following table

```
x | 1   2
1 | 1   2
2 | 2 1'1
```

Where 1'1 is only temporarily used in place of 121 during the calculations

Thus

```
 121
x 12      (2 x 1 = 2, 2 x 2 = 1'1, i.e. 1 and a carry
 112       of -1, & 2 x 1 = 2 + the carry of -1 is 1)
121
0022      (or 4 x -1 = -4)
```

Again

```
          2 0 0 1 2
x             2 1 0
        2 0 0 1 2 0
    1'1 0 0 1 1
  1 2 1 2 0 1 2 2 0       (or 115 x 13 = 1493;)
```

1.4 Division.

As with multiplication, so also with division, the old grade school algorithm still works. However a difficulty in applying it arises from our lack of familiarity with the magnitude of numbers when expressed in base -3.

Example, divide 122 by 120. Proceeding in the usual fashion we use 2 as our first divisor since 120 divides 122 about twice.

```
       2
120)122
   -110
```

To subtract we use the shortcut mentioned previously, that is we add 1'1'0'. This gives

---

```
           2.1
1 2 0)1 2 2
    + 1'1'0
        1 2 0
        1 2 0
```

Thus 122 / 120 = 2.1   (or 5 / 3 = 2 - 1/3)

From all of the above we can see that negative integers less than -1 can be employed as a base of a positional number system using the digits 0, 1, 2, ..., up to the absolute value of the base -1, and that they satisfy the criteria:

1. All numbers can be expressed in the system, and
2. There exist algorithms for the four fundamental operations of arithmetic.

1.5 Changing Bases.

To change a number from base -3 is easy enough. Simply expand the number in the usual way, for example 1 101 121 equals

$$1(-3)^6 + 1(-3)^5 + 0(-3)^4 + 1(-3)^3 + 1(-3)^2 + 2(-3) + 1 = 327;$$

To change a number into base -3 we have a two step procedure. First change the number into base +3. (For information regarding converting numbers from one base to another see Manual of the Dozen System, page 1*.) Then change every other digit into a temporary negative digit. Thus to change 327; into base -3 we proceed as follows:

```
3 )327
3 )10*     1
 3 )43     1
 3 )15     0
  3 )5     2
  3 )1     2
     1
```

Continued...

STRANGE BASES (I), Continued

---

or 327; = $122\ 011_{+3}$ = $1'22'\ 01'1_{-3}$

= 2'01 121 = 1 101 121

An application of base negative two appears in Colson News (two-way numbers), Vol. 1, No. 3, page -44'- (-36-). (That's using the digit -4 in base dek.)

They write 2 as 110_{-2} which equals 4 - 2 + 0, and
3' as 1101_{-2} which equals -8 + 4 - 0 + 1.

In future articles we will continue our investigation of possible number bases from positive integers (such as twelve) thru negative integers (such as -3) to fractions (1/2), algebraic irrationals (the square root of 2), transcendentals (pi), and imaginary numbers.

Part II of this series will appear in the Fall 1985 issue of the DSA Bulletin

A gentle reminder -- the DSA Annual Meeting is October 11 and 12, 1985!

DOZENAL JOTTINGS

News from or about the dozenal activities of members and friends....

The **NAVAL WEAPONS CENTER** at China Lake, California recently contacted us for information on how to change between tolerances given in inches, twelfths of inches or lines, twelfths of lines, etc. and decimal tolerances. Essentially they needed to know how to change numbers such as 2;345 into decimals and conversely. (There are three methods explained in our Manual of the Dozen System which is available for $1;00.)...We welcome several new LIFE members to our Society: **ALBERT DE VALVE**, member number 32, of Alaska; **EUGENE 'SKIP' SCIFRES**, number 11, of Colorado, and **BILL LEONHARDT**, number 18*, of Canada. We greatly appreciate their generosity...ALBERT writes, reminding us of the meetings he attended years ago in New York City. Like many others, he discovered the convenience of duodecimals and invented his own notation before he found out about the Society. That would have been back in the late thirties...SKIP's wife passed away recently, and we send our condolences to him. A former member of our Board of Directors and an officer of the Society, he is retired now, and starting on a new career as a consultant, using his home computer. He hopes to attend some of our annual meetings in the future, and we look forward to seeing him again...BILL complains about using the asterisk and the octothorpe. He prefers the Pitman symbols used by our sister organization, the DSGB. He says that they are easier to write, and that the rotated 2 looks like a script T (for ten), while the rotated 3 looks like a script E (for eleven), and hence that they are easier to read and interpret. (The question of symbols has bothered our members since our founding almost three and one half dozen years ago. It is clear that no choice of symbols will ever satisfy all of us. We welcome your ideas. -Editor). BILL also sent some info on on the PPC ROM plug-in firmware module for the HP-41C (-CV, -CX) calculator, which has several valuable conversion routines. BD and TB (base to decimal and ten to base) handle only integers, but fractions can be easily accomodated by multiplication, then redivision. But the same amount must

Continued...

be factored each time, and since machine arithmetic on other than ten is unavailable, the easiest is to move the other out of the fraction category by an even power of base (simply treat it as an integer), then redivide it by the integer decimal value of that power of the other base. For conversion in the other direction, reverse the divide and multiply operations. Bill's favorites are PR and UR, pack and unpack registers, which is done by a simple polynomial to a given base. (Use twelve to unpack a ten number or to pack a number into tens, because digits packed to ten merely reproduce the ten number.) Finally, he says, there is QR, quotient remainder, the "complete mod function, essentially a poor man's base conversion.")...

HAYDEN STEELE, member number 258; writes from New Zealand. He says that the ten based metric system is forced upon them, and that it is very confusing. He inquires about our sister society, the DSGB. Well, the DSGB has just printed their third issue of The Dozenal Journal, Number 3, Summer 1984/1194. By the way, if any of you have old copies of the DSGB Journal Number 2, or old copies of their Dozenal Review we have received some requests for them from libraries. If yours are just gathering dust on a shelf, how about passing them along to us? It might be the source of some new converts!...Our annual cocktail party for people in the NY area was a great success. We hosted exactly one dozen guests, but we would have preferred more. A great time was had by all who attended, so if you can be present next year please let us know....**A. ADLER HIRSCH**, number 231; wrote asking for copies of our pamphlet so that he could distribute them at a meeting of the Louisiana Academy of Sciences. Do you know some person or persons who might enjoy reading some of our literature?...Your editor, **PAT ZIRKEL**, member number 251; recently attended a workshop at State University of NY at Stony Brook, Long Island. It was a day well spent learning how to edit, lay out, and produce a better publication...Scholarly and Professional Journals are usually reviewed. Thanks to the efforts of a few volunteers, articles in our BULLETIN will now be reviewed prior to publication. Our thanks to these members for volunteering. Our BULLETIN will be better because of their efforts....**FRED NEWHALL**, number 279; has volunteered once again to speak about Dozenals at the Annual Student

Symposium conducted by the Nassau County Association of Mathematics Supervisors and by the Nassau County (LI) Mathematics Teachers Association. The Symposium this year, the seventh annual, will be held on May 31st. Our thanks to Fred for his efforts to spread the word.

FRED also reports that his book on dozenals is growing apace. He hopes to have a copy in the Dozenal Library soon, for anyone's use... Heard from **DON HAMMOND**, the Editor of DSGB's Journal . Their last issue was a gem. They use a rotated 2 for dek and a rotated 3 for el, and pronounce them as ten (or decim) and elf. They say zen for dozens and hence what we would pronounce dek do el, they would call tenzen elf (or decimzen elf)....**ARTHUR WHILLOCK**, number 262; writes about ternary arithmetic and base three computers. Sounds fascinating. He asks for a half dozen copies of our Manual. He keeps spreading the word. Arthur uses both elf and el for eleven and dec or dek for ten, and states that there is no firm preference over there....Congrats to **TONY** 291; and **ANNETTE CATANIA** on the birth of their new son, **MICHAEL**. **TONY** is a member of the Annual Meeting

Continued...

Committee. Congratulations also to TONY for his recent promotion to Assistant Professor at Nassau Community College... Congratulations, too, to **BOB FOLEY** for his recent promotion to Associate Professor at the same institution...**KENNETT LOVE** 268; writes that he has moved to Sag Harbor, Long Island, NY. **KENNETT** is also a member of Americans for Customary Weight and Measure, and he vows that he will be present at our next Annual get-together. We look forward to meeting him. He is interested in symbols and words and appreciated the ease of using * and # when using his typewriter. He agreed with **FRED NEWHALL's** review of Metric Madness, and expresses interest in **DUDLEY GEORGE's** symbols and words....We caught that squirrel that got into our archives, and we plastered up the hole he used for access. A few old papers were chewed up and destroyed, but it doesn't appear that he got at anything essential. Fortunately, most of our material is stored in steel cabinets....**MRS. F. EMERSON ANDREWS** 257; the widow of one of our founders, is moving to Connecticut. She has offered us some back copies of the BULLETIN which we will be very happy to receive. Many thanks!....Mail to **VIVIAN LINTON** 244; has been returned by the post office. We have also lost contact with **B. KLAUSEN** of Copenhagen Denmark. Does anybody know their new addresses?...**DAVID SINGMASTER** 28*; showed our BULLETIN to **CEDRIC SMITH** in London, who in turn wrote to us. **SMITH** is one of the editors of the Colson News (Two-way Numbers), a journal dedicated to the use of negative digits in the style of the late **J. HALCRO JOHNSTON**. He has sent us back copies of Colson News which we have put in our Dozenal Collection housed at the Nassau Community College Library. Thanks to both of them....**A. ADLER HIRSCH** writes that the sixteenth-century Papal decision to decalate the calendar every century (?) year, except when divisible by 400 was in opposition to a recommendation by one of his own churchmen. Cardinal Pierre d'Ailly of Paris had pointed out that 132 years would be the proper interval. The Cardinal's figuring was close enough to present knowledge, which says 128 years is the proper interval. As Adler says: "This damnable second power of 10 somehow had the same lure on humanity as did the Serpent in Eden."....**CHARLES and MIRIAM BAGLEY** write from New

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Mexico that they were very pleased to learn that JUX HANDY had been honored by conferral of the Annual Award, since he has given so many years of faithful service to the Society. Charles also writes: "Your concern about our health is sincerely appreciated, and we are pleased to report that we are doing quite well in spite of our age. Here there is an advantage in dozenals -- Miriam is 69; and I am only one year older! But Father Time still thinks we are 81 and 82, at least until next birthday...**PAUL RAPOPORT** writes from Ontario that he is still interested in getting a dozenal analog watch created to go along with the dozenal clock he has, which has been running for 15 years. The watch preferably should have four hands in hierarchical ratios of a dozen, although Paul will settle for two or three hands. Paul asks: "Is there anyone out there who is interested along with me?"...**DR. ANTON GLASER** writes from the U. of Pennsylvania that he is sending us a number of duplicates from among his dozenal materials which he recently discovered while sorting out his collection. Thanks, Tony...

Welcome to new members:

293; **ISAAC ASIMOV** (NY), author of numerous books and articles on both scientific and science-fiction topics, has accepted Honorary Membership in the DSA...

294; **GERARD ROBERT BREST**, of Florida...

We are also pleased to announce the re-instatement of member number 37; **THOMAS M. O'NEILL**, of Washington...Welcome to all!

End

Remember - your gift to the DSA is tax deductible.

SYMBOLS -- *A REPLY TO DUDLEY GEORGE*

Dr. Paul Rapoport
McMaster University
Hamilton, Ontario, CAN

In Bulletin 50; page 17; I was interested to see Dudley George's symbols. (I think the spellings and nomenclature are problematic, but that doesn't matter for this little story.)

Many years ago (perhaps 20) I decided that I had to create symbols for ten and eleven which would not look like any other numerals or commonly used symbols, nor like any letters of the alphabet. Part of the reason for the latter was that I believed the new numerals should not betray an affinity to English any more than to any other language. Curiously, I came up with virtually the same things Dudley did for ten and eleven. Since, however, the eleven looked like the written letters *e* and *l*, I placed a horizontal bar through both ten and eleven. That is the only essential difference between my symbols and his.

Here are my symbols in an enlarged but rather square version, along with the other numerals in a monofont:

As you can surmise, I have always found things like * and # quite unusable because of their prior use in other ways, and the altered X and E unusable for all the reasons stated earlier. As for Bell Telephone, their placing of the additional symbols on either side of the zero makes at least the # useless for counting!

ON THE COVER

OCT 31 = *DEC 25* when octal 31 = decimal 25.

THANKS TO PROF. ROCHELLE MEYER, NASSAU COMMUNITY COLLEGE, NY.

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition—("Who needs a symbol for nothing?")—the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accomodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used, and as early as 1585, Simon Stevin stated that the duodecimal base was to be preferred to the base ten.

The parallel seems tenable. The notation of the dozen base better accomodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis**FACTOR**y because it has **NOT ENOUGH FACTORS.**

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurings. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions (1/3 = 0;4) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

COUNTING IN DOZENS

1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*✱*	*#*	*10*
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10,* and is called *do,* for dozen. The quantity *one gross* is written *100,* and is called *gro. 1000* is called *mo,* representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5,* and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3#2*	Two ft. eight in.	*2;8'*
19#	*1000*	Eleven ft. seven in.	*#;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53.* Using this *"which is"* step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2#,* which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

12) 365
12) 30 + *5*
12) 2 + *6*
0 + *2* Answer: *265*

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by ✱, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or ✱.

For more detailed information see *Manual of the Dozen System* ($1;00).

DETACH HERE

We extend an invitation to membership in our society.
Dues are only $12 (US) per calendar year; the only requirement is a constructive interest.

Application for Admission to the Dozenal Society of America

Name ______ LAST ______ FIRST ______ MIDDLE

Mailing Address (for DSA items) ______

______ (See below for alternate address)

Telephone: Home ______ Business ______

Date & Place of Birth ______

College ______ Degrees ______

Business or Profession ______

Employer (Optional) ______

Annual Dues .... $12.00 (US)

Student (Enter data below) .... $3.00 (US)

Life .... $144.00 (US)

School ______

Address ______

Year & Math Class ______

Instructor ______ Dept. ______

Other Society Memberships ______

Alternate Address (indicate whether home, office, school, other)

Signed ______ Date ______

My interest in duodecimals arose from ______

Use space below to indicate special duodecimal interests, comments, and other suggestions, or attach a separate sheet:

Mail to: Dozenal Society of America
c/o Math Department
Nassau Community College
Garden City, LI, NY 11530