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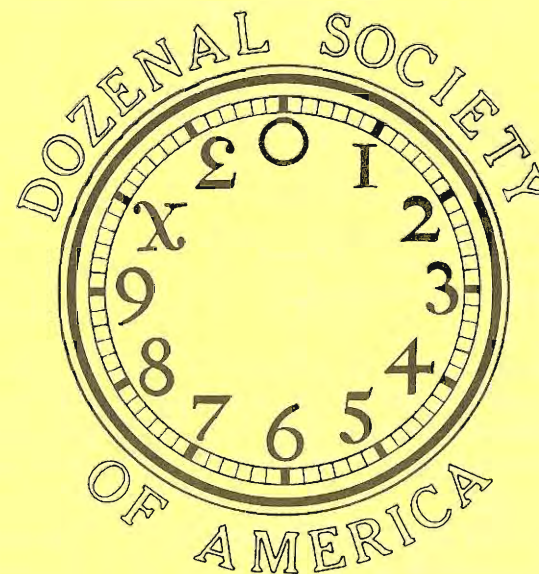
# The Duodecimal Bulletin

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Whole Number 48

Volume 28, Number 3

Fall 1983



**THE DOZENAL SOCIETY OF AMERICA**

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*c/o Math Department  
Nassau Community College  
Garden City, LI, NY 11530*



**DOZENAL SOCIETY OF AMERICA**  
*c/o Math Department  
Nassau Community College  
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## THE DOZENAL SOCIETY OF AMERICA

(Formerly: *The Duodecimal Society of America*)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are \$6.00 for one year. Student membership is \$3.00 per year.

*The Duodecimal Bulletin* is an official publication of the DOZENAL SOCIETY OF AMERICA, Inc., c/o Math Department, Nassau Community College, Garden City, LI, NY 11530.

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# The Duodecimal Bulletin

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*Whole Number 48*

*Volume 28, No. 3  
Fall 1983*

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## DOZENAL SOCIETY OF AMERICA

Annual Meeting

Friday to Sunday  
October 14 to 16, 1983

Nassau Community College  
Garden City, LI, NY 11530

Schedule

Friday evening, October 14, 1983

Our usual social gathering with friends and spouses at a theatrical performance at the College, or at a local sporting event. (Schedules for both are yet to be announced.)

Saturday, October 15, 1983

I 10 AM Business of the Society (Lower Level -  
Nassau Hall, Room B-10)

Tentative Agenda

1. Minutes of the 1982 Annual Meeting -- DeSanto
2. President's Report -- G. Zirkel
3. Treasurer's Report -- Malone
4. Editor's Report -- P. Zirkel
5. Reports by other officers
6. Report of the Nominating Committee -- Zirkel, Earnest,  
Handy

Election of the Class of 1986 of the Board of Directors. (Charles Bagley has declined to run for re-election. Henry Webber is deceased.)

SCHEDULE, 1983 ANNUAL MEETING, *Continued*

Election of Officers by the Board.

7. Appointment of new Committees, including Nominating and Awards.
8. New Business

LUNCH 12 - 2 PM

II 2 PM (Room B-10, Nassau Hall)

1. Guest Speaker (to be announced)
2. Panel discussions including Symbols, Calculators and Computers; other panels of interest to those attending.

III Evening

Our Annual Banquet with spouses, guests and friends.

Sunday, October 16, 1983

Sightseeing and departure, at leisure.

Please let us know if you are coming so that arrangements can be made for refreshments, tickets, etc.

For further information, call President Gene Zirkel at:

(516) 222-7611/7388 (College)  
(516) 669-0273 (Home)

*Continued.....*

## HOTELS CONVENIENT TO DSA ANNUAL MEETING

Friday, October 14 to Sunday, October 16, 1983

## I

The Long Island  
MARRIOTT HOTEL at  
Nassau Coliseum

BRAND NEW

About 3/4 mile from  
College center

101 James Doolittle Blvd.  
Uniondale, NY 11553  
(516) 794-3800

Single \$73 - 90<sup>1</sup>  
Double 88 - 105  
Each additional  
guest 5  
Children under  
18 free.

## II

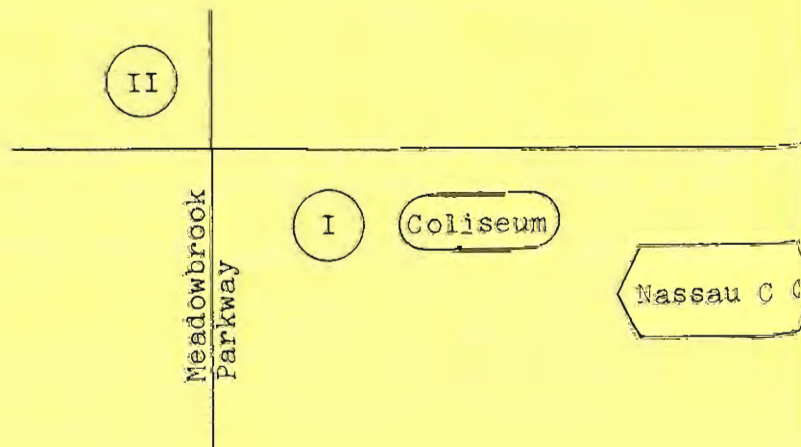
The COLISEUM MOTOR  
INN

Previously used  
by members

Less than 2 miles  
from College center

1650 Hempstead Tpke.  
East Meadow, NY 11554  
(516) 794-2100

Single \$40  
Double \$48  
Each additional  
guest \$ 8  
Children under 12  
free with parents



## III

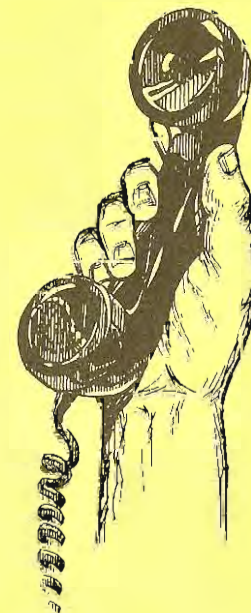
The HEMPSTEAD PLAZA  
HOTEL  
(Formerly: The  
Holiday Inn)

Previously used  
by members

About 2 1/4 miles from  
College center

80 Clinton Street  
Hempstead, NY 11550  
(516) 486-4100

Single \$45  
Double \$50  
(Advise hotel that  
you are with the  
Dozenal Society)



Hempstead  
Turnpike

1. All prices were quoted by hotels for the autumn.  
However, the DSA cannot guarantee rates.

## CYCLIC DUODECIMAL INTEGERS

Charles W. Trigg  
2404 Loring Street  
San Diego, California

An integer is said to be cyclic if every cyclic permutation of its digits produces a multiple of the integer.

2497 is cyclic since  $2497 \times 1 = 2497$

$$\times 2 = 4972$$

$$\times 3 = 7249$$

$$\times 4 = 9724$$

To progressively move from the first permutation to the others requires clockwise rotations of 3, 2, and 1 steps, respectively. The cyclic 2497 is a period of the repeating decimal  $1/5 = 0;\overline{2497}$ , and  $24 + 97 = \#\#$ .

The other duodecimal cyclic integer is  $186*35$ , which has no digits in common with 2497. Indeed,

$$186*35 \times 1 = 186*35$$

$$\times 2 = 35186*$$

$$\times 3 = 5186*3$$

$$\times 4 = 6*3518$$

$$\times 5 = 86*351$$

$$\times 6 = *35186$$

To move progressively from the first permutation to the others requires clockwise rotations of 2, 5, 3, 1, and 4 steps, respectively. The cyclic  $186*35$  is a period of the repeating decimal  $1/7 = 0;\overline{186*35}$ , and  $186 + *35 = \#\#\#$ .

CYCLIC DUODECIMAL INTEGERS, *Continued*

No integer can properly begin with zero, so the period of the repeating decimal  $1/15 = 0;\overline{08579214\#36429*7}$  is not a proper cyclic integer. However, it is of interest since it contains 15 - 1 digits, including each of the 10 distinct duodecimal digits plus the digits of 2497. Furthermore, each of its cyclic permutations is a multiple of the period P. Thus,

$$P \times 1 = 08579214\#36429*7$$

$$\times 2 = 14\#36429*7085792$$

$$\times 3 = 214\#36429*708579$$

$$\times 4 = 29*708579214\#364$$

$$\times 5 = 36429*708579214\#$$

$$\times 6 = 429*708579214\#36$$

$$\times 7 = 4\#36429*70857921$$

$$\times 8 = 579214\#36429*708$$

$$\times 9 = 6429*708579214\#3$$

$$\times * = 708579214\#36429*$$

$$\times \# = 79214\#36429*7085$$

$$\times 10 = 8579214\#36429*70$$

$$\times 11 = 9214\#36429*70857$$

$$\times 12 = 9*708579214\#3642$$

$$\times 13 = *708579214\#36429$$

$$\times 14 = \#36429*708579214$$

*Continued...*

## CYCLIC DUODECIMAL INTEGERS, *Continued*

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To progressively move from the first permutation to the others requires clockwise rotations of \*, 1, 9, 3, 12, 4, 5, 8, #, 10, 2, 11, 7, 13, and 6 respectively. This involves each integer from 1 to 13.

In each of the three cases, the next multiple would be a string of #'s. —

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### "EACH ONE TEACH ONE"

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*A letter from Brian Bishop in England reminded us of the above words which were often uttered by the ever zealous Ralph Beard. We should never forget that our most precious asset is YOU, our members, and that your most important activity as a member is to explain duodecimals to ONE other person.*

*Our Society is on the upswing. We have dozens of active members both in the U.S. and abroad. If, in the course of the next twelve months, each one of us secured only one new member our Society would double!*

*You should always have some copies of our brochure available to give to an interested person. Remember that we will gladly send a copy of the Excursion or a copy of our Bulletin to anyone who desires one. Now, it's up to you!*

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## AN UNAMBIGUOUS NOTATION FOR NUMBER BASES

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*Gene Zirkel  
Nassau Community College*

Most books that cover some number-base theory use a subscript notation to indicate which base is being used. Thus, we see such things as:

$$14_{12} = 16_{10} = 10\ 000_2$$

This causes no confusion to a person who ordinarily uses base ten and only infrequently dabbles in other bases. However, the anomaly in the symbols

$$10_2 = 2_{10}$$

can become confusing to people such as those in the computer field or to number theorists, some of whom must do extensive work with bases other than ten. Note for example, not only does the symbol 10 have two meanings in the expression

$$10_2 = 2_{10}$$

but also, a novice might have assumed that the second 10 meant ten. In fact, it could correctly stand for any base other than base two! To make matters worse the equation

$$100_{10} = 10_{10}$$

is true in many bases. For example, we could interpret the 10 on the left side in base three and both 10's on the right side as base nine.

A simple notation used by those in the computer field removes these ambiguities. It is a common practice to indicate hexadecimal numerals by enclosing them in single quotes and prefixing an X, thus we write

$$X'73' \text{ instead of } 73_{16}.$$

*Continued...*

UNAMBIGUOUS NOTATION, *Continued*

Binary numerals are preceded by a B making

$$B'101' = 101_2.$$

This notation replaces the subscript 16, which could have more than one interpretation, with the unambiguous symbol X. Some people also use 0'134' for the octal base numeral 134<sub>8</sub>.

I would like to suggest that those of us who work with different number bases extend this simple idea, using symbols such as those listed below. (Most of the symbols that I am suggesting derive from the Greek prefixes we learned in geometry for naming polygons.)

*Continued...*

*Professor Bronowski in his BBC Lectures on "The Ascent of Man" discussed crystalline structures and observed that spatial relationships are related to simple numbers 2, 3, 4 and 6. No crystal has a 5 times rotation of surfaces.*

*- Arthur Willock, 262;*

*DSGB*

Are your dues paid? Why not mail that check today?

UNAMBIGUOUS NOTATION, *Continued*

Base	Proposed symbol
Two	B (Binary)
Three	T (Triangle)
Four	Q (Quadrilateral)
Five	P (Pentagon)
Six	H (Hexagon and Half dozen)
Seven	S (Septagon - admittedly a variation of heptagon)
Eight	O (Octagon)
Nine	N (Nonagon)
Ten	A (Honoring the ancient Arabs who gave us our present day Hindu-Arabic numerals and the idea of a positional notation in the first place. A is also the usual hexadecimal digit for ten.)
Eleven	E (Eleven)
Twelve	D (Dodecagon and Dozen)
Sixteen	X (heXadecimal)

*Continued...*

*Celebrate National Metric Week (9 - 15 October, 1983) by attending the DSA Annual Meeting, 14 - 16 October, 1983, in New York!*

UNAMBIGUOUS NOTATION, *Continued*

Thus, the ambiguous expressions above could be rewritten as follows:

I  $14_{12} = 16_{10} = 10\ 000_2$  would become  $D'14' = A'16'$   
 $= B'10\ 000'$

II  $10_2 = 2_{10}$  might become  $B'10' = A'2'$  if we meant base ten, but we could write  $B'10' = D'2'$  if we were counting in dozens.

III  $100_{10} = 10_{10}$  might become  $T'100' = N'10'$  or  $B'100' = Q'10'$ , depending on what we intended. It would no longer be a vague equation open to interpretation.

It certainly would be clearer that  $14=4^2$  is true (provided that we are working in the dozen base) if we wrote it as  $D'14' = 4^2$ .

The subscript notation may be fine for those who do very little work in other bases, but the  $B'101'$  notation has proved very useful and clear to those who are constantly required to handle binary numerals. Extending this notation to other bases offers a similar utility and clarity to all numerals in whatever bases we wish to express them.       



*Remember - your gift to the DSA is tax deductible.*

## THE AXIOLOGICAL ANALYSIS OF ARITHMETIC BASES

*Rod Glasgow*

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The aim of this article will be to develop a methodology that will concisely determine the most efficient arithmetic base system within the entire range of natural numbers. Formulae will be developed in the methodology which will be used for testing the efficiency of arithmetic bases.

I am submitting this article as a way of introducing myself as a new member of the D.S.A. in the hope that The Duodecimal Bulletin will provide a forum for others to comment upon the following analytical method and as a response to Jay Schiffman's interesting article in Volume 27, Number 3, "A Dozen Properties of the Number Twelve". In addition to being an amateur mathematician, I am an avid neologist. I have substituted the new term "multivision" for the traditional term "division" in an effort to embrace a more appropriate description for this basic arithmetic operation. In fact, it is this operation which is of initial importance to the above mentioned methodology. So, I ask that the reader allow me the indulgence of using the function  $m(N)$ , the quantity of multivisors of a positive integer (or more simply, the multivisors of a number), instead of the traditional  $d(N)$ , the divisors of a number.

In view of this article's aim, the arithmetic base which most efficiently accomodates multivision will automatically accomodate the more simple operations of addition, subtraction, and multiplication. This is due to the fact that multivision has the greatest limitation in operating wholly. Thus, our methodology begins with the premise that whole multivision is the fundamental operation upon which numerical base systems are critically dependent. To not choose efficient multivision as fundamentally important to the task of determining the best numerical base is indeed arbitrary, if not negligent.

*Continued...*



AXIOLOGICAL ANALYSIS, *Continued*

The term "axiological" hereafter refers to logical criteria which have been deemed to have important value in the analytical method, while the term "arcanon" represents a specific principle which has axiological status. The table at the end of this article contains the values for the first 2½ dozen natural numbers in terms of the axiological relationships which shall be considered below. The table is written in duodecimal notation.

"Multivisibility" shall now be designated as the first arcanon in the axiological analysis of arithmetic bases and represented by the abbreviation AAA1, which is read "triple A one". The AAA1 projects the quantity of multivisors of a positive integer, N, to be tested for its quality of efficiency as a numerical base in the form of a function of N taken as a proportion of N. This proportion will represent a measure of the efficiency in terms of the criteria of the axiological analysis. Let the arrow in the notation that follows be read "projects the efficiency ratio", or more simply "projects":

$$\text{AAA1} \longrightarrow m(N)/N$$

Referring to the table, if  $N=4$ , then  $m(N)=3$ , representing the three multivisors of 4, namely 1, 2, and 4 itself. Then, also,  $m(N)/N=0;9$ , the efficiency ratio for Base Four strictly in terms of multivisibility. What will be quickly noticed in the table is that 1 and 2 are the only numbers which have the maximum efficiency ratio of  $m(N)/N=1;0$ .

As initially important as the AAA1 is, the determination of the efficiency of a numerical base number will require a refinement in the axiological criteria. What criterion would most properly serve as the second arcanon? I suggest that it is directly evident that multivision by 1 and N itself is of little value in determining the most efficient base on the grounds that every number possesses this capacity. With this consideration in mind, 1 shall be called the universal multivisor and N shall be called the holoversal multivisor; all other multivisors will be particular to the base being tested. "Particularity"

AXIOLOGICAL ANALYSIS, *Continued*

shall therefore be designated as the AAA2 and the particulate multivisors of a number shall be indicated by  $\mu(N)$ , such that  $\mu(N)=m(N)-2$ , for  $N > 1$ . Thus our formula becomes refined as:

$$\text{AAA1} + \text{AAA2} \longrightarrow \mu(N)/N$$

Referring again to the table, if  $N=4$ , then  $\mu(N)=1$ , representing the only particulate multivisor of 4, namely 2. And  $\mu(N)/N=0;3$ , a more refined efficiency ratio for Base Four in terms of particulate multivisibility. From the table it will be noticed that the maximum value for  $\mu(N)/N$  is  $0;4$  and is limited to two numbers, 6 and 10. Although the table does not go beyond  $N=26$  it will be recognized that the ratios become progressively smaller after  $N=10$  as N approaches infinity (not in natural sequence, but in a "flip-flopping" manner that reflects the random sequence of the prime numbers and their incorporation into the composite numbers).

At first sight it is tempting to say that 10 is a better base than 6 because it is the larger quantity and has a larger quantity of particulate multivisors. Although this is a fact, it will prove helpful if the spirit of axiological analysis is maintained in an effort to determine the qualitative characteristic that will distinguish whether 6 or 10 is the best base for a number system, while the use of the analytical principle of quantity is maintained for the purpose of characterizing the efficiency of each numerical base.

The third and final arcanon, however, can be derived if we do utilize our intuition that 10 is a better base than 6 due to its quantity, or more accurately its quantity of multivisors. What does having more multivisors provide 10 in a qualitative sense? I submit that by comparing the respective multivisors of 6 and 10 that, although 10 is multivisible by 6 and by the two particulate multivisors of 6, it is because 6 is only particulate multivisible by primes that 10 is the better base. That is to say, 10 is also particulate multivisible by the first two composite numbers in the natural number sequence, and it can not be

*Continued...*

AXIOLOGICAL ANALYSIS, *Continued*

---

emphasized too highly how important the numbers 4 and 6 are in geometry (which I prefer to call holometry, but that's another article in itself). We must be careful at this point, however, in the further refinement of  $\mu(N)/N$ . If we merely isolate those particulate multivisors which are composite, then 20 and 10 have the maximum efficiency ratio 0;2; although only 20 competes with 10 in this regard, the qualitative characteristic that was deemed important in the axiological difference between 6 and 10 has not been found. I have indicated the composite particulate multivisors of a number in the table as  $\mu_c(N)$ .

It is at this point in the axiological analysis that intuitive judgment becomes critically important, and I shall proceed by taking an assertive leap: Some composite numbers are focalized between the two primes of a prime doublet, such that if  $p$  and  $(p+2)$  were the members of a prime doublet for  $p \geq 3$ , then the focalized composite,  $c_f$ , would be defined as  $c_f = p+1$ . 10 and its two composite particulate multivisors, 4 and 6, are focalized by prime doublets! 20 and one of its composite particulate multivisors, namely 8, are not.

This characteristic quality succeeds in isolating a single solution for  $N$  having the maximum efficiency ratio when combined with the AAA1 and the AAA2. I have therefore designated the AAA3 as "prime-focalization of composite numbers", or more simply "prime-focality". The prime-focalized, composite, particulate multivisors of a number shall be indicated by  $\mu_f(N)$ , such that:

$$AAA1 + AAA2 + AAA3 \longrightarrow \mu_f(N)/N$$

Thus, referring to the table, if  $N=4$ , then  $\mu_f(N)=0$ , since there are no prime-focalized, composite, particulate multivisors of 4. And  $\mu_f(N)/N=0;0$ , the fully refined efficiency ratio of Base Four in terms of prime-focalized particulate multivisibility.

*Continued...*

AXIOLOGICAL ANALYSIS, *Continued*

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The formula for the maximum efficiency of an arithmetic base can now be maximized:

$$\max(\mu_f(N)/N) = 0;2$$

The formula for the base number which has this maximum efficiency can be written:

$$N(\max(\mu_f(N)/N)) = 10_{\text{Twelve}}$$

Knowing that Base Twelve is the only number from 1 through infinity having the maximum efficiency ratio of 0;2 which is determined by only three highly distinctive criteria will greatly aid in our explanation to others why it is not only better than Base Ten, but the "best" base of all. The arcanons of "multivisibility" and "particularity" are very straightforward, but the main task in the explication of this axiological analysis of arithmetic bases lies in the adoption of "prime-focality" as the third arcanon. In my next article, I will give a more full account for why composites focalized by prime doublets constitute the only reasonable criterion for the AAA3. In the meantime, may a dozen joys come your way soon.

*Continued...*



We'll be *very disappointed* if you're not with us for the DSA Annual Meeting -- October 14 - 16, 1983 in New York.

Call (516) 669-0273 for further information.

(maximum values are indicated by "max")

N	m(N)	m(N)/N	$\mu(N)$	$\mu(N)/N$	$\mu_c(N)$	$\mu_c(N)/N$	$\mu_f(N)$	$\mu_f(N)/N$
1	1	1;0 max	0	0;0	0	0;0	0	0;0
2	2	1;0 max	0	0;0	0	0;0	0	0;0
3	2	0;8	0	0;0	0	0;0	0	0;0
4	3	0;9	1	0;3	0	0;0	0	0;0
5	2	0;4972...	0	0;0	0	0;0	0	0;0
6	4	0;8	2	0;4 max	0	0;0	0	0;0
7	2	0;35186X...	0	0;0	0	0;0	0	0;0
8	4	0;6	2	0;3	1	0;16	1	0;16
9	3	0;4	1	0;14	0	0;0	0	0;0
X	4	0;4972...	2	0;2497...	0	0;0	0	0;0
E	2	0;1...	0	0;0	0	0;0	0	0;0
10	6	0;6	4	0;4 max	2	0;2 max	2	0;2 max
11	2	0;1X...	0	0;0	0	0;0	0	0;0
12	4	0;35186X...	2	0;186X35...	0	0;0	0	0;0
13	4	0;32497...	2	0;17249...	0	0;0	0	0;0
14	5	0;39	3	0;23	2	0;16	1	0;09
15	2	0;149X7...	0	0;0	0	0;0	0	0;0
16	6	0;4	4	0;28	2	0;14	1	0;08
17	2	0;131X89...	0	0;0	0	0;0	0	0;0
18	6	0;37249...	4	0;2497...	2	0;12497...	1	0;07249...
19	4	0;235186...	2	0;1186X3...	0	0;0	0	0;0
1X	4	0;2	2	0;1	0	0;0	0	0;0
1E	2	0;106316...	0	0;0	0	0;0	0	0;0
20	8	0;4	6	0;3	4	0;2 max	3	0;16
21	3	0;15343X...	1	0;059153...	0	0;0	0	0;0
22	4	0;1X1X1X...	2	0;0EOEOE...	0	0;0	0	0;0
23	4	0;194	2	0;0X8	1	0;054	0	0;0
24	6	0;26X351...	4	0;186X35...	2	0;0X3518...	1	0;05186X...
25	2	0;09E209...	0	0;0	0	0;0	0	0;0
26	8	0;32497...	6	0;2497...	3	0;12497...	1	0;0497...

## MORE ON FELLOWS OF OUR SOCIETY

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In the Summer, 1983 issue, we listed a few members who had been designated as Honorary Members, Life Members or as Fellows of the DSA. Going thru some of our old records, we came across a list of 29 Fellows from about the year 1964. Unfortunately, some of these have since passed away or are now out of touch. The following are those active members who are Fellows:

Bagley, Charles S.	*3
Baumeister, Theodore	140
Churchman, Henry C.	72
Cunningham, George S.	107
Godjevatz, Velizar	#56
Gray, Nelson B.	#9
Handy, Jamison, Jr.	19
Johnston, J. Halcro	55
Lien, Dallas H.	14
Moon, B. A. M.	1*7
Schumacher, William C.	84
Scifres, Eugene M.	11
Valevski, I.V. Colonna	#7

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*Why not give some of our literature to a friend? Pamphlets, Excursions, and Bulletins are available.*

## DOZENAL JOTTINGS

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We all know about F. EMERSON ANDREWS' famous article in the October 1934 Atlantic Monthly which was the instigation of the birth of our Society. But did you know that he also published another article in the February 1935 Atlantic Monthly entitled "Revolving Numbers"? It is to be found on Pages 208-211....GENE ZIRKEL was rummaging through old files recently and came across a clipping from the NY Herald Tribune, dated 1952. It concerned a remote village in rural Spain, the majority of whose inhabitants had six or more fingers on each hand. They had developed their own dozenal counting system as a direct result of counting on their fingers....ARTHUR WHILLOCK of the DSGB writes: "Your note on nomenclature in Bulletin No. 46 ("How Do You Pronounce 32?", Gene Zirkel, p.7) raises an aspect second

*Continued...*

### INTER-LIBRARY LOAN

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*Now that the Dozenal Collection has been permanently established at the Nassau Community College Library, it is possible for anyone to borrow books through inter-loan service with your local public or school library. Out-of-print books such as New Numbers by Andrews, The Dozen System by Terry, and others are available.*

DOZENAL JOTTINGS, *Continued*

only in importance to the digits themselves. Whereas the initiated can (and do) discuss matters of mutual interest in terms that are handy without misunderstanding, we should be in the business of explaining our subject to those who are naturally suspicious of anything new. Neologisms may sound sweet to their initiators, but tend to be off-putting to others; so we must tread carefully. There have not been quite as many methods of verbalising numerals in bases other than ten but it seems that some effort should be made to distinguish them from ordinary usage for the sake of ordinary folk. The principle of Ockham's Razor applies to all things -- ("Entities should not be multiplied unnecessarily.") -- and it is thus safer to do no more than say the numbers in order to stress that a denary sequence is not assumed. Between ourselves, however, just as three-tens-two has been shortened and euphonised (?) to thirty-two, the present three-do-two will, with use, become thirdy two. No? Another point is that care should be exercised when writing numbers. Figures are often used as abbreviations when words would be more appropriate. It is particularly irritating to read 10 when it is not clear whether ten or do is meant. Figures should be reserved strictly for calculations with words for adjectival and descriptive use.....".....Health and happiness to MIRIAM and CHARLES BAGLEY who celebrated their eightieth and eighty-first birthdays over the summer.....On Monday, August 29th, an air conditioner malfunctioned in the room above our Society's storeroom at Nassau Community College. Some of our papers and booklets got soaked. As we go to press, the full extent of the damage has not yet been ascertained, but we will keep you posted.....

*end*

## DAFFYNITION

*Douzaine Dont's: The 10; commandments*

## DOZENAL CALCULATOR

*Texas Instruments has responded favorably to our request for a calculator that would work in dozens. Their Research and Development Department is looking into the idea.*

*"There is a principle which is a bar against all information, which is proof against all arguments and which cannot fail to keep a man in everlasting ignorance - that principle is contempt prior to investigation."*

*- Herbert Spencer*

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*The following are available from the Society*

1. Our brochure (free)
  2. "An Excursion In Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, Oct. 1934. (Single copies free. Bulk orders 40¢ each)
  3. *Manual of the Dozen System* by George S. Terry (\$1;00)
  4. *New Numbers* by F. Emerson Andrews (\$10;00)
  5. *Douze: Notre Dix Futur* by Jean Essig in French (\$10;00)
  6. Dozenal Slide rule, designed by Tom Linton (\$3;00)
  7. Back issues of the *Duodecimal Bulletin* (as available) 1944 to present (\$2;00 each)
-

## PUZZLE CORNER

26 DAYS HATH SEPTEMBER

In the September 1982 issue of the Mathematics Teacher, there appeared a calendar which asked this question:

- September - Can you express the dates from 1 to 30 with six 9's?  
 October - Can you express the dates from 1 to 31 with six ten's?  
 and so on for each month.

We are moved to ask: Given the dozenal digit,  $d$ , how many numbers can you express with six such digits?

For example:

$$1 = (d+d+d)/(d+d+d)$$

$$6 = [(d+d+d)/d]!+d-d$$

$$\# = d/d;d - [(d+d)/(d+d)]$$

$$11 = d/;\bar{d}+(d/d)+(d/d)$$

You are invited to send us your solutions to or your extensions of these problems. Also, send us other problems which are related to dozenals or to number bases.



## PUZZLE CORNER

## SOLUTION

One of the more interesting solutions to the question proposed in our last issue was

$$2(\overline{1510}) = \overline{1510};$$

For those readers who are not familiar with this Reverse Notation invented by J. Halcro Johnston, 55; of Scotland, a bar over a digit indicates a negative digit and one counts from one to a dozen as follows:

1, 2, 3, 4, 5, 6,  $\overline{15}$ ,  $\overline{14}$ ,  $\overline{13}$ ,  $\overline{12}$ ,  $\overline{11}$ , 10.

Thus  $\overline{1510}$  in base  $b$  is equivalent to  $1 \cdot b^3 - 5 \cdot b^2 + 1 \cdot b + 0$ .



*"And Neolithic man was counting, and falling under the spell of numbers....He was beginning to use tallies, and wondering at the triangularity of three, and the squareness of four, and why some quantities like twelve were easy to divide in all sorts of ways, and others, like thirteen, impossible. Twelve became a noble, generous, and familiar number to him, and thirteen rather an outcast and disreputable one."*

- H. G. Wells  
*The Outline of History*

## IN NEXT ISSUE

DUODECIMAL PRIMES

*Charles W. Trigg*

MINUTES: 1983 ANNUAL MEETING

ORIENTAL DOZENS



SEE YOU

AT THE

DSA ANNUAL MEETING

*Friday through Sunday, October 14 through 16, 1983,**at Nassau Community College, Garden City, Long**Island, New York.**Call (516) 669-0273 for further information.*

## WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Hindu-Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numerals then universally used.

Yet, although it took D years, and despite much opposition—"Who needs a symbol for nothing?"—the new notation did come into popular use. Released from the drag of Roman notation, man's thinking leapt forward dramatically, and mathematicians discovered a new dimension in mathematical symbolism. Working with Hindu-Arabic numeration, they found that the new system better accommodated mathematical statements and facilitated the working out of ideas. Re-examining their fundamental concepts of numbers, they made advances in arithmetic, algebra, logarithms, analytic geometry and calculus, and thus contributed to the explosion of human thought which later became known as the Renaissance.

In a related development, man awoke to the fact that different number bases could be used, and as early as 1585, Simon Stevin stated that the duodecimal base was to be preferred to the base ten.

The parallel seems tenable. The notation of the dozen base better accommodates mathematical statement and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base is preferred for the very same advantages which led the carpenter to divide the foot into twelve inches, the baker and the grocer (one who deals in *grosses*) to sell in dozens, the chemist and the jeweler to subdivide the Troy pound into twelve ounces. And yet, this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatisfactory because it has **NOT ENOUGH FACTORS**.

Then should we change? Yes, but no change should be forced, and we urge no mandated change. All the world counts in tens. But people of understanding should learn to use duodecimals to facilitate their thinking, their computations and their measurements. Base twelve should be man's second mathematical language. It should be taught in all the schools. In any operation, that base should be used which is the most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity because they simplify the all-important problem of the correlation of weights and measures, the expansion of fractions ( $1/3 = 0;4$ ) and give an advantage in calculations involving time and our twelve-month calendar. Perhaps by the year 2000, (or maybe by 1200; which is 14; years later!) duodecimals may be the more popular base. But then no change need be made, because people will already be using the more convenient base.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have erroneously thought staid and established and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is most cordially invited.

## COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, 265 represents 5 units, 6 dozen, and 2 dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5;9'
31	694	Three ft. two in.	3;2'
96	3#2	Two ft. eight in.	2;8'
19#	1000	Eleven ft. seven in.	#;7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is 5 dozen and 3*; so set down 53. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only 2#, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

$$\begin{array}{r} 12 \overline{) 365} \\ \underline{12} \phantom{0} \\ 30 \phantom{0} \\ \underline{12} \phantom{0} \\ 18 \phantom{0} \\ \underline{12} \phantom{0} \\ 6 \phantom{0} \\ \underline{0} \\ 0 + 2 \end{array} \text{ Answer: } 265$$

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12<sup>2</sup> (or 144) times the third figure, plus 12<sup>3</sup> (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by #, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or #.

For more detailed information see *Manual of the Dozen System* (\$1;00).

We extend an invitation to membership in our society.  
Dues are only \$6 per year; the only requirement is a constructive interest.

## Application for Admission to the Dozenal Society of America

Name \_\_\_\_\_

Mailing Address (for DSA items) \_\_\_\_\_  
LAST FIRST MIDDLE

(See below for alternate address)

Date & Place of Birth \_\_\_\_\_

College \_\_\_\_\_ Degrees \_\_\_\_\_

Business or Profession \_\_\_\_\_

Employer (Optional) \_\_\_\_\_

Annual Dues ..... \$6.00

Student (Enter data below) ..... \$3.00

School \_\_\_\_\_

Address \_\_\_\_\_

Year & Math Class \_\_\_\_\_

Instructor \_\_\_\_\_ Dept. \_\_\_\_\_

Other Society Memberships \_\_\_\_\_

Alternate Address (indicate whether home, office, school, other)

Signed \_\_\_\_\_ Date \_\_\_\_\_

My interest in duodecimals arose from \_\_\_\_\_

Use space below to indicate special duodecimal interests, comments, and other suggestions:

Mail to:  
Dozenal Society of America  
c/o Math Department  
Nassau Community College  
Garden City, LI, NY 11530

DETACH HERE