

# The Duodecimal Bulletin

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Whole Number 43

Volume 27, Number 1

Winter 1982



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THE DOZENAL SOCIETY OF AMERICA

*c/o Math Department  
Nassau Community College  
Garden City, LI, NY 11530*

**DOZENAL SOCIETY OF AMERICA**  
*c/o Math Department  
Nassau Community College  
Garden City, LI, NY 11530*

## THE DOZENAL SOCIETY OF AMERICA

(Formerly: The Duodecimal Society of America)

is a voluntary, nonprofit, educational corporation, organized for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Membership dues are \$6.00 for one year. Student membership is \$3.00 per year.

*The Duodecimal Bulletin* is an official publication of the DOZENAL SOCIETY OF AMERICA, Inc., c/o Math Department, Nassau Community College, Garden City, LI, NY 11530.

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# The Duodecimal Bulletin

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## TOM B. LINTON



Born: May 13, 1910  
Died: March 5, 1981

Tom B. Linton, a member of the DSA Board of Directors for the past 23 years, died at the age of 70 in March of this past year. As a devoted Dozenalist and a tireless worker and organizer, Tom will be sorely missed by his many friends and associates in the cause of duodecimal counting.

By profession, Tom was an engineer whose education and experience included both the mechanical and electrical arms of his discipline. Within the profession he was active in research and development and, early in his career, was issued several patents. Perhaps the most noteworthy of these was on his design of aircraft hydraulic apparatus.

For several years in the late 1950's Tom worked at the California Institute of Technology (Pasadena) as a Project Engineer, responsible for the design and construction of scientific apparatus used in various research projects within the Institution. Later he participated in the design and manufacture of electronic instruments, and in 1962 went to work for the U.S. Naval Weapons Station at Seal Beach, CA, where he quickly became Supervisory General Engineer.

In late 1977, Tom retired from Federal Service in order to devote his full attention to the Dozenal Society. He had been elected to the Board of the Society in 1958 to fill a vacancy created by the death of Paul E. Friedman during that year. In 1963 Tom became Executive Secretary of the Society, sharing the duties of the post with Society co-founder Ralph Beard, who became Corresponding Secretary at that time. In 1964 Tom assumed both duties, which he held through 1973. It was during these years that Tom designed and made master plates for a log log trig circular slide rule in base twelve for the use of the Society. He was also honored with the Society's Annual Award in 1968. In 1974 he became Society President, a post he retained until his death. Tom's wife Vivian has also served the Society. She was elected to the Board in 1977 and served as Secretary from 1977 to 1979.

As President, Tom corresponded regularly with members of the Dozenal Society of Great Britain and was prime mover in the twin causes of revitalizing the Society and re-issuing the Bulletin.

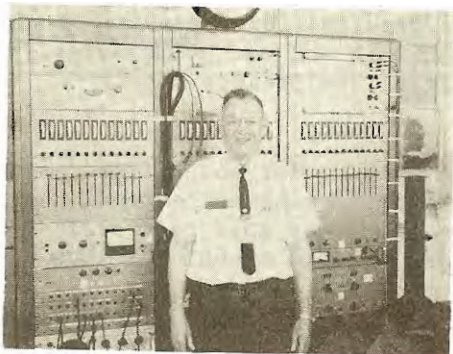
Throughout his life Tom was an avid reader and writer, and published many articles on engineering and dozenal topics. Following his retirement he was able to travel rather extensively, and attended Society annual meetings in many parts of the country.

Tom will be remembered as a man of large vision and prolific accomplishment. His many friends, both here and abroad, are a testimony to the breadth of his character and involvement. His life was rich in family -- he leaves three sons and a daughter -- and in the depth of his devotion to his varied interests. He cannot be replaced, but his spirit will certainly survive among those with whom he came in contact, both Dozenalists and others, and his influence will continue to be felt in their accomplishments.

*Many, many thanks to Vivian Linton for her kind assistance in providing material for the preceding article.--Ed.*

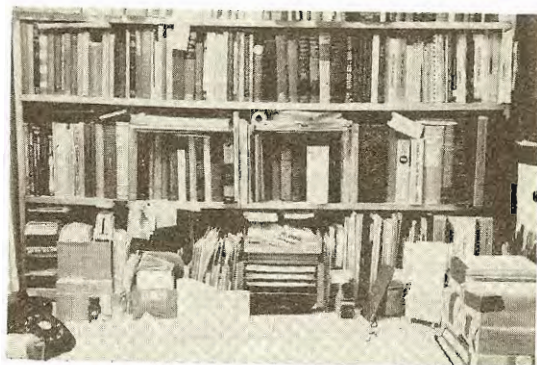


*Tom Linton, Henry Churchman, Charles Bagley and Kingsland Camp shown at the 1971 Annual Meeting, Royal Inn, Boulder, CO.*



*Tom Linton--the man and his machine.*

*Tom's office  
(den? lair??)*



*The following are available from the Society*

1. Our brochure (free)
2. "An Excursion In Numbers" by F. Emerson Andrews. Reprinted from the *Atlantic Monthly*, Oct. 1934. (Single copies free. Bulk orders 20¢ each)
3. *Manual of the Dozen System* by George S. Terry (\$1;00)
4. *Duodecimal Reciprocals 4/6 Places* by James M. Dixon (\$1;00)
5. *New Numbers* by F. Emerson Andrews (\$10;00)
6. *Douze: Notre Dix Futur* by Jean Essig in French (\$10;00)
7. Dozenal Slide rule, designed by Tom Linton (\$3;00)
8. Back issues of the *Duodecimal Bulletin* (as available) 1944 to present (\$1;00 each)

DOZENAL SOCIETY OF AMERICA

Annual Meeting

*Friday and Saturday  
May 12; and 13; 1192  
(May 14 and 15; 1982)*

*Nassau Community College  
Garden City, LI, NY 11530*

Schedule

Friday, May 12; 1192 (May 14, 1982)

Group will attend "On the Move" a dance production, presented by the Theatre and Dance Department of the College--in the Recital Hall, (Building T-412)

Saturday, May 13; 1192 (May 15, 1982)

I 10 AM. Business of the Society (Lower level-Nassau Hall, Room B-10)

Tentative Agenda

1. Minutes of the 1981 Annual Meeting --John Earnest, Secretary.
2. The Treasurer's Report--James Malone, Treasurer.
3. Report of the Committee to Update the Constitution and Bylaws--Chmn. Selfridge, Earnest, Scordato.
4. President's Report (Archives; library; correspondence; progress; Annual Awards; appointment of new committees for Nominations, Awards, Annual Meeting) --Zirkel
5. Report of the Nominating Committee--Chmn. Selfridge, Malone, Scordato.

Election of the Class of 1985 of the Board of Directors. (Miriam Bagley declines to run for re-election.) Election of officers by the Board. (John Earnest and Charles Bagley decline to run for re-election.)

Appointment of the Editor of the Duodecimal Bulletin.

6. New business.

II 2 PM. (Room B - 10, Nassau Hall)

1. Guest Speakers
2. Discussion of Dozenal ideas and activities by all present.
3. Visits to DSA Archives and Library Collection.

III Evening

Annual Banquet with spouses, guests and friends.

*Please let us know if you are coming, so that arrangements can be made for refreshments, tickets, etc.*

*For further information call President Gene Zirkel at:*

(516) 222-7611 / 7383 (College)  
Or (516) 669-0273 (Home)

#### A CALL FOR PAPERS AND ARTICLES

Anyone who wishes to present a paper at the annual meeting or who wishes to submit an article for publication should contact us as soon as possible.

Papers and articles need not be lengthy nor advanced. Some of the best ideas are short and simple. So don't keep that nice idea to yourself. Speak out at the annual meeting or in a future issue of this Bulletin. Share your good ideas with the rest of us. --GZ

Suggested accommodations in the area of Nassau Community College:

The Island Inn  
Old Country Road  
Westbury, LI, NY  
(516) 741-4500

Holiday Inn,  
Westbury  
369 Old Country Rd  
Westbury, LI, NY  
(516) 997-5000

Coliseum Motor Inn  
1650 Hempstead Tpke  
East Meadow, LI, NY  
(516) 794-2100

Holiday Inn,  
Hempstead  
80 Clinton Street  
Hempstead, LI, NY  
(516) 486-4100

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11530  
U.S.A.



By secret ballot a near unanimous vote of the Board of Directors bestowed the Society's Annual Award for 1982 on our outgoing Chairman of the Board, Charles S. Bagley. (It was a secret because we didn't tell Charlie, and it wasn't unanimous because we didn't give him an opportunity to vote.)

Chairman Bagley was a Geodesist with the Air Force Missile Development Center at the Hollman-White Sands Range before he retired, and he is a Bishop in the Church of Latter Day Saints. He had published his dozenal paper "Redivivus Reckoning" before he ever heard of the Duodecimal Society of America. After he became a member it was reprinted in this Bulletin (v.12; no. 1; p.19; Aug. 1958).

Charlie joined the Society in August of 1957 and has served the Society on the Board of Directors since 1959 when he was first elected to the Class of 1962. He was elected the fourth Chairman of the Board following Terry, Andrews, and Camp in 1971 at Boulder, Colorado. Prior to that he was the fifth President of our Society serving from 1961 to 1970 .

Chairman Bagley has spent many years working tirelessly for the advancement of duodecimals. He wrote an article entitled "Duodecimal Society Urges Use of Base Twelve in Various Branches of Science" which was published in April 1966 in the Hollman Monthly News Bulletin, a publication



*Miriam and Charles Bagley,  
shown at Boulder, CO, in 1971.*

#  
of the American Rocket Society. He has been written up in a New Mexico newspaper and appeared on the radio there. He also wrote "Remarks on the Metric & English Measures", this Bulletin (v. 15; no. 1; p. 2#; Spring 1961) and "New Symbols" (v. 15; no. 2; p.34; Dec. 1961).

Those who work for the cause of spreading dozenal arithmetic and measurement owe Charlie a great debt of gratitude for his efforts on their behalf and for his leadership over the years. It was indeed a fortuitous circumstance that led the Directors to bestow this award on him just before he announced his decision not to run for re-election as Chairman of the Board. It is fitting that we thank him in this small way for his dedication and service to the Society.

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#### YOUR OFFICERS

On January 11; 1982 the four local officers (Gene, Tony, Jim, and John) and the acting editor of the Bulletin (Pat) met at Nassau Community College. This was our second meeting--the first being last September--since the 1981 annual meeting. We discussed plans for the coming annual meeting this May, some budget items, the Bulletin, and our archives which are slowly being put into order. All four dozen cartons from Iowa and from California have now been opened and a treasure of material is heaped around us. Several large files were given to those officers who volunteered to go through them and put them in order. More books including some computer generated Trig functions by Charles Bagley, were sent to the Dozenal Collection in the College Library.

We have acquired an IBM typewriter and can now use the elements containing the special symbols which the Society had made in the past. We obtained a Scriptomatic addressograph machine from California, and Pat and John succeeded in figuring out how to operate it, which will save us a lot of time in future mailings. Our archives contain some microfilm copies of books by Andrews and Terry, and Tony is looking into whether or not the college machines will accept the films. --GZ

Jay Schiffman

Kean College of New Jersey

Union, New Jersey 07083

Introduction. The number twelve plays a significant role in finite group theory. To briefly illustrate, 12 is the smallest cardinality in which one has a divisor  $d$  of 12 yet no subgroup of order  $d$  exists (6 divides 12 in  $A_4$ ) demonstrating the invalidity of the direct converse of Lagrange's Theorem on finite groups. Twelve is additionally the minimal order separating the classes of solvable and supersolvable groups again in lieu of the alternating group of degree four,  $A_4$ . The purpose of this paper is to present a diverse and somewhat novel application of twelve. The relationship between the number of divisors of a positive integer and the order of a non-abelian simple group is tied to the following theorem which yields a sharp lower bound:

**THEOREM.** The minimal number of divisors for the cardinality of a nonabelian simple group is 12. The remainder of this paper is devoted to building up the proof of the above theorem via its component stages. It is somewhat fitting that a proof is formulated in this manner. Indeed the simple group is considered to be the "atom" of finite group theory. To be more precise for the non expert, the following background information is useful.

The mathematical concept of a group is essential in the disciplines of geometry, analysis, the theory of equations, chemistry, and physics in addition to pure algebra, serving as a useful yet basic mathematical system. A group consists of a pair  $(G, ')$  where  $G$  is a non empty set and  $'$  a binary operation satisfying the following four axioms:

(I)  $(G, ')$  is closed in the sense that if  $a, b \in G$ , then  $a' b \in G$ . This connotes  $a' b$  is a member of  $G$ .

(II)  $(G, ')$  is associative: If  $a, b, c \in G$ , then  $(a' b)' c = a' (b' c)$ .

(III) There exists  $e \in G$  ( $e$  is known as the identity or neutral element) such that  $a' e = e' a = a$  for each  $a \in G$ .

(IV) For each  $a \in G$ , there exists  $a^{-1} \in G$  ( $a^{-1}$  is known as the inverse of  $a$ ) such that  $a' a^{-1} = a^{-1}' a = e$ .

Moreover, if the equation  $a' b = b' a$  is satisfied for each  $a, b \in G$ , then  $(G, ')$  is called an abelian group in deference to N.H. Abel, one of the chief pioneers of group theory.

Examples of groups abound in literature. The familiar number systems such as clock arithmetic, and the sets of integers, rationals, reals, and complex numbers constitute groups under the binary operation of addition, while with respect to the binary operation of multiplication one has the groups of rationals, reals, and complex numbers after deleting the element zero. Furthering our group theoretic vocabulary,  $(G, ')$  is cyclic if for some  $a \in G$ , every  $x \in G$  is of the form  $a^m$  where  $m$  is chosen from the set of integers. All groups of prime order are cyclic and abelian where the order or cardinality of a group is precisely the number of elements it contains. A subgroup  $(H, ')$  of a group  $(G, ')$  denotes a nonempty subset which itself forms a group relative to the same binary operation  $'$ . Furthermore, a normal or invariant subgroup  $H$  of  $G$  is a subgroup satisfying the condition  $gHg^{-1} = H$  for each  $g \in G$ , or what is the same,  $gH = Hg$  for each  $g \in G$ . Every subgroup of an abelian group is normal. The sets  $gH = \{gh: h \in H\}$  and  $Hg = \{hg: h \in H\}$  are called the respective left and right cosets of  $H$  in  $G$ . If  $H$  is a normal subgroup of  $G$ , then the set of all distinct right (left) cosets of  $H$  in  $G$ , denoted  $G/H$ , is termed the quotient or factor group.

A key arithmetic theorem in finite group theory yields the relationship between the possible orders of a subgroup given the order of the original group. Namely, the order of a subgroup of a finite group is necessarily a divisor of the order of the group. This result is known as Lagrange's Theorem. The direct converse of this theorem (1771) is invalid and the minimal counterexample depicting this is the (nonabelian) alternating group,  $A_4$ , consisting of all the even permutations of four symbols. A permutation is a one-to-one mapping of a set onto itself. For more on permutations, one is invited to consult [6], Chapters 4 and 5. (I.e. reference number 6 in the bibliography at the end of this article.)

One of the major problems still being researched entails conditions under which the direct converse of Lagrange's Theorem holds; namely when does a group of

cardinality  $n$  possess subgroups of cardinality  $d$  for those divisors  $d$  of  $n$ ? The Norwegian Mathematician A. Sylow (circa 1871) formulated three theorems known as the Sylow Theorems dealing with subgroups of prime power order yielding essential partial converses. The precise statements of these theorems can be found in references [2], [6], and [9] cited below.

A second problem dealt with finite simple groups and their classification. The groups of prime order fully classify the abelian simple groups. The most difficult classification problem for nonabelian simple groups has now been resolved. The solution was in no small measure partially attributed to the work of W. Burnside and P. Hall on solvable groups. The problem was initiated with the range problem (working with groups in a certain cardinality range seeking their simplicity). It appeared to Burnside from various arithmetic criteria that no group of odd order could be the cardinality of a nonabelian simple group (see [7]). This conjecture was proven to indeed be so in the comprehensive 255 page paper by W. Feit and J. Thompson in 1963 on the odd order theorem which is listed as Lemma 2 in this paper. Equivalently, any nonabelian simple group is necessarily of even order. We thus view the skeleton of the role of solvable groups in the spectrum of deep algebraic problems.

In passing, we should mention the work of E. Galois and the theory of equations (see [6], chapters 46-49). This theory is dependent upon the theory of solvable groups in the sense that a polynomial equation  $f(x) = a_0 + a_1x + \dots + a_{n-1}x^{n-1} + a_nx^n$  is solvable by radicals if and only if its Galois Group is solvable. All symmetric groups,  $S_n$ , (groups of permutations) on at least 5 symbols are not solvable due to the simplicity of  $A_n$  for  $n$  at least 5 (see [2]); and consequently the polynomial equations of degree at least 5 are not solvable by radicals, meaning in general one has no formula for the solutions of such polynomial equations in terms of the coefficients  $a_i$  ( $a_i$  integers) via a finite number of operations involving additions, subtractions, multiplications, divisions, and the extraction of roots. For the standard quadratic equations  $a_2x^2 + a_1x + a_0 = 0$ , we have the famous quadratic formula:

$$x = \frac{-a_1 \pm \sqrt{a_1^2 - 4a_0a_2}}{2a_2}$$

See GROUP THEORETIC, page 21

## A Dozenal Calendar 1192 (1982)

By Gene Zirkel,  
Nassau Community College, Garden City, New York

					1 ONE	2 TWO
3 THREE	4 FOUR	5 FIVE	6 SIX	7 SEVEN	8 EIGHT	9 NINE
*	#	10 DO	11 DO-ONE	12 DO-TWO	13 DO-THREE	14 DO-FOUR
15 DO-FIVE	16 DO-SIX	17 DO-SEVEN	18 DO-EIGHT	19 DO-NINE	1* DO-DEK	1# DO-EL
20 TWO-DO	21 TWO-DO-ONE	22 TWO-DO-TWO	23 TWO-DO-THREE	24 TWO-DO-FOUR	25 TWO-DO-FIVE	26 TWO-DO-SIX
27 TWO-DO-SEVEN						

JANUARY

	1 ONE	2 TWO	3 THREE	4 FOUR	5 FIVE	6 SIX
7 SEVEN	8 EIGHT	9 NINE	* DEK	# EL	10 DO	11 DO-ONE
12 DO-TWO	13 DO-THREE	14 DO-FOUR	15 DO-FIVE	16 DO-SIX	17 DO-SEVEN	18 DO-EIGHT
19 DO-NINE	1* DO-DEK	1# DO-EL	20 TWO-DO	21 TWO-DO-ONE	22 TWO-DO-TWO	23 TWO-DO-THREE
24 TWO-DO-FOUR						

FEBRUARY

	1 ONE	2 TWO	3 THREE	4 FOUR	5 FIVE	6 SIX
7 SEVEN	8 EIGHT	9 NINE	* DEK	# EL	10 DO	11 DO-ONE
12 DO-TWO	13 DO-THREE	14 DO-FOUR	15 DO-FIVE	16 DO-SIX	17 DO-SEVEN	18 DO-EIGHT
19 DO-NINE	1* DO-DEK	1# DO-EL	20 TWO-DO	21 TWO-DO-ONE	22 TWO-DO-TWO	23 TWO-DO-THREE
24 TWO-DO-FOUR	25 TWO-DO-FIVE	26 TWO-DO-SIX	27 TWO-DO-SEVEN			

MARCH



## APRIL

				1 ONE	2 TWO	3 THREE
4 FOUR	5 FIVE	6 SIX	7 SEVEN	8 EIGHT	9 NINE	* DEK
# EL	10 DO	11 DO-ONE	12 DO-TWO	13 DO-THREE	14 DO-FOUR	15 DO-FIVE
16 DO-SIX	17 DO-SEVEN	18 DO-EIGHT	19 DO-NINE	1* DO-DEK	1# DO-EL	20 TWO-DO
21 TWO-DO-ONE	22 TWO-DO-TWO	23 TWO-DO-THREE	24 TWO-DO-FOUR	25 TWO-DO-FIVE	26 TWO-DO-SIX	

## MAY

						1 ONE
2 TWO	3 THREE	4 FOUR	5 FIVE	6 SIX	7 SEVEN	8 EIGHT
9 NINE	* DEK	# EL	10 DO	11 DO-ONE	12 DO-TWO	13 DO-THREE
14 DO-FOUR	15 DO-FIVE	16 DO-SIX	17 DO-SEVEN	18 DO-EIGHT	19 DO-NINE	1* DO-DEK
1# DO-EL	20 TWO-DO	21 TWO-DO-ONE	22 TWO-DO-TWO	23 TWO-DO-THREE	24 TWO-DO-FOUR	25 TWO-DO-FIVE
26 TWO-DO-SIX	27 TWO-DO-SEVEN					

## JUNE

		1 ONE	2 TWO	3 THREE	4 FOUR	5 FIVE
6 SIX	7 SEVEN	8 EIGHT	9 NINE	* DEK	# EL	10 DO
11 DO-ONE	12 DO-TWO	13 DO-THREE	14 DO-FOUR	15 DO-FIVE	16 DO-SIX	17 DO-SEVEN
18 DO-EIGHT	19 DO-NINE	1* DO-DEK	1# DO-EL	20 TWO-DO	21 TWO-DO-ONE	22 TWO-DO-TWO
23 TWO-DO-THREE	24 TWO-DO-FOUR	25 TWO-DO-FIVE	26 TWO-DO-SIX			

## JULY

				1 ONE	2 TWO	3 THREE
4 FOUR	5 FIVE	6 SIX	7 SEVEN	8 EIGHT	9 NINE	* DEK
# EL	10 DO	11 DO-ONE	12 DO-TWO	13 DO-THREE	14 DO-FOUR	15 DO-FIVE
16 DO-SIX	17 DO-SEVEN	18 DO-EIGHT	19 DO-NINE	1* DO-DEK	1# DO-EL	20 TWO-DO
21 TWO-DO-ONE	22 TWO-DO-TWO	23 TWO-DO-THREE	24 TWO-DO-FOUR	25 TWO-DO-FIVE	26 TWO-DO-SIX	27 TWO-DO-SEVEN

## AUGUST

1 ONE	2 TWO	3 THREE	4 FOUR	5 FIVE	6 SIX	7 SEVEN
8 EIGHT	9 NINE	* DEK	# EL	10 DO	11 DO-ONE	12 DO-TWO
13 DO-THREE	14 DO-FOUR	15 DO-FIVE	16 DO-SIX	17 DO-SEVEN	18 DO-EIGHT	19 DO-NINE
1* DO-DEK	1# DO-EL	20 TWO-DO	21 TWO-DO-ONE	22 TWO-DO-TWO	23 TWO-DO-THREE	24 TWO-DO-FOUR
25 TWO-DO-FIVE	26 TWO-DO-SIX	27 TWO-DO-SEVEN				

## SEPTEMBER

			1 ONE	2 TWO	3 THREE	4 FOUR
5 FIVE	6 SIX	7 SEVEN	8 EIGHT	9 NINE	* DEK	# EL
10 DO	11 DO-ONE	12 DO-TWO	13 DO-THREE	14 DO-FOUR	15 DO-FIVE	16 DO-SIX
17 DO-SEVEN	18 DO-EIGHT	19 DO-NINE	1* DO-DEK	1# DO-EL	20 TWO-DO	21 TWO-DO-ONE
22 TWO-DO-TWO	23 TWO-DO-THREE	24 TWO-DO-FOUR	25 TWO-DO-FIVE	26 TWO-DO-SIX		

DOZENAL JOTTINGS...*News from or about the dozenal activities of members and friends.....*

OCTOBER

					1 ONE	2 TWO
3 THREE	4 FOUR	5 FIVE	6 SIX	7 SEVEN	8 EIGHT	9 NINE
* DEK	# EL	10 DO	11 DO-ONE	12 DO-TWO	13 DO-THREE	14 DO-FOUR
15 DO-FIVE	16 DO-SIX	17 DO-SEVEN	18 DO-EIGHT	19 DO-NINE	1* DO-DEK	1# DO-EL
20 TWO-DO	21 TWO-DO-ONE	22 TWO-DO-TWO	23 TWO-DO-THREE	24 TWO-DO-FOUR	25 TWO-DO-FIVE	26 TWO-DO-SIX
27 TWO-DO-SEVEN						

HENRY WEBBER, Class of 1983, has been very busy seeing that our listing in various publications and almanacs is current and correct. We are grateful to him for his research and efforts...

Heard from NELSON GRAY in Arizona. He reminded us of his simple method to change from any base to any other (See this Bulletin December 1959) and conjectured that octothorpe may have started as a cartographer's symbol indicating a central square with 8 streets emanating from it...

Is the science fiction writer, Peter J. Andrews of N.J., who contributed to Beyond, Vol. 1, Fall 1981, the same Peter Andrews who was once on our Board of Directors, the son of our founder, F. Emerson Andrews?...

THE MATYC JOURNAL Fall 1981, Vol. 15, No. 3 printed a dozenal calendar for 1192; (1982) as a pullout center-fold. (A reprint is included in this Bulletin.) In the same issue there appeared "An Extension to Changing Bases" by GENE ZIRKEL, pages 224 and 230...An article by JOHN SELFRIDGE (Class of 1982) appeared in the September 1981 issue of Mathematics Magazine (Vol. 54, No. 4, p. 211)...PAUL RAPOPORT wrote from McMaster University, Hamilton, Ontario, Canada and told us about his operational dozenal clock: "...Two hands in the dozen-to-one ratio, with the slowest hand going around once per day. 1 divide the day into 1000; units and each 30° on the actual clock face into six." He would like to work with someone on a dozenal analog watch of similar design. Is anyone interested?...

BILL SCHUMACHER wrote from Cherry Hill, N.J., and included a chart comparing conventional with Duodecimal indication of time. His "mins" = 0;02 of a standard hour (vs. conventional minutes). Therefore, every two standard hours a gro of "mins" occurs. This allows him to denote time with a 3 digit display. For example:

midnight = 000;  
1 a.m. = 060;  
2 a.m. = 100;

The pattern recurs every two hours hereafter, up to #60 for the conventional 23:00 hours...Bill also had thoughts

NOVEMBER

	1 ONE	2 TWO	3 THREE	4 FOUR	5 FIVE	6 SIX
7 SEVEN	8 EIGHT	9 NINE	* DEK	# EL	10 DO	11 DO-ONE
12 DO-TWO	13 DO-THREE	14 DO-FOUR	15 DO-FIVE	16 DO-SIX	17 DO-SEVEN	18 DO-EIGHT
19 DO-NINE	1* DO-DEK	1# DO-EL	20 TWO-DO	21 TWO-DO-ONE	22 TWO-DO-TWO	23 TWO-DO-THREE
24 TWO-DO-FOUR	25 TWO-DO-FIVE	26 TWO-DO-SIX				

DECEMBER

			1 ONE	2 TWO	3 THREE	4 FOUR
5 FIVE	6 SIX	7 SEVEN	8 EIGHT	9 NINE	* DEK	# EL
10 DO	11 DO-ONE	12 DO-TWO	13 DO-THREE	14 DO-FOUR	15 DO-FIVE	16 DO-SIX
17 DO-SEVEN	18 DO-EIGHT	19 DO-NINE	1* DO-DEK	1# DO-EL	20 TWO DO	21 TWO DO-ONE
22 TWO-DO-TWO	23 TWO-DO-THREE	24 TWO-DO-FOUR	25 TWO-DO-FIVE	26 TWO-DO-SIX	27 TWO-DO-SEVEN	

similar to Nelson Gray's above, on the derivation of "octothorpe"...A. ADLER HIRSH (Shreveport, LA) writes that DSA members can help "...eradicate the intrusion of metrification by supporting H.R. 1660, The Metric Conversion Repeal Act, (sponsored) by Rep. Eldon Rudd, 4th Dist. Arizona, currently pending before the House Sub-committee on Science, Research and Technology..." He urges all interested persons to write to their Congressional Representatives...JOHN CHURCHMAN writes that his Dad, HENRY CHURCHMAN, was very pleased with the plaque presented to him by the Society in honor of Henry's receiving the 1980 DSA Annual Award. HENRY resides at the Bethany-Lutheran Nursing Home in Council Bluffs, Iowa, and the plaque now occupies a place of honor in his room...We also heard from JAMISON HANDY (CA) and ROBERT McPHERSON (FL) by phone...



*President Gene Zirkel shows off his custom-printed DSA tee and a felicitous birthday present (made up of a dozen dollars in dimes!) from Treasurer Jim Malone and his wife Mary.*

*We continue our reports on past Annual Meetings.....*

DUODECIMAL SOCIETY OF AMERICA  
ANNUAL MEETING REPORT -- 1976

4728 Cielo Drive  
Huntington Beach, CA

HELP

*The following report includes all data on the 1976 meeting which is available from records here at Nassau Community College. Much information is missing (e.g. The date of the meeting); and some of the reportage contained herein seems to contradict the 1975 Report (i.e. Frieda Butler resigned as Secretary at that time, but again assumes the responsibility at this meeting.) If any member can supply us with missing facts or insights relating to the Annual Meetings of 1976-1979, we would be pleased to publish some in the next issue.*

Accomplished at the Meeting:

Henry Churchman was elected Vice President.

Re-elected were:

Charles Bagley, Chairman of the Board  
Tom Linton, President  
Frieda Butler, Secretary  
Eugene Scifres, Treasurer

Henry Churchman was re-appointed as Editor of the Bulletin.

Miriam Bagley was elected to the Board of Directors, as a member of the class of 1979. Re-elected to this group were Kingsland Camp, Tom Linton and John Selfridge.

The class of 1977 consists of Robert McPherson and Charles Bagley (due to the deaths of Ralph Beard and Paul Beaver in 1976).

The class of 1978 consists of F. Emerson Andrews, Henry Churchman, Jamison Handy and Eugene Scifres.

1\*  
DUODECIMAL SOCIETY OF AMERICA  
ANNUAL MEETING REPORT -- 1977

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5631 Trinetta Avenue  
Garden Grove, CA

December 29 and 30, 1977

Accomplished at the meeting:

Vivian Linton was elected to the Board of Directors (Class of 1980), and was elected Secretary.

Re-elected were:

Charles Bagley, Chairman of the Board  
Tom Linton, President  
Henry Churchman, Vice President  
Eugene Scifres, Treasurer

Henry Churchman was re-appointed as Editor of the Bulletin.

Robert McPherson and Charles Bagley were re-elected to the Board of Directors, and together with Vivian Linton, constitute the Class of 1980.

The Class of 1978 consists of F. Emerson Andrews, Henry Churchman, Jamison Handy and Eugene Scifres.

The Class of 1979 consists of Miriam Bagley, Tom Linton, Kingsland Camp, and John Selfridge.

The agenda for the above meeting was as follows:

1. Summarize our status; elect board members and officers.
2. Review microcomputer developments; discuss availability of equipment with dozenal capability; display microcomputer (IASIS, 8080A/Cassette).
3. Discuss the dozenal metric system.

The agenda mentions a future (Spring 1978) meeting to plan action on the following:

1. Enlist the active aid of more members to enable the Society to function more effectively.
2. Develop a dozenal electronic digital calculator system.

*end*

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ADDITIONS TO  
THE DOZENAL COLLECTION

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We continue to sort out the dozenal materials we have gathered at Nassau Community College. Among them we have discovered many valuable books and pamphlets. We gave over three dozen items to the College's Dozenal Collection, including an original copy of Duodecimal Arithmetic by George Terry, four bound volumes of past issues of our Bulletin, and a copy of New Numbers by F. Emerson Andrews published in 1944 by Essential books. (I understand that there were previous editions of New Numbers published by Harcourt, Brace in NY in 1935, and Faber & Faber in London in 1936, but I have never seen copies of these.) We also gave the library Our World In Space And Time by Kingsland Camp as well as bound reprints of many of Camp's magazine articles. Other donations included material by Andrews, Terry, Dr. Anton Glaser, J. Halcro Johnston, Jean Essig, J.C. Aitken, Louis P. d'Autremont, and Charles H. Brittain, as well as publications by Bell, Hall and Knight, Achelis, Cunningham and Woodall, Swedburg, Ingalls and Zirkel.--GZ

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DUES STILL DUE!

*Thanks to the many members who sent us their dues (which were due on January first), as a result of our request in the last issue. How about it? Send us those dues checks now and save us the time and cost of billing you. Dues are a minimum of a half dozen dollars, and all donations to the Society are tax deductible.*

## DSA ANNUAL AWARDS TO DATE



1944	F. EMERSON ANDREWS
1945	GEORGE S. TERRY
1946	F. HOWARD SEELY
1947	RALPH H. BEARD
1948	HARRY C. ROBERT JR.
1951	J. HALCRO JOHNSTON
1956	JEAN ESSIG
1957	HERBERT KAY HUMPHREY
1959	BRIAN R. BISHOP
1967	BRIAN R. BISHOP <i>on retiring from DSGB</i>
1968	TOM LINTON
1970	KINGSLAND CAMP
1980	HENRY C. CHURCHMAN
1981	CHARLES S. BAGLEY

## GROUP THEORETIC, continued from page 12

while the standard third and fourth degree polynomial equations possess such formulas although exceedingly more complex.

With the preliminary ideas clarified, we can initiate our discussion incorporating the pertinent definitions and lemmas.

Definition 1. An  $n$ -square matrix is unimodular if its determinant equals one. (a matrix is a rectangular array of elements taken from a field. A matrix having an identical number of rows as columns is termed a square matrix. If a matrix has  $n$  rows and  $n$  columns, then one is describing a  $n$ -square matrix. The determinant is a field element arising from square matrices. The precise definition can be found in [1], P. 182.)

Definition 2. A group  $G$  is simple if  $G$  possesses no proper normal subgroups. To cite examples, the alternating groups,  $A_n$ , for  $n \geq 5$  are simple as well as the multiplicative groups  $PSL(2, p^n)$  of two-square unimodular matrices modulo their centers over a field of prime power cardinality which furnish simple groups of Lie Type if  $p^n > 3$  and  $n \geq 1$ . (A subgroup apart from the full group  $G$  and the trivial group consisting of the identity element alone is called proper.) Groups under the binary operation of multiplication are said to be multiplicative groups.  $PSL(2, p^n)$  is a class of groups called the Projective Special Linear Groups of  $2 \times 2$  matrices under multiplication where the determinant is unity over fields of prime or prime power order. Modulo is a synonym for factoring out, while the center of a group consists of those elements which commute with all elements of the group. It is easy to see that all abelian groups coincide with their centers. In contrast, some groups do not possess centers apart from the identity ( $A_4$ ) and are described as centerless. The center of a group is an abelian normal subgroup. A field consists of a triple  $(F, ', *)$  where  $F$  is a non empty set with two binary operations  $'$  and  $*$  satisfying the following axioms:

- (I)  $(F, ')$  is an abelian group.
- (II)  $(F, *)$  is closed.
- (III)  $(F, *)$  is associative and commutative.
- (IV)  $(F, *)$  has an identity element.
- (V) Apart from the identity element for the operation  $'$ ,

every element of  $(F, *)$  has an inverse.

- (VI) The left and right distributive laws hold for  $*$  with respect to  $'$ : For each  $a, b, c \in F$ ,  $a * (b ' c) = (a * b) ' (a * c)$  and  $(a ' b) * c = (a * c) ' (b * c)$ .

Please see [7], P. 169 for a brief discussion of Lie Groups.

Definition 3. The identity is a technical term for the trivial group having only one element, the identity element. The terms abelian and cyclic factors signify the factor groups are respectively abelian and cyclic. A group  $G$  is solvable if there exists a descending sequence of subgroups starting with the full group  $G$  and stretching down to the identity with each subgroup normal in its predecessor and having abelian factors. Moreover,  $G$  is supersolvable if there exists a sequence of descending subgroups commencing with  $G$  and terminating with the trivial group with each subgroup normal in  $G$  and having cyclic factors. It is easily seen that all abelian groups are solvable and all supersolvable groups are likewise. The first two lemmas should rightfully be classified as major theorems. These celebrated results of William Burnside and Walter Feit and John Thompson paved the road for the study of finite simple group theory and undoubtedly led to the solution of The Classification Problem for finite simple group theory.

Lemma 1. W. Burnside (1904): Every group of order  $p^a q^b$  ( $p$  and  $q$  distinct primes;  $a$  and  $b$  counting integers) is solvable. [3]

Lemma 2. W. Feit and J. Thompson (1963): A sufficient condition for the solvability of  $G$  is that  $o(G)$  be odd where  $o(G)$  connotes the order of  $G$ . [5]

Lemma 3. A group  $G$  such that  $o(G) = 2n$  for  $n$  odd is necessarily not simple. [4]

Definition 4. An integer  $n$  is termed square-free if  $n$  is not divisible by the square of any prime. For example, the integers 15, 35, and 105 is each square-free, while 12 is not since it contains  $2^2$  in its prime factorization.

Lemma 4. Any group of square-free composite order is not simple.

Proof: If  $o(G)$  is odd then  $G$  is necessarily solvable and

hence not simple since  $o(G)$  is not a prime by Lemma 2. If  $o(G)$  is even, then  $o(G) = 2 \cdot p_1 \cdot p_2 \cdot \dots \cdot p_r$  where all the  $p_i$  are distinct odd primes. None of the  $p_i$  could be 2; for otherwise  $o(G)$  would be the product of a power of 2 with odd primes contradicting its square-free character. In identical fashion all the primes are necessarily distinct. The proof is now completed by appealing to Lemma 3 and the closure with respect to multiplication of the odd counting integers. One can actually demonstrate more; namely every group of square-free order is indeed solvable.

Lemma 5. Any finite group of prime power order is supersolvable and hence solvable [2].

The proof of Lemma 5 easily follows from the definition of the concepts involved.

Lemma 6. If  $o(G) = p^2$  where  $p$  is a prime, then  $G$  is abelian [2]. We are now in a position to achieve our basic goal.

THEOREM: The minimal number of divisors for the cardinality of a nonabelian simple group is 12.

Proof: The proof will be resolved into cases with the aid of the following number theoretic fact whose proof is readily accessible in any book on elementary number theory.

If  $n = p^a q^b r^c \dots$ , then  $n^* = (a + 1)(b + 1)(c + 1) \dots$  (where  $n^*$  denotes the number of divisors of  $n$ ;  $p, q, r, \dots$  distinct primes in their natural order;  $a, b, c, \dots$  counting integers.) The proof now proceeds on cases with  $1 \leq n^* \leq 12$ .

If  $n^* = 1$ , then  $o(G) = 1$ . Hence  $G$  is the abelian trivial group.

If  $n^* = 2$ , then  $o(G) = p$ . Therefore  $G$  is cyclic and hence abelian.

If  $n^* = 3$ , then  $o(G) = p^2$ .  $G$  is abelian by Lemma 6.

If  $n^* = 4$ , then either  $o(G) = pq$  or  $o(G) = p^3$ . In either case  $G$  is solvable by Lemmas 1 and 5 respectively.

If  $n^* = 5$ , then  $o(G) = p^4$ . Now  $G$  is solvable by Lemma 5.

If  $n^* = 6$ , then  $o(G) = p^2q$ ,  $o(G) = pq^2$ , or  $o(G) = p^5$ . In any case  $G$  is solvable by Lemmas 1, 1, and 5 respectively.

If  $n^* = 7$ , then  $o(G) = p^6$ . The solvability of  $G$  is immediate from Lemma 5.

If  $n^* = 8$ , then  $o(G) = pqr$ ,  $o(G) = p^3q$ ,  $o(G) = pq^3$ , or  $o(G) = p^7$ .  $G$  is solvable by Lemma 4, 1, 1, and 5 respectively.

If  $n^* = 9$ , then  $o(G) = p^2q^2$  or  $o(G) = p^8$ . Thus  $G$  is solvable in turn by Lemmas 1 and 5.

If  $n^* = 10$ , then  $o(G) = p^4q$ ,  $o(G) = pq^4$ , or  $o(G) = p^9$ .  $G$  is solvable in turn by Lemmas 1, 1, and 5.

If  $n^* = 11$ , then  $o(G) = p^{10}$ .  $G$  is solvable by Lemma 5.

Finally, if  $n^* = 12$ , then  $o(G) = p^{11}$ ,  $o(G) = p^3q^2$ ,  $o(G) = p^2q^3$ ,  $o(G) = p^5q$ ,  $o(G) = pq^5$ ,  $o(G) = p^2qr$ ,  $o(G) = pq^2r$ , or  $o(G) = pqr^2$ . In the initial case  $G$  is solvable by Lemma 5. Lemma 1 assures the solvability of  $G$  in the succeeding four cases. However,  $o(G) = 60 = 2^2 \cdot 3 \cdot 5$  where  $G = A_5$ .

This is precisely the factorization in the sixth case corresponding to  $p = 2$ ,  $q = 3$ , and  $r = 5$ .  $A_5$  is known to be the initial nonabelian simple group. We have therefore completed the proof of our desired goal. The reader is invited to find other group applications of the integer 12.

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2. B. Baumslag and B. Chandler, Theory and Problems of Group Theory, Schaum's Outline Series, McGraw Hill, New York, 1968.
3. W. Burnside, "On Groups of order  $p^a q^b$ ", Proc. London Math. Soc. 2 (1904) 388-392.
4. G. Cornell, N. Pelc, and M. Wage, "Simple Groups of Order less than 1000", J. of Undergraduate Mathematics, 5, 1973, P. 77-86

5. W. Feit and J. Thompson, "Solvability of Groups of Odd Order", Pacific J. Math., 13 (1963) 775-1029.

6. J. Fraleigh, A First Course in Abstract Algebra, Addison-Wesley, Reading, Mass., 1967.

7. J. Gallian, "The Search for Finite Simple Groups", Math. Magazine, Vol. 49, No. 4, September 1976, P. 162-179.

8. D. McCarthy, "A Survey of Partial Converses to Lagrange's Theorem on Finite Groups", Trans. of The New York Academy of Sciences, Series 2, Vol. 33, No. 6, P. 586-594, June 1971.

9. J. Rotman, The Theory of Groups-An Introduction, Allyn and Bacon, Boston, 1965.

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#### THINK METRIC?

*Then down with every metric scheme  
 Taught by the foreign school;  
 We'll worship still our father's God  
 And keep our father's rule-  
 A perfect inch, a perfect pint.  
 The Anglo's honest pound,  
 Shall hold their place upon the earth  
 Till time's last trump shall sound.*

--A popular song of the late 19th century

## COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	*	#	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal—based on 10. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, 265 represents 5 units, 6 dozen, and 2 dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

We use a semicolon as a unit point, thus two and one-half is written 2;6.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5;9'
31	694	Three ft. two in.	3;2'
96	3#2	Two ft. eight in.	2;8'
19#	1000	Eleven ft. seven in.	#;7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is 5 dozen and 3*; so set down 53. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only 2#, which  $12 \overline{) 365}$  is two dozen and eleven. For larger numbers,  $12 \overline{) 30} + 5$  keep dividing by 12, and the successive remainders are the desired dozenal numbers.  $12 \overline{) 2} + 6$   
 $0 + 2$  Answer: 265

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus  $12^2$  (or 144) times the third figure, plus  $12^3$  (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by #, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or #.

For more detailed information see *Manual of the Dozen System* (\$1;00).

We extend an invitation to membership in our society.  
Dues are only \$6 per year; the only requirement is a constructive interest.

## Application for Admission to the Dozenal Society of America

Name \_\_\_\_\_

LAST FIRST MIDDLE

Mailing Address (for DSA items) \_\_\_\_\_

(See below for alternate address)

Date & Place of Birth \_\_\_\_\_

College \_\_\_\_\_ Degrees \_\_\_\_\_

Business or Profession \_\_\_\_\_

Employer (Optional) \_\_\_\_\_

Annual Dues ..... \$6.00

Student (Enter data below) ..... \$3.00

School \_\_\_\_\_

Address \_\_\_\_\_

Year & Math Class \_\_\_\_\_

Instructor \_\_\_\_\_ Dept. \_\_\_\_\_

Other Society Memberships \_\_\_\_\_

Alternate Address (indicate whether home, office, school, other)

Signed \_\_\_\_\_ Date \_\_\_\_\_

My interest in duodecimals arose from \_\_\_\_\_

Use space below to indicate special duodecimal interests, comments, and other suggestions:

Mail to:

Dozenal Society of America  
c/o Math Department  
Nassau Community College  
Garden City, LI, NY 11530

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