

COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	X	E	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written 10, and is called *do*, for dozen. The quantity *one gross* is written 100, and is called *gro*. 1000 is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozal counting. For example, 265 represents 5 units, 6 dozen, and 2 dozen-dozen, or gross. This number would be called 2 *gro* 6 *do* 5, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5;9'
31	694	Three ft. two in.	3;2'
96	3E2	Two ft. eight in.	2;8'
<u>19E</u>	<u>1000</u>	Eleven ft. seven in.	E;7'

You will not have to learn the dozal multiplication tables since you already know the 12-times-table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, which is 5 dozen and 3; so set down 53. Using this "which is" step, you will be able to multiply and divide dozal numbers without referring to the dozal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozal you are only 2E, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozal numbers.

12)	365	
12)	30	+ 5
12)	2	+ 6
0)	2	+ 2

Answer: 265

Dozal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12² (or 144) times the third figure, plus 12³ (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by X, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or X.

Numerical Progression

1	One	
10	Do	:1
100	Gro	:01
1,000	Mo	:001
10,000	Do-mo	:000,1
100,000	Gro-mo	:000,01
1,000,000	Bi-mo	:000,001
1,000,000,000	Tri-mo	and so on.

Multiplication Table

	1	2	3	4	5	6	7	8	9	X	E
Edo	2	4	6	8	X	10	12	14	16	18	1X
Egro	3	6	9	10	13	16	19	20	23	26	29
Emo	4	8	10	14	18	20	24	28	30	34	38
Edo-mo	5	X	13	18	21	26	2E	34	39	42	47
Egro-mo	6	10	16	20	26	30	36	40	46	50	56
Ebi-mo	7	12	19	24	2E	36	41	48	53	5X	65
	8	14	20	28	34	40	48	54	60	68	74
	9	16	23	30	39	46	53	60	69	76	83
	X	18	26	34	42	50	5X	68	76	84	92
	E	1X	29	38	47	56	65	74	83	92	X1

The Duodecimal Bulletin

Whole Number 38

Volume 20

April 1969 (1181)



THE DUODECIMAL SOCIETY OF AMERICA

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is a voluntary nonprofit organization for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

The forms of membership include Honorary, Life, Fellow, and Senior Members, as well as Members, and Student Members. Members and Student Members are not required to pass aptitude tests in base twelve, but are encouraged to do so.

Senior membership with voting privileges requires passing of elementary tests in the performance of twelve base arithmetic. The lessons and examinations are free to those whose entrance application is accepted. Remittance of \$6, dues for one year, must accompany application. Forms free on request.

The Duodecimal Bulletin is an official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island, New York 10304. Kingsland Camp, Chairman of the Board of Directors; Charles S. Bagley, President; Jamison Handy, Jr., Editor; Henry C. Churchman, Associate Editor. Permission for reproduction may be granted upon application. Separate subscription rate \$3 per year.

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MEETING OF THE BOARD
AND
ANNUAL MEETING OF 1969

Proceedings of 1181 March 26;460

The Duodecimal Society of America was called to order by President Charles S. Bagley in the Conference Room at Francisco Torrez student residence of the University of California at Santa Barbara, 6850 El Colegio Road, Goleta, California 93017, at 0900 Hours, 30 March 1969, in keeping with written notice mailed to the whole membership at least ten days prior to that meeting.

A roll call disclosed the following among the active members present:

- Charles S. Bagley, of Alamogordo, N. M.
- Ralph H. Beard, of Staten Island, N. Y.
- Kingsland Camp, of New York, N. Y.
- Henry C. Churchman, of Council Bluffs, Iowa.
- Jamison Handy, Jr., of Pacific Palisades, Calif.
- Tom B. Linton, of Garden Grove, Calif.

Other members included Edwin J. Baker, 2215 Crenshaw Blvd., Paul H. Beaver, 2825 Hyans Street, and Donald Simpson, 745 North Spaulding Avenue, all of Los Angeles, California.

A fruitful forenoon was enjoyed when Ralph H. Beard advanced an array of ideas for advancement of base twelve and expansion of our work in other areas of the earth, followed by an encouraging talk by Kingsland Camp in favor of developing more dozenal equipment.

Recessed for lunch at the fine Francisco Torrez cafeteria, and promptly at 1230 Hours the meeting was again called to order by President Bagley.

Donald Simpson was recognized and told of his effort to develop a dozenal electric clock by combining a 24-hour on-off electric timer plus the defunct windup clock hour and minute pinions and gears, so that the hour hand should describe one circle in a 24-hour period and the moment hand a dozen times in that same period of time. The face would look like Big Ben's in London, or contain our dozenal symbols perpendicular to the base, from 0 to £ around the circle, with 0 at the bottom and £ at the top.

Motion by Tom Linton, seconded by Beard, authorizing purchase by the society of one dozen such clocks from Donald Simpson to retail at about \$10 each, during next eight months. Carried.

Jamison Handy, Jr., was recognized by President Bagley and spoke on the subject of computer symbols and computer language.

Paul Beaver was recognized by President Bagley and gave an exceptionally interesting 'book review' of an article by Dr. John D. Clark appearing in a recent issue of Analog Science Fiction-Science Fact, bearing the enticing title DIMENSIONS, ANYONE? If you can get your hands on the entire article, look not only for

entertainment but tintinnabulation for fundamental quantities derived from universal constants. The natural unit of time, for instance, is that waiting period required for a photon to travel the natural unit of length or h/mc^2 , where h = Planck's Constant, m = the electron's rest-mass, and c = velocity of light. This does not give you time to turn around, being $0.809295 \text{ second} \times 10^{-20}$, but multiplying it by 10^{20} yields a unit of time about 0.81 second long. We could, says Dr. Clark, call it the scientific second, as contrasted with the civil or payroll second which will presumably vary from planet to planet. Might not one or more of our members find time to transmute these constants into dozenals?

At 1425 Hours Tom Linton, recognized by President Bagley, gave a significant talk on the Economy of Digits. He also described an attachment permitting the use of base ten while working in base twelve, both input and output, on integrated circuits.

President Bagley advanced a suggestion that we promote Dozenal Micrometers of high precision, possibly acquiring two dozen. He intends to pursue the matter further and hopes to find a manufacturer at a competitive price.

Edwin J. Baker was recognized by President Bagley and advanced in turn several worthwhile ideas for advancement of the use of dozenals. Highly appreciated.

Election of Board Members

Moved by Ralph Beard, seconded by Tom Linton, that F. E. Andrews, Henry C. Churchman, Jamison Handy, Jr., and Eugene Scifres be nominated as members of the Board of Directors DSA, class of 1184 (1972); that nominations be closed, and that the Secretary be directed to cast the unanimous vote of the members present for the motion. Carried. The President declared the following elected:

Class of 1184 (1972)

F. E. Andrews,
Henry C. Churchman
Jamison Handy, Jr.,
Eugene Scifres.

At 1645 Hours (0;848) adjourned until after completion of the meeting of the board of directors to begin at 0900 Hours, on 31 March 1969. After a completely satisfying evening meal in the Francisco Torrez cafeteria, the society met informally at the conference chamber and enjoyed more than two hours of general and open discussions and exchange of ideas.

Proceedings of 1181 March 27;460

Chairman Kingsland Camp now called to order the Annual Meeting of the Board of Directors of DSA, at half past four (460 Moments or 0900 Hours) in the Conference Room at Francisco Torrez student residence of the University of California at Santa Barbara, 31 March 1969, in keeping with prior announcements.

A roll call disclosed more than one half the active members of the board of directors present, as follows:

Charles S. Bagley, of Alamogordo, N. M.

Ralph H. Beard, of Staten Island, N. Y.
Kingsland Camp, of New York, N. Y.
Henry C. Churchman, of Council Bluffs, Iowa
Jamison Handy, Jr., of Pacific Palisades, Calif.
Tom B. Linton, of Garden Grove, Calif.

Chairman Camp noted that a working quorum was present and legally able to conduct all proper business to come before it.

The minutes of the last meeting were read, discussed, and approved, with an amendment to complete the record of those events as follows:

In the minutes of 8 April 1968, meeting in the Chicago area, there is absent the following resolution: "Resolved, that the Treasurer of DSA is hereby authorized by the Board of Directors of the Duodecimal Society of America to deposit any funds of the society coming into his hands, in the Bank of Denver, Colorado or in any other bank in that city at the discretion of the treasurer."

Resolved that the Secretary of the Duodecimal Society of America is hereby authorized and directed to add the foregoing paragraph to the record of minutes of the 1968 meeting. Carried.

Report of the Secretary

Secretary Tom Linton reported the dropping in January 1969 of all lapsed members. We now have 122 in our total membership including 22 Fellows, four of whom are honorary. In 1968 three adult and one student member accepted. In February and March, 1969, three new members were accepted.

Announced the printing of 500 booklets "Duodecimal Reciprocals 4/6 Places," from a computer output programmed and run by James Dixon at Purdue University, on their IBM 7094. The booklet is described as DSA 101, March 1969, and may be obtained through the secretary's office.

The secretary reported additional activities expected from the 'computer group' which includes Stan Bumpus of the University of Illinois, James Dixon and Peggy Dwyer of the University of Purdue, Thomas Goodman who has access to an IBM 360 with ALC language, Robert McPherson who has a laboratory in Florida, and a highly experienced, trained, and productive Fellow of DSA, Dr. B. A.M. Moon, Director of the Computer Laboratory, University of Canterbury, Christchurch, New Zealand.

Tom B. Linton reported progress on the 6-inch duodecimal ruler and pocket scale. Work already authorized. Will be completed before yearend and appear in the Bulletin classified ads perhaps in December 1969.

Report of secretary approved. Recessed 1145 to 1330 Hours for lunch.

Board of Directors met at 1330 Hours. Treasurer's report presented by Tom Linton, in the absence of Eugene Scifres. The report was duly approved.

Editor's Report

Henry C. Churchman, Associate Editor announced the publication

and complete mailing of over 500 September and December 1968 Bulletins, Numbers 36 and 37. Also, by agreement with Jamison Handy, Jr., one bulletin for December 1966 and one for 1967 will be published and put in the mail by Churchman in 1970, to smooth out the complete line of whole numbers now suffering from a lack due to the inability of the Editor to devote the necessary time. In addition, current editions will be put out for April of 1969, and 1970.

Election of Officers

It was moved by Ralph H. Beard, seconded by Jamison Handy, Jr, that the following members of the Board of Directors be nominated and stand elected to the following offices:

Kingsland Camp, Chairman of Board;
Charles S. Bagley, President;
Henry C. Churchman, V. President;
Tom B. Linton, Secretary;
Eugene Scifres, Treasurer;

that nominations be closed, and that the Secretary be instructed to cast the unanimous vote of the directors for the motion. The motion carried.

Moved by Ralph H. Beard, seconded by Tom Linton, that Jamison Handy, Jr., be appointed Editor and Henry Churchman Associate Editor of the Bulletin for the year 1969, and authorized to publish the bulletins specified in the editor's report. Carried.

Dr. Kenneth Mears, of Oakville, Ontario, was duly elected as a Fellow of the Duodecimal Society of America, in recognition of his work in advancing a greater understanding and employment of base twelve.

Ralph H. Beard moved, seconded by Linton, the following Resolution:

Resolved that the Board of Directors of The Duodecimal Society of America hereby expresses its profound regret over the recent death of one of its Founding Fathers, George S. Terry, recalling his many and outstanding contributions to the founding and development of the Society, and hereby extends to his surviving widow, Martha Gay Terry, its deepest sympathy.

That this Resolution be spread on our permanent records, that the Secretary be instructed to express our deepest sympathy to Martha, and that one of our members be appointed by the chairman to prepare a suitable obituary for next issue of the Bulletin. Carried.

New Business

Henry Churchman moved adoption of the following resolution:

Resolved, that we endorse in principle the improved meter as advanced by M. Jean Essig of France in Douze Notre Dix Futur and urge adoption by the more advanced nations of one or more pairs of the following units of dimension for voluntary use, to-wit:

Dozenal Units	Dimension in Krypton 86 light waves	Approximate Inches and Feet
Métron ¹	75 000;0	44 lines (3-2/3 inches)
Mètre duodécimal ¹	750 000;0	44 inches (3-2/3 feet)

{ Duodécamètre duodécimal 7 500 000;0 44 feet
Edon 75 000 000;0 528 feet

{ Navinaut 750 000 000;0 6336 feet
Dominante 10 000;0 Navinauts Great Circle of Earth

¹The mètre duodécimal and the metron might be described in the French language, in base ten usage of today, as follows:

Le mètre duodécimal est la longueur égale à 1 845 504 longueurs d'onde dans la vide de la radiation correspondant à la transition entre les niveaux $2p_{10}$ et $5d_5$ de l'atome de krypton 86.

Le mètre duodécimal = le métron.
douze

These dimensions are not adopted as the final units of dimension of the DSA in either form or content, and DSA is still open wide to suggestions and solutions in keeping with tomorrow's advancements in science, presently known constants, etc.

Discussion. Motion lost for lack of a second.

The following motion was made by Henry Churchman and seconded by Ralph H. Beard:

Resolved, that members of the society make a study of comparisons of the Jean Essig units of dimension, area, and capacity, with the proposed units set forth in the Do-Metric System by Ralph Beard, the Redivivus Reckoning System by Charles Bagley, and the Metronic System by Henry Churchman, as well as other dozenal metric systems heretofore or hereafter proposed to the society, and analyze their possibilities in advancing base twelve as a universal number base. Carried.

Discussed interchange of efforts with DSGB. Moved by Tom Linton, seconded by Beard, that the Duodecimal Society of Great Britain be furnished by the editor with a number of Bulletins of all future issues equal to that society's membership, to be sent postpaid to its headquarters in the United Kingdom. Carried.

Erection of a Duodecimal Society of Canada discussed, urged by Beard. Preliminary work will be undertaken by Beard. Carried.

At 1640 Hours, 31 March 1969 (1181 March 27;840), the board of directors meeting was adjourned sine die.

Further Session of the Society

At 0900 Hours 1 April 1969 (1181 April 1;460) the society was called to order by President Charles S. Bagley at Francisco Torrez.

Meeting place for 1970 was discussed. Moved by Jamison Handy, Jr., seconded by Ralph Beard, that the 1970 meeting be held in or near Florida, at a time and place to be determined by Tom Linton, and the membership advised in due time. Carried.

It was moved and seconded that the DSA express its appreciation to the President and administration of the University of

(Concluded on page two-de-four)

METRIC TIMEPIECES, AND THE MOON

By Henry C. Churchman

One of the things to be done by the team of astronauts, after APOLLO 8 sets down on the moon, is to emplace a laser reflector within a stone's throw of the touchdown point. This should become a historic fact in June or July of 1969. If they meet with some accident or collision between their craft and asteroids another daring trio will make the attempt. Natural disasters, we need hardly say, do not block man's inquisitiveness.

The plan calls for a laser beam to be thrown at the moon from the earth to strike the reflector and immediately start earthward, to be picked up by a precise timing device at the observatory whence it shall have momentarily fled.

Since one Dot of a metric timepiece equals 25/72 part of a second of time, and reflections of sun light from the moon reach the earth, on an average, in about 4 dots, it can be seen quick as lightning that the roundtrip from earth-to-moon-to-earth can occur well within nine dots or three seconds of time.

It is hoped the starter-gate controlling this race will open and close, and open again when ordered, in one of one hundred million parts of a second of time, so that the laser beam will travel to the moon much as bubbles rise to the surface of water separately, but infinitely faster---at the velocity of light.

Let us visualize, if possible, the speed expressed by the fraction of 1/100,000,000ths of a second of time, by stating it in the base-twelve metric time of dots.

Computations indicate that 1/100,000,000 second of time approximately equals 1/35,831,808 Dot, or 1 Dot times 10^{-7} , or 1/10 000 000;0 Dot. Since 1/10 000 000;0 dot is exactly equal to 1/103,195,607.04 second of time, might it not be said that 1/10 000 000;0 dot is an infinitesimal, but finer subdivision of time than is 1/100,000,000 part of a second of time?

As scientists become more familiar with the capabilities of the metric timepiece, with its finer subdivisions of time, it is possible that more experiments involving the element of time might be stated in dozenal fractions of the Dot. Especially so when it dawns upon more minds of science that one gross of Dots are the equal of one Moment and that twenty Moments are the exact equal of one kilosecond of time.

-o-0-o-

Fewer every day work in base-ten. More efficiency is found in other bases.

-o-0-o-

More professors of mathematics every year are discovering that a free movement by Americans to a dozenal metric system will prove much more practicable than the 20th century movement to force the base-ten metric system above feet and inches.

-o-0-o-

An absurd, unsystemic 1852 meters equal a nautical mile--sixty seconds of angle of a great circle of the earth. Navigators find nothing metric about it save the word "meters."

COMMENTS ON STANDARDS

By Tom Linton

The search for better standards of measurement is a never ending activity, some of which is discussed in these bulletins, and in DSA correspondence. The following notes are given to help place the search in perspective.

PRODUCT ENGINEERING magazine for January 13, 1969, reports a talk by Jack C. Riley, technical consultant to Electro-Scientific Industries, Inc., at an Instrument Society of America meeting in New York City: "Measurement accuracy is defined as agreement between true and measured values. But true values are unknown, unknowable and unstable, and they change with environment. A measured value becomes a fixed quantity, but it is almost certainly not the true value and we do not really know how close the true value is to the measured value." Riley sees hope for improving the assessable accuracy of our national measurement system with improvements being made on the broad front of instruments, handling, and methods of use.

From Bowditch, AMERICAN PRACTICAL NAVIGATOR, H. O. #9 (1958), pages 26 and 27: "Finer instruments and new methods make increasingly more accurate determinations of the size of the earth an ever present possibility. Hence a unit of length defined in terms of the size of the earth is undesirable. Recognition of this led, in 1875, to a change in the definition of the meter from one ten-millionth of the distance from the pole to the equator of the earth to the distance between two marks (39.37 inches) on a standard platinum-iridium bar---." (Now the inch-meter relation is accepted as 2.54 centimeters per inch, yielding the exact ratio of 127 cm to 50 inches.)

From the National Bureau of Standards TECHNICAL NEWS BULLETIN of February 1963, the length of the International meter is defined as 1,650,763.73 wavelengths of orange-red krypton 86 light. Since 1962 the wavelengths of certain other materials are recognized as suitable for length standards. The international inch (2.54 cm) equals 41,929.3987 kr 86 wavelengths.

In 1964 the Twelfth General Conference of Weights and Measures defined a second of time as 9,192,631,770 (1 946 716 076) cycles of the transition between two superfine levels of a fundamental cesium 133 (21) atom. The tropical year 1900 contains 31,556,925.9747 (~~X 69X 139; 2843~~) such seconds of time, and the solar day lasts 86,400 seconds.

The "cesium second" and the "krypton meter" are related thru the speed of light; that is, $f = \frac{c}{\lambda}$, where c is the speed of light, f is the frequency in cycles per second, and λ is the wavelength, all in relatable units. The current I A U velocity of light is 299,792.5 Km/sec or 186,281.7 mps.

-o-0-o-

A map has been produced of the whole Western Soviet Union on a scale of one thousand ¹Navinauts on the ground to one foot on the chart. Atlas Plate 45, National Geographic, Sept 1959.

¹"Redivivus Reckoning" by C. S. Bagley, DSA Bulletin, Aug 1958.

METRIC TIME

How a Modern Scientist Tells Time Today

By Henry C. Churchman

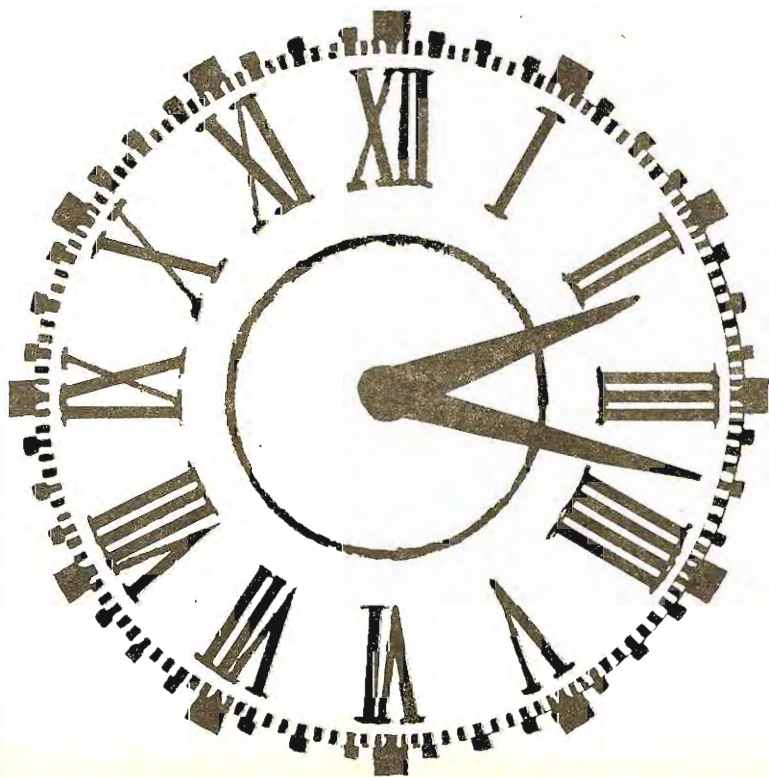
The Greenwich Mean Time shown on the Metric timepiece Roman face, Figure 1, may be read, not as two (or fourteen) hundred seventeen or eighteen hours, as in base ten, but simply as Two, three, six. Merely adding the word "Julian" to the digits will indicate Greenwich Mean Time, which begins at noon. Otherwise, the three digits refer to civil time, starting with midnight.

When the small hand points at Two (or between Two and Three), a scientist sounds the first digit as Two.

When the longer hand points to Three (or between Three and Four), we sound the second digit as Three. Always choose the lesser of the two bracketing digits, treating XII as zero in the beginning of a twenty-four hour (or twelve duor) period.

The distance the longer hand has moved between Three and Four determines whether the third digit is called "aught," "three," "six," "nine," "eleven," or a number between them. This third digit to be sounded in stating metric time is determined by interpolation alone (when only two hands are employed on the metric timepiece).

Figure 1



For dramatic effect in distinguishing between Greenwich Mean Time, and civil or U.T., the Roman metric timepiece face may be inverted, showing VI at the top in lieu of XII, since Universal Time begins at midnight. But the three digits are determined in exactly the same manner as stated heretofore.

On the average, using the face inverted for six days is required to acclimate oneself to the changed position---you may determine your own acclimating period by wearing the face of your wrist watch inverted for one week, telling time normally.

We can picture in our mind four principal divisions of the arc between XII and I, I and II, II and III, III and IV, etc. And each of these quarters we imagine divided into thirds. One does this every day on a tower clock such as Big Ben in London, where the face is high above the pavement. Today the average layman bows in the direction of base ten and divides the arc, not into four principal divisions, but, into five equal parts which are called "minutes."

It is unnecessary to tie a name to any one of these three digits in telling metric time. Hours, minutes, moments, seconds, are a waste of effort, generally. Every scientist, eventually, may learn to tell unequivocally the division of a day reached at any particular moment if we merely sound three digits and nothing else, whether he be an advocate of base ten or of base twelve, in other fields than time.

In Figure 2 are shown a few comparisons between telling time by Hours (beginning of the 20th Century) and by a simple metric timepiece. One "moment" is the exact equal of fifty seconds of time, as now defined by scientists. By a happy coincidence ten minutes and twelve "moments" are of exact duration. There are one dozen gross moments in a twenty-four hour period of time; in the same period there are 1440 minutes.

Figure 2

Scientists'		Scientists'	
Metric	Base Ten Method	Metric	Base Ten Method
000	= 0000 ^h (new day)	090	= 0130 ^h
006	= 0005 ^h	096	= 0135 ^h
010	= 0010 ^h	020	= 0000 ^h + 6 kiloseconds
016	= 0015 ^h	026	= 0145 ^h
020	= 0020 ^h	020	= 0150 ^h
026	= 0025 ^h	026	= 0155 ^h
030	= 0030 ^h	100	= 0200 ^h
036	= 0035 ^h	200	= 0400 ^h
040	= 0040 ^h	300	= 0600 ^h
046	= 0045 ^h	400	= 0800 ^h
050	= 0000 ^h + 3 kiloseconds	500	= 1000 ^h
056	= 0055 ^h	600	= 1200 ^h
060	= 0100 ^h	700	= 1400 ^h
066	= 0105 ^h	800	= 1600 ^h
070	= 0110 ^h	900	= 1800 ^h
076	= 0115 ^h	200	= 2000 ^h
080	= 0120 ^h	200	= 2200 ^h
086	= 0125 ^h	000	= 0000 ^h (new day)

X = ten. ℓ = eleven. 0 = aught. 010 = aught one aught.

ENGLISH E-Z LOCATOR EXPOSITION

By Brian R. Bishop, Fellow DSA

Location Exact Within 1/3 Inch	Conventional		E-Z Locator Symbols	
	Latitude	Longitude	Easting	Northing
Charing Cross, London	51°30'28.5"N	00°07'25.5"W	522 4X5 699-487 2X3 321	
London Airport centre	51°28'11"N	00°27'08"W	529 9E5 108-487 07X 569	
Southend Airport	51°34'15"N	00°42'00"E	603 43X 026-487 659 155	
Salisbury Cathedral	51°03.86'N	01°47.75'W	5E3 468 782-486 138 002	
Framingham	52°34'26".8915N	01°20'21".1080E	606 517 9E2-490 432 203	
C'ter Water Tower	52°39'27".2531N	01°43'04".5177E	608 2E5 167-490 90X 250	
Southend Pier Head	51°30'51"N	00°43'27"E	603 58E 177-487 327 130	
Exact Within 44 Ft. 155, Leighton Avenue	51°32.766'N	00°39.31'E	603 18X-487 506	

X is called ten. Z is equal to and called eleven.

Above tabulation was produced by Brian R. Bishop, one of the Founding Fathers of DSCB and forwarded to Associate Editor Henry Churchman, 13 December 1968, on a Season's Greeting card. Its receipt was delayed 60 days (less 6) by the U. S. Longshoremen's strike. We are happy to publish data herewith, and are grateful for the meticulous preparation of this exposition of the advantages of the use of base twelve in navigation. Kingsland Camp of New York City, Chairman of DSA Directors, has pioneered in this.

In all E-Z measurements, the initial half of the total quantity of symbols states the dozenal fraction of a circle easting from 180° longitude, and the second half gives the fraction northing from the South Pole. The South Pole, for instance, may be shown as 0000-0000 (a point within one Aeromile of the Pole); and the North Pole may be stated as 0000-6000, thus conjuring up the Eskimo name of Awtawt-awtawt Siksawt-awtawt for the arctic center point.

The Bishop description indicates by use of only one dozen and six digits the location, for instance, of Charing Cross (center of London?) within a precision equal to one-third inch, or less, north, south, east or west, on the ground.

And the location of 155, Leighton Avenue (requested earlier by the editor), Leigh-on-Sea, Essex, England, shown by only twelve symbols or digits, is pointed out on this earth within an exactness equal to forty-four feet, or less, north, south, east, or west---a splendid example of a second or auxillary direction to the Bishop home, without mention of United Kingdom, Essex, Leigh-on-Sea, Leighton Avenue, or even 155.

The tiniest island can no longer be said to be nameless. Both its name or Zip and the location of its highest point could be spelled out with twelve symbols or digits.

(Concluded on page twodo-four)

STUDY OF A MODERN METER
AND THE OBSOLETE 18TH CENTURY METER

Henry C. Churchman

If one Metron equals 75 000 krypton 86 light waves (decimally 153,792 such wave lengths) and if one Navinaut¹ (kilomètre duodécimal² or kmd) equals 750 000 000 kr. 86 light waves, then 10 000 Navinauts (kmd) will equal 7 500 000 000 000 krypton 86 light waves, or ONE great circle of the earth. The assumed-length of a Great Circle is vital to the art of terrestrial navigation.

If one conventional 18th century decimeter remains equal to 165,076.373 krypton 86 light waves and if a Canadian inch³ were enlarged by not quite 14 kr. 86 light waves to equal the sum of 41,943-3/11 EXACTLY, then the following comparisons will result ---stated in terms of a DUODECIMAL kilometer, hectometer, decameter, meter, decimeter, centimeter, millimeter, or decimillimeter standard, to-wit:

A Dozenal or Duodecimal Metric System	Obsolete Meter Employed in U.S.	Canadian or international System
1 kmd = kilomètre duodéc. (750 000 000 kr. 86)	= 1.93 kilometer = (1 Navinaut)	= 6336 feet = (6/5 mile)
1 hmd = hectomètre duodéc. (75 000 000 kr. 86)	= 1.61 hectometer = (1 edon)	= 528 feet = (0.1 mile)
1 damd = decamètre duodéci. (7 500 000 kr. 86)	= 1.34 decameter = (1 remetron)	= 44 feet
1 md = mètre duodécimal (750 000 kr. 86)	= 1.118 meter = (1 dometron)	= 44 inches
1 dmd = décimètre duodéci. (75 000 kr. 86)	= 0.93 decimeter = (1 Metron)	= 44 lines
1 cmd = centimètre duodéc. (7 500 kr. 86)	= 0.77 centimeter = (1 edometron)	= 3-2/3 lines
1 mmd = millimètre duodéc. (750 krypton 86)	= 0.64 millimeter = (1 eremetron)	= 11/36 line
1 dmmd = deci-mm duodéci. (75 krypton 86)	= 0.53 decimillimeter = (1 emimetron)	= 11/432 line

For EXACT scientific comparisons of old with new, it would seem desirable to employ the defined DECIMAL number of krypton 86 wave lengths in one Metron (dmd), that is to say 153,792 and relate them to 165,076.373 such wave lengths, the defined dimension of one 18th century decimeter. Metrons are multiplied by powers of twelve, either positive or negative, as we increase or decrease these dimensional standards to their next duodecimal unit of denominative measurements.

If the length of a Canadian inch (41,929.3987) be increased by not quite 14 krypton 86 light waves (which would be required most likely by perfectionists), then a carefully surveyed distance across North America equal to 3001 Canadian miles would be quite precisely equal to 3000 Canadian INDEPENDENT krypton 86 defined miles. And 30,010 Canadian feet would be equal to

30,000 krypton 86 defined feet---not a very drastic change.

The current Canadian inch length (41,929.3987 kr. 86 light waves) need not be increased by some 14 wave lengths for nearly one or more centuries, depending perhaps on when the base-ten millimeter or centimeter, on which it is based, joins the cubit and the ancient Roman mile.

In any event we retain our old Congressional surveys in the United States, one old square mile or SECTION of farm lands being defined as equal to one hundred duodecimal hectares or Gardens,⁴ each being the equal of a one-tenth statute mile square.

And there would be 144 duodecimal hectares (Gardens) in the Field or Champ.⁴ In each Congressional Township (originally three dozen square miles in size) there would be 25 such Champs by definition. The Congressional Township today might contain 23,040 acres of land, or 3600 duodecimal hectares (Gardens), but due to the curvature of the earth, and human errors in surveys under pioneer conditions, it more often fails to equal these sums.

A city lot surveyed in Early American practice to equal 44 by 132 feet, now found frequently in town and city title descriptions, would be equal to three 'Parks'⁴, each such dozenal park being equal to a surface quite precisely 44 feet square.

¹See Redivivus Reckoning, August 1958 Bulletin, by C.S. Bagley.

²See Douze Notre Dix Futur, by M. Jean Essig, 1955, Dunod, Paris. (French language)

³Canadian inch and International inch are same dimension.

⁴See Doremic Surfaces, December 1960 Bulletin, H. C. Churchman.

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The 1000 Candy Bar

By definition and use, one Metron = 1 dozen (10) édometrans, 1 gross (100) éremetrans, or one great gross (1000) émimetrans. Also by definition:

1 Metron (75 000 kr. 86 light waves) = 153,792 krypton 86.

1 Decimeter (100 millimeters) = 165,076 " 86.

If a candy bar were produced not one decimeter but one metron in length, it might be advertised as the 1000, which is to say, the "Great Gross" candy bar, being one great gross émimetrans in length.

True, the Great Gross would be 153,792 kr. 86 wave lengths (3-2/3 inches) long, while the One Hundred would be 165,076 kr. 86 wave lengths (3.9 inches) long, but the flash of 1000 conveys the idea of superiority over 100 which no amount of talk can overcome short of an entire treatise. Well, equally 100 mm suggest superiority over 4 inches, but are smaller, as we know. If Hindu-Arabic digits be written as italic symbols, thus 10, they are by habit presumed to represent base-twelve numerals.

If you make a cigarette 3-2/3 inches long, let your advertising agency determine how to apply the metronomic definitions.

CHROMATIC MUSICAL SCALES AND NOTATIONS

By Erich Kothe

Part Two

Musical Scales

So long as man performed only simple vocal music, a systematic selection of tone pitches may have been less important. However, as soon as he started to include musical instruments into his musical activities, he had to start fixing the tone pitches into tone systems---he had to develop some system of musical scales.

One method of creating a musical scale was by dividing a certain length of a musical instrument (such as the length of a string) into equal parts, as in ancient China; but this method proved to be unsatisfactory. All other methods used so far, divide the pitches (or frequencies) proportionally, and they start out by dividing the pitches into so-called "octaves" or "homosonances." Then they subdivide an octave either arbitrarily (as

Figure 13

Degree of Consonance

frequency ratio	conventional term of tone combination
2 : 1	octave or 8th or "homosonance"
3 : 2	quint or 5th
4 : 3	quart or 4th
5 : 4	large or major 3rd
6 : 5	small or minor 3rd
5 : 3	large or major 6th
8 : 5	small or minor 6th
9 : 5	small or minor 7th
15 : 8	large or major 7th

in some Oriental music) or according to the principle of "consonance" (as within the Western Civilization). Since the term "octave" can be misleading and the other term "homosonance" is long (although very descriptive), the author proposes to shorten the second term into "honance," which we will use henceforth.

Based on the principle of "consonance" or pleasant tone combinations (see Fig. 13), four notes can easily be inserted within a "honance": the quint or 5th, the quart or 4th, the large or major 3rd, and the large or major 6th. However since these notes can be taken either as steps upwards or as steps downwards, two different scales can be formed, as shown in Fig. 14---a falling, or a rising, pentatonic scale.

Figure 14

conventional musical notes	falling scale			rising scale							
	C	E ^b	F	G	A ^b	c	e	f	g	a	c ¹
frequency ratios to note "c"	$\frac{1}{2}$	$\frac{3}{5}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	$\frac{1}{1}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{2}{1}$

Figure 15
Diatonic C-Scale

Musical note	C	D	E	F	G	A	B	C'
Frequency as related to the tonic	1	$\frac{9}{8}$	$\frac{5}{4}$	$\frac{4}{3}$	$\frac{3}{2}$	$\frac{5}{3}$	$\frac{16}{9}$	2
Frequency ratio for successive intervals	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{16}{15}$	$\frac{9}{8}$	$\frac{10}{9}$	$\frac{9}{8}$	$\frac{16}{15}$	

For our Western Civilization the just mentioned 'rising' pentatonic scale became very important, because it led directly to the development of the diatonic scale, set forth in Fig. 15. As shown, an additional note "D" has been inserted between the "C" and the "E" which can be explained as a step-down from the "G" (also called "Dominant") by a 4th; and another additional note, "B", has been inserted between the "A" and the "C'" which can be explained as a step-up from the "G" by a large or major 3rd.

From the listed successive intervals, it can be seen that the diatonic scale consists of five large or major 2nd (also known as whole note) intervals, with ratios of 9/8 or 10/9, and of two small or minor 2nd (also known as half-note) intervals with a ratio of 16/15; and these two half-note intervals were fixed between the "E" and the "F" and between the "B" and the "C'". During the Middle Ages the position of these two half-note intervals with respect to the starting tone or "initialis" was used to differentiate several diatonic scales into types, called "modes", as shown by Fig. 16.

All the medieval scales were included in a tonal system known as the "Greater Perfect" system (in itself a misunderstood interpretation of antique musical theory), which is shown in a more modern form in Fig. 17. It consisted of 20 notes ranging from "g" to "e²", where each note of the diatonic scale (7 notes with a "honance") has only one letter---with the exception of two additional "b-flats" (or "b^b"). Apparently these two "b-flats" were not regarded as a half step down from the "b-natural" (as they were later), but rather as variants of the note "b" and note "b¹".

Figure 16

The church modes were scales used principally in the Middle Ages, although the last two pairs were additions of Renaissance theorists. The starting tone, or *initialis*, of a melody in each scale is shown as a whole note; the reciting tone for declamatory church chants is shown as a half note. Corresponding authentic and plagal modes had the same *initialis*, but differing ranges and reciting tones.

Figure 17
Greater Perfect System

The Greater Perfect system comprises all the notes used in medieval music. It was the basis for all the medieval scales.

An application of the just given Greater Perfect system is illustrated by Fig. 18, which shows a full keyboard of a clavichord.

With the dawn of polyphonic music at the end of the Middle Ages attempts were made to expand the diatonic C-scale in order to gain greater flexibility. One such expansion was the enharmonic scale, shown by Fig. 19, where each note of the diatonic C-scale is extended by an additional half-step up and an additional half-step down.

Today the described enharmonic C-scale is being used by the double-action pedal harp. The mechanism of the double-action pedals (7 pedals) is shown schematically in Fig. 20. In the up-

Figure 18

DIATONIC CLAVICHORD KEYBOARD (GUIDO'S SCALE) FROM VIRDUNG, BEFORE 1511, SHOWING THE SHORT KEYS ■ FLAT

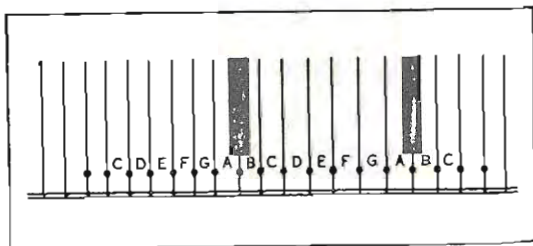
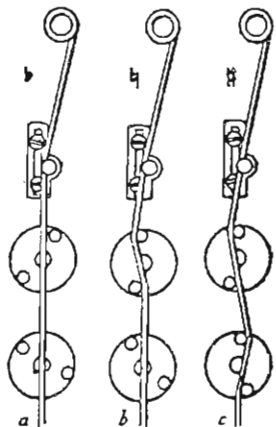


Figure 20

Diagram of the action of a Double-action Pedal Harp



per position (rest) a string is unshortened (as shown in Figure 20a); in Fig. 20b the pedal is depressed half-way down and the shortened string is tuned a semitone higher; in Fig. 20c the pedal is fully depressed and the double-shortened string is tuned a whole tone higher.

Although the enharmonic scale described by Fig. 19 is still in use today, it has to be regarded as an unsatisfactory solution, since it did not resolve the inherited problems of the diatonic scale. Due to its containing two different kinds of whole-note intervals with the ratios of 9/8 and 10/9, and having two half-note intervals with the ratio of 16/15, which is a half step of neither 9/8 nor 10/9, the use of a note other than "C" as a tonic

Figure 19
Enharmonic C-Scale

C	1	1
C#	25	24
D	16	15
D#	75	64
E	5	4
E#	6	5
F	4	3
F#	25	18
G	36	25
G#	3	2
A	5	3
A#	125	72
B	15	8
B#	9	5
C'	125	64
C''	2	1

results in another scale. Two successive half-note intervals of 16/15 equal 256/225 or 1.13777..., which is larger than either 9/8 (1.125) or 10/9 (1.111...). This is illustrated in Fig. 21, which gives in its second row the frequencies of the C-scale, basing them on a standard "A" with 440 cps., and in its third row the frequencies of the diatonic D-scale. As seen, four new notes (marked with asterisks) would have to be added in order to maintain the exact, or just, relationship. If all possible musical notes were to be provided for, 72 keys would be needed within each "honance".

Figure 21

Musical note	C	D	E	F	G	A	B	C'	D'
Key of C	264	297	330	350	393	440	495	528	
Key of D		297	*334	*371	396	*445	495	*557	594

Since many musical instruments are being tuned in fifth intervals, or quints, like the violin (tuned in g, d¹, a², e), the viola (tuned in c, g, d¹, a¹), or violoncello (tuned in C, G, d, a), an attempt has been made to employ the fifth interval in combination with the "honance", or octave, for developing a scale. The so-called "Pythagorean" C-scale, as shown in Fig. 22, is such a scale.

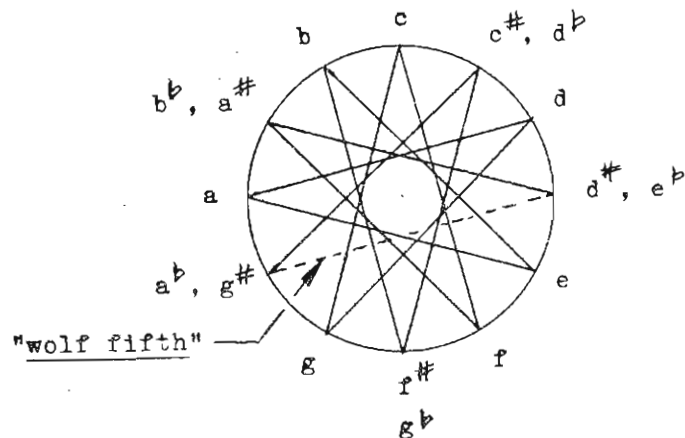
Figure 22

Pythagorean C-Scale

Musical note	C	D	E	F	G	A	B	C ¹
Frequency as related to the tonic	1	9/8	81/64	4/3	3/2	27/16	243/128	2/1
Frequency ratio for successive intervals	9/8	9/8	256/243	9/8	9/8	9/8	9/8	256/243

While comparing the Pythagorean C-scale (Fig. 22) with the diatonic C-scale (Fig. 15) it might be observed that in both scales the c, d, f and g are identical, but that the e, a and b are slightly higher in the Pythagorean scale (e/c = 81/64 = 1.265625; a/c = 27/16 = 1.6875; b/c = 243/128 = 1.8984375) than they are in the diatonic scale (e/c = 5/4 = 1.25; a/c = 5/3 = 1.666...; b/c = 15/8 = 1.875). There is obviously one major gain by the Pythagorean scale having only two ratios for successive intervals, the ratio 9/8 for whole note intervals and the ratio 256/243 for half note intervals. However, that ratio for the half note interval is too small, since two successive intervals of 256/243 are equal to 65536/59049 = 1.109857.. (for comparison: 9/8 = 1.125; and even 10/9 = 1.111.. is larger).

Figure 23



If the succession of the fifth intervals (exact tuning with the ratio of $3/2$) is being carried out to the end as shown schematically in Fig. 23, there will occur at one point a discrepancy, since 12 fifth intervals are slightly higher than 7 "honances" (12 fifth intervals equal $531441/4096 = 129.746337890625$ and 7 honances equal 128). This discrepancy became known as the "Pythagorean Comma" ($531441/524288 = 1.013643264770\dots$) and it was usually reserved for a seldom used interval like between "g#" and "a#" (or between "a#" and "b") as indicated by a dotted line on Fig. 23. Such intervals, shortened by a "Pythagorean Comma", were called "wolf" (like any other intervals unpleasantly out of tune).

The same discrepancy would occur if 6 whole note intervals with the ratio of $9/8$ are being used in sequence (6 whole note intervals with the ratio of $9/8$ equal $531441/262144 = 2.02728652954\dots$) and, on the other hand, if 6 whole note intervals with the ratio of $10/9$ are being used in sequence, their total would be slightly smaller than a honance (6 whole note intervals with the ratio of $10/9$ equal $1000000/531441 = 1.84404289\dots$).

Similarly, 3 large or major thirds with the exact ratio of $5/4$ used in sequence are slightly smaller than a honance (3 large or major thirds with the ratio of $5/4$ equal $125/64 = 1.953125$); 4 small or minor thirds with the exact ratio of $6/5$ used in sequence are slightly larger than a honance (4 small or minor thirds with the ratio of $6/5$ equal $1296/625 = 2.0736$) etc.

Realizing that it is numerically impossible to combine the different tone combinations (see Fig 13) with their exact frequency ratios into one musical scale, Andreas Werckmeister (1645 - 1706) proposed in 1691 in a treatise on "Musical Temperament" to divide a honance (or octave) into 12 equal half-steps. This musical

scale became known as the equally tempered or well-tempered scale and, today, is usually referred to as the "chromatic scale" (or more precisely the "12-note chromatic scale"). Its twelve half-intervals have only one constant ratio which is the twelfth root (basing the frequencies on an "a" with 440 cps). In 1722 Johann Sebastian Bach (1685 - 1750) demonstrated as the first composer in his "Well-tempered Clavichord" the feasibility of the "Chromatic scale." Because of its simplicity and its sufficiently good tuning it became the most representative tone system of the Western world.

Since a standard pitch "a" of 440 cps. has been used in Figure 21 as well as in Table 1 and Table 2, a few notes on this subject seem appropriate. As far as we know about a standard pitch in ancient times which would be equivalent to our present "a", it may have varied as much as from 375 to 500 cps. Not until the invention of the tuning fork in 1711 by Handel's trumpeter, John Shore, could more reliable data be obtained about standard pitches in the various musical centers. Handel's and Mozart's forks had an "a" of 422.5 cps., which is lower than our present standard. But during Beethoven's lifetime (1770 - 1827) the standard pitch in Vienna rose close to the modern pitch, and by 1850 it even reached 456 cps.

In England during the 1820's a standard pitch of 433 cps. had been adopted, but twenty years later the "high Philharmonic pitch" of 452.4 cps. took over, which was almost identical with the "sharp Vienna pitch", and this high standard pitch continued to be used for British bands until 1928. In the United States during the 1880's the standard pitch went as high as 458 cps., and this "concert pitch" was still common forty years later.

In 1858 (actually February 1859) the French agreed upon a standard pitch of 435 cps. (actually 435.4 cps.) at 15°C (59°F). At the Vienna Conference of 1885 the standard pitch of 435 cps. became "International Pitch". However the British adopted in 1896 a "new Philharmonic pitch" of 439 cps. at 20°C (68°F); and in 1939 they adopted the "British Standard Concert Pitch" of 440 cps., which is the standard pitch adopted about twenty years earlier by the American Federation of Musicians.

Today the frequency of 440 cps. is regarded as the standard "a" pitch. Yet, on one hand, singers would prefer, for understandable reasons, to return to the older "International Pitch" of 435 cps.; and, on the other hand, Philharmonic orchestras and bands show again tendencies to raise the standard pitch. (Ref.: G. Révész, "Einführung in die Musikpsychologie", Basel 1946; "Encyclopedia Britannica", 1962).

As mentioned, the "chromatic scale" (that is, the 12-note chromatic scale) is today the most representative tone system of the Western world. Certainly it is a compromise, because only its "honances" (or octaves) are tuned exactly (that is, they have a $2/1$ frequency ratio), and all its other intervals are slightly out of tune---they are not well-tempered or equally tempered.

But such problems can be found in many other areas whenever standardization is undertaken (see for example "Preferred Numbers" in "Machinery's Handbook" by E. Oberg and F. D. Jones). The most frequent objections to the "chromatic scale" seem to stem from

the assumption that a diatonic scale is the perfect musical scale---an attempt will be made to explain briefly where this assumption might have come from; and a critical analysis will be given those claims.

One major factor which contributed to implant the diatonic scale (or scales) into the Western mind as being "the musical scale" was a mystical belief in numbers during the first half of the Middle Ages, initiated probably by the Roman philosopher Boethius (c. 480 - 524). His philosophy exhibits Neoplatonism and Stoicism, and his writing "De Institutione Musica" is an adaptation and re-elaboration of the Greek handbook of Nichomachus of Gerasa. It regarded music as an audible example of abstract mathematical concepts of ratios and of proportions--of the ratios between the intervals, and of the proportions between long and short notes.

The Pythagorean theory of intervals was often demonstrated by a monochord; and the diatonic scale was supposed accordingly to be a magical key to the new world of music (world of harmony).

Three other contributing factors will be considered: the association of the plain Latin letters "a" through "g" with the tones of the diatonic scale; the original tuning of the white keys of keyboard instruments according to the diatonic C-scale; and the original basing of pitch notations on the diatonic C-scale. (See Part III).

As a final contributing factor, one should mention a mnemonic technique known as "solmization". It was apparently introduced by a monk, Guido d'Arezzo (died c. 1050), who noticed that the pitches on which the initial syllables of the first six lines of a hymn to St. John the Baptist were sung, a petition for protection against hoarseness, formed the ascending scale, C, D, E, F, G, and A (see Fig. 24).

Figure 24

C <i>Ut</i> queant laxis	F <i>famuli</i> tuorum
D <i>resonare</i> fibris	G <i>solve</i> polluti
E <i>mira</i> gestorum	A <i>labii</i> reatum

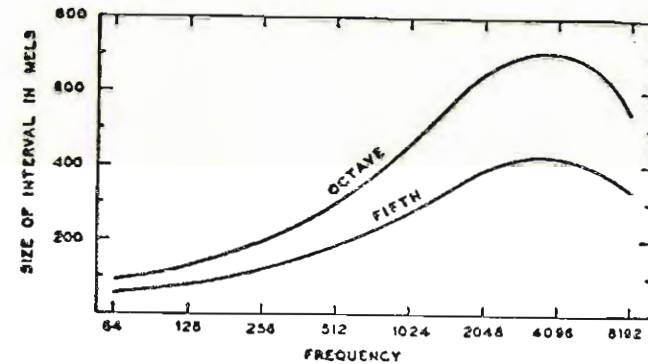
By applying these syllables "ut, re, mi, fa, sol, la" with their associated pitches to the notes of other melodies, a singer could learn new melodies quickly. These syllables, with the minor change of "ut" to "do" and the addition of "si" (or "ti") for the seventh degree, are still used today.

The diatonic scale (Fig. 15) was supposed to be based upon the principle of "consonance" (see Fig. 13), that is, of pleasant tone combinations (the "vertical principle").

This ignores another function of the musical scale, which is to provide intervals for building melodies, and for transposing them (the "horizontal principle"). It assumes that when a musical scale satisfies the first function, the second function will automatically resolve itself. Unfortunately, this does not occur, partly because the diatonic scale has varying successive inter-

vals, and partly because the sensed distances between successive tones, the sizes of successive intervals, vary according to their frequency levels. This was indicated by Fig. 7. Also, Fig. 25 shows the sensed sizes of two successive intervals, the "honances" (or octaves) and the "fifths" (or quints) expressed in mels.

Figure 25



The pitch-extent of musical intervals depends upon where the intervals are located on the frequency-scale. The values on the abscissa represent the mid-points of the intervals whose subjective extents are determined from the ordinates.

(Stevens and Volkman, *Am. J. Psychol.* 53, 329 (1940))

Of course, if a melody is being transposed over one "honance" or more, about the same distortion of the melody will occur by all known musical scales. However, if a melody is being transposed over a small interval only, and it uses a chromatic scale, no noticeable distortion of the melody will take place due to the equal successive intervals (equally tempered) of a chromatic scale. Therefore, a chromatic scale better satisfies the "horizontal principle".

Recognizing the advantages of a chromatic scale with regard to the "horizontal principle", yet realizing the shortcomings of the "12-note chromatic scale" with regard to the "vertical principle" (although relatively small), a few more complex chromatic scales have been proposed in the past.

Two of them are worth mentioning, because they have been tested and were found conducive to very good tuning: the "53-note chromatic scale" by Nicholas Mercator (1675); and the "41-note chromatic scale" by Paul von Jankó (1856 - 1919).

However, their greatly increased complexity and thus their much more difficult handling outweighs their relatively small gain in tuning. Moreover, probably due to the shortcomings of man's hearing, as mentioned in Part One of this writing, no singer or virtuoso was found who used consistently one particular musical scale. Therefore, it would seem that the "12-note chromatic scale" (today usually referred to as the "chromatic scale") will

be the most satisfactory musical scale so long as man himself is to continue to perform in the world of music.

With the upcoming of pre-programmed and of electronic music, the question of a "musical scale" might require a review. There a more complex chromatic scale could be considered, since man's handling is eliminated. Some experimental approaches have gone so far that recognizable tonal fragments are avoided ("athematism"). However, since such music does not require a "written" musical notation in the usual sense, it goes beyond the scope of this work. ---(Ref.: M. Bauer and E. Peyser, "Music Through The Ages", third edition, 1967, New York).

Part Three

Chromatic Notations

Music can be regarded as a system of signs made by sounds, similar to speech: it is a form of communication which is received auditorily. Therefore, it is for momentary transmission and it has the advantage of leaving the channel of communication open immediately after the end of a transmission. But at the same time it has the drawback that its communication is gone right after its transmission. In order to stabilize, to fix such kind of communication, it has to be somehow connected with a material. Only very recently man was able to develop devices which in various ways record sounds on materials and which reproduce those sounds from recordings, such as phonographs, tape recorders, etc. Another, much older, way of recording music, as well as speech, is "writing".

"Writing" in its many different forms represents systems of visual signs which are fixed on a material. These visual signs can be used as substitutes for auditory signals, as the signs of music, speech, etc. Then anyone who is more or less initiated into that particular system of "writing" can understand it and, if wanted, can reproduce the original auditory signals.

The "writing" which is used for recording music is usually referred to as "musical notation", in a broad sense. In developed form it indicates essential features of music: like the pitch and duration of a note, loudness, tempo (speed), etc. However, this article deals mainly with the indication or notation of pitch.

Three major types of pitch indications are known: 1) pitch indications by letters or numbers; 2) notations by a staff, "musical notation" (in a stricter sense) as usually referred to and as presently used in the Western world; 3) indirect notation of pitches by indication of fingering or handling. Since this last type of pitch indication is a specialized "musical notation" restricted to certain musical instruments, it falls outside the scope of this writing.

The first type of pitch indications suggested above is being employed with a great variety of "musical notations" in many parts of the world. It uses either letters or ordinal numbers: they indicate in absolute or relative terms the proper place of a note, with regard to its pitch, within a tone system or tone scale. This lends itself to "naming of the notes" as it is largely done in the Western world.

These employed letters or numbers which may be either native or foreign to the users are sometimes so modified that the original symbols are hardly recognizable. In the upper row of Fig. 26 is shown such a "musical notation"; it had been used during the 9th century and was known as "Musica Enchiriadis". Its symbols have probably been derived from an ancient Semitic alphabet.

Figure 26

τ	ρ	N	γ	δ	ϕ	ι	ϕ	ϑ	η	ζ	ε	ε	γ	κ	σ	ς	
G	A	B♭	c	d	e	f	g	a	b	c ¹	d ¹	e ¹	f ^{#1}	g ¹	a ¹	b ¹	c ^{#2}

The first important "musical notation" which influenced our presently used one (our "naming of the notes") was the "musical notation" by Boethius (c. 480 - 524). In the upper row of Figure 27 is shown this "musical notation", while in the lower row are indicated the equivalent conventional notes.

Figure 27

a	b	c	d	e	f	g	h	i	k	l	m	n	o	p
A	B	c	d	e	f	g	a	b♭	c ¹	d ¹	e ¹	f ¹	g ¹	a ¹
								b _γ						

Another important "musical notation" became known as the "Odo-
nic notation". It had been named after Odo, Abbot of Cluny from 927 to 942, to whom the treatise "Enchiridion Musices" (Handbook on Music) had been attributed until recently. However, this notation in its original form as shown in the upper row of Fig. 28 was not accepted until the Greek letters in lower case had been replaced by Latin double-letters in lower case as shown in the second row. In the third row are indicated the equivalent conventional notes.

It should be noted that the "b" in lower case could have represented either a "b-flat" or a "b-natural" note. And two forms of the b-letter have been used as double-letters, a rounded soft "b" (rotundum) for "b-flat" and a square hard "b" (quadrum) for "b-natural".

Using two forms of the b-letter as just mentioned led to an interesting development which is still common in central Europe. While modifying the square hard "b" (quadrum) to "q" for easier

recognition, it started to look like the Latin lower case letter "h". Since within the Greater Perfect system (see Fig. 17) only the b-notes had two variants of "b-flat" and of "b-natural", the letter "b" became restricted to "b-flat" and the letter "h" has been adopted for the "b-natural" notes. In this way it became possible to express every note of the Greater Perfect system by only one letter.

Figure 28

A	B	C	D	E	F	G	a	b	c	d	e	f	g	α	β	γ	δ	ε
A	B	C	D	E	F	G	a	b	c	d	e	f	g	a	b	b	c	d
A	B	c	d	e	f	g	a	b ^b ₄	c ¹	d ¹	e ¹	f ¹	g ¹	a ¹	b ¹ ₁	b ¹ ₂	c ²	d ²

Very soon the two forms of the b-letter were applied to other usages. The rounded soft "b" (rotundum) became the flat-sign "b" (bemol) which lowered an associated note by a half-step. Similarly, the square hard "b" (quadrum), modified as "b^h" (diesis), became the natural-sign which restored an associated note to its original pitch value.

In a later development the square hard "b" (quadrum), modified as "b^h" (cancellatum), became the first sharp-sign. By being crossed it indicated that the associated note is neither flat nor natural and thus it raised the associated note by a half-step. Its newer form is "b^h". Its invention is ascribed to Josquin Des Prés (1450 - 1521).

The double-flat sign "bb" and the double-sharp sign "x", as well as the combined signs "bb" and "x" are conventions of recent times.

As shown in Fig. 28, the Odonic notation grouped its notes into honances (or octaves) from the letter "e" to the letter "g", and it indicated the different honances by using Latin or Greek letters in upper or lower case, as single or as double letters. Today the notes are conventionally grouped into honances from the letter "c" to the letter "b", and the different honances are usually indicated by employing Latin letters in lower or upper case (always single), either with or without additional numbers.

Two additional "musical notations" which used the first type of pitch indication gained some acceptance during the last century, the Tonic Sol-fa system and the Chev  system. However, both systems were conceived primarily for teaching music, and thus they fall outside the scope of this writing.

Although very proper for "naming the notes", the first major type of pitch indication fails to show graphically the flow of

melody. Therefore, an early form of "musical notation" developed from accent marks and chirographies of chorus leaders, which was used all over Europe by the ninth century. It is known as "neumatic notation", and it showed graphically the direction of a melodic line.

Figure 29

APPROXIMATE DATES			
10-11 CENTURY	14-15 CENTURY	16th CENTURY	17-20 CENTURY
F	F	F	F
C	C	C	C
G	G	G	G

Clefs, which assign a given note to a given line of the staff, developed from letters which named the fixed notes. From top to bottom, they are: the F (bass) clef, which fixes the note F on the next-to-the-top line of the bass staff; the C (alto or tenor) clef, a movable sign which fixes the note C between the curving brackets, and the G (treble) clef, which fixes the note G on the next-to-the-bottom line of the treble staff.

Initially the neumes were very vague with regard to their pitch values, and a single neume could indicate one, two or more notes. Therefore, toward the end of the tenth century and after the association of Latin letters with pitches, anonymous scribes conceived the idea of tracing a red line horizontally above the text and to assign the pitch of the note "f" to it--the line which today carries the bass clef. This method was so successful that soon a second line, usually green or yellow in color, was employed to signify the middle "c".

The monk Guido d'Arezzo (died c. 1050) perfected a four-line staff which utilized both the lines and the spaces between the lines. Each line was separated by the interval of a third (such as the interval "c-e"). Thus, a definite position was given to each scale degree. Furthermore, the use of a clef (French for "key") on the staff, which originated at this time, fixed a given pitch to a line and made it unnecessary to draw colored lines. Fig. 29 shows the development of the three most commonly employed clefs, C, F, and G, which at first were ordinary letters and later developed into abstract symbols.

-o-o-o-

(The remainder of Part III will be set out in the next issue of the Bulletin. It clearly demonstrates the advantages of twelve-base symbols in music and contains examples of Chopin's Etude op. 25, No. 9, written in the conventional notation, the Notation Godjevatz, and the author's notation. M. Essig, in Douze Notre Dix Futur, demonstrates the disadvantages of a base-ten metric system. So Mr. Kothe shows the many disadvantages suffered in music before its introduction to base-twelve symbols.)

GEORGE S. TERRY

1887 - 1969

In the passing of George S. Terry we mourn the loss of an able mathematician, a generous patron, and to many of us a personal friend. We may say now what would have been difficult while this modest man was among us: without George S. Terry it is doubtful that The Duodecimal Society of America would have come into existence.

My first acquaintance with Mr. Terry was in a letter in October, 1934. Commenting on an article of mine on duodecimals which had just appeared in the *Atlantic*, he asked whether serviceable logarithmic tables to this base were available. I reported in the negative, and suggested that a very real contribution to the future might be made by developing such a table.

Mr. Terry rose to that challenge with his monumental *Duodecimal Arithmetic*, published by Longmans, Green and Company in 1938, making available for the first time in history logarithms to the twelve base, trigonometric functions in terms of the duodecimal circle, and other needed apparatus.

In 1939 he wrote again, this time suggesting that our informal corresponding group of four which humorously called themselves "The Duodecimal Society of America" might be organized into a genuine society which he was willing to finance, if necessary. A substantial endowment and other money gifts came from Mr. Terry for the founding and operation of the Society, added to the great intellectual contributions he had made and continued to make.

It took some years, and the organizing genius of Ralph H. Beard, before the Society held its first off-

icial meeting on 5 April 1944. Mr. Terry was induced to serve as chairman of the board of directors from 1944 through 1949, continuing thereafter as a board member until the time of his death. He was given the Society's Annual Award for 1945 in recognition of his imposing work, *Duodecimal Arithmetic*, and other papers including the popular brochure, *The Dozen System*; also his generosity in establishing the Endowment Fund for the Society. He served as editor of this *Bulletin* from 1950 through 1954, and contributed many papers to enliven its pages.

Though he urged responsible posts for younger members, so long as his health permitted he continued to attend the annual meetings, usually with his charming wife, Martha Gay Terry; and a few of us knew that he personally furnished travel funds for members in need of this aid.

George S. Terry was born in East Ilsley, Berkshire, England, on 27 February 1887. He graduated from London University in 1908 with a B.S. in Engineering. He worked his way around the world, financing himself with engineering stints. In World War I he was Captain of the Northumberland Fusiliers, and was then sent to the United States to inspect steel for British shells. During this duty he met his first wife, Ruth Kellogg, in Buffalo. He moved to Hingham, Massachusetts where he resided during most of his many services to the Society. But with full retirement he moved to the better climate of Sonoita, Arizona, and his death occurred there on 12 February 1969.

At its recent annual meeting the Society recognized his immense services to the Society with an appropriate resolution, and extended its deep sympathy to Mrs. Terry.

---F. Emerson Andrews

ENGLISH E-Z LOCATOR EXPOSITION (Cont'd from p. X)

Let no one doubt the complete ascendancy of navigation by able use of base twelve throughout the earth. Mr. Bishop points the way with clarity and his usual mastery of the complex.

H. C. C.

A point by Courtesy of



Sam is perhaps a dozen years ahead of his time! (See page X)

-o-0-o-

ANNUAL MEETING (Continued from page 5)

California at Santa Barbara and the Manager of Francisco Torrez student hall for the manner in which they received us as guests, and the many courtesies extended to us during our short stay. Carried.

At 1181 April 1;500 (1969 April 1, 1000 Hours) it was moved by Camp, seconded by Linton, that the 1181 Annual Meeting of the membership of DSA at Santa Barbara, California, stand adjourned sine die. Carried.

In a matter of moments the members present were checking out of their rooms and heading for New York, New Mexico, Iowa, or areas of California, some by way of local points of interest.

-o-0-o-

EXCERPTS FROM LETTERS AND CORRESPONDENCE

BRUCE A. M. MOON, Director of the Computer Centre at the University of Canterbury, Christchurch, New Zealand, writes: "Take computers. Whether we like it or not, we are on the threshold of the computer age. The University of Canterbury has an old computer--one of the first in the country--and it works in decimals. The University has just installed one of the most modern computers in the world. It will be the fastest calculating machine in the country *and it cannot do decimal arithmetic*. It uses instead twos and sixteens. Decimals are abandoned because they were too slow, inefficient, and expensive!

Mr. Moon further writes: "This is a clear warning. While we follow a blind alley into which the all-too-dubious advantages of (a decimal metric system) are leading us, those who possess the awareness of non-decimal number systems and their advantages ... will advance, and we shall become obsolescent."

STAN BUMPUS, 2215 College Avenue, Mount Vernon, Ill. 62864, has written: "In my opinion, the computer may be the jinni for many future conversions of mathematical tables & physical constants."

S. FERGUSON, 58 Scotby Village, Scotby, Nr. Carlisle, Cumberland, England, writes: "A member of the DSA has suggested the subscript $n - 1$ in base n , causing the base seven number 123₇ to appear as 123₆. There is danger of confusion here. In the text books on 'Modern' Math used in English schools there are two methods in current use--123₇ means '123' in base seven. The second practice is 123_{seven}. I would suggest the second is preferable to the first."

Note: American and English customs differ occasionally. To many in the U.S. the FIRST floor of an office building is the ground floor, and their SECOND floor is that which the English call the FIRST floor---the first above the ground level. Thus, in Charles Dickens' BLEAK HOUSE, Chapter XXII, the author describes the location of the Tulkinghorn rooms with respect to the ground floor gate through which three of his characters have momentarily passed: "As they ascended the dim stairs (Mr. Tulkinghorn's chambers being on the first floor), Mr. Bucket mentions that he has the key to the outer door in his pocket and that there is no need to ring." The English custom of counting floor levels is more logical, in the editor's opinion. More Americans are now starting the upcount from zero in all manner of common procedures, and eventually no doubt, the year of B.C. 1 will be designated by an aught or zero, and 2 B.C. will become calendarwise the year of 1 B.C., or simply -1, in history books, Old Testaments, archeology, etc., as all specific years B.C. reject one calendar year of distance in time away from this common era. This would have enabled us to add 45 B.C. (-44) to A.D. 1956 (ignoring the negative sign) to see that the year in which Julius Caesar died was a leapyear under the Julian Calendar reform, and that Caesar had been struck down exactly 2000 years before 15 March 1956. Not 1955, as some Latin classes claimed in that year!

H.C.C.