COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	X	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3Ɛ2*	Two ft. eight in.	*2;8'*
19Ɛ	*1000*	Eleven ft. seven in.	*Ɛ;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
12 ) 30 + 5
12 ) 2  + 6
     0  + 2     Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *X*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *X*.

Numerical Progression

1	One		
10	Do	*;1*	Edo
100	Gro	*;01*	Egro
1,000	Mo	*;001*	Emo
10,000	Do-mo	*;000,1*	Edo-mo
100,000	Gro-mo	*;000,01*	Egro-mo
1,000,000	Bi-mo	*;000,001*	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	Ɛ
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
Ɛ	1X	29	38	47	56	65	74	83	92	X1

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THE DUODECIMAL SOCIETY OF AMERICA

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THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of base twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

The forms of membership include Honorary, Life, Fellow, and Senior Members, as well as Members, and Student Members. Members and Student Members are not required to pass aptitude tests in base twelve, but are encouraged to do so.

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All figures in italics are duodecimal.

*CHROMATIC MUSICAL SCALES AND NOTATIONS

By Erich Kothe

Part One

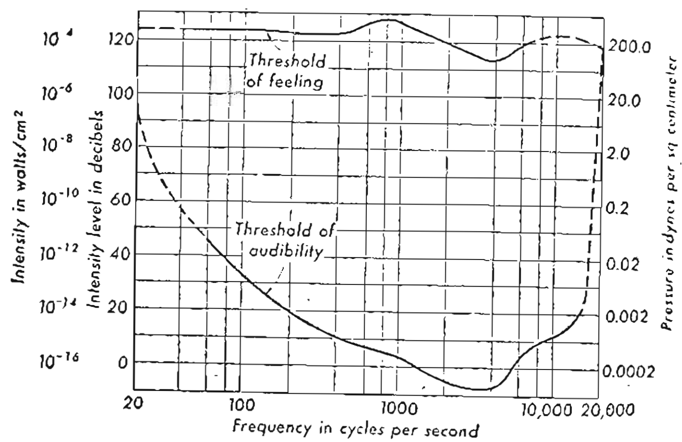
Hearing and Sound

One of the most important sensory organs of man is his sense of hearing. The usual physical stimuli for hearing are successive pressure waves of the surrounding air. Such pressure waves are called (transversal) vibrations of the air. However, in order to be perceived by the human ear as a sound, these vibrations have to be within the frequency response of man's ear, between approximately 20 and 15000 cycles per second. And their intensity has to be above the so-called "threshold of audibility," yet below the "threshold of feeling" (also called "threshold of pain").

This range of intensity to which man's ear is sensitive is about a millionfold. The graph of Fig. 1 shows these limits. It also indicates that man's ear is most sensitive to sounds of about 2700 cycles per second, which is the approximate resonance frequency of the air within the ear canal (see Fig. 2).

The frequency of a sound is sensed by the ear as its pitch: the higher the frequency, the higher the pitch. Similarly, the intensity of a sound is sensed as its loudness: the higher the

Figure 1.



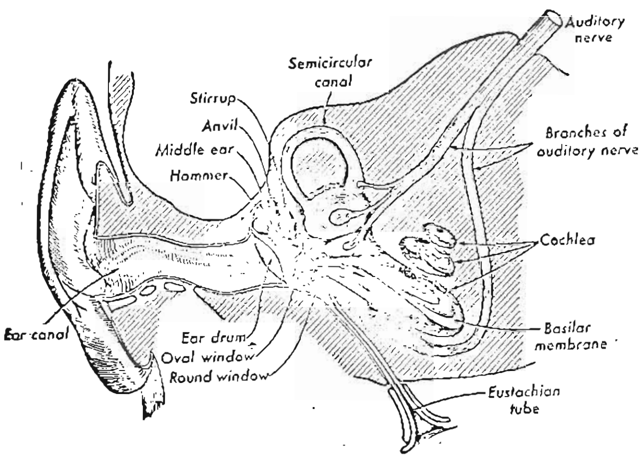
Range of frequencies and their intensities which are perceived by the human ear.

(After H. Fletcher, *Reviews of Modern Physics*, January, 1940.)

*This interesting article must be run in three parts, and the first is almost entirely introductory, containing illustrations employing ten-base notation and metric terms quite exclusively.
---Ed.

intensity, the greater its loudness. However, there is not necessarily a one-to-one correspondence between the absolute intensity of a sound and its sensed loudness. A simple test can prove this point.

Figure 2.



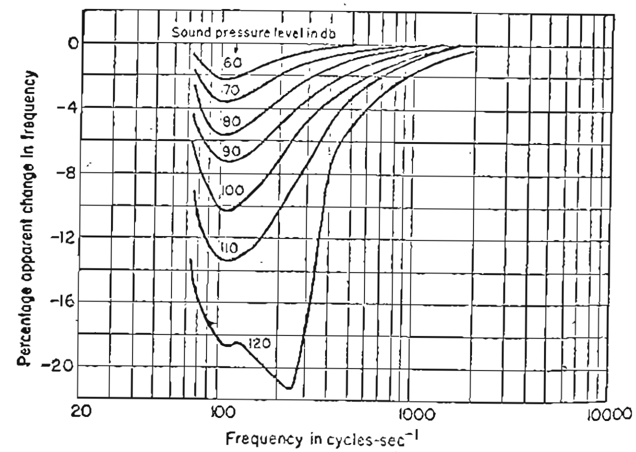
Semidiagrammatic section of the right ear.

If someone exposes only one of his ears to a sound of about 800 cycles per second and of medium loudness for about 2 *minutes*, and then he exposes independently each ear to another sound of about 1200 cycles per second and of the same intensity for about 2 *seconds*, he will sense different pitches and loudness of this second sound by his two ears. The ear which was exposed to the first sound will sense the second and shorter sound higher (approximately a half tone higher) and distinctively less loud than the other ear which was exposed to the second sound only.

Besides the just mentioned influence of pre-exposure on the sensed pitch and loudness of a sound, another influence on the sensed pitch has been observed which was due to high intensity. This influence becomes usually noticeable as a decrease of the sensed pitch, and the amount of this apparent change is being affected by the frequency as well as by the intensity of the sound. The graphs of Fig. 3 are from data by Snow.

Many hearing tests have been performed which disregard the already mentioned influences of pre-exposure and of high intensities, and which ignore the widely differing hearing characteristics of different individuals. Therefore, their results have to be interpreted cautiously, and they are valid only for the (statistically) average ear. Nevertheless, those results are helpful guides, as for instance the curves given in Fig. 4 which show sensed loudness levels (expressed in phones).

Figure 3.

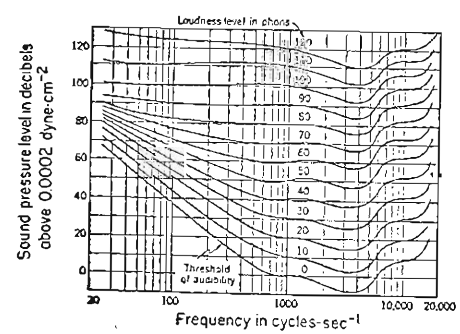


Apparent shift in frequency with high sound pressures. (*After* Snow)

Snow, *Jour. Acous. Soc. Amer.* 8, 14 (1936).

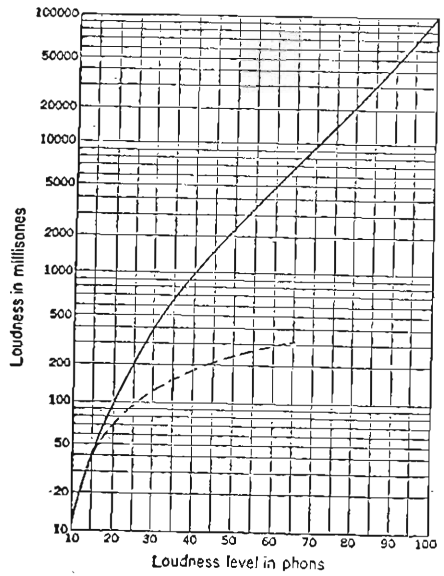
These curves as shown by Fig. 4 indicate equally sensed loudness at different frequencies; and their numerical values expressed in phons coincide always with the numerical values of the equivalent sound pressure levels expressed in decibels at 1000 cycles per second. Consequently, this means that a sound of 40 decibels at 1000 cycles per second will appear approximately as loud as a sound of 70 decibels at 60 cycles per second, or a sound of 37 decibels at 3000 cycles per second.

Figure 4.



Loudness level contours.

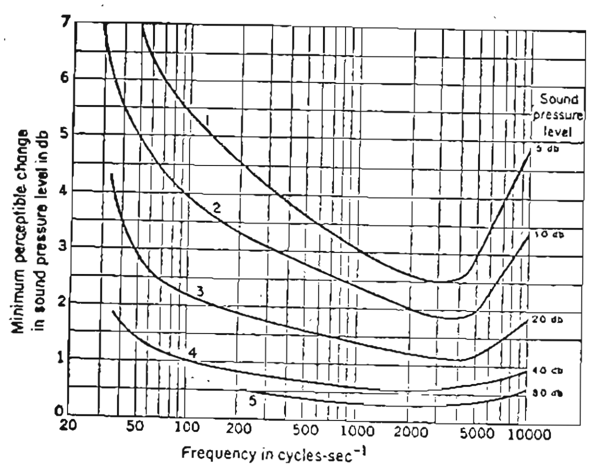
Figure 5.



Relationship between loudness and loudness level.

Fletcher and Munson, *Jour. Acous. Soc. Amer.* 9, 1 (1937)

Figure 6.



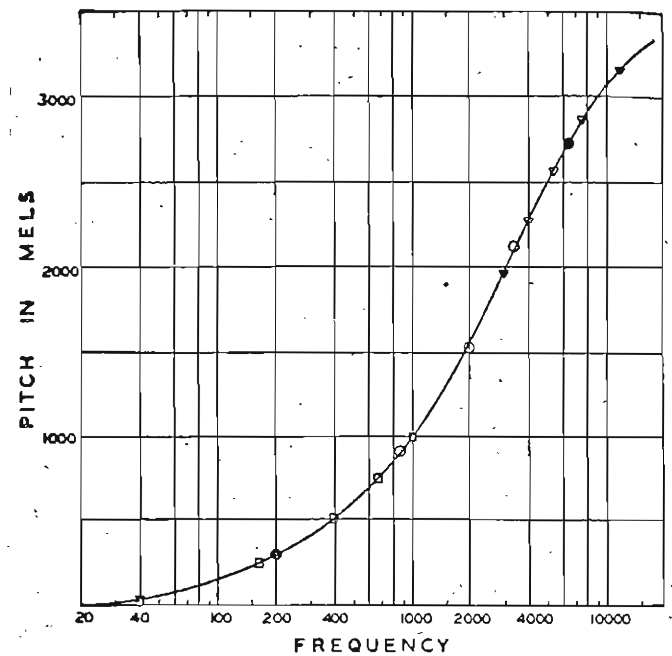
Curves showing sensitivity of the ear to changes in sound pressure.

(*After* Fletcher)

Unfortunately, the sensed loudness of a sound does not increase linearly with an increased sound pressure level. Therefore, another scale based on the average listener's reaction has been devised which uses a "sone" as its fundamental unit; and a sone is defined as the loudness of a simple (pure) tone of 40 decibels at 1000 cycles per second. According to this scale, a sound which appears to be two or three times as loud as a sound of one sone would have its loudness classified as of two or three sones. The actually measured relation of the sensed loudness expressed in millisones (one millisone equals .001 sone) against the loudness levels expressed in phons is indicated by a heavy line on the graph of Fig. 5. The broken line shows a hypothetical, strictly linear relation.

For musical purposes, the relative intensity of one sound to another one, the change of the intensity level, is much more important than the absolute intensity of each sound. Therefore

Figure 7.



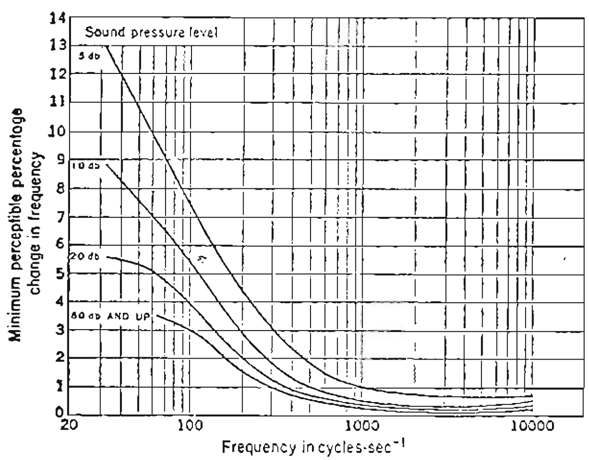
The curve shows how pitch, scaled in subjective units (ordinates), varies with frequency. The circles, squares and triangles represent data obtained in the experiment on equal sense-distances. The filled figures mark the ends of three frequency-ranges and the hollow figures show the points arrived at when the Os divided the ranges into four equal intervals of pitch.

(Stevens and Volkman, *Am. J. Psychol.* 53, 329 (1940)) the minimum detectable change in the intensity level is significant, even though it varies greatly with the intensity itself, the frequency, and to some extent with the complexity of the sound. For simple (pure) tones, curves have been drawn (see Fig. 6) which indicate the minimum detectable change in intensity level as a function of intensity and of frequency.

Similarly as to the loudness, the sensed pitch of a sound does not increase linearly with the increased frequency. Therefore, some scales have been proposed which are based on the average listener's reaction. The best of these scales seems to be the one devised by S. S. Stevens and by J. Volkmann which uses a "mel" as its measuring unit. A "mel" (derived from the word "melody") is defined as one thousandth of the pitch of a simple (pure) tone at 1000 cycles per second and of 55 phones (see Fig. 4) loudness. The curve appearing in Fig. 7 shows the sensed pitches of various frequencies in the range between 40 and 12000 cycles per second.

For musical purposes, the relative frequency of one sound to another one is much more important than the absolute frequency of each sound. Therefore, the minimum detectable frequency shift is significant, even though it varies greatly with the intensity, the frequency itself, and with the complexity of the sound. For simple (pure) tones, curves have been drawn (see Fig. 8) which indicate the minimum perceptible change in frequency as a function of frequency and of intensity.

Figure 8.



Curves showing sensitivity of the ear to changes in frequency.

(Shower and Biddulph, *Jour. Acous. Soc. Amer.* 3, 275 (1931))

(Continued on page fourdo-five)

COUNTING AND TALLYING

B. A. M. Moon, Director,
Computer Centre, University of Canterbury.

We are often told that the reason most people count in tens most of the time is because we possess ten fingers. This is probably true. We are sometimes told that this is the 'natural' way to count. This is a rather more questionable assertion. Two widely different modern stone age cultures use the parts of their bodies to count in quite different ways. The practice is recorded amongst the Eskimos of using the *gaps* between their fingers to count, which naturally leads them to a scale of eight, that is, an octal scale, which will undoubtedly be helpful to the Eskimos when they start using digital computers.

By contrast, the natives of the Sibil valley of the mountainous central region of Western New Guinea use a much greater portion of the body as an abacus. They count up to twentyseven beginning with the left little finger, moving along the arm, over the head, and down to finish at the right little finger. The words for the numbers are the same as the corresponding parts of the body. Thus 'sirong' is the left ear and also twelve, and 'sirong tabar' is the right ear and also sixteen. This quantity of numbers seems to be sufficient for the business of these remote tribesmen. It may be compared with the number language of the Tasmanian aboriginals 'one, two, plenty' or that of the ancient Greeks to whom the word 'muriod' meant ten thousand and also 'an innumerable quantity'. From it our own word 'myriad' is derived.

If it be further asserted that the natural way to count is to use the parts of the body, we may perhaps ask why, hitherto, much greater use of the binary scale has not been made. After all, we have two eyes, two ears, arms, legs, in fact we have two of any organ much more frequently than we have ten! However, apart from rudimentary development of a binary scale amongst some Australian aboriginal tribes there does not seem to be much evidence of a number language based on two. The practical value of the scale of two has shown itself in other developments, notably our scales of weight and liquid measure, the practical subdivision of the inch, and, of course, more recently in the electronic computer.

Important though it is, counting is the most primitive of our number skills. We all learn early that it is bad practice to count on our fingers since it arrests the development of more advanced number skills, but how many of us still do it? This is perhaps a good argument for using a number base other than ten--to break the persistent temptation to use the fingers!

However, there are other ways of using the fingers to aid in counting. If (now advanced by Dr. Kenneth Mears) we use a bent finger to represent 1 and a straight finger to represent 0, and use each finger to represent a binary place, then we can count on one hand alone to 31, and on two up to $2^{10} - 1$, which is more than a thousand. Try it. It takes a little practice to master but it does develop skill with binary numbers.

It is interesting to observe the development of counting habits in children. At kindergarten age they can nearly all count 'by ones'---'one, two, three' and so on, up to twenty, thirty, or perhaps further. At six or seven they know how to count by fives 'five, ten, fifteen, twenty--' or by tens, or even, (getting sophisticated) 'to count to a hundred by hundreds'. This kind of counting is often practiced in such games as 'hide and seek' as 'it' counts while the others hide. We may ask what this practice achieves apart from an illusion of getting somewhere fast. The answer is not very much except practice in the 'five times' table, which is easy in decimals. As much is achieved by counting in ones to twenty as in fives to a hundred.

The situation is rather different however when we employ our counting skills for purposes of tallying, that is, for counting things. If you were asked for instance to count the number of words in this article or to assist with the stock-taking in a shop (both practical needs from time to time), how would you set about it? The chances are that you would find groups of five awkward and rather too big to count at a glance. It is quite possible you might even count them in ones, certainly the slowest and most tedious way, but nevertheless nearly always used by younger children. You might do better counting in twos 'two, four, six, eight...'. Would you count in threes or fours (quite likely), which might be suitable groups to see at a glance, and quite a lot quicker to tally?

The answer, if you are working in decimals, is probably no, because the three and four times tables are not easy to use in this way.

People who have to tally with speed and accuracy have found a way to overcome this problem---by counting in the scale of twelve or dozens. Thus, by 'threes' it becomes 'three, six, nine, a dozen; (one and) three, (one and) six, (one and) nine, two dozen, (two and) three...,'; or by 'fours' we have 'four, eight, a dozen...' and so on, where the running count of the dozens themselves (bracketed above) may be omitted in a rapid verbalization.

This latter scheme also makes it easy to count by 'sixes' if the pattern of objects is sufficiently regular to permit it, or to fall back to pairs if use of larger groups is inconvenient. It is this fact, just as much as the greater economy in packaging by multiples of a dozen and their greater convenience for handling or other purposes, which has led to the use of dozens groupings of goods for a very high proportion of commercial transactions.

If indeed we were to make greater use of base twelve for arithmetic, counting could be a substantial aid to the learning of multiplication tables and also to a better understanding of their nature than sometimes seems to be the case at present. For those who still feel the need for an anatomical aid to calculation, even one of these is available--use the twelve joints

(Continued on page fourdo-ten)

OUR MISERABLE MESS OF MEASURES?

"If a slow-moving country such as England can finally change its monetary system, it ought to be more than time for the U.S. to move our whole system of weights and measures from the Middle Ages into modern times." I quote above what is being said by the philadecimetrics every day now. But let us go further.

Is it not time for the whole world to improve its Eighteenth-century, outdated, base-ten metric system, and, eventually, the international inch and mile, to bring both opposing systems into ONE WORLDWIDE, TWENTY FIRST CENTURY wholly correlated system of weights, measures, time, angle, and base? The base-twelve metric system could integrate all of these!

These United States of America have been reluctant to adopt an eighteenth century metric system until that concoction shall have been brought to date and so improved as to include time, angle, and navigational metric units--not the half-baked, indigestible cake sometimes offered to us at an undetermined price.

The French metric system of measurements is pointing the way. But, at a time when any scientist could understand how tenuous is the tie of his head to his torso, with the guillotine standing in Concorde Place to welcome anyone, erudite or ignorant, whose conduct offended a powerful rabble, thoughtful mathematicians, sitting in Paris, considered and abandoned base-twelve (the necessary ingredient to create a universal metric system) as too advanced for the people of France.

Fortunately, Frenchmen today do not live in fear of that ancient anesthetic, and some are working out a complete system to change their own measuring scheme into a base-twelve potentially worldwide metric system. These efforts have produced a system in which time, angle, and navigational units of measurement are correlated with base-twelve millimètre, mètre, and kilomètre, and all other metric units now in use, to form a complete, thus far unachieved, modern metric system.

When the international metric union adopts these changes, and takes steps to implement them, the people of the United States and Canada, without any statutory coercion, might reasonably be expected not only to live with the improved metric system, but to embrace it eventually in its entirety.

In fact, every day, Americans are employing multiples and submultiples of the metron (*75 000;0* krypton 86 atomic wave lengths) and the edon (*75 000 000;0* kr. 86 wave lengths), which are the equals of the French newly suggested duodécimètre and the hectomètre* duodécimal, under the cloak of 3-2/3 inches and the Canadian one-tenth land mile, respectively. The edon is only two and a fraction inches greater than the length of 528 international feet. The metron is 1/3000th part greater than 3-2/3 International or Canadian inches---requiring a microscope to see the difference.

Navigationally, one edon multiplied by the fifth power of twelve is the equal of one great circle of the earth. Edons (one-tenth of an English land mile) are not unsuitable for motor vehicle use, ships at sea, and air jetliners---an identical unit for land, sea and air, all three employing the aeromile (twelve edons), also called the Navinaut& or Kilomètre Duodécimal.* The single, universal unit of length could replace kilomètres, land miles, and nautical miles completely, all out the window together when people begin to think and act.

If pounds and shillings are obsolete, then so are degrees and minutes of angle, and minutes and seconds of time, since all are compound denominate numbers alike. In any properly improved metric system, degrees and minutes and seconds must follow pounds and shillings to Sheol. The Eighteenth Century, base-ten, metric system has seemed unable to supplant the second of time, which is quite unmetric. Today we find the second in the metric MKS and cgs formulas. Nothing is metric about seconds in their relation to the hour or the day, in base-ten.

In the United States of America, in one square statute mile (a Congressional Section of farm lands) there can be said to be one hundred square edons or 100 square hectomètres duodécimaux. The variation is one in three thousand parts---about the same dissimilarity as between one section of land and perhaps an adjoining section in the same township. The U. S. Department of Agriculture might be able to work in percentages instead of 1/640 parts of a Section of farm lands. Iowa could be said to have rested 5937.50 Sections or square miles (3,800,000 acres) of crop land in 1968. An inadequate conservation of farm land!

One dozen metrons are equal to the dimension of forty-four inches (one mètre* duodécimal) in a framework of reference differing by one in 3000 parts. By definition, a metron is exactly equal to seven dozen and five great gross (*75 000;0*) kr. 86 wave lengths. The international meter equals 1,650,763.73 kr. 86 wave lengths today. The N.B.S. can measure either exactly.

The costs of a changeover, voluntary or involuntary, in the United States of America from our present inches and miles to the metric system, either ten-base or twelve-base, is entirely prohibitive. But, if we let Frenchmen lead in reforming their present metric system* into base-twelve units of length, volume, weight, angle, time, and all related or derived measurements, and join with France in setting up the base-twelve system on a voluntary, or auxilliary, worldwide scale, the costs might be half as much as they must be if we change first to the base-ten metric system and later to the base-twelve metric.

But if we move in now and embrace the base-ten metric system, then when nearly the whole world is *using a twelve-base metric system* we might be the only nation, plus a few aboriginal Pygmies in outback Australia, stuck with a ten-base metric system ---simply because of the stubborn resistance of any people, *any people*, to two changes in their measures in ten generations.

*Read DOUZE NOTRE DIX FUTUR, by Jean Essig, 1955, Dunod, Paris, with a foreword by M. Albert Caquot, Membre de l'Institut.
&Duodecimal Bulletin, page 22, Aug 1958, "Redivivus Reckoning," by Charles Stuart Bagley. *H.C.C.*

MORE EVIDENCE OF BASE TWELVE ECONOMY

By Tom Linton

The increasing fraction of our membership active in computer work brings to mind the superiority of base twelve over ten in relation to the binary workings of the computer. The binary fractions, representing successive halving, are relatable to dozenal numeration, using much fewer digits than when relating to base ten, as follows:

Common Fraction	1/2	1/4	1/8	1/16	1/32	1/64
Binary Fraction	.1	.01	.001	.0001	.00001	.000001
Decimal Fraction	.5	.25	.125	.0625	.03125	.015625
Dozenal Fraction	*;6*	*;3*	*;16*	*;09*	*;046*	*;023*

As you see, we have as many digits in the decimal as in the binary to express a given binary fraction! This is in contrast to the integers required generally where the larger the number base, the fewer the digits to express a given number. For instance, $64_9 = 1000000_1$ (in this comparison I use the subscripts to indicate the number base minus one). When such binary numbers are multiplied, the disparity in what we might call digit efficiency is even greater, and is accumulative in a succession of multiplications:

$$1/64_9 = 1/54_{\varepsilon} = .015625_9 = ;023_{\varepsilon}$$

$$(1/64_9)^2 = .000\ 244\ 140\ 625_9$$

$$(1/54_{\varepsilon})^2 = ;000\ 509_{\varepsilon}$$

So we see greater efficiency in the computer, in higher speed and less required capacity, when processing from binary to base twelve, or back, over that possible in base ten.

-o-O-o-

In the above article Mr. Linton is not only demonstrating the great waste of digits required when we employ base ten instead of base twelve to process from, or to, the computer's binary base, but he also introduces us to his suggestion on how to indicate any number base from binary to twelve unmistakably, and avoiding the use of any number symbol which might mean two or more different quantities. Thus, base two is indicated by the subscript $100_{1}(+1)$, base ten by the subscript $100_{9}(+1)$, and the base of twelve by the subscript $100_{\varepsilon(+1)}$.

After we become accustomed to indicating the number base less one, it is quite natural to omit (as Mr. Linton has done) the (+1) following the subscript base number. --Editors.

CONVERSION OF DECIMAL CONSTANTS

By Stan Bumpus, 2215 College Avenue,
Mount Vernon, Illinois 62864.

Modern computers have made it possible for man to extend the decimal representation of mathematical constants by a factor of about six gross. Longer decimal representations provide a better basis to analyze the randomness of these fractions and also to show where these fractions nearly terminate. Because fractions are easier to analyze when they are expressed duodecimally, there is a similar need to extend the duodecimal representations of these constants. The subsequent computer conversions are the first step in this analysis.

Because the IBM 360 Model 75 computer that I used can not accurately store more than a ten digit number, the doubly-checked decimal fractions were stored in eight digit blocks in a linear array. To multiply the fraction or array by twelve, the computer was instructed to multiply the last block of eight digits by twelve, add a carry (initialized to zero for the last block), and store the result in a dummy variable. Because the computer absolutely truncates to integers in division, that is to say, 1199999999 divided by 100000000 gives simply 11, the computer was then instructed to divide the dummy variable 10^8 and store the quotient in the carry to be used for next block up in the array. Finally, to get the last block ready to be multiplied again by twelve, the product of the carry and 10^8 was subtracted from the dummy variable and the result stored back in the last block of the array. The computer then carried out the whole set of above operations successively up to the blocks of the array until the initial block was completed. The carry was then the first digit in the duodecimal representation.

Mechanically carrying out this procedure, one sees that it is essentially multiplying the array by twelve. After eight such "multiplications by twelve," the computer was told to ignore the last block of eight digits because these digits had lost all significance in computing the result and because this procedure saved computer time. The "multiplications" were repeated until all decimal significance was gone.

The conversions shown on the following five pages are Pi, Euler's Constant, Khintchine's Constant, e, and 1/e. The duodecimal representation of Khintchine's constant is as accurate, I found, as the decimal representation. All of the rest of the constants have been truncated to save space. Euler's constant was converted to *1X£4;0* duodecimal places; 1/e to *1448;0*, pi and e to *10,068;0*, and I have the natural log of two converted to *1£88;0* places.

I would like to thank Robert Childress, James Roman, James Dixon, Janette Birkner, and Bruce Moon for their help in assisting me with my program. A special thanks goes to Bruce Moon for testing the program and suggesting a programming technique which saved *50;0* e/g computer time.

[In the following five tables X equals ten and E represents el]

Computer Work by Bumpus

DUODECIMALLY PI EQUALS

3;184809	493E91	866457	3X6211	EE1515	51X057	29290X	7809X4	927421	40X60X	55256X	0661X0
3753X3	XX5480	564688	0181X3	683083	272EEE	X0X370	E12265	529X82	8903E4	E256E8	403759
X71626	E8X546	876218	49E849	X82256	16E442	796X31	737E22	9E2391	489853	943E87	637256
164472	36E027	X421XX	17X38E	52X18X	338E01	514X51	144X23	315X30	09X890	6E61E8	E48X62
253X88	X50X43	EX0944	572315	933664	476E3X	XEE775	839751	206835	26E75E	462060	EE03E4
325519	137727	29X214	755353	179384	8X0402	E999E5	058535	374465	X68806	716644	039539
X84319	351985	27E939	9E1129	90XEE0	383E10	764542	4577X5	1601E3	624X88	E7X676	X39929
12121X	213887	E92873	946X61	332242	217XX7	354115	357744	939112	602EX4	E88881	8X3269
222E52	848774	783999	4XE223	E65E87	626954	228226	69EX00	X58609	7842X5	175036	2073E5
X76836	3E21EE	1X97X4	X19444	774939	980492	2175X0	68X467	394619	90X206	5EE0X3	0EEXE7
024X58	5E1X84	428195	489784	X07X33	1X7E0X	157456	5E373E	05E03X	5X80X1	3XE878	577346
799855	58X537	3178X7	E28271	992X38	94X577	608508	3E9E23	8E2220	542462	888641	X2EXE8
E3083X	E49659	172X31	2E7851	865449	4X0686	62586X	181835	X64440	E2970X	122813	975898
815367	208905	801032	881449	223841	428763	329617	531239	E9X657	405584	014534	390E58
762560	6EE809	237959	44E437	57X431	E03955	628297	8X6X49	590553	490EX1	844947	175637
X90824	7E5012	772246	444138	0X852E	0847E5	813019	EE70X6	7663E4	265654	340698	844761
321933	44EX55	X2128X	038389	74606E	851E29	79321X	408067	225X5X	X4E346	4X1X17	473595
333909	XE9127	079655	E3164E	68E9E2	8X9E81	8X220X	025XE0	934203	995E7X	62X7XX	739355
340539	EX3182	905E19	390560	3X43E6	60E942	6X9229	469714	4X896X	5E2339	358EE2	E7294E
E89635	E071X6	351211	360E82	0E1882	XE8433	E54757	E87X37	3284E1	EX182X	103264	76E369
X4X636	5E58E8	018994	EE1525	567654	75X704	EE94E6	E2X394	58971X	8E9051	2786E5	029404
818644	323552	916170	E3XEE7	363496	427E08	8E6872	5X6857	004061	794928	9077E2	78069X
09E559	324E8X	66828E	40549E	029606	5E2300	330592	569X7E	76E92E	X12935	85E6X9	E60456
7X0901	362856	373E4E	568979	46256E	4172E1	E50474	351364	749X33	996X81	EX8847	347X84

ACCURATE TO 1728.0 DUODECIMAL PLACES

Computer Work by Bumpus

Duodecimally Euler's Constant Equals

```
0;6E1518 8X6760 E381E7 543345 20434X 22560X 590X6X 56X386 76055E 475E81 E68078 743649
  72X465 351992 5697X2 X843E5 0EE071 557641 368417 196E7X 4672E2 52E4EX 865571 X14731
  4741E4 678809 8X5308 82X53E 255234 E46262 919378 892E99 512811 019623 77E83E 594433
  32XE5E 588008 059350 X45X15 1E1154 90E27E 149XEE 618111 843E04 7991E2 34194E 143X35
  77E066 708303 841992 985535 E70435 2X7577 988823 1E374X X59957 6E2X43 EE41E9 495079
  7X3080 559X76 998923 6E2352 316450 86X967 E2X89E 897721 60X83E 99550E 952316 1X1960

  6775X8 7X27E2 505199 088022 E92140 X7193E 12360X E16X64 5X83X3 15X468 72612X 552037
  111938 19625X 902EEX X5X24E 027975 E4X972 7523X7 84XX8E XE7741 379319 483545 971X71
  X13295 38EE4E 63E939 08789E 674293 614E17 864133 729554 X5XX09 919677 8789XE 2465X5
  835463 E489E9 4X0XX4 136357 3X5447 28190E 0892E6 8810X5 6655E9 49174E 58E21E 40291E
  E63977 532414 484E3X 626092 632445 8X0709 48X2EX 41EX09 60X268 E38905 6371X7 X76441
  36547X E3X1XE 227030 XX6740 X0186E 9656E8 208X15 383X84 3X7421 6E6XE6 036147 43918X

  944763 628844 3E8248 304485 051E35 2159X2 623360 84XXE6 52E008 X82468 177331 772XE7
  30E32X 06E030 8XX55E 7EE25X 1160X4 EX3808 5E348X 183709 4X5398 0317X0 879042 XXX279
  2X7133 570166 EE9E37 300605 24E91E 9E7822 3X24X3 X87299 113412 462X22 958988 83340E
  4388E7 511788 1EE051 48X075 5486X4 55X6X2 16XXE8 X81733 644502 EE8E51 491E32 057270
  7E2E3E 785350 494549 071726 296X3E 04X548 7E618E 217076 E09370 969188 066160 789X1X
  E01110 XX1963 E48256 04508X 072208 2EX04E X02066 693326 040X33 878XE3 8104XX 4748X2

  848211 467289 844989 388142 513X19 1E3783 06E713 3X0965 3839E1 4524E5 6622X5 332596
  47EXX5 1799X1 686X9E 39X155 98X940 X0608E 988774 7274XX 9412E6 901767 357618 576176
  77X298 06X871 6X3X04 1419E7 2869X1 2346X2 61X13E 305890 768E61 E62181 133401 862280
  796X9X 5477E8 871X67 X41E0E 4X6X63 6410E4 X9X6XX 182995 632617 38XX49 987XE1 326402
  616622 015132 264X5E 538X87 751863 728X8X 879522 932X33 E4E614 2XX694 075315 711E69
  3647X4 39403E 679390 05236E X1582E X5X28X XX8257 E6E753 86731X 585072 06EX86 8913X8

  646354 3172E4 6635E5 X2E334 X80208 489917 07X17E 96659E EE0710 653983 X2926X 8EE867
  2X3805 94E893 X21161 E5435X 629110 216469 803825 95674X 52012X 836085 3564XE 839618
  512EE7 82537X 08299X 1X5327 475922 982888 831401 496261 E837E0 202868 04571X 583X02
```


Computer Work by Bumpus

Euler's Constant Concluded.

```
  862285 1021E7 4648X3 5E3585 X3E609 9E2E85 E2E818 959327 257425 946562 2706E4 1E7502
  35E5X5 9X9336 731E85 E68568 493643 229352 37040E 469189 628023 X70154 0X0E37 519264
  E05587 3XE050 598845 380X48 65X721 487771 623707 692X62 817329 X8E390 7627E5 72X770

  231248 4488X5 XE5009 3963E4 222349 169340 0521X9 490365 00X880 06653E 8E0X9X 214880
  480609 3X0X21 X2E21X 7X9679 381650 5291X5 96E47E 078898 672325 16E885 24X295 664E81
  732492 809EX9 08376E 2721E9 64E6EX X28XX9 E33547 E3E720 119678 5X4321 658100 0812E8
  555122 9E0660 EX87X1 26E8EE 9091X1 19E754 189412 579963 01EE19 63527X 8244X2 33E128
  0XEX91 134587 390676 533658 E8X67E 826E66 6736EE 525EE9 594EX6 E99748 X010E3 973116
  23E2X0 991E72 37021E 5E80E9 186552 E43474 5X3761 090931 5476EE 4XE757 639E32 X3184E

  514116 177X05 8X7782 4E3X82 4645E3 421395 7XX82X 2E0211 84X948 7E6424 71X174 760852
  490X94 774646 8553X7 41448X 147412 9051E8 893X1X 00926X X5040E 828893 9XE760 027460
  531538 X61919 X78X29 949957 8X88XX 775852 97X154 448489 X3E209 19718X 429163 812534
  5608X5 7X5230 499853 087266 954E36 2621X5 087871 21E05E 1X2100 X264E5 5078XX 09X464
  67EE73 41EXX0 XEX346 X46462 6E0210 974X66 84862E 781XXE X60192 621534 4X51E8 677748
  346E03 512712 41E652 500987 000754 016410 1535E0 7X2EX9 703790 19X400 998717 E957XX

  4283X5 186X40 001390 1E1494 817741 72258X 9E350E 4373E7 364134 670543 60859X 897582
  13X134 X9E830 241118 3967X1 89X2XX 8X3253 155366 1E8016 26846X E300E3 E02284 6091X9
  942827 361320 0X1462 88E3E3 194E4E 8E47EX 11096E 047XX2 3045E9 X9EE26 7530X3 452136
  3635E9 233E28 7X8057 076845 125XX7 5416E5 046423 525049 058969 5512XX 13E4
```

ACCURATE TO 3304.0 DUODECIMAL PLACES

Khintchine's Constant

DUODECIMALLY KHINTCHINE'S CONSTANT EQUALS

```
2;828564 85E404 426608 2X7290 9684EE 96341X E36275 953X1X 0EE299 811X86 2X2X04 X06066
  E33073 E8779E 780349 9E6EE4 174654 X59244 855281 642EX3 247107 078572 062222 418952
```

ACCURATE TO 144.0 DUODECIMAL PLACES

Computer Work by Bumpus

Duodecimally e Equals

```
2;875236 069821 9EX719 71009E 388XX8 766760 256427 2786EE 923E31 032566 054257 348716
  X1759X 1X9985 631X46 051X68 72106X 888554 3X2410 165435 646X75 971X05 0X9310 654772
  407374 X55X36 0623EE E44557 5X4975 555EX6 E9513E 498521 X2873X 48909X 1X4E13 95670X
  9572E9 X86567 59XE64 EX6214 171794 330X86 736905 6EF060 657X19 436000 X5783E 4289E3
  775355 730065 252X2E EX706E 919270 91290X F430E7 008606 10X278 428950 6323X6 10E756
  863537 316850 721EE4 31EE37 7EE041 068217 686765 0X005X 1640X6 8F7765 34X27E 1XE317

  76XX17 469X86 E81268 0X8140 664788 019115 2696XE 2748X9 2648E2 1E0E33 25E0E2 564190
  222366 E96840 98X789 5429X8 X90E0E 3XX944 069E65 3E54X7 2EE090 786655 967146 698336
  X76691 005E09 547806 0EE539 X350E3 4878E0 537748 204X76 8E78EE 4XE47E X3X2EE 284232
  858645 8233X2 85X21X 635X17 130010 60X2X9 771098 630476 03E397 918291 256088 911X18
  594119 E21E72 82X342 57EX1E 415106 54XE59 38598E 2X694X 01X540 4885E7 954830 140078
  279161 735733 X15837 04E09X 704325 E56224 25EE49 9707X1 0321XE 09458E X43699 1E0719

  054337 X88916 3961X5 E73963 584842 9E650E E8009E 252302 051513 772020 X70741 33960X
  X96039 816827 X9E470 417X87 85E536 854364 721594 4E7X4E 841598 365867 310261 4X53E7
  E92158 E88669 92200E 358308 386753 561415 28E345 153X42 738846 E27946 946X46 414051
  038581 33221X 273055 5X452E 75X967 7869E1 214X26 405487 853E32 361E64 05243E 61E655
  581E8X 6036E9 577426 X8938X 97X9XE E785EX 633056 439089 4F4423 X06828 553469 834050
  939209 37X877 76X403 644077 869049 193280 3359EX 592490 8X26E7 5X0933 EX4292 153EX2

  8468E9 461891 711249 540149 657X73 26E42E 725001 X140X1 E2F953 4X590F 262265 283757
  XX6179 325491 2822X7 981121 952952 8628E9 557E6X 686176 2897E2 172EEE 471024 9910X6
  751277 147137 315211 71X991 276129 67X303 33X732 341787 X23263 4421EE X76395 4X8697
  5X3640 F3730E 101547 69X894 543E86 521845 5X3580 85X044 272751 203310 6X141X 1X8X92
  9E3822 80201E 929991 6X9453 769545 070378 8X2167 255X72 X64770 313423 577948 329879
  984570 9E97X8 X416X5 610E59 E74XXX E9373X X75331 9XX618 F16921 356400 854E19 286181
```

ACCURATE TO 1728.0 DUODECIMAL PLACES

Computer Work by Bumpus

Duodecimally 1/e Equals

```
0;44E842 160564 X444E6 X08E59 279584 0071E4 XX1563 4037X0 473169 092423 590857 506962
  EX0XE0 261106 0E7041 965513 602132 633736 543524 27X295 X29X71 6E7677 25X1X9 E36284
  55514E X09203 19476E 698X70 3E3395 134352 19X8E7 116X21 575E79 3566E9 11X34E 2EX473
  97760E 5X8442 X34062 940465 75X774 E43X97 303266 4E1041 5E52E9 E68E53 620819 74787X
  E24162 540E12 00X92E 4X1985 775294 0X2324 0X2754 272989 255475 846E60 1E6419 E40377
  1X5641 4E1404 482082 79137E 1X181E 10X175 696E0X 9E2673 X36X22 096255 240581 844368

  E57250 2XX1X4 387680 414001 6X9127 E4519X 219X24 6E5977 2222X7 549398 E96757 X66368
  X7222E 755866 811249 745331 821X29 03X6E4 3X0490 351458 842959 44833X 268294 E2X1EE
  X10172 6253X1 8071E9 818703 0585E3 168X97 700797 E85029 485753 393328 320911 X48X17
  2895EE 878331 114525 145113 216061 X74E69 640E30 94E9E4 814XX9 69013X 33863X 983086
  624153 1C7324 74EE87 11606E X062XE 135315 837728 7426EE 150097 193669 883059 9317E1
  92331X 909930 568818 277371 E52936 399336 EX80XE 090093 5EE282 5E4379 2414X8 95926E

  359676 993306 3X7523 249959 E01232 E261X3 920668 605547 X45X50 179197 7X7X6E 553119
  84X832 EX0X66 X87589 EX5545 586E5E 71XXX8 796E58 4E6551 995235 2903X6 X52X57 X43349
  X05939 789X75 6X467X 8X9X04 E0X184 297868 078172 7E0019 0X090X 0X8342 67X389 215758
  9390XX 509465 X152E3 712287 X23E62 52X586 860X72 604450 916510 912E30 791607 646869
  69E211 XE2949 754900 E8X73X 57430X 896976 5E0469 382E96 7416X7 27E851 274766 707X39
  761X79 863050 6E9217 E39538 90E840 E32259 42924E 655967 EX1X89 52108X 777X66 93X796

  685866 586559 829290 309X3E 1590X4 X31675 7501EE 450296 XEX940 5X9428 3E4703 6282XX
  9XX089 402342 581450 856311 6X6279 4E391X 00104X 4395E7 552329 2X1021 189583 977360
  X3871X E859X2 963EEX 868257 180075 E98417 122995 8449X1 679630 979129 828890 X4X752
  814027 758759 413415 172957 394498 5X4158 872182 E35905 33EE19 3797EE 735928 330068
  376910 E89XX3 2373EE X62600 8X7X09 1649E2 X75025 09324E 4E3049 5X3102 8X5X06 556146
  X69563 XX8398 57E048 134175 X5E159 343408 47354X 817964 44XE43 39X335 86XX15 2X1989
```

ACCURATE TO 1728.0 DUODECIMAL PLACES

NUMBER-BASE ODDMENTS

By Shaun Ferguson

I

"A number, of three digits, in base seven, when expressed in base nine has its digits reversed; find the number."

I found this question in an old algebra book, circa 1910, and solved it (the number is 503 base seven). While thinking out the solution, I wondered if there were any other numbers in any other bases which could be treated in the same way. I present the following results for your interest.

Number	Base		Number	Base
201	Five	is equal to	102	Seven
503	Seven	"	305	Nine
302	Nine	"	203	Eleven
705	Eleven	"	507	Thirteen.

Might one find a pattern? For example, in base five, if 201 obeys the rule, then so should 402.

Whence, we find:

Number	Base		Number	Base
402	Five	is equal to	204	Seven
604	Nine	"	406	Eleven

II

"A number consisting of three digits is doubled by reversing the digits. Show that the same will hold for the number formed by the first and last digits."

I knew one solution (in base eight) to be 2 x 25 = 52. For three digits the solution, in base eight, is 2 x 275 = 572.

And upon investigating other bases, I found, for example:

In base five, 2 x 143 = 341.

In base eleven, 2 x 3X7 = 7X3.

-o-0-o-

(Mr. Ferguson will have more on this subject in the next issue of the Bulletin. His delight is to be with figures and to work among digits in any base, purely in exploratory manner and without limiting himself to base twelve or base ten alone. He is an asset to any duodecimal society. His address is 58, Scotby Village, Nr. Carlisle, Cumberland, England. He will be pleased to hear from you, if you are unable to reach same results as he, or when you find other examples. ---Editors.)

MORE ON DOUZE NOTRE DIX FUTUR

On page 106 of DOUZE NOTRE DIX FUTUR*, loosely translated, we find the following:

"Now note that one sees at once, in dozenal numeration, how all these new units of length are derived one from another, by an extremely simple formula:

$$\frac{\text{A Great Circle of Earth}}{\mathit{10\ 000;0}} \quad \text{equals}$$

One kilomètre duodécimal	(*1;0* kmd) =
One dozen hectomètres duodécimaux	(*10;0* hmd) =
One gross duodécamètres "	(*100;0* damd) =
One dozen gross mètres "	(*1 000;0* md) =
One dozen great gross duodécimètres	(*10 000;0* dmd)..."

Since there could be as many different Great Circle lengths as there are physicists or authors (each great circle divided by *100 000 000;0* will equal *75 000;0* krypton 86 wave lengths, MORE OR LESS), it is suggested that one duodécimètre duodécimal be legally defined by the world's governments as the exact equal of *75 000;0* krypton 86 light waves to achieve familiarity.

This dimension could be determined by the National Bureau of Standards of the United States of America, or, for that matter, anywhere in the heavens or on earth, now or in any regroup of years hereafter, in the most exact manner and in whole numbers of krypton 86 light waves (now used to define the present international meter as equal to 1,650,763.73).

Moreover, a stick in that dimensional unit (or one dozen such units) might be held in the palm of one's hand (or seen at your arm's length) in a most practical manner.

Iowa farmers, in moving from decimal to dozenal dimensions at some future date, might be surprised to learn that one hundred hectares duodécimaux and one congressional section (one square statute mile) of farm lands are sisters under the skin.

H.C.C.

*Published by JEAN ESSIG, 1955, Dunod, Paris.

EXHIBIT OF A COMPUTER PROGRAM

By Robert R. McPherson.

Here is shown on a printer page an 80-80 listing of (1) onedofive data input data cards, and (2) onedofive data output data cards. These two items constitute an exhibit of a computer program which accepts data in decimal format and punches an output card containing a statement of equivalent decimal and duodecimal numerals. In item (2), ten or dek is shown by the symbol X and el or eleven by the symbol E. Also note that base twelve numbers are preceded by an asterisk in lieu of a duodecimal point. The decimal numbers, excepting the tens, hundreds, thousands, etc., have been chosen at random, and include whole numbers, decimal fractions, and mixed decimal numbers.

Item (1)

```
123.45698
  4758.456
9876543.12
.00001234
.00000045
.00000008
11111111.
10.
100.
1000.
10000.
100000.
1000000.
10000000.
```

Item (2)

```
        123.45698000 =  *X3.5597E2
       4758.45600000 =  *2906.557E
    9876543.10000000 =  *3383713.1
           .00001234 =  *0.000030X
           .00000045 =  *0.0000014
           .00000008 =  *0.0000002
   11111111.00000000 =  *387X05E.0
         10.00000000 =  *X.0
        100.00000000 =  *84.0
       1000.00000000 =  *6E4.0
      10000.00000000 =  *5954.0
     100000.00000000 =  *49X54.0
    1000000.00000000 =  *402854.0
   10000000.00000000 =  *3423054.0
```

-o-0-o-

Senior Member Robert McPherson operates the McPherson Laboratories for employment of Hindu-Arabic-American Base-Twelve Numerals, at 1030 N. E. 3rd Street, Gainesville, Florida 32601. Space permits the showing of only two of his exhibits. --Eds.

EXCERPTS FROM LETTERS AND COMMENTS of our correspondents.

BRIAN R. BISHOP, 155, Leighton Avenue, Leigh-on-Sea, Essex, England, writes: I was absolutely delighted to receive the [September] Bulletin. ...I feel honoured by the flattering references to me on page six [but] I did not resign "for reasons of health." I found myself quite unable to give proper attention to my duties, and I saw no point in half-doing a job. At that time I found I had a lot of work at home, and, being recently married, with our first home to set up, I thought my prime duty lay there. My job was also changing and becoming more exacting and, being my bread and butter, could not be allowed to suffer. Please let those concerned know, lest the incorrect documentation be accepted.

Equally concerned, Mr. Bishop further writes: I read with interest your page *15*. You may be interested to know that I most certainly dislike "the printers' lower case letter l to represent one" [and] I am not alone in this. This is mainly a typewriting convention to save a character. Although characters on my machine were at a premium because I had to cater on it for dozenal ten and eleven, and all the accents in French and Spanish, nonetheless I insisted on having a *1*.

S. FERGUSON, 58 Scotby Village, Scotby, Nr. Carlisle, Cumberland, England, writes: In your Dozenal Essays you have used ! for the base sixteen point. I think this is a mistake. The use of the exclamation mark for the factorial function makes this confusing. Factorial 6 = 6! and I read 10! to mean factorial 10, whatever base the 10 is.

STAN BUMPUS, 534 Townsend ISR, Urbana, Illinois 61801, writes: I strongly disagree on the use of (!) for the hexadecimal identification point. This symbol is now almost exclusively used to mean a factorial. Thus, 3! = 3 · 2 · 1. I think (!) as the h.i.p. would cause too much confusion.

[Editors' note: There was a time when |6 was the factorial sign to indicate the continued product up to 6. The use of ! by so many younger mathematicians simply indicates that the science of mathematics is not standing still.]

THOMAS H. GOODMAN, 3218 Shelburne Road, Baltimore, Maryland 212 08, writes: The article [see September 1968 Bulletin] on hexadecimal system particularly interested me because I just finished IBM orientation to the relatively new #360 computer, ALC language, which heavily used hex. ..the more these binary-type computers take over, the less obvious will be the lacks of the ten-system of counting. The big use I see for the duodecimal system is in the elementary mathematical curriculum---it will enhance the idea of number systems and give better conceptions of quantities.

F. EMERSON ANDREWS, 34 Oak Street, Tenafly, New Jersey 07670, writes: Many thanks for Dozenal Essays of 1968, and all your efforts to do that most needful---get our publications going again. One suggestion, in the light of historic accuracy. The use of E for duodecimal value eleven derives from my original Atlantic Monthly article, October 1934, where I said (pp 462-3) "First, we must invent the two additional symbols which the Hindus and Arabs forgot. For 11, let us use E, and call it elf." I had the printer use an italic E, and was already distinguishing duodecimal values by italicizing. On the typewriter we used simply capital E. It was never a reversed 3. As a capital E it stood for Eleven, or the German Elf. See also the first (1935) edition of NEW NUMBERS, Appendix B, Notes on Nomenclature. "The E, which is not an epsilon, but a 'fancy' form of the italic E known to printers as 'swash E', seemed to avoid confusion with the regular letter E and at the same time be easily identified as a symbol for the old 11. For names, for the same reasons of simplicity, I adopted "dec"---a one syllable word coined from Latin decem--and 'elf'--German for eleven."

Equally interesting, Mr. Andrews further writes: When the Society adopted its official seal (I cannot be sure of precise date, but before 1945) that eminent typographer Dwiggins designed the "flat" E which has been put on some of our typewriters and generally used where possible. See also the William S. Crosby letter on this subject in the 1945 Duodecimal Bulletin, Vol. I, No. 2, page 9.

RALPH H. BEARD, 20 Carlton Place, Staten Island, New York 10304 writes: I wish I had the courage to explore the 24- and the pentadozenal 60-base. Harry Robert said that he had done a lot of work in it. We really should have some one become fully familiar with its advantages and faults. I have enough on my plate. I may have to learn the Boolean Algebra of the Ternary Base.

And the farseeing Mr. Beard further writes: I am fully convinced that the ultimate computer base will be the 12-base, and that the computer language will be Esperanto.

GEORGE S. TERRY, Box 101, Sonoita, Arizona 85637 writes: Also a tribute to Ralph Beard for his discovery in 1963 that $10^{1228} + 1$ has the factor *6Ɛ1Ɛ15* (Duo figures). Since the power has *1229* figures in Duo and 12^{2049} has about three thousand or more it was no small discovery.

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If we did not have these pages, this periodical would lose much of its usefulness. Speak your piece. Without your cooperation serious faults might creep into any man-created article and any machine-created work as well. Of course we look for constructive comments particularly and bits of interesting and informing dozenal culture, as well as individual preferences. We reserve our right to cut without damage to an idea. --The Editors.

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The March 1969 bulletin is being put together. It will contain many scintillating subjects of interest to all dozeners. And begin to plan now to attend, in the Los Angeles area in the Spring of 1969, the annual membership meeting.

CHROMATIC MUSICAL SCALES (Cont'd from page threedo)

These curves reveal clearly that the average listener's ear is most sensitive to the shifts, expressed as fractions of the frequencies, of those above about 1000 cycles per second. However, the ability of the ear to detect an off-pitch note in the lower register of the piano is not necessarily as poor as the data might suggest, since the low piano tones are rich in harmonics.

Although the frequency response of man's ear stretches from about 20 to about 15000 cycles per second, a musical sound (or tone) ranges only from about 60 to about 4000 cycles per second. Below and above these frequencies a sound (or tone) loses its musical character. Within this musical range man can recognize more or less accurately the frequency region of a given sound without any pre-exposure to another one ("regional hearing"). Similarly, he can sense the distance (interval) between two consecutive sounds ("relative hearing").

Such a recognition, within a modest degree of accuracy, is universally necessary for understanding human speech, and man quite generally possesses this ability. However, for musical purposes, especially for musical performance, a greater than usual accuracy is required, and many persons need additional hearing training. While practicing this hearing, some people can memorize the pitch of one particular tone (or even more than one); and then with the aid of this reference they can identify quite accurately the pitch of any given sound by judging the interval between the reference pitch and the given one ("relative hearing with reference pitch").

Another rare form of musical hearing is the "absolute pitch" (or "perfect pitch"), where the listener can recognize immediately and very precisely the pitch of a given tone. Certainly, these last two forms of musical hearing are very helpful for musical performance as well as for musical analysis. However, they have no obvious bearing on musical appreciation, since this is a "conditioned response." (Ref.: G. Révész, "Einführung in die Musikpsychologie," Basel, 1946).

When a vibrating body produces a sound, especially a musical instrument, this sound is usually not a tone of a single frequency but is a complex sound consisting of several different frequencies, which are called "partial frequencies." Depending upon the distribution of the partial frequencies (when the partial frequencies are whole-number multiples of a fundamental frequency and thus follow the simple harmonic series, they can

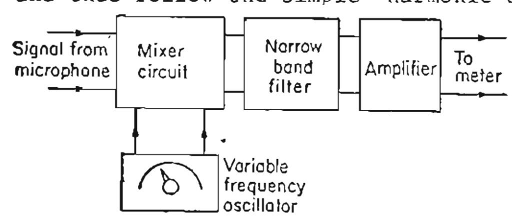


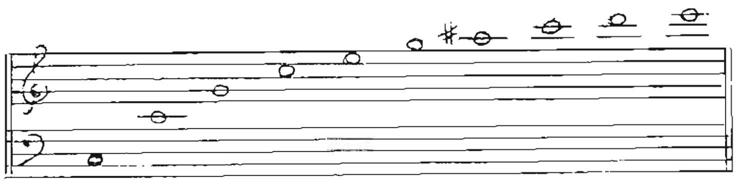
Fig. 9

Functional diagram of the heterodyne type of complex wave analyzer.

be called "overtones" or "harmonics") and their relative intensities, the sound of a particular body or musical instrument assumes its characteristic quality, its timbre.

With the aid of a frequency analyzer such as the heterodyne type analyzer shown schematically by Fig. 9 (or similar instruments such as an arrangement of tuned mechanical reeds, etc.) or with the Fourier's series applied to the oscillographic record of a wave shape, a complex sound can be broken down into its partial frequencies.

Figure 10



a)	c	c^1	g^1	c^2	e^2	g^2	a^2	c^3	d^3	e^3
b)	fundamental	1st. overtone	2nd. overtone	3rd. overtone	4th. overtone	5th. overtone	6th. overtone	7th. overtone	8th. overtone	9th. overtone
c)	1	4	2	8	3	7	5	9	6	10
d)	1 :	2 :	3 :	4 :	5 :	6 :	7 :	8 :	9 :	10
e)	octave or 8th.	quint or 5th.	quart or 4th.	large or major 3rd.	small or minor 3rd.	augmented 2nd.	diminished 3rd.	large or major 2nd.	large or major 2nd.	

While listening to a complex sound man may hear, in addition to the existing partial frequencies, some frequencies which are not present in the vibrating air. They could have been created within his ears due to some distortion or due to mixing of existing partial frequencies, called "combination frequencies" (which are the sum and difference frequencies).

The easiness with which different partial frequencies can be recognized, out of a complex sound, varies widely. For the harmonic overtones of the c-tone, Figure 10 indicates, in row c, the approximate order by which they are recognizable. Row a expresses the usually given notes of the fundamental and its harmonics as conventional letters; row b gives their names; row d lists their theoretical, exact frequency ratios; and row e names the successive intervals. (Subharmonics can usually not be heard and are therefore omitted.) (Ref.: G. Révész, "Einführung in die Musikpsychologie," Basel, 1946).

As already mentioned, the tones of most musical instruments are rich in partial frequencies, especially in harmonics, some of which may be even more prominent than the fundamental. Now, if the correct harmonics are presented, man will hear the pitch associated with the fundamental frequency, even though the fundamental may be missing. This phenomenon is probably a result of the difference in frequency between the various terms in a harmonic series which, of course, is identical with the fundamental of that series.

When two sounds of nearly the same frequency and the same intensity occur simultaneously, they alternately reinforce and cancel each other, as shown in row b of Figure 11. The sensation is of a tone associated with the slightly higher frequency and with periodic variations of its intensity known as beats.

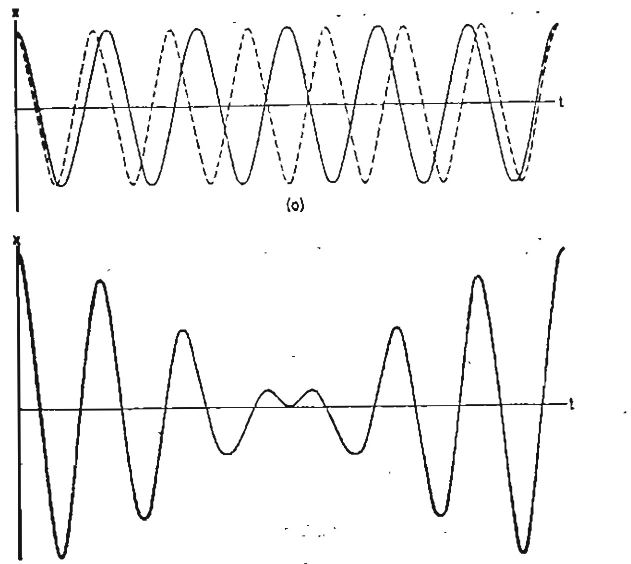
If two simultaneously occurring sounds are fairly strong, and if their frequencies differ by 50 or more cycles per second, other sounds may be heard besides the primary ones. These are the combination sounds, or "combination frequencies," which were heretofore mentioned in describing "complex sounds." They are either the sum or the difference of frequencies between the one primary frequency or its harmonics, and the other primary frequency, or its harmonics. This relation can be expressed by the following two formulas, where "f_1" and "f_2" are the two primary frequencies, "a" and "b" are whole numbered factors, and "f_{cs}" and "f_{cd}" are the combination frequencies:

$$f_{cs} = (a)f_1 + (b)f_2$$

$$f_{cd} = (a)f_1 - (b)f_2$$

Combination frequencies between the primary frequencies (where the factors "a" and "b" equal one) are of the first order, and they are usually the strongest and the most easily noticed ones. They are best heard when they are well below or

Figure 11

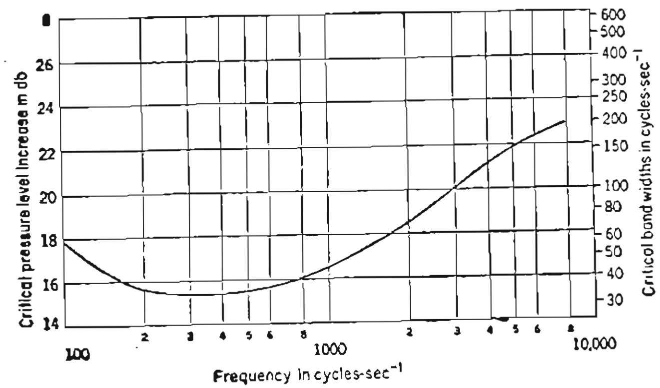


Beats.

well above the primary frequencies; otherwise, the masking effect of the primary frequencies makes it difficult to recognize these combination frequencies. For example, frequencies of 2000 and of 2500 cycles per second will give a frequency difference of 500 cycles per second, which can be readily recognized. However, frequencies of 1000 and of 2500 cycles per second will give a difference frequency of 1500 cycles per second, which can be noticed only by a careful observer.

The just mentioned phenomenon of masking a useful sound by another sound is a familiar one. Studies of this phenomenon with simple tones show that the masking effect is greatest for tones whose frequencies are close to the disturbing tone; it falls off rapidly for more distant frequencies. Thus a loud tone of 1000 cycles per second will have a profound effect upon a tone of 900 cycles per second. A very similar effect results from background noise on a useful simple tone. However, although noise usually consists of a heterogeneous mixture of many frequencies, only a certain band width of frequencies adjacent to the desired simple tone frequency causes masking, and this band width varies with the frequency of the desired tone as shown by Figure 12.

Figure 12



Masking effects. (*After* French and Steinburg)

[FRENCH, N. R., and STEINBERG, J. C., *Jour. Acous. Soc. Amer.*, **19**, 90–119 (1947)]

Furthermore, in order to be audible, the intensity of the desired simple tone has to be higher by a certain amount than the effective pressure of the interfering noise band (spectrum level). Since this amount is related to the band width of the interfering background noise, that critical pressure level increase has also been indicated on Figure 12.

Figure 13

Degree of Consonance

frequency ratio	conventional term of tone combination
2 : 1	octave or 8th. or "homosonance"
3 : 2	quint or 5th.
4 : 3	quart or 4th.
5 : 4	large or major 3rd.
6 : 5	small or minor 3rd.
5 : 3	large or major 6th.
8 : 5	small or minor 6th.
9 : 5	small or minor 7th.
15 : 8	large or major 7th.

As a last item of Part One, we should mention the pleasantness of tone combinations, their consonance or harmony. Both musicians and physicists have long discussed what constitutes a pleasant combination of frequencies (called consonance) and what is an unpleasant combination (called dissonance). However since there is a strong individual subjective element that often determines the final impression, and since this subjective element is influenced by the changing musical fashions, it is questionable if this problem will ever be solved in absolute terms.

Nevertheless, Figure 13 lists the major combinations of two simultaneous musical sounds expressed in conventional terms (although these terms may be misleading), it arranges them according to their degree of consonance, and it gives their frequency ratios as related to the lowest tone (called tonic). (Ref.: G. Révész, "Einführung in die Musikpsychologie," Basel, 1946).

Of course, "hearing and sound" (the topic of Part One of this discourse) could include many more items than mentioned thus far, like the transient features of a sound (its change in loudness or timbre) or the different theories of hearing, etc., but that would go beyond the scope of this writing.

(In the next issue will appear Part Two, Musical Scales, by Mr. Kothe, based upon this introduction. Lack of space makes this interlude necessary. Ed.)

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COUNTING AND TALLYING (Continued from page 32)

of the fingers (not thumb) with the hand palm up. This method is used today in the bazaars of the Middle East where theoretical knowledge of arithmetic may be limited today but where hard practical experience had led to the discovery of the practical usefulness of dozens.

The lesson these examples can teach us is not to feel bound to use our improving knowledge of the theory of arithmetic to bolster up the use of a structure for numbers based in the most primitive way on any particular anatomical feature. We should, on the contrary, *examine the many practical uses of number in daily life, and then aim to learn the number skills best suited to making those practical tasks simpler or more efficient.* We may find as a result that many of the assumptions upon which current teaching of arithmetic is based are indeed themselves questionable and subject to improvement.

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Look at the Bankers in line ahead
of us--the best seats will be sold out!

⑆0713⑈0025⑆ 080⑈812⑈1⑈31

DEDICATED TO AN UNKNOWN ENGLISH MATHEMATICIAN

Let it be clearly understood, once and for all time, that inches and miles will never disappear from the face of this earth so long as it remains in orbit! Why? Because an Englishman (I do not, sad to admit, know his name), skilled in both ten-base and twelve-base arithmetic, counselled an English king and parliament to abandon five thousand feet (1000 Roman paces), once the length of the English land mile, and substitute a dimension of 5,280 feet in their place. (Add 56 paces!)

May we divert to a nostalgic period? From the law school, in a time of national emotion, we enlisted in the Aviation Section of the U. S. Army Signal Corps at Washington Barracks, on the Potomac river, and were shipped to Kelly Field, Texas. It was, of course, necessary (to avoid the appearance of a mob scene) to place us in small groups and to teach us how to march.

The drill sergeant of our outfit was a man named Riley, who had theretofore served in the British Army in the Boer War. In elementary training he told us that a soldier, at the command "March!" steps off with his left foot, lifting, moving and placing his left heel exactly thirty inches in front of his right heel; then smartly lifting, moving, and placing his right heel exactly thirty inches in front of his left heel; and so repeating until he receives another order and command of execution.

Here we had a former British Private teaching American troops a skill employed by Roman Legions more than two milleniums earlier. Two steps equal one pace, or sixty inches, and one mille (Roman mile) was the equal of 1000 *paces*! Refused flight training, we transferred to light field artillery COTS and on French 75s learned about plateau, drum, and 1000 *yards*.

One-tenth of 5280 equals 528 feet, or the length of one edon in the metronic system of measurements. And one edon, multiplied by twelve has been said to be the equal of one aero mile or navinaut, or nante, or kilomètre duodécimal. Any one of the last named units of distance, all substantially the same in dimension, when multiplied by the fourth power of twelve may be said to equal a Great Circle of the Earth. Nevertheless---

The human eye unaided is unable to distinguish between the length of one metron and the dimension of three and two-thirds inches. Twelve metrons, the equal of one dometron, are so precisely the equal of forty-four inches they are interchangeable by cabinetmakers. One metron is defined as the equal in length of seven dozen and five great gross krypton 86 atomic light waves (*75 000;0*), and in this manner may be compared with the international meter which equals 1,650,763.73 of the same.

The inch and mile, and decimal or dozenal fractions and multiples of either, may continue to be used with no confusion by those who prefer them for the remainder of their lifetime, whether a duodécimal metric system, or the dozenal metronic system of weights and measures, time, angle, and navigation be employed; or our present decimal system of measures continues. Flowers, and a solemn salute, to the ashes of The Unknown English Mathematician! H.C.C.