COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	Χ	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5;9'*
31	*694*	Three ft. two in.	*3;2'*
96	*3Ɛ2*	Two ft. eight in.	*2;8'*
19Ɛ	*1000*	Eleven ft. seven in.	*Ɛ;7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
12 ) 30  + 5
12 )  2  + 6
      0  + 2     Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *Χ*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *Χ*.

Numerical Progression

1	One		
10	Do	*;1*	Edo
100	Gro	*;01*	Egro
1,000	Mo	*;001*	Emo
10,000	Do-mo	*;000,1*	Edo-mo
100,000	Gro-mo	*;000,01*	Egro-mo
1,000,000	Bi-mo	*;000,001*	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	Χ	Ɛ
2	4	6	8	Χ	10	12	14	16	18	1Χ
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	Χ	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5Χ	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
Χ	18	26	34	42	50	5Χ	68	76	84	92
Ɛ	1Χ	29	38	47	56	65	74	83	92	Χ1

The Duodecimal Bulletin

Whole Number *33*

Volume *19*; No. *1*
June, 1966 *(117Χ)*

THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of BASE TWELVE in numeration, mathematics, weights and measures, and other branches of pure and applied science.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island, New York 10304. Kingsland Camp, Chairman of the Board of Directors; Charles S. Bagley, President; Tom B. Linton, Executive Secretary; Ralph H. Beard, Editor. Separate subscriptions $1.00 a copy.

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SPECIAL COMMENT. The contents of this entire issue are the results of the co-operation with the Society by Mr. Bruce Moon (who is now one of our members). He is a Senior Lecturer in charge of the Mobil Computer Laboratory at the University of Canturbury, Christchurch 1, New Zealand. Dr. Mears is a medical practitioner in the same city, and a graduate of Edinburgh, Scotland, and we are glad to gather from his listing as co-author that Mr. Moon wasn't alone on his outside civic work—that he has attracted some local help and support in leading or helping others to help themselves to find the way to easier ordinary arithmetic along the lines particularly first pointed to by people like A. C. Aitken for that part of the world. Thanks also for air-mail service which made such co-operation possible within reasonable time for nearly half-way around it.

(Oni korespondas Esperante)

Decimal Currency and INFLATION

THE introduction of dollars and cents in Australia last month went off more smoothly than most people might have anticipated. What started out on February 14 as a massive, many-faceted operation to reorganise the nation's financial system became, overnight, an exercise in community togetherness. Not since Federation had a single issue affected so many people.

The first official pronouncements proclaimed the changeover an unqualified success. The Prime Minister (Mr Holt) told his fellow Australians that they deserved "a pat on the back" for the essentially good-humoured way they accepted the change. Of course, there were exceptions.

In Melbourne, tram passengers refused to accept pennies and became irate with conductors when cents ran out. In Sydney, some small retailers, fearing chaos, remained closed for the day. In the country, a woman storekeeper swore she had never heard of decimal currency and refused to accept a dollar bill. And a barmaid who did found later it was a "cut-out" from a colour magazine.

But that was merely the crazy side of C-day. Sir Walter Scott, chairman of the Decimal Currency Board, agreed that there had been isolated and minor problems, but nothing had emerged to cause any real concern. Disenchantment came later, when shoppers discovered that they were now committed to paying more for such essentials as milk, bread, most daily newspapers, some rail and tram fares, bottles of beer, and numerous groceries.

The elimination of the half-penny was a prime mover behind the increases, though many people took a different view, accused shops of short-changing. By week's end, complaints were pouring into the Decimal Currency Board in Melbourne at the rate of a thousand a day; the good humour of C-day had begun to wear thin.

In Sydney officials were concerned at reports of blatant cheating by some small traders as customers rang newspapers and radio stations to complain about "decimal robbery".

But it wasn't only customers who were complaining. In Queensland, service-station operators reported that motorists were using two-cent coins instead of florins, in self-service pumps. They lost thousands of gallons of petrol. In Adelaide, motorists used two-cent coins to operate parking meters.

Parking meters can be adjusted more finely, but it could take some weeks for the machine-conversion programme to catch up with automatic petrol pumps. Conversion of Category A equipment, cash registers, adding and accounting machines, is now under way in all States, but it will be more than a year before all machines are serviced. Conversion may be quicker and cheaper than originally expected. The current estimate puts the cost of Australia's conversion at around £28 million.

(over)→

from *BETTER BUSINESS, March, 1966*
(Box 793, Auckland, N. Z.)

from
BETTER BUSINESS, March, 1966, (Auckland, New Zealand) 23

The changeover was watched closely by New Zealand observers who, no doubt, learned much from Australia's experience. Whether these lessons can be put to work in New Zealand is, of course, another matter.

Technically, Operation Decimal Conversion could hardly be faulted. Millions of man-hours went into its preparation with all sections co-operating to explain how the new currency would affect the community. In the weeks preceding C-day public utilities and essential services bought newspaper space to advertise new rail and tram fares, bread prices, discuss gas-meter readings and electricity charges. Banks distributed booklets showing how to make a deposit and write a cheque. Nothing was left to chance. The Decimal Currency Board, using singing commercials to convey the simplicity of the new system, got through to millions of people on television and radio. The Board even set up special switchboards manned by "Dollar Jills" — young women who stood by ready to answer telephone enquiries.

Still to be Felt

An operation on this vast scale cannot fail to have weaknesses. While the full impact of the change has yet to be felt, the problems that have surfaced so far are largely a matter of interpretation and can be rectified by the parties involved.

Some large business firms, for instance, instituted comprehensive staff training programmes, but failed to carry through and group-train executives. On C-day departments found they were working at cross purposes. There was no administrative integration, as a department had no conception of how its procedures affected the others. One retail store trained staff but did not train the executives responsible for implementing the changeover. The memos sent out to cashiers, pay clerks and ledger machinists not only compounded the confusion, but showed that some executives had no idea what these jobs entailed.

Can be Remedied

These are technical weaknesses and can be remedied. The most disturbing aspect of the changeover is an ethical weakness — the way decimal conversion lends itself to price increases. Take milk, for example. Before decimal currency milk cost 10½d per pint, or 1s 8½d for two pints. The changeover pushed the price to the nearest equivalent, 0.9 cents, or 11d. Two pints now cost 1s 10d, or 1½d more.

In Victoria, industrialists are backing the Premier (Sir Henry Bolte) in calling for the addition of a half-cent coin. "It is not too late to halt price rises if a half-cent coin is brought in," says A. N. Curphey, general manager of the Victorian Chamber of Manufacturers. But some firms have taken advantage of the prevailing outlook of uncertainty and instability to hike prices regardless. A spokesman for the Decimal Currency Board said he was satisfied that "about 50 shops in Sydney" had used the changeover to increase prices. "Some shops which have not even changed to decimal currency have increased their prices in £sd," he said.

One thing is clear. The changeover to dollars has given a lift to Australia's general pattern of inflation. Readers' letters in the newspapers show that some families are now paying 3s 6d a week more for milk. And this is only one item. Increases due to the new currency could add as much as an extra ten shillings on to the living costs of some families.

DECIMAL COINAGE

A LOST CAUSE?

B A. M. MOON
K. P. G. MEARS

AUTHOR'S NOTE: -

This article is a preliminary communication or pilot study. A full research report, now overdue, would cost something like £50,000; it could not possibly be made by two professional men in their spare time.

All cost and other figures for which sources are not given are our own best estimates. Actual total overall costs for decimal currency or the metric system might be half our estimate—or half again as much additional. As an appendix, we include some calculations for those who wish to believe that the costs of decimal coinage might be less than our estimates.

Without a full report there can be no full statistics. Existing statistics do not cover large parts of the problem. We welcome any facts which would help fill the gaps, as well as questions and suggestions.

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* Edited and printed in this publication by the Duodecimal Society of America for the interest of its members and friends. 20 Carlton Pl., Staten Island,4, New York, U.S.A.10304

FOREWORD

Before you remember what you know about "duodecimals", we suggest that you consider this:

Greater use of dozens will be an investment yielding at least ten and perhaps more than twenty times the best possible return from a change to Decimal Currency and the Metric System.

The return is so great that it can make a most significant difference to national income, productivity, and prosperity. On the other hand, the return on investment of a change to Decimal Currency and the Metric System is so small that it is statistically insignificant. Even South Africa, four years after making its change to Decimal Currency, has no statistics showing the return from the considerable cost of the change.

It is only four years since Professor A. C. Aitken first drew public attention to the fact that using dozens saves, on average, one third of the time spent in ordinary calculations (THE CASE AGAINST DECIMALIZATION, Oliver & Boyd, 1962).

We present here what we believe is the first assessment on a national scale of the implications of Professor Aitken's discovery.

The reader's natural response may be to look into it for difficulties - "surely it's wrong somewhere?"

You are invited <u>either</u> to read the whole <u>or</u> to read only the first page of proposals and then consider them in relation to the REPORT OF THE DECIMAL COINAGE COMMITTEE (N. Z., 1959). *It will be found that <u>only</u> in relation to paragraphs* 59(5)(c), 60(1), 119, *and* 120 (part) *is our proposal less satisfactory. <u>For all of the rest</u>, our proposals are equal to or better than those of the Committee, or else their paragraph is not relevant because, by adopting our proposals, the difficulty does not even arise.*

The proposals and arguments in this pamphlet are not made carelessly or thoughtlessly, but only after serious consideration of the actual practical problems involved and in the light of the results of the lifetime's work of a scholar who is one of the most brilliant and experienced in calculation in our age.

The sooner we act in this way, the greater will the advantages be.

B.A.M.M., K.P.G.M., May 31st, 1965

"If you want to succeed, you should strike out on new paths rather than travel the worn paths of accepted success."

- John D. Rockefeller.

MEN or MACHINES?

It has been said:-

"<u>If it will save human labor</u>, <u>waste anything else.</u>"

[Of course, if the savings are real, the "waste" is not real waste....simply a catchy label for the cost of the savings.]

Where the savings are substantial, this is the key to higher production. Let us examine some labor saving and waste:

A. Decimal coinage would cost something like £30,000,000.

Metric system " " " " £500,000,000

The promised savings approximate to about 0.2% of national income, or £1,000,000 per year.

B. Decimal calculating machines were worth £10,000,000. (as of 1957)

Dozens arithmetic machines, costing £6,500,000, do the same work <u>AND</u> release 1/3 of the labor force used in calculation. Other comparable savings are made in idle time and elsewhere, giving a total saving of the order of 1% of the national income, or £10,000,000 per year.

C. It appears that around £1,000,000 SPENT WISELY NOW WOULD GIVE NOT ONLY ALMOST ALL OF THE MARGINAL BEBEFITS OF DECIMAL COINAGE, BUT ALSO BRING SYSTEM TO THE USE OF DOZENS AND *GREATER* SAVINGS FROM IT!

This is how:-

1. Enact a decimal coinage based on the shilling unit (retaining present pence as fractions) simply by making shillings legal tender to any amount; and enact that, for all currency purposes except coinage, decimal or any other fractional notation may be used.

2. Enlarge present knowledge of dozens arithmetic and of its practical uses, both in and outside of schools.

3. Encourage the systematic use of dozens wherever it is profitable.

These proposals are simple; the savings of human labor are substantial; there's <u>no</u> unwanted waste, only improvement!

WHAT would be the SAVINGS for AUSTRALIA, the UNITED KINGDOM, the COMMONWEALTH, the U.S.A., and the WHOLE WESTERN WORLD?

COINAGE, CURRENCY, WEIGHTS, & MEASURES

£200,000,000 INVESTMENT or £1,000,000,000 LOSS?

If the ULTIMATE COST to New Zealand of adopting both decimal currency and the metric system would be in the neighborhood of a half a million pounds (or to an American, a billion dollars), then the returns from this massive expenditure are actually tiny in comparison - it may be regarded as a billion dollar total loss for practical purposes.

Its adoption can only be brought about compulsorily by legislation, judging by the example of every other country which uses decimal currency or the metric system exclusively. In many countries legal penalties have had to be imposed to compel people to use them.

If New Zealanders cared to learn dozens arithmetic, it would bring them advantages in everyday calculation equivalent to an investment of £200,000,000.

To reap the ultimate benefits of using dozens, it is vitally important that we keep our present familiarity and understanding of the use of twelve in our present system of money.

WE CANNOT AFFORD TO ABANDON THE SHILLING OF TWELVE PENCE.

FOR NEW ZEALAND TO GAIN THESE BENEFITS only three things are now needed.

1. Official recognition of a decimal coinage based on shilling units, divided into twelve pence; and equal recognition, for other currency purposes, of decimal or other fractions of shilling units.*

2. Encouragement of better understanding of dozens arithmetic and its uses, especially in schools.

3. Encouragement of use of dozens arithmetic by those to whom it brings practical advantages.

* Using modern terminology for an old practice, the United States uses binary divisions of the dollar for stock market quotation in the press and octal base numerals for the fraction column of both manual and automatic quotation boards. i. e. the fraction bar and the denominator "eighths" is understood without the need of writing it. A bill with "three bits" is actually settled at 38¢ if that happens to be the fractional part of a dollar due, because ½¢ coins have been out of circulation (last minting was in 1857).

The first step simply legalizes an existing widespread practice of expressing sums of money in shillings and pence alone - e.g. "117/6d" for "£5 - 17 - 6". It permits continued use of £.s.d by those who prefer it.

We do **not** advocate **any compulsory** changes

We are opposed to the compulsory changes which are proposed to our system of currency following an inadequate official study of the issues and their consequences.

Even with legal recognition of decimal or other fractions of shilling, we recognize that our first proposal is a little less convenient for use with wholly decimal calculators than a full decimal currency; but it is fully usable with them, since only eleven decimal equivalents of pence need be memorized - it demands no compulsory changes or expensive conversion costs - and it leaves the way open for us to gain the very much greater benefits which are offered by dozens arithmetic.

COMPULSION or CHOICE?

We shall quote from the *Joint Report* of the British Association for the Advancement of Science and the Associated British Chambers of Commerce (Butterworths, 1960) and from *Mechanical Engineering*, journal of the American Society of Mechanical Engineers, issue for March, 1965:

From the foreword to the *Joint Report:*

> "gradualness is the keynote"
>
> "evolution and not revolution is the way to progress"

From the *Mechanical Engineering* editorial:

> "The A.S.M.E. favors evolution rather than economically disastrous revolution."
>
> "If the metric system turns out to be the more practical system from a competitive standpoint evolutionary change, without disastrous revolution by legal fiat, will brint it about. If our inch system seems better, Europe will evolve to it."

WITH SUCH VIEW POINTS WE WHOLEHEARTEDLY AGREE.

WHAT are the trends, so far?

The Metric System was first adopted officially in France in 1801, following the French Revolution.

In 1837, thirty-six years later, it was necessary to introduce legal penalties in France to force people to use the metric system instead of other units.

The ONLY other country to use metric units exclusively (excepting some of its satellites) is Russia, by whom it was adopted in 1918 during the Russian revolution. Communist methods of compulsion are well-known.

Japan decided to adopt the metric system in 1924. Recently - forty years later - they have begun to enforce its use.

From evidence in the *Joint Report:*

> "In all countries where the metric system is [in general use] it has been imposed by legislation and has never been adopted by a process of evolution or voluntarily."

In a recent step in this process of compulsion, the South Africans have told the Australians how to force people to accept decimal currency.

From *Choice*, Feb. 1964:

> "The National Federation of Consumer Associations of South Africa has recommended that Australia should make it illegal to quote in £.s.d. soon after C-Day."

Where gradual evolution, not compulsion, is permitted, what are the trends?

Continental Europe officially uses the metric system only, and foot-pound standards have always been foreign. Yet, between a quarter and a half of European industrial production is to foot-pound standards and in Sweeden more than one half is to British standards. Foot-pound units are used instead of metric, not only for export, but for the domestic market as well.

In Great Britain and the United States of America, where the customary standards are foot-pound, but the metric system is permitted, adoption of the metric system, except in the pharmaceutical and chemical industries, has been negligible. Almost the entire productive capacity of the U.S.A. is based on foot-pound standards.

> "Production of goods in the world today is almost equally divided under the inch and the metric systems, with the inch system having the edge."
>
> *(Mechanical Engineering)* March, 1965, p. 31

It is not inevitable that we shall be forced to adopt the metric system sooner or later. WHEN FAIRLY TRIED IN PRACTICAL SITUATIONS, THE FOOT - POUND SYSTEM GAINS GROUND OVERALL AGAINST THE METRIC SYSTEM, *EVEN WHEN USED WITHOUT DOZENAL ARITHMETIC!*

COMPETITION or CO-OPERATION

Proponents of decimal currency and the metric system have made much of the point that they promote international co-operation, encourage the tourist industry, and improve our competitive position. What are the facts?

The British Travel and Holiday Association reported that it

> "had no evidence that the introduction of the metric system or decimal coinage would directly increase the volume of tourist traffic" to Britain.

> "From the point of view of our international trade or even from the point of view of tourism we find that the benefits to be derived from decimal coinage are slight."
>
> *(Joint Report)*

Does adoption of the metric system increase our ability to export to metric countries?

> "From the point of view of U. K. export trade, there would be little advantage to us in changing to the metric system to improve our trade with metric countries."
>
> *(Joint Report)*

and "Importers and customers in foreign countries will accept British products if the price and value are right, whatever their units of manufacture."

"Can the U. S. remain with the inch system and still export to metric-based countries? Yes, if our labor costs are competitive, if our products' quality and design features make them desireable and if they are not excluded by any economic-political barriers. It is these factors and not whether a country uses an inch or metric system which primarily determine a country's export potential."

(Mechanical Engineering)
March, 1965, p. 32

RETAINING FOOT-POUND UNITS DOES NOT RESTRICT OUR EXPORT POTENTIAL. How would our competitive position be affected by our adopting the metric system?

We find in the *Joint Report*:

"Countries using inch-pound measurement realize that adoption of the metric system will throw practically the whole burden of change upon them."

"The expense and trouble of a change to the metric system, during the period of changeover, adversely affect the competitive efficiency of manufacturers."

and "U. K. engineers have comparitively recently changed over thread dimensions to a unified form to standardize with America and an attempt to make a further change would be disastrous."

"There is pressure on the U. S. to undertake the entire changeover itself, going to a metric system of threads at huge costs."

As one machine tool company reported:

"The cost that would be incurred if we converted to the metric system .,.. would decrease our ability to compete in the world's markets."

(Mechanical Engineering)
March, 1965, p.33

Apart from the massive capital cost of adopting the metric system, which can hardly be recovered except by increasing prices, productive capacity would be seriously and adversely affected during a changeover. It is on productivity that success in competitive situations depends, because lower production means greater costs. Not only is productivity lost, but duplicated and carefully separated stocks, spare parts, and tools must be held for many years.

"Anything which increases the cost of our products would make them less competitive in the export market."

"If the metric system turns out to be the more practical system from a competitive standpoint, evolutionary change, without disastrous revolution by legal fiat, will bring it about. If our inch system seems better, Europe will evolve to it."

"...let the metric countries do their share of the adjusting..."

(Mechanical Engineering)
March, 1965, p.29-37

TO ADOPT THE METRIC SYSTEM UNDER COMPULSION WOULD *LOWER* OUR PRODUCTIVE EFFICIENCY AND *REDUCE* OUR ABILITY TO COMPETE IN INTERNATIONAL MARKETS.

WE BELIEVE in international co-operation, but not if all the cost is on our side. While there may be ways of improving international co-operation and understanding, international economic competition will always continue. If we adopt the metric system for the sake of co-operation we shall only give a competitive advantage to our all-metric competitors. It is the Communist countries which are the main all-metric countries with no capacity for production to inch-pound standards.

WE DO NOT BELIEVE that the current moves to encourage decimalization and the metric system are part of a Communist plot, but we do believe that if these moves are successful, it is the Communist nations who will reap an unearned gain simply because we will have incurred costs arbitrarily without sound justification that they have not.

COST CONSIDERATIONS

In any enterprise, expenditure of large sums of money is normally related to the returns expected.

The cost of the officially sponsored decimal currency proposal for 1967 is £4,500,000 directly to the taxpayer. (Figures quoted by Minister of Finance.)

The indirect costs of unproductive work necessary to enforce this changeover may be £20,000,000 to £30,000,000. (Various estimates of this figure have been made in other quarters - some greater than these.)

There is no official estimate of the savings in return for this expense!

> "Indeed there are no statistics of any kind reflecting the savings achieved by South Africa through decimalizing its monetary system."
>
> (Secretary to the Treasury, South Africa, May, 1965)

If there are any real financial advantages to machine users in converting to decimal currency, the taxpayer should not be required to bear the conversion cost.

COST OF THE METRIC SYSTEM

From a study of what material is available, we reckon that the direct cost to New Zealand of adopting the metric system would be of the order of £200,000,000, That is between six and ten times the cost of decimal currency.

Take one example; an investigation by the General Electric Company of America found that it would cost them $200,000,000 in direct costs to adopt the metric system excluding the cost of lost production, even if conditions of change were optimal for G. E. This figure is thus the least possible cost, not the likely cost, and they were:

> "...forced to conclude that practically none of the $200,000,000 cost would be recoverable.
>
> *(Mechanical Engineering)*
> March, 1965, p.34

High though it is, this figure does not appear to take account of the consequential costs in distribution, marketing, and in homes. Furthermore, G. E. is not an extreme example for U. S. A.

> "for the country as a whole, the result would have to be at least multiplied by a factor of two or more due to the less mechanical nature of G. E. products as compared with, for example, to machine tools, automobiles, railroad equipment, ships, and the vast supply industries such as tools and gages, fasteners, and so on."
>
> *(Mechanical Engineering)*
> March, 1965, p.35

General Electric Company has a gross turnover of $4,200 million; earnings of $272 million; assets of $2,700 million; and working capital of $670 million. Thus the rock bottom net cost of the metric system to General Electric is about one year's earnings or one third of its working capital; either way, a complete loss.

These figures show that General Electric is comparable in size to New Zealand, if New Zealand is considered as a business. General Electric makes millions of a few thousand different products and New Zealand, so to speak, makes a few thousand of each of a million products. Our investment in mechanical equipment is larger than that of General Electric. The value of motor vehicles alone in New Zealand is about the same as that of General Electric's total assets. Thus, even though New Zealand is less highly industrialized than the United States of America, costs of change would be likely to be substantially greater than those of General Electric.

Including factors for loss of production and other contingencies, it is justifiable to estimate the total direct cost to New Zealand of adopting the metric system at £200,000,000.

To help visualize this amount, £200,000,000 (at 1959 values) is approximately equal to any one of the following:

New Zealand's capital investment in State Hydroelectric Works

1½ times the value of New Zealand manufacturers' stocks

20 times the annual sum invested in building homes and shops

1½ times the annual total of industrial salaries and wages

2/3 year's gross farming income

or £400 FROM EVERY FAMILY IN THE COUNTRY

(derived from data in N. Z. Official Yearbook)

Practically none of this cost is recoverable, and the confusion in homes, offices, shops, and factories could last for years,

BUT THIS IS NOT THE TOTAL COST.

These are some of the implications of what, on the face of it, seems to be a simple, though quite expensive change for the better - a change to decimal currency.

These are some of the reasons for the instinctive distrust, almost inarticulate, but nonetheless strongly felt, of the ordinary citizen, and his increasing disturbance and dissatisfaction with the proposed changes.

The British system of currency, weights, and measures has been developed over many generations of practical experience and was not designed to make life difficult. From it, decimal relations are notably absent; this can only be explained by the limited practical value of decimals.

Decimal arithmetic and the metric system are comparatively recent innovations, based largely on the needs of an uneducated mass of population, ignorant of any practical means of reckoning beyond counting on its fingers. It is an Oriental graft on European civilization, adopted after the Renaissance, when the calculating methods of the ancient priests, astronomers, and engineers had not been rediscovered.

We would not defend every vagary of the British system, but it may be shown that nearly all of its units and the relations between them (and certainly all of the most important ones) have been chosen as practical means to solve practical problems and the importance of most of these practical situations remain to this day.

Indeed, they have been shown to be superior in new situations which have arisen. For control of aircraft, metric units and instruments were adopted internationally, only to be expensively abandoned when it was found that flying to metric scales of height brought more difficulties than it was worth.*

* The JOINT REPORT (Appendices, p. 94) quotes the British European Airways as stating that many countries which originally agreed to the full I C A.O. (mainly metric) units had in fact reverted to the Blue Table (feet for altitude and nautical miles for distance). Many metric countries also use feet for vertical distance as meters were not convenient for quadrantal separation of flight levels where used. The expense of converting (again?) all of their aircraft and ground equipments would be considerable, and would apply to most other airlines all over the world. ....possibilities of delays and danger.... "unlikely to be covered by savings".

Some Continental architects have abandoned meters in favour of feet and inches.

It is only when people insist in interpreting the British system in terms of decimal numbers and regard the need to calculate in decimals as of over-riding importance in any system of units, that it appears to be full of anomalies.

Not only does this represent an unduly restricted appreciation of the properties of an efficient system of units, but *IT IS NOT EVEN NECESSARY TO CALCULATE IN DECIMALS ANYWAY!*

It has long been known that calculation using numbers to other bases can give complete and satisfactory methods of arithmetic and that THERE ARE NO SPECIAL PROPERTIES OF TEN WHICH FAVOUR IT AS A BASE FOR ARITHMETIC.

In fact, the greater simplicity of bases two, eight, and twelve, especially for processes involving multiplication and division and the representation of the popular common fractions that have been known from the dawn of Western civilization.

It was not however, until the quite recent work of Aitken that we have had an expert assessment, permitting us to measure this advantage, especially for arithmetic using a base of twelve, that is, dozens arithmetic.

Aitken's words (in THE CASE AGAINST DECIMALIZATION Oliver & Boyd, 1962) are:

> "But the final quantitative advantage, in my own experience, is this; in varied and extensive calculations of an ordinary and not unduly complicated kind, carried out over many years, I come to the conclusion that the efficiency of the decimal system might be rated at about 65 or less, if we assign 100 to the duodecimal (i.e. dozens method) of arithmetic."

> "If such a waste of time and effort (about 350 hours lost in every 1000) were found to be trickling away in any department of a modern production unit, a time-and-work study would at once be set up."

"Is it to be doubted that such time, saved and turned to more productive ends, social and economic, would give an advantage much outweighing any advantage assumed to accrue now, at this late stage of decision from moving over to the decimal system?"

The consequences of this discovery are profound. Not only is dozens arithmetic intrinsically simpler than decimal, but it is ideally suited to calculations using all our units based on twelve and twice as efficient for those based on multiples of two. Interpreted thus, it becomes evident that, taking all the relevant factors into account, British (foot-pound) units are superior to metric ones for most practical purposes, even without any reforms at all.

It becomes apparent further, that only now is any nation in a position to make an accurate assessment of the disadvantages of increased decimalization and the advantages of greater use of dozens arithmetic. The South Africans may be considered unlucky to have made their choice just before this discovery was made. We of all people are the ones with the opportunity to gain most from it - if we firmly reject decimalization at this stage.

Even the Americans, naturally inclined to favor decimalization since their revolutionary gesture of defiance towards King George III by adopting decimal currency, are now beginning to realize the advantages of working with dozens. *Mechanical Engineering* reports the recommendations of an investigating committee. We are well aware that this committee raises the possibility of decimalizing the inch-pound system, but it also states that -

"due consideration be given to duodecimalization (i. e. use of dozens relationships) where advantageous"...

and comments -

"Nor should we overlook inherent advantages in the duodecimal system, where 1/2, 1/3, 1/4, 1/6, and 1/8 are much more convenient to use than their decimal equivalents."

HOW MUCH could this discovery save us?

We reckon that it would not be an overestimate to say that individuals spend, on average, one hour in thirty (two minutes per hour) in calculating, or in idleness awaiting the results of calculations. Professor Aitken has shown that a third of this time can be saved. At 1959 rates this is worth £10,000,000 annually, which capitalized at 5% is equal to a capital sum of £200,000,000.

This is a saving we cannot make by choosing decimalization instead!

Thus, in round figures, at 1965 rates, a total of £500,000,000 seems a fair estimate of the ultimate costs of decimalizing - a billion dollars - if we permit the change.

Something like £1000 **from every family in the country is the ultimate cost.**

Apart from the massive cost of adopting the metric system, which cannot be recovered except by increasing prices, productive capacity would be seriously and adversely affected during the changeover. It is on productivity that success in competitive situations depends, because lower production means greater costs. Not only is productivity lost, but duplicated and carefully separated stocks, spare parts, and tools may have to be held for many years.

"Anything which increases the cost of our products would make them less competitive in the export market."

"If the metric system turns out to be the more practical system from a competitive standpoint, evolutionary change, without disastrous revolution by legal fiat, will bring it about. If our inch system seems better, Europe will evolve towards it."

"Let the metric countries do their share of the adjusting."

(Mechanical Engineering)
March, 1965

To adopt the metric system under compulsion would lower our productive efficiency and reduce our ability to compete in international markets,

DOZENS, DOZENS ARITHMETIC, DUODECIMALS and DUODECIMAL SYSTEMS

These names range from the obvious, through the little understood, to the theoretical fancies of earnest and often brilliant minds.

DOZENS we all know. They are an ever-present necessity of civilized life and appear in innumerable places in remarkably diverse forms. (There are dozens of misunderstandings of the disadvantages of decimals, for instance.)

Counting in dozens is often convenient. Packing in dozens is often essential. The flexibility and variety which dozens permit make this so. Likewise, our coinage, our foot-rules, our clocks, and calendar are based on dozens and indeed our way of thought is trained in dozens and their fractions and multiples. If we count in dozens we can conveniently counts by *2*'s, *3*'s, *4*'s, or *6*'s, thus:

"three, six, nine, a dozen, three, six, nine, two dozen ..."

or "four, eight, a dozen, four, eight, two dozen ......"

By contrast, in using decimals it is only convenient to count by twos or the usually less convenient fives.

To count double or treble columns or rows, for example, using dozens can be extremely rapid, with clear advantages in tallying, stocktaking, and packing.

In packaging, the variety of simple factors of the dozen and its multiples give a wide choice of shapes permitting economy both in materials and handling costs.

In costing and stock valuation there are the same advantages if the currency has a base of twelve, and of course this brings added simplicity through being compatible with the normal and efficient process of recording stock quantities in dozens.

DOZENS ARITHMETIC is the skill of reckoning or calculating in dozens - a complete, powerful, and general method of arithmetic, which, owing to the pre-eminence of dozens in practical affairs, is particularly suitable for practical calculations. It obeys all the general laws of arithmetic such as those for placing or shifting the point. The addition and multiplication tables are different from those for decimals but owing to the superior divisibility of twelve to ten, are notably simpler, as is the representation of common fractions, for the same reason.

Dozens arithmetic must not be confused with DUODECIMALS, which may be an ill-defined or blanket term covering whatever the user wishes. It may, for example, be used to describe numbers expressed using some particular version of duodecimal notation (characterized perhaps by the symbols used for the numbers *ten* and *eleven*), or, one of the still more numerous possible systems of weights and measures based entirely on the dozen which have been proposed from time to time.

There is no doubt that suitable symbols for ten and eleven will become internationally recognized, but anyone familiar with nationally differing representations of the symbols for one and seven (consider the German, for instance) will be well aware of the marked variations which do in fact exist for the first nine number symbols.

So far as DUODECIMAL SYSTEMS are concerned, we may be certain that the foot and inch, clockface and twelve-month calendar will maintain their positions, but no one can foresee which other units will be the standards of the future.

We seek only to let them evolve gradually from our existing units, under the test of time and in the light of a fuller knowledge and appreciation of the use and practical advantages of dozens arithmetic. It is even possible that some non-dozenal units will remain because they have other practical advantages and are easily expressed for use in dozens arithmetic.

EVOLUTIONARY PROCESSES

Evolution of currency.

As part of this process of evolution or gradual change, we suggest that:

FOR COINAGE, SHILLINGS SHOULD BE LEGAL TENDER TO ANY AMOUNT, AND PENCE AS FRACTIONS SHOULD CONTINUE. BUT FOR ALL OTHER CURRENCY PURPOSES, DECIMAL OR ANY OTHER FRACTION NOTATION SHOULD BE LEGAL ALSO. £.S.D. SHOULD ALSO REMAIN LEGAL.

The only concrete change this would imply would be the expression (decimally) on banknotes of their value in shillings as well as in pounds. No other expense is involved, except perhaps to replace our lost half crowns.

This modest change is clearly a development of an evolutionary change now taking place. It is widely used at present in virtually every department store and by wholesale grocers and drapers in quoting prices, and it is well understood by almost everybody.

Existing decimal calculators may then be used for money sums, and only eleven decimal equivalents of pence (to as many places as needed) need be memorized for use with them.

PENCE ARE SIMPLE SHORTHAND EXPRESSIONS FOR THE MOST COMMONLY OCCURING FRACTIONS.

No coinage need be changed and there is no chance of inflation from round-up of prices or unproductive expenditure. In fact no conversion tables and no machine conversions are needed. The Decimal Currency Board and its offshoots, present and proposed, can be disbanded, a substantial pruning of bureaucracy, and the people saved can work on far more profitable jobs.

£.s.d. may remain in use in parallel (as at present) for those who prefer to use them, or have machines which do not need replacing.

Many existing accounting machines may be used for either system with no modifications at all. Computers are actually being used in this country today in the manner suggested.

New Zealand would not thereby have a curious and untried system of currency, but on the contrary can make even better a very cheap and remarkably efficient system that everybody can understand. We can approach "eating our cake and having it too" by applying the principle of well ordered consistency of the decimal-metric system to our own favorite measures.

Evolution of arithmetic teaching.

Revision of the school curricula to include binary and dozens arithmetic instead of the drudgery of the cumbrous methods now used for sums with money, weights and measures, in which they are converted to decimals for calculation and back again, would, by the simplicity and practicality achieved, give a great stimulus to the teaching of arithmetic.

This is fully in harmony with modern trends in school mathematics to generalize the concepts of arithmetic.

Development of use of dozens arithmetic.

Dozens arithmetic will automatically come into use because of the savings inherent in its use.

No change of number system is needed, except, where convenient, by users of dozens arithmetic for their own purposes.

Dozens slide rules are already available.

Dozens calculating machines will become freely available as they are called for.

Computers can freely accept quantities expressed in dozens now.

No change will ever be necessary, for the sake of dozens arithmetic, in:

Telephone numbers
Motor registration plates
House numbers
Stock number codes
Serviceman's numbers

.... and such like, as these are merely labels, as their frequent admixture with letter symbols testifies.**

Even where the expression of such labels in an ordered sequence (as for house numbers) is required, this does not imply any association with decimal arithmetic. For instance the addresses on the street neighboring the home of one of us are labelled:

1, 3, 9, 11, 15, 17, 19, 21, 25, 25A, 27, 31, 37.

2, 4, 6, 10, 12, 14A, 14B, 16, 22, 24, 28, 30, 32, 34, 34A

** [Even though the existing number assignments do not need to be changed, the fact that ten and eleven are expressed in single numerals for dozens arithmetic (and are mechanically internally single numerical digits on many computer cards and key-punches, despite their being set to print-out 10 or 11 compressed-into-a-single-space when not used for some other arbitrary characters for human readers) opens opportunities to add two to a set (or field) of ten, 44 to a field of a 100, 728 to a field of 1000, and to double a field of 10,000 stock or identification numbers without having to go to alphabetical characters, when wanted.]

EDUCATION

The most important area in which knowledge of dozens arithmetic may develop is in the schools.

Teaching dozens arithmetic at the elementary level is justifiable not only for its own sake, but because of the positive advantages it brings to the understanding of arithmetic and skill in its application.

Specifically these advantages are:

1) ...that it bears the simplest relationship to the most abundant examples of the structured use of number a child may observe in his environment, thereby stimulating his interest in arithmetic as a practical subject and a means for him to acquire mastery over his environment;

2) ...that it breaks the association of arithmetic with unit counting on the fingers, a primitive skill which retards the child's development of maturity in ideas of number, and skill in their use;

3) ...that it is intrinsically easier than decimals, especially for multiplication, division, and the expression of fractions;

4) ...that it is very much easier to apply to practical sums involving weights and measures, especially those most important such as telling time, feet and inches, shillings and pence, pounds and ounces, which form a large part of the practical sums undertaken at the elementary level.

DOZENS ARITHMETIC BRINGS POSITIVE ADVANTAGES IN ARITHMETIC EDUCATION SUBSTANTIALLY GREATER THAN ANY WHICH DECIMAL CURRENCY MAKES POSSIBLE.

Here we would caution that although dozens arithmetic is child's play for children, (and we know of at least one child who had professed herself "hopeless at arithmetic", and who mastered dozens arithmetic in an afternoon with the aid only of a copy of the dozens multiplication tables), this is not necessarily the case for adults.

To master dozens arithmetic (or indeed any other variety) requires a freshness or flexibility of outlook which those who have become more fixed in their attitudes may find difficult to attain. In older persons therefore it may require some distinct intellectual effort to overcome those restrictions or prejudices imposed by many years of habit.

Fortunately, the use of a dozens slide rule, for example, reduces the amount of effort needed.

When dozens arithmetic is used in conjunction with our present weights and measures, it shows clearly that they are superior to any decimal-based system and removes forever the threat of our being forced to accept the metric system and the enormous costs associated with it.

Learning dozens arithmetic is a voluntary undertaking - there are advantages now for those who care to do so. Those who do not wish to learn it may still continue to calculate in decimals if that is what they prefer.

FINALLY ....

We wish to make it quite plain that New Zealand can have both dozens and the short-term benefits obtained from decimals.

We are aware of the pressures which are being brought to bear on the Government - both internal and external - to enforce changes in currency and in weights and measures.

We are only too well aware that beliefs and habits of thought which have developed over the last three hundred years or so constitute a considerable force directed towards a change to decimals.

What we seek is no breakaway from present practice, but a development which is based on a wider view of relevant practical factors than decimalists have taken, and which seeks to preserve much of what is best in our heritage.

We do not ask the Government to spend large sums of money; on the contrary we have shown that enormous expense and waste of effort can be avoided at the same time as the development and use are encouraged of methods more efficient and economical than those which even unlimited expense on decimals would provide.

Indeed it is the very possibilities inherent in dozens arithmetic, and the smallness of the expense required, which make them seem unreal and therefore impractical. No compulsion is needed, no bureaucracy and inspectorate are required, simply a few minor changes in the law to adjust it to present practice, sounder teaching in arithmetic, and adoption of the practice of dozens arithmetic by individuals only as they are convinced of its advantages to them personally.

Any further changes need be only those which a well - informed public calls for, after developments of which these changes are a part, perhaps in fifteen or twenty years and when dozenal arithmetic is in use naturally.

ECONOMIC CONSEQUENCES OF CHANGES IN CURRENCY AND SYSTEMS OF MEASUREMENT

The calculations following are hypothetical. They are intended simply to show the inherent limitations of decimal arithmetic and coinage and the metric system. Though these are consistent with each other, they are not compatible with arithmetic operations so generally used; particularly that the need to figure by *3* (or thirds) and *4* (or easier fourths) arise generally far more often than 5 (or fifths). Consider the clockface, packaging, and numerous other practical uses. These restraints are so severe that, as the calculations show, two rules arise:

1. RETURNS FROM INVESTMENTS WHICH INCREASE USE OF DECIMAL ARITHMETIC ARE MARGINAL AT BEST.

2. RETURNS FROM INVESTMENTS WHICH INCREASE USE OF DOZENS ARITHMETIC INCREASE IN PROPORTION TO ITS USE, AND RAPIDLY AS THAT PART OF INCOME EARNED FROM CALCULATION INCREASES.

Even metric countries can profitably adopt a shillings/pence currency! Even if New Zealand completes conversion to decimal currency a return to shillings and pence would be profitable, but a change back from the metric system would be slow and costly.

The following examples suppose that:

1. Dozens arithmetic will be used fully, when profitable.
2. National income is £1,000 million.
3. 1/33 of this income is earned through computation - £30 million.
4. 1/3 of computation is costing - £10 million.
5. Cost to change to Dicimal Currency is £30 million.
6. Cost of machines and adult education for Dozens is £15 million.
7. The ruling rate for loan money is 5%.
8. Decimal Currency saves 10% of the time in costing - £1 million.
9. Dozens arithmetic saves 1/3 of time on all kinds of computations

Changes proposed by Decimal Coinage Committee (1959):

I. I. TO DECIMAL CURRENCY, only (decimal arithmetic for all purposes): Return on investment 3.3% - capital never repaid.

II. POSSIBLE CHANGE TO FULL METRIC SYSTEM (decimal arithmetic only): Cost something like £250 million - minimal necessary return of £12½ million needed to justify doesn't appear possible. No predictable income increase, judging by South Africa experience.

Now, consider change to other alternates than I or II:

III. TO SHILLING/PENCE CURRENCY: No appreciable change for those who don't want to be bothered other than more practice in halving or doubling significant figures when comparing with £.s.d. figures. Use of the old is not outlawed, and the way is open for up to 66% return for those wanting to develop the use of dozens arithmetic in the future.

IV. CHANGE TO SOME "COMPLETE DUODECIMAL SYSTEM": Cost may approach that of converting to full metric, but is NOT NECESSARY since main benefits come from dozens arithmetic we already know from handling shilling/pence money in our daily lives, and extending it to our popular measures - twelve hours to the clock, twelve inches to the foot, twelve eggs to the dozen, twelve months to the year, and so forth. In due time, as convenience may dictate, new units may be tried and catch popular acceptance to take place along side of or even take the place of some of our existing measures that represent the "survival of the fittest" so far.

For Existing Metric Countries:

V. CHANGE TO SHILLINGS/PENCE AND FOOT/POUND MEASURES : Cost £250 (as for II), savings of £10 million annually --4%-- barely profitable but unlike II is not a complete loss.

VI. CHANGE TO SHILLINGS/PENCE; RETAIN METRIC MEASURES: Cost as in I, savings in costing one third --11%-- worthwhile.

VII. CHANGE £.S.D. TO DECIMALS AND BACK AGAIN: Change to decimal currency - £30 million, change back another £30 million: yearly savings £10 million. Both the cost of the mistake and reconversion can be paid off with interest in under ten years, but with no profit for eight extra years!

APPENDIX

Comparitive figures

Part of G.N.I. due to calculation	Value of calc. £	Value of cost calc. if 1/3 £	Annual Saving change £.s.d. to:	£	Saving % of G.N.I. %	Cost of Change	Annual Return (less 5% capital charge)	Advantage of DOZENS over DECIMALS	Case
1/16	62	21	s,d,(III)	21	2	15	135%		
			dec. (I)	2.1*	0.2	10	16	8	I
						20	5.5	25	II
						30	2	67	III
1/33	30	10	s.d.	10	1	15	62		
			dec.	1	0.1	10	5	12	IV
						20	0	∞	V
						30	-1.6	∞	VI
1/66	15	5	s.d.	5	0.5	15	28		
			dec.	0.5	.05	10	0	∞	VII
						20	-2.5	∞	VIII
						30	-3.3	∞	IX

Gross National Income (G.N.I.) £1,000 million (all £ figures above are millions).

* Estimated at 10%...no official statistics available - may be too high.

(If the reader doesn't like our figures because of lack of solid cost information of compulsory changeover, (even from South Africa's recent experience) he is invited to make his own choice of a high, medium, or low speculation from above and select the advantage accordingly. Decimals are barely profitable under optimum figures. Dozens pay under the widest variety of conditions. Closer to optimum, the bigger the savings, and nobody has to change except to the extent he sees it would pay him to.) WE KNOW OF NO OTHER ESTIMATE OF THE RESULTS OF AITKEN'S DISCOVERY APPLIED TO A NATIONAL ECONOMY. WE HOPE WE HAVE SHOWN CLEARLY THAT THE GAINS FROM HIS DISCOVERY COME FROM USING DOZENS ARITHMETIC, and that sweeping changes of units are expensive and wasteful.

A BASE-CONVERSION ALGORITHM

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In a recent paper (1), Gladwin generalizes a little known algorithm, used in the Boeing Company Transport Division for converting whole numbers from base eight to base ten, and he gives a proof for the general case.

Since this method is clearly applicable to the conversion of whole numbers between base twelve and base eight, ten, or indeed any other base, a demonstration of its application in some such cases will be of interest.

1. Conversion from base Ten to base Twelve

1.1. Method.

Starting from the left of the decimal number and working throughout in dozens arithmetic:

(i) Multiply the first digit by *2*, align the product one place to the right, and subtract.

(ii) Take all the digits in the result generated by the subtraction operation, (i.e. from the left, all those up to and including the right-most digit above which the previous product was aligned), multiply by *two*, shift one place right and subtract again.

(iii) For an n-digit decimal number, repeat this procedure up to a total of $n-1$ steps. In the last step the units digit of the original number is always included.

The only arithmetic procedures required are multiplication by *two* and subtraction in dozens arithmetic.

1.2 Examples.

We apply this process to the examples given in the *MANUAL OF THE DOZEN SYSTEM* (2), pp. *1X - 1Ɛ*.

Change to dozenals, the decimal numbers: (i) 20735
(ii) 13579

(i)

```
 20735
 -4          (2 times 2)
 18735
 -34         (18 times 2)
 15335
 -2X6        (153 times 2)
 12495
 -2496       (1249 times 2)
  ƐƐƐƐ  answer
```

(ii)

```
 13579
 -2          (1 times 2)
 11579
 -22         (11 times 2)
  Ɛ379
 -1X6        (Ɛ3 times 2)
  9519
 -16X2       (951 times 2)
  7X37  answer
```

2. Conversion from base Twelve to base Ten

2.1. Method.

The previous procedure is repeated except that we add instead of subtracting, and work in decimal arithmetic throughout instead of dozens.

The procedure is a little less obvious than in the last case since the digits 𝒳 and Ɛ which may occur in the base twelve number do not normally occur in decimal arithmetic. Clearly a similar situation arises in converting a decimal number into octal.

No difficulty arises if we remember to treat 𝒳 and Ɛ as single digits in selecting the multiplicand, but treat them as 'ten' and 'eleven' in the subsequent decimal arithmetic.

2.2. Examples.

As examples we reconvert the previous two examples, in which either 𝒳 or Ɛ is present, and also give a related example in which neither of these digits is present.

Change to decimal the dozenal numbers: (i) 7𝒳37
(ii) 8037
(iii) ƐƐƐƐ

```
(i)    7𝒳37                      (ii)   8037
     + 14      (7 times 2)            + 16      (8 times 2)
       9437                             9637
     + 188     (94 times 2)           + 192     (96 times 2)
      11317                            11557
     + 2262    (1131 times 2)         + 2310    (1155 times 2)
      13579    answer                  13867    answer
```

```
(iii)  ƐƐƐƐ
     + 22      (Ɛ times 2)
      143ƐƐ
     + 286     (143 times 2)
      1727Ɛ
     + 3454    (1727 times 2)
      20735    answer
```


3. Conversion from base Eight to base Twelve

3.1. Method.

As an indication of how the procedures given may be generalized, we give two examples of conversion of octal numbers to dozenal.

The rules are identical with those given in ¶1.1 for conversion of decimal numbers to dozenal, except that the factor used for multiplication is *4* rather than *2*. More generally, this factor is the difference between the two bases.

3.2. Examples.

Convert to dozenal, the octal numbers: (i) 440
(ii) 2002

```
(i)    440                       (ii)   2002
     - 14    (4 times 4)              -  8      (2 times 4)
       300                              1402
     - 100   (30 times 4)             -  54     (14 times 4)
       200   answer                      𝒳82
                                      - 368     (𝒳8 times 4)
                                         716    answer
```

4. Conclusion

A method has been described for converting whole numbers between base twelve and other bases. In some cases this requires less work than methods considered previously.

It is also of interest that the arithmetic operations carried out use the same base as that to which conversion is required.

References

(1) GLADWIN, H. T. "An Algorithm for Converting Integers from base α to base β." *A.C.M., Communications, 7,* 4, April, 1964.

(2) .... *Manual of the Dozen System.* Duodecimal Society of America (1960) 20 Carlton Pl., Staten Island, New York U.S.A. 10304