COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	X	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5;9'
31	694	Three ft. two in.	3;2'
96	3Ɛ2	Two ft. eight in.	2;8'
19Ɛ	1000	Eleven ft. seven in.	Ɛ;7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
  12 ) 30 + 5
     12 ) 2 + 6
          0 + 2     Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *X*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *X*.

Numerical Progression

1	One		
10	Do	;1	Edo
100	Gro	;01	Egro
1,000	Mo	;001	Emo
10,000	Do-mo	;000,1	Edo-mo
100,000	Gro-mo	;000,01	Egro-mo
1,000,000	Bi-mo	;000,001	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	Ɛ
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
Ɛ	1X	29	38	47	56	65	74	83	92	X1

The Duodecimal Bulletin

Whole Number *32;* Volume *18;* No. *1*
August 1965 *(1179)*

THE DUODECIMAL SOCIETY OF AMERICA
20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of BASE TWELVE in numeration, mathematics, weights and measures, and other branches of pure and applied science.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island, New York 10304. Kingsland Camp, Chairman of the Board of Directors; Charles S. Bagley, President; Tom B. Linton, Executive Secretary; Ralph H. Beard, Editor. Separate subscriptions $1.00 a copy.

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Note: The articles, LET'S NOT GO METRIC, and WEIGHTS AND VOLUMES: STREAMLINED AND CO-ORDINATED, by Dr. Robert C. Gilles, have been published earlier in the Duodecimal Newscast of the Duodecimal Society of Great Britain.

Oni korespondas Esperante

The Duodecimal Bulletin

All figures in italics are duodecimal.

MEETING OF THE BOARD, 1965

Chairman Kingsland Camp called a Meeting of the Board for 28 April 1965 in New York City. This meeting failed of quorum, and it was decided that there should be a Mail Meeting of the Board, in accordance with our established procedure, and this Mail Meeting was completed 14 May 1965.

Minutes of the Denver Meeting of the Board were approved as published in the Duodecimal Bulletin, Vol. *17*, No. *1*, December 1964 *(1178)*. Secretary Tom Linton, reporting on the activities of the year, comments on the congressional efforts in favor of the metric system. Bills had been introduced in the Senate and House of Representatives, and public hearings on these measures were being planned, – but adverse public reaction led to these hearings being indefinitely postponed. There has been a wave of interest in duodecimals in Canada, Australia and New Zealand. Dr. Moon, Senior Lecturer in Mathematics at the University of Canterbury, Christchurch, New Zealand, is including our duodecimal material in his short course on non-decimal arithmetic.

Our membership continues its satisfactory growth. Present enrollment is 174, consisting of 26 Fellows, 48 Senior Members, 27 Members, and 73 Student Members. This is an increase of 20 for the year, especially noticeable in the numbers of student members.

The major item of publicity value this year was the publication in the F. B. I. Review, the journal of the Federation of British Industries, of a paper, "Let's Not Go Metric," by Dr. Robert C. Gilles, who has long been a member of our Society from West Germany. Dr. Gilles has supplied us with reprints of this fine article, which we have distributed to our members. We plan to include this paper in a future issue of the Bulletin, to make it conveniently accessible in our records.

Also valuable to us is the publication by Fawcett of the book, "More Fun With Mathematics," by Jerome S. Meyer, which devotes a chapter to The Duodecimal or Dozen System. Problems and material on other number systems are included.

The National Referral Center for Science and Technology has listed the Duodecimal Society of America in its Directory of Information Resources in the United States, – available through the Government Printing Office.

The Secretary has reprinted a modest quantity of the first issue of the Duodecimal Bulletin, as our supply of this issue had been completely exhausted. He has also secured a 16 mm. microfilm of the complete series of the Duodecimal Bulletin to date, to have this material referrable for our members.

We receive about six requests a day for the free duodecimal literature that we supply on individual requests. Larger quantities of the pamphlet, An Excursion in Numbers, are supplied at 10¢ a copy. These requests continue to be handled at our official mailing address of 20 Carlton Place, Staten Island, New York 10304.

Our Treasurer H. K. Humphrey reports as follows:

Balance in checking account	$1175.20
Balance in Savings Acct. (Slide Rule)	94.75

These funds are in the Winnetka Trust and Savings Bank, and the Society has 2M U.S. 2 ½% Treasury Bonds, 1972-67, and 3M U.S. 3 ¾% Treasury Bonds, 1968, whose current prices total $4776. These bonds are the residue of the endowment originally contributed by George S. Terry, later augmented by the bequest from the estate of Lewis Carl Seelback. The checking account contains the balance of about $750 from a gift by George S. Terry to finance the meetings in Alamogordo and Denver. This balance is subject to disposition arranged between Mr. Terry and Tom Linton.

Secretary Linton also maintains a Supplemental Account for the Treasury in Los Angeles for handling current funds such as dues, sales, and current expenses. Present balance in this account is about $197.

Treasurer Humphrey has asked that he be relieved of the duties of the Treasury, and that for the present he continue to serve only as Custodian. Consequently, the Board of Directors has elected Eugene M. Scifres, 1580 S. Milwaukee St., Denver, Colo. 80210, as Treasurer. Other officers have consented to continue in their present assignments.

Directors of the Class of 1965 have been re-elected to serve as the Class of 1968. They are:

Charles S. Bagley,
Ralph H. Beard,
H. K. Humphrey, and
George S. Terry.

There was a review of our promotional ideas. The Secretary has been working on the improvement of the circular duodecimal slide rule. The corrected plates are better than the original, but there is still room for more refinement. President Charles Bagley has expressed concern over the load that Tom Linton has been carrying for the Society at the sacrifice of personal advantages in the important professional work that he is doing. For this reason it is decided that perfection of this instrument await his convenience, with our heartiest thanks.

The production of a 6" plastic rule, carrying the scales of the millimeters, the standard U. S. inch, and the duodecimal palm has been deferred for lack of demand for this gadget now. However, the plastic 6" slide rule of the simple Mannheim type with duodecimal scales was considered valuable for school use, and favored by many of our Student Members. And steps toward its provision seem justified.

Periodically, the suggestion arises that we make available an emblem, badge or pin. It is now proposed that we explore the costs that would be involved in some device of this sort in bright colors and an attractive design.

These matters of promotional methods within our organization are of relatively minor concern, compared with the recent developments toward further decimalization of the world's standards. In London on Empire Day, – the 24th of May, – Douglas Jay, President of the Board of Trade, made the announcement that it had been decided to switch Britain to the use of the Metric System.

In recent years, the trend in the Commonwealth toward the adoption of decimal currencies has been general. And we were aware that the trend included surveys and polls of the general magnitude of the costs of changing to the Metric weights and measures as well. But we had thought that it was admitted that the terrific expense of such change as well as the general public aversion to the use of these measures in place of the familiar British and American units made this change inadvisable.

We are reminded of the provisions of Herbert Spencer's Will, which provided for the distribution of reprints of his pamphlet, "Against the Metric System," among members of both houses of the Parliament, if official action were taken towards the decimalization of the British currency. (See the Duodecimal Bulletin, Vol. *Ɛ*; No. *2*; page *27*)

Now Britain takes this further step toward the adoption of the Metric System – and takes it seemingly without formal action of the

Please turn to page 25

AN ETYMOLOGICAL STUDY OF BASES OF NUMERATION

by William H. Leonhardt

One precious but difficult resource for the evocation of the history of a people is the detritus of former locutions within the written and spoken language of the present, — a tool too little applied hitherto to the problem of the evolution of folk numeration. In addressing persons directly involved in the extension of present systems, — obliged to use and even invent new designations, — I feel that an idea of the etymology and growth of present usages could be helpful, and might even be thought necessary.

Somewhere in the almost limitless plains lying between the Indus Valley and the Rhine-Danube system, arose in antiquity a group of people speaking inflectional languages we now style Indo-European. Their vocabularies, scant in words of seafaring, of mountain-valley and forest, — but rich in pastoral, hunting and food-gathering terms, showed numerous affinities with those of the ethnic groups variously referred to as Semitic, Cino-Burmese, Hamitic, etc. That theirs was ever — in any modern sense — one homogenous tongue is extremely doubtful. Dialectically, they were quite split.

Little is known of these folk whose origins are loosely assigned to Iran, the Russian steppes, and northern India. It has been said that a major division of the group occurred which sent eastward a group classified as "Santum," and westward a "Kentum" portion. But even this has been ably challenged. About all we know of this language group is that our people today, along with many others of the world, speak a tongue descended from theirs by diverse and devious paths; the exact parentage is in question as often as not.

In the same realm of scholarly conjecture, I should like to set out a few of my own conclusions, — as much the product of studying man's most ancient document, the alphabet, as it is of linguistics proper. They may help to fill out the picture.

Apparently the first sound man learned to make was a voiceless gasp, and the second a mute velar-dental. These and succeeding word-sounds and their pictographs served as the first alphabet and a first collection of number symbols.

Thus "one" — Old English "unnu", German "ein", Latin "unus", Greek "oinos" and "hies, hen", Russian "odna", Hebrew "ehhod", Sanskrit "Aika", and so thru the roster of languages, — stands for the first count, a single object. So also "two", whose original Indo-European form the Larousse "Dictionnaire des Racines", gives as "dwi" or "duwo", (guided perhaps by the Sanskrit "dvih" which means "twice",) can be traced thru the Indo-European langaguages as well as the non-Indic tongues, an example of which is the Hebrew "sheni, stei".

It is far from disallowed that an original binary count was used. Simply, however, if such a usage ever existed, it has been obliterated by the overlying strata of time and change. But the similarity between "dwi" and "tri", the root of the third number, hints at the probability of a ternary system. Above all, it makes us want to look for signs of former calculations to any other bases in the linguistic history of our tongue.

We have not far to look. In English, four and five, six and seven, very much resemble each other. So do they in many other languages, not all of them close relatives of English. Gothic "fidvor", "before five", (4), and "five", tempt us with the suggestion that either four or five or both had in some past period served as bases of reckoning. Certainly, six and seven both had. The wealth of numerations to six and seven, and the symbols for them, provide us with hardly questionable evidence of this. Sanskrit "shash", and "sapta", Hebrew "shishi", and "shevighi", the six-pointed star, and the seven-day week, all lead to the same conclusion. Alphabetic research arrives also at six basic assylabic sound-words and their symbols, and one rest sign.

An eight sign seems to have been developed later, before the proliferation of tongues in the Middle East. I may have led you to believe that I meant to exclude eight as having been itself a basis of count. Far from it. A period when octal arithmetic was used must not only have existed, but must have been a long one, continuing until near historic times.

Nine is called the new number: Latin "novus", new; "novem" nine. German, new "neu", nine "neun". Greek, nine "ennea", new "nea", (and "en" or "hen" one.) Ten, "zehn" in German, is short for "zwein-neun" or two new ones; "decem" Latin, "duo-cum"; "deka" Greek, "duoin-co". To prove that ten has served as a base ought to be like bringing coals to a proverbial Neuchatel, or ice to the Eskimos. But nuclear coals do go to Newcastle, and deep-freezes have their place in the arctic. In proving the similar development of the tens-count, we'll learn to view that more recent development in perspective. Bear with us.

By this time, wide territorial spread had made intercommunication less likely, and less and less effective when it occured. In the Mediterranean region, the writing of one-ten, two-ten, as we still write thirteen, fourteen, etc., was never displaced. This is how we got our word "duodecimal", which because of the French word "douze" for twelve, and "douzaine" for around twelve, gave us "dozen". In Germanic lands things went the same way at first, perhaps, and then deviated. Or, the northern forest people began to count, "ain-lif, twa-lif", meaning of course "left over", and then counted no farther, twelves being the most convenient amount. They gave us the gross too, – gross twelve, big twelve, or twelve of twelves; and we've continued the pattern by speaking of the great gross; tho to be precisely alike we should say merely the "great". However we do speak of great-grandmothers and of ten grand in money. In these matters, tradition almost deserts us.

It is clear, that had the Germanic peoples not been exposed so early to the binding forces of recorded tradition, they were moving on to a twelve-base arithmetic. The amount of influence that the number of fingers on our two hands had on the adoption of ten as our basis of calculation seems to be debatable. On the contrary there has been a progressive trend to a larger base. This was halted and offset by the vast overhaul of numeration occasioned by the introduction of the Arabic symbols, with the old Assyrian zero, transmitted thru Moslem India.

It is clear that the pressures toward the larger base, tho much abated by modern methods and the mechanization of arithmetic, must continue slowly to grow. It is clear that only an accident stopped the progression at this temporary phase. I confidently reassure you of the millenium-long trend toward duodecimal numbering, that this is a mere momentary repose at the last cairn before lifting our rucksacks, now filled and carrying the two final figures ten and eleven, down to the oasis of the arithmoi.

I hope, that in writing this, I have not deceived you about the complexity of the material. To do justice to the subject would demand much more. Even my sketchy notes contain many times the material I have presented. But if it helps you to teach or to evolve duodecimals, or merely to sustain your own guttering torch, this paper is well given.

LET'S NOT GO METRIC!

By Dr. Robert C. Gilles

Buy a Volkswagen tyre in Germany and it is sized in inches–buy almost any canned food in Europe and it is marked up primarily in ounces and only nominally in grams. Yet still people argue that it is a decadent nation which will not adopt the metric system. Dr. Gilles sees no reason for such a changeover–even in export business.

UNLESS YOU AGREE forthwith that the future of the world is in some way dependent on the universal adoption of the metric system, certain people write you off as too old to learn, or they look at you in disbelief as if you were not quite right on top. If you counter that perhaps they are only a messianic tribe like the thirteen-month-calendar cult, they invite you condescendingly to stand one side and not obstruct the chariot of enlightenment and progress.

To avoid indictment for old fogeyism or dull-wittedness, let us see whether the case for metricism is such a one-way street as its proponents would make it appear.

To begin with–and this was odd, as Lewis Carroll might say–the metric system was not even decimal at birth. It was quadridecimal. The metre was intended to be one forty-millionth of the polar distance around the earth. (To say one ten-millionth of the distance from the pole to the equator is only evading the question). Assuming that circumference to be 25,000 miles, this would give 1,600 metres to the mile. Why the designers did not take one four-hundred-millionth, or better one billionth (U S) of the distance to the moon, is not as facetious a question as it may sound. Even in the late eighteenth century it was easier to calculate by plane trigonometry how far it was to the moon, under rigidly specified conditions, than it was around the earth. Be that as it may, the scientists discovered in due course that their measuring rod had slipped a couple of times and that the metre was not a metre after all, at least in the sense first planned. By that time it was too late. They could not very well ask everybody to change again. Today we know that the earth is slightly pear-shaped and not an oblate spheroid, so that repeating the estimate would produce a third value.

NOT A NATURAL MEASURE

It is curious, almost perverse, how narrowly the metre has missed the boat to achieving what might be termed a natural length. Thus a pendulum in the latitude of New York at 760 mm. barometric pressure swings exactly once a second if 0.9932 metres long. At the equator the length is somewhat less, but even at the poles it does not reach unity. The acceleration of gravity varies likewise, but averages 9.801 metres a second, nowhere becoming ten. The speed of light, which Einstein stated was the only constant in the universe, is not computed at 299,792,000 metres per second. As matters stand, the metre has no more validity in nature than the yard or the ancient Persian parasang.

The decimal system came from God insofar only as He gave us four fingers and a thumb on each hand. If our savage forbears had had five fingers and a thumb with each arm, we would undoubtedly be operating in this day and age under a duodecimal system, that is to say, one employing twelve as its base. Viewed objectively, ten must be acknowledged to be a poor base. It is divisible evenly only by two and five, where twelve has the much handier factors of two, three, four and six. This advantage is not great in mathematical computations, but stands forth in practical affairs. So long as you have ten of something you cannot partition them alike among three, four or six persons without splitting one or more units, and that is not always feasible. One example is packing in crates or boxes. Neither ten nor one hundred lend themselves to easy arrangement. Both the dozen and the gross, or lower multiples of the dozen such as 24 or 60, are much more flexible and are consequently favoured in wholesale trade.

Other instances of duodecimal realities are the sixty-second minute, the sixty-minute hour, the twenty-four-hour day, and the twelve-month year, all of which seem to be holding their own in metric countries. The division of the circle into 360 degrees, and the resultant angles of 120°, 90°, etc., enjoy world-wide acceptance. With these angles go latitude and longitude, and the nautical mile or knot, also in universal use. The English table of weights and measures is of course a multiple illustration.

So far in our analysis it has been made clear that the metre can make no pretence to superiority on natural or philosophic grounds which should compel us to accept it, and that the arithmetical principle which underlies it deals faultily with a formidable list of numerical facts and entrenched practices which it can hardly hope to survive. If we must have uniformity (a stupid deity without recognition in nature) let us insist on a scheme of measures and a scheme of arithmetic which conform with each other and with practical experience.

PROVING THE METRIC PUDDING

Before scrapping our own system, with all its faults, let us examine briefly how the metric system works where it has been longest established. It is at once apparent that both the metre and the kilogram have proven too large as units for daily use in retail trade. Even with dry-goods one is more likely in Europe to hear 160 centimetres, for example, than one metre, sixty centimetres. The litre is technically a cubic decimetre, but is always referred to as 1,000 cubic centimetres. The decimetre, one-tenth of a metre, is well-nigh obsolete. The English yard, foot and inch are much better spaced for practical purposes than the metre, decimetre and the upstart centimetre.

Such articles as coal and cement are sold in bulk by the metric ton (1,000 kg.) and the kilogram, according to amount—and Germany has a hybrid unit, the zentner, which however means 100 pfund (lbs.) and not 100 kilos. And thereon hangs a tale. For almost all retail trade is conducted there— for weight—by the pound. Belgium and Luxembourg do the same except that they call the pound the livre. In France itself, it is true, the situation livre v. kilogram is somewhat mixed. In Holland, the Dutch ounce or ons of 100 grams prevails in retail stores and prices are based on that.

The metric "pound" is 500 grams. Scales are all graduated to the kilogram; but when prices are quoted in pounds, there goes half, at least, of the vaunted superiority of the metric system out the window. To smooth out the arithmetic, the purchaser must double either the price or the weight. If the butcher gives you 325 grams of pork at DM 4.40 a pound, you must multiply 0.650 by 4.40 or 0.325 by 8.80. You ponder a second which is easier. Then while you are working out the problem in your head, the customers behind you signal their impatience and you usually wind up by accepting the butcher's pontification. You'd be just as well off at home with 11½ ounces.

Thus in lands where it is known the best, the metric system has not worn too well.

INCHES AND POUNDS IN EUROPE

It will startle many people to learn that the English system is better known on the Continent than the metric system is in English-speaking countries, apart from scientific circles. Leading off with a mild example, all plumbing and heating pipe in Germany is measured by the number of inches forming the inside diameter. "Inch" in German means "zoll", and may be rounded off by workmen as 25 mm., but an inch it remains.* Tyres

* The "zoll" used here should not be confused with the Rhenish zoll, which is slightly larger than the English inch but is outmoded.

for all types of motor vehicles and bicycles are sized in inches, whether built in Italy, Germany or France. An apparent exception has been the French Citroen, controlled by the Michelin tyre firm; but the latest Citroen tyres are branded both in inches and in millimetres, and the more exact number of millimetres now given shows that the inch is the controlling measure. No German seems to have protested that the national dignity is being impugned by the Volkswagen cruising happily on tyres that have never heard of the metric system.

Exporters to Germany of fresh fruit make no effort to use other boxes than they would at home. I have seen cakes and cookies from Holland and France on sale in Berlin stores with no other indication of net weight on the outside of the package than six and seven ounces, respectively. If the buyer wants to know the weight in grams, the grocer has only to lay the package on the scales.

Coming to canned goods, it is the rare exception when one finds on a grocer's shelves in Germany a can of anything meant to hold a round weight in grams, or even a multiple of 50 grams. It is safe to say that 80 per cent of grocery sales there in cans are in ounces, converted in most cases of course to grams. Easily the most popular can in Germany is the American No. $2\frac{1}{2}$ can, $4\frac{1}{16}$ in. in diameter and $4\frac{11}{16}$ in. high, which has been adopted as one of the standard (DIN) German cans. In it come fruits and vegetables from all over the world, including Germany herself. It holds the improbable weight of 822 grams of peaches or 1 lb. 13 oz., also printed on many labels.

More significant than can-dimensions is the variety of eatables packed in cans in ounce weights. A few examples are canned beef from France, 340 gm. or 12 oz.; German condensed milk, also 12 ounces and 6 ounces; beef and ham, 14 ounces; dog food, 16 ounces. Holland sends pears in cans marked both 15 ounces and 425 grams. Even Czecho-Slovakia sends raspberries to Germany marked exactly the same, and strawberries 1 ounce lighter. Small cans of peas from Jugoslavia weigh (net) ten ounces. It would be well to add at this point that dry cereals such as flakes and puffs from German millers, who are mostly subsidiaries of American and English companies, to be sure, are regularly merchandized in six or eight ounce packages.

Shot guns, regardless of source, are known by gauge, and rifles (except military) by calibre, though bore measurements in millimetres also occur. The American army's renaming its rifle the 7.62 mm. appears to have been a gratuitous action that discommodes everyone. There is probably not a rifle user on earth who does not know what 0.30 calibre

means—and I suspect there are plenty of American soldiers who still call their weapon just that. Organ pipes the world over are also known by their English names of 16 foot, 8 foot, etc.

In none of the cases referred to, directly or indirectly, in this section do exporters from English-speaking countries need to alter their manufacturing weights or sizes one iota to penetrate Continental markets. All they must do is to make sure that metric equivalents are plainly available to the consumer. Still less occasion is there to drop the use of English units in selling at home.

NO CAUSE FOR CHANGE

Reversing the picture, considerable point has been made of the fact that 35 mm. film is sold and asked for as such in all English-speaking lands—and that this may be the metric camel's entering head. In what way? Even a child can be sent to the store to buy 35 mm. film without a qualm; and the salesman knows perfectly well what to look for. Actually, the so-called 35 mm. film is not 35 mm. but $1\frac{3}{8}$ in., the size originally determined by the American inventor Thomas A. Edison for projector film. We thus have another illustration of an article fabricated according to English-system standards being marketed under a metric size. That does not mean that the family must change the speedometer on the car to read kilometres, or that the grocer must start packing sugar in metric pounds. Nor is the film-store owner under any compulsion to sell liquids from now on in cubic centimetre bottles.

The advantages and disadvantages of having one or two systems of measurement current in foreign trade have alike been greatly exaggerated. Both the English and the metric systems have been around long enough for those on each side of the counter to know what they mean. Buyers and sellers, whether at home or abroad, have shown much more intelligence in coping with the problems involved than the lawmakers and the scientists—who rarely think beyond their laboratories—seem to realize. Most of the necessary adjustments have already been made. Nor are the problems so great as some persons seem to imagine. They are probably less irksome than those which recur annually in the U S A because of daylight saving.

Where we must conform to the metric system in order to sell in other countries, it is usually no great difficulty for our factories to re-set the necessary machines for that part of the run. Even the Common Market countries are obliged to make daily calculations in dealing with each other because no two of them have the same currency.

LET ENGLISH MONEY ALONE

For English firms the problems caused by the tripartite unit of English currency—pounds, shillings, pence—are confined almost entirely to quoting prices to prospective customers abroad and collecting payment for exports. There are doubtless cases where the English firm could avoid difficulty here by pricing the transaction in the currency indigenous to the buyer. But the most effective solution in general would seem to be the rendition of bids and invoices in a decimalized shilling, forgetting pounds and pence for that particular purpose only. Not only are European countries accustomed to single monetary units and percentage fractions thereof, but the shilling has a similar order of value, promoting easy comparison.

FORCED 'REFORMS' UNPOPULAR

History records that in France it was necessary to compel citizens by a special law to use the metric system, over forty years after it had been officially proclaimed. Its use was made permissive in England in 1864 and in America in 1866. One might think that a century was quite long enough for its virtues to have won it full triumph by acclamation. On the contrary, its gains in both lands have been next to negligible.

The trouble is, that however kindly your intentions, people seldom do what you want them to. One historian comments rather mildly on the French experience as follows: "The use of familiar weights and measures is so engrained in the lives of a people that a radical change meets strong objections."

And there is the case in a nutshell. There simply is not enough to be gained in the development of foreign trade by more intensive use of the metric system to justify the gigantic upsets that would accompany its compulsory adoption in domestic business. Equally certain is it that no English-speaking nation needs to change its present scheme of weights and measures—including the British monetary system—to work with the Common Market, the E F T A, or any other metric aggregation, in perfect freedom and without handicap.

A DIMENSIONING CHALLENGE

by Tom Linton

The binary base of two is so much better than the base ten for electronic digital computors that practically all such computors use the base two for internal computations. The base ten is so seriously inadequate for high speed counting that all programming of the computor is either done in base two, or input and output means are used to convert from the awkward base ten and back again.

Is there a parallel to this awkwardness in our everyday work? The difficulty of switching a drafting department from the fractional – decimal methods to uniform decimal usage led me to examine other possibilities.

One difficulty in using decimals in engineering drafting is the exasperating division by three where one divided by three yields the continuous decimal .333 . . . . I think it was Randolph Churchill who commented on "those damned dots," and with the frequency of needing thirds, they do indeed seem to be damned, but a necessary evil in the base ten. Another difficulty lies in the frequent basic need of binary division; that is, successive divisions by two. The logic of the binary division is clearly seen on the draftsman's scale dividing inches into halves, quarters, eighths, sixteenths, and thirty-seconds. The difficulties arise in conversion from fractional to decimal values. Conversion for 1/32 = .03125 and 1/64 = .015625 are particularly obnoxious since "rounding off" to three or four decimal places is usually required, thus necessitating inaccurate conversions. These difficulties present a real challenge.

Obviously if the number base could be chosen such as to be divisible by more factors than is ten, some of the awkwardness of the base ten could be avoided. A brief scan shows twelve to be such a number. Twelve is evenly divisible by 2, 3, 4, and 6. Since ten is divisible only by 2 and 5, we have doubled the number of divisors by the selection of twelve as a possible number base. This apparently minor advantage is seen to have a large cumulative effect in our daily engineering and other number requirements.

As an example of the relative simplicity of the common binary divisions (successive halving), in the base twelve, note the following comparision with the base ten:

TABLE I: BINARY FRACTIONS

(expressed in decimals and dozenals)

	DECIMAL	COMMON	*DOZENAL*	
five tenths	.5	1/2	*;6*	six twelfths
twenty five hundredths	.25	1/4	*;3*	three twelfths
hundred twenty-five thousandths	.125	1/8	*;16*	dozen six per gross
six hundred twenty-five ten thousandths	.0625	1/16 *1/14*	*;09*	nine per gross
three thousand hundred twenty-five hundred thousandths	.031,25	1/32 *1/28*	*;046*	four dozen six per great gross
fifteen thousand six hundred twenty five millionths	.015,625	1/64 *1/54*	*;023*	two dozen three per great gross

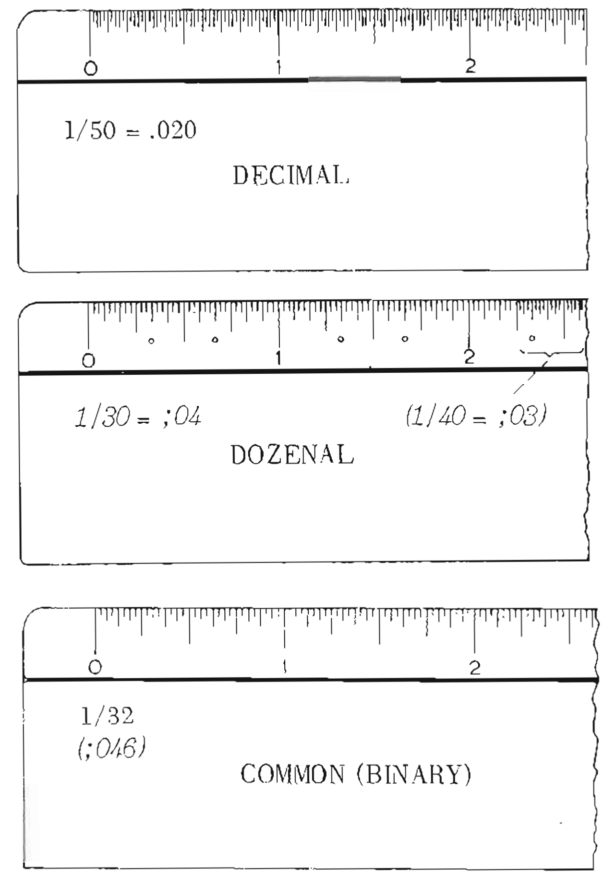
This relative simplicity and exactness of the dozenal twelve-base equivalents is striking. How many hours are wasted in fractional – decimal drafting in the dropping or adding the decimal tag-endings to make dimensions add up to a given value?

The natural next thought is to avoid the use of fractions entirely by using decimals only, and I have tried that "natural" step with a group of designers. It proved to be about as natural as walking sideways!

Among the drawbacks of decimal dimensioning were these: The decimal scale was slower to read, especially when spotting a succession of points; associated with the slowness were more frequent errors; checking a drawing was more tedious and subject to error and differences of opinion; more time was needed to correlate with tooling, vendors, and the material and production control departments. The fine decimal divisions, .020 on the scale, and the division by 5's instead of the more natural division by 2's, are physical handicaps which may be reduced but not eliminated, by practice and fimiliarity.

Look at two drafting scales, one divided fractionally down to 1/32 and the other divided decimally to .020 as in the SAE preferred scale.

The greater legibility of the fractional scale is unquestionable; the resulting speed and accuracy of reading follows inevitably.



Then note the "dozenal" scale of Figure 1. Divisions are as legible as the fractional scale but illustrate the dozenal equivalent of the decimal scale. Reference to the binary equivalents of Table 1 shows the easy conversion from the fractional to the dozenal scale values; it follows that THE GREAT ARGUMENT between the fractional and decimal adherents has disappeared in the versatile base twelve usage. In its place we have the compatibility of the fractional-dozenal scales. On the dozenal scale the small circles indicate the 1/3 and 2/3 place, or dozenally .4 and .8 (since 4 and 8 are respectively 1/3 and 2/3 of twelve). At the right end of the dozenal scale the binary fivisions of twelfths are shown as an alternate smallest division; such flexibility is denied us on the decimal scale.

I once firmly believed the ultimate answer lay in the metric system. Most if not all science students come first to this erroneous conclusion after a period of working simple problems in the metric units. Here are obvious advantages based almost totally on one characteristic: the units are related by exponential expressions of the base ten. A kilometer is a thousand meters; a kilogram is a thousand grams; and this pleasant relationship holds for many, but by no means all, metric units. Only two other advantages are unique to the metric system, and while minor, they are helpful and significant.

The nomenclature is tidy: From a given unit such as the meter, most others are derived by adding Greek prefixes to designate larger units, as kilometer, while Roman prefixes are used to designate smaller units, as centimeter. The other advantage is in the numerical equality of specific gravity and density at standard conditions of temperature and (sea level) gravity.

Why not the metric system? Mainly because it is based on what we have indicated is the awkward ten-base. Because of this awkwardness, the metric system is not yet a homogeneous system. Metric countries use the 24 hour day, and the minutes and seconds of time. People buy eggs by the dozen. The year has twelve months. The astronomical parsec and light year measures of distance may be expressed in either kilometers or miles, but are not exponentials of the ten base. The derived physical units of the cgs centimeter gram second and of the mks meter kilogram second system fail to live up to the simple relationship arguments of the metric proponents.

While physical relationships involving the transcendentals π and ϵ are not significantly affected by a change of the numerical base in which they are expressed, the ease of handling calculations including them may be greatly affected. If we want to avoid the obnoxious pitfalls of the base ten and the metric system, and maybe we should want to, we may further examine the base twelve.

To figure by twelves we need two new symbols for ten and eleven. Modified and used here are the Roman symbol X for ten and the capital script Ɛ for eleven.

While good unique names have been suggested and used for the two extra digits, such as dek for ten and el for eleven, no great objection exists to retaining the present names ten and eleven for the names of the quantities they represent. For twelve I personally find it easy to use the name "dozen" to designate the 10 spot in the dozenal number series,

since it seems to be a little more flexible than twelve, and perhaps less ambiguous. "Two dozen", for instance, can mean only one number. Decimal and dozenal counting compare thusly:

TABLE II: COUNTING COMPARISON

DOZENAL		–	DECIMAL
one	1	1	one
two	2	2	two
	…		
nine	9	9	nine
ten	X	10	ten
eleven	Ɛ	11	eleven
dozen	10	12	twelve
dozen one	11	13	thirteen
dozen two	12	14	fourteen
	…		
dozen eight	18	20	twenty
dozen nine	19	21	twenty one
dozen ten	1X	22	twenty two
dozen el*	1Ɛ	23	twenty three
two dozen	20	24	twenty four
two dozen one	21	25	twenty five
	…		
eight dozen four	84	100	one hundred
	…		
one gross	100	144	one hundred forty four
	…		
six gross el dozen four	6Ɛ4	1000	one thousand
	…		
seven gross	700	1008	one thousand eight **
	…		
one great gross	1000	1728	seventeen hundred twenty eight

* Short for "a dozen and eleven".
** A convenient close approximation for a thousand, or "kilo" units.

Fractions of ten (decimals) and fractions of twelve (dozenals) are seen to compare thus:

DECIMAL

one tenth	two tenths	three tenths	four tenths	five tenths	six tenths	seven tenths	eight tenths	nine tenths
1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10
.1	.2	.3	.4	.5	.6	.7	.8	.9

DOZENAL

one twelfth	two twelfths	three twelfths	four twelfths	five twelfths	six twelfths	seven twelfths	eight twelfths	nine twelfths	ten twelfths	eleven twelfths
1/10	2/10	3/10	4/10	5/10	6/10	7/10	8/10	9/10	X/10	Ɛ/10
;1	;2	;3	;4	;5	;6	;7	;8	;9	;X	;Ɛ

To help understand this number form, it is important to see that numbers (as a quantity concept) do not necessarily belong to the numerals (digits or other symbols) used to represent them. For example, eleven is eleven, whether written as XI, 11. or Ɛ. It took nearly a millenium for European people to generally accept Arabic 11 for Roman XI, and even now, since the latter is not outlawed (as the Arabic once was) many still find employment for it where identification rather than computation is the purpose (such as for outlining or organizing material, and such inertial use as for chapter headings or cornerstone dates for buildings).

What we have learned from the Arabs is not limited to the base ten. The electronic computer engineers found this out to their great advantage when they cultivated the ability to switch from decimal to binary thinking, and then, finding that while straight binary was good for optimum use of computer-insides but clumsy for pencil and paper, they went to a more convenient derivative called octal or base eight (count to seven; write 0 and carry on eight) arithmetic for detail work. Inasmuch as mechanical computers (e.g. ordinary adding machines) could be made to any reasonably wanted base ——— just a matter of things like how many teeth on a gear or rack ——— it made good sense to make them fit their ten-fingered human users. But, small and inexpensive electronic basic components for digital counting are intrinsically binary (such as "on" or "off", or, + or –). Human inertia was so great in this matter that even in this day of rapid progress, a lot of possible computer capacity was wasted for some twenty years before, during, and after World War II before it dawned on many beyond a few pioneers that the savings of straight binary on big jobs more than paid for the in-and-out conversion, and an intermediate hybrid; binary-coded-decimal (which can count to sixteen per unit, but deliveratively "wastes" eleven through sixteen to avoid conversions) was well worth it for other applications.

So, to go on with this dimensioning challenge, if we realize that two in the base two is written 10, three in the base three is 10, eight in the base eight is 10, besides ten in the base ten being 10, we are ready to find the advantages of writing twelve in the base twelve as 10, and read and think of it as one dozen.

Similarly for fractions; .1 indicates one half in the base two, one third in the base three, one eighth in the base eight, etc. Again we separate the quantity represented from a particular form or way of representing it. For example, if someone says "one tenth", you may write either "1/10" or ".1" and could either read it back the same, or distinguish by reading digitally "one per one-oh" or "point one" respectively. (The digital or "telephone style" reading is obviously safer in this day and age if the radix is not obvious or understood!) So, in the dozenal fraction form, the respective denominators of twelfths, per gross, per great gross, etc., may be implied by the use of the "point" and successive "places".

When we compare Table I with the scale drawings we see the compatibility of the dozenal-fractional scales compared with the awkwardness of the decimal-fractional combination. On a typical opposite bevelled 2-faced scale, the dozenal divisions on one side and fractional divisions on the other side make a smoothly functioning pair.

TABLE III: RECIPROCALS

DOZENAL		DECIMAL
.6	*1/2*	.5
.4	*1/3*	.33333....
.3	*1/4*	.25
.2497....	*1/5*	.2
.2	*1/6*	.16666....
.186X35....	*1/7*	.143857....
.16	*1/8*	.125
.14	*1/9*	.11111....
.12497....	*1/X = 1/10*	.1
.11111....	*1/Ɛ = 1/11*	.090909...
.1	*1/10 = 1/12*	.083333...

In the above Table III two advantages of the base twelve are apparent; fewer continuing "dozenal" fractions, and simpler equivalents for the commonly used fractions 1/3, 1/4, and 1/6.

Among our present system of units, in the base twelve.

1 month = *;1* year (instead of .08333.... decimally)
1 hour = *;06* day or *;1* the clock (half) day.
1 inch = *;1* foot
1 troy ounce weight = *;1* pound (decimally .08333....)
1 avoirdupois ounce = *;09* pound (decimally .0625)

In accurate machine protractors, the vernier usually reads to 5 minutes of arc. This is 1/12 degree, or in the base 12, 5 minutes (arc) = *;1* degree.

The 30-60 degree triangle is frequently used; in the base 12, 30° is *;1 (1/10)* circle, or *;2π* radians.

In a larger number base, the same number of digits can express high-integers. For example:

> *1000* (a great gross) in the base twelve is 1728 in the base ten, so there is a saving of one digit for all numbers between it and 999.

Similarly, for fractionals (if we can coin this word as a noun to generalize the term "decimals") in a larger number base the same number of places to the right can express a smaller number. For example:

> *.0001* in the twelve base is $\frac{1}{20,736}$, or about .000,0482 in base ten.

Stated another way, the successive digits representing twelfths, per gross, per great gross, represent finer as well as more useful divisions than tenths, hundredths, thousandths, etc. In a very convenient special case (from table I), six hundred twenty five ten-thousandths exactly equal nine per gross. (.0625 = *;09)*

A comparison of the multiplication tables shows interesting advantages in the base twelve:

In the light of the singularly usable numerical characteristics of the base twelve perhaps I shouldn't have been surprised to find the idea is not new.

The number 60, as the lowest number divisible by both ten and twelve, and its use in arc and time division, was used by the Babylonians and probably by the Sumerians before that. As trade and attendant commercial computations developed beyond the finger counting (certainly the only reason we have the base ten at all) the invention of the abacus preserved the unwieldy base ten through at least the later stages of the 2000 years of Roman numeral usage.

When the Arabs introduced their superior Arabic notation with the zero into Spain, it took some five hundred years for that usage to spread over Europe, and the spreading was in the face of hysterical arguments against the strange new symbols. Everywhere, it seemed, decent citizens banded together to stamp out these unholy numbers with their alien cipher, clearly instruments of the devil.

Gradually, however, the Arabic numbers were accepted, then helped to beget a resurgence of mathematical activity reminiscent of the earlier Greece, but continuing on to and including our day. It seems quite certain that the numerical facility afforded by the Arabic notations was a necessary for many of the calculations of that day, nor are numbers necessary to large segments of modern mathematics. But the change served first as a strong catalyst, then as an essential ingredient to the total mathematical and scientific efforts to follow.

Simon Stevin published the first decimal notation (late 16th century) and is reported to have recognized the superiority of the base twelve, but as today, was faced with overwhelming odds, so went along with finger counting based ten. A few years later (1614) Napier published his tables of logarithms (but the natural logarithms sometimes called Naperian were published a little later, by others) Almost certainly the logarithms would never have been invented without first having the decimal (or dozenal!) notation with its zero.

When Jean Picard, and later James Watt of steam engine fame suggested the basis of the metric system, no rush to acceptance occured, but in the wake of the French Revolution the method was adopted and improved upon by a committee appointed by the French Assembly. It is reported that Lagrange, president of the committee, argued eloquently and successfully against the adoption of the base twelve, so it may be presumed that some of the other committee members, such as Lavoiser and Laplace, must have been in favor of the twelve base.**

For the century and a half (or shall we say "gross years plus"?!) after the adoption by France of the Metric system we have had many arguments favoring the base twelve, culminating in the publications: F. Emerson Andrews's NEW NUMBERS, a very readable account; George S. Terry's DUODECIMAL ARITHMETIC, a monumental volume of tables; and Jean Essig's DOUZE, NOTRE DIX FUTUR (=: TWELVE, OUR FUTURE "TEN"). The fact that this last came rather recently from metric France is particularly noteworthy.

The base ten will not be displaced at all soon, maybe never, but perhaps the increasing complexity of computations and the increasing volume of them will eventually cry out for that simplification which counting by the dozen most certainly offers, especially to those of us who are NOT always able to live in constant access to a computor.

Some 900 years have elapsed since the Arabic numerals with their "cipher" catalyzed mathematics and science. Do we need another catalyst? My decimally divided drafting scale seems to say YES.

** BELL; *Men of Mathematics*, p. 169.

MATHEMATICAL NOTE

by George S. Terry

PROBLEM: To prove that 2^{770} is divisible by 771^2.

The only factors of 770 other than 2 and 3 are 7 and 11.

We note that $3^7 \equiv +1$ modulus 771, and that $\mathcal{E}^{11} \equiv +1$ mod 771;

i.e., $1 + 3 + 3^2 + \ldots + 3^6 = 771$, and $1 + \mathcal{E} + \mathcal{E}^2 + \ldots + \mathcal{E}^{10} = 771k$.

Also $2^{22} \equiv -3^5$ mod 771. Also $2^{12} \equiv -\mathcal{E}$ mod 771.

The powers 22 and 12 being singly even, suggest a singly even power of 2 to be divided by 771^2: namely 2^{132}; 2^{770} being $(2^{132})^6$.

So $2^{132}+1 = (2^{22}+1)(1-2^{22}+2^{44}-..+..-2^{XX}+2^{110})$. Seven terms.

$\equiv (-3^5+1)(1+3^5+3^X+3^{13}+3^{18}+3^{21}+3^{26})$ mod 771,

and also $\equiv (-3^5+1)(1+3^5+3^3+3^1+3^6+3^4+3^2)$, since $3^7 \equiv +1$ mod 771.

Also $2^{132}+1 = (2^{12}+1)(1-2^{12}+2^{24}-..+..-2^{10X}+2^{120})$. Thirteen terms.

$\equiv (-\mathcal{E}+1)(1+\mathcal{E}+\mathcal{E}^2+\mathcal{E}^3..+..+\mathcal{E}^{\mathcal{E}}+\mathcal{E}^{10})$.

Thus $2^{132} \equiv -1$ mod 771^2; 2^{264}, also $2^{770} \equiv +1$ mod 771^2.

Check for $\mathcal{E}^{11} = +1$ mod 771:

$\mathcal{E}^4 = 8581 \equiv 300$ $\quad \mathcal{E}^{10} \equiv 300^3 \equiv 563$ $\quad \mathcal{E}^{11} \equiv 563(\mathcal{E}) \equiv 5089 \equiv +1$ mod 771.

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FIRST PROOF. We have $1 + 3 + 3^2 + .. + .. + 3^6 = \frac{3^7 - 1}{2}$

$3^7 \equiv +1$ mod 771

Also $2^{22} \equiv -3^5$ $\quad 2^{44} \equiv 3^X \equiv 3^3$ $\quad 2^{66} \equiv -3^1$ $\quad 2^{110} \equiv 3^9 \equiv 3^2$ $\quad 2^{132} \equiv -3^7 \equiv -1$ mod 771.

Thus $2^{770}-1=(2^{264})^3-1=(2^{132}-1)(2^{132}+1)(2^{508}+2^{264}+1)$

and $2^{132}+1$ is divisible. (a)

Also $2^{770}-1=(2^{110})^7-1=(2^{110}-1)(2^{660}+2^{550}+2^{440}+2^{330}+2^{220}+2^{110}+1)$

$\equiv (3^5 + 3^3 + 3^1 + 3^6 + 3^4 + 3^2 + 1) = 771$

Two different factors divisible by 771.

$\therefore$ $2^{770} - 1$ is divisible by 771^2.

SECOND PROOF. $2^{770}-1=(2^{396}+1)(2^{396}-1)$

$=(2^{66}+1)(2^{330}-2^{286}+2^{220}-2^{176}+2^{110}-2^{66}+1)(2^{396}-1)$

$\equiv (3^6 + 3^5 + 3^4 + 3^3 + 3^2 + 3^1 + 1) = 771$

which with equation (a) in First Proof gives two different factors, but does not prove that $2^{132}+1$ is itself divisible by 771.

THIRD PROOF. $2^{770} = (2^{132})^6$

$(2^{132}+1) = (2^{22})^7+1 = (2^{22}+1)(2^{110}-2^{XX}+2^{88}-2^{66}+2^{44}-2^{22}+1)$

and $-2^{22} \equiv +\underline{3}^5$ $\quad +2^{44} \equiv +3^X \equiv +\underline{3}^3$ $\quad -2^{66} \equiv +\underline{3}^1$

$+2^{88} \equiv +\underline{3}^6$ $\quad -2^{XX} \equiv +3^{21} \equiv +\underline{3}^4$ $\quad +2^{110} \equiv +\underline{3}^2 + 1$.

Thus the seven term bracket $\equiv (1+3^2+3^4+3^6+3^1+3^3+3^5)$ i.e., 771, and is divisible. Also $2^{132}+1$ itself is divisible as shown above.

$\therefore$ $2^{132}+1$ is divisible by 771^2.

To check the result by straight arithmetic is not laborious, and it is good practice, in preparation for the interesting question to follow.

TO SHOW THAT $2^{770} - 1$ is divisible by 771^2 or 497421,

i.e., $2^{770} \equiv +1 \bmod 771^2$ BY SIMPLE ARITHMETIC.

```
2^30      =   1139X01Ɛ854(
               972842
               3871 7ƐƐ
               3726169
                1476528
Divn. by        124X063
497421           2284855
                 20008X5
                  283Ɛ704
                  2973527
r_30              -133X23
                  -133X23
                   39Ɛ669
                  267846
                70925X6
                39Ɛ669
               39Ɛ669
              133X23
r^2           17688533509
              1725484
                4338Ɛ85
Divn. by        400158X
497421           3375670
                 324X948
                  1269429
                  124X063
r_60               1Ɛ1X6
```

```
2^17      =        2134X8
r_60      =         1Ɛ1X6
                  1078540
                 190X0X8
                 2134X8
                1Ɛ21594
                2134X8
p_77            4095940000(
                400158X
                  943720
                  497421
Divn. by          4682ƐƐ0
497421            44989XƐ
                   1X62010
                   20008X5
r_77               -15X895
                   -15X895
                    7557Ɛ1
                 1150709
                 ƐƐ1X34
                12XƐ3X2
                7557Ɛ1
(r_77)^2       15X895
               22825X36081(
               20008X5
                2818ƐX6
                2498106
                 340XX00
Divn. by         324X948
497421            1800748
                  17254X4
                    972841
                    972842
r_132     =            -1
```

Tables

497421	
972842	
124X063	
1725484	
2000X85	
2498106	
2973527	
324X948	
3726169	
400158X	
44989XƐ	
133X23	1
267846	2
39Ɛ669	3
647Ɛ23	5
10925X6	X
2134X8	1
63X280	3
1078540	6
190X0X8	X
1Ɛ21594	Ɛ
15X895	1
5Ɛ6Ɛ18	4
7557Ɛ1	5
ƐƐ1X34	8
1150709	9
12XƐ3X2	X

Thus $2^{132} \equiv -1 \bmod 771^2$, $(2^{132})^6 = 2^{770} \equiv +1 \bmod 771^2$,

or $2^{770} - 1$ is divisible by 771^2.

PROEM: In the sums of series of the powers of 3,

$1+3+3^2 = 11$ is the first prime sum;

$1+3+3^2+3^3+3^4+3^5+3^6 = 771$ is the second;

The next is $1+3+3^2+3^3+3^4+3^5+3^6+3^7+3^8+3^9+3^X+3^Ɛ+3^{10} = 3253X1$.

The factors of 3253X0 are $2 \cdot 3 \cdot 4 \cdot 5 \cdot 7 \cdot 11 \cdot 61$.

QUESTION: Is it known that $2^{3253X0} \equiv +1 \bmod 3253X1^2$?

WEIGHTS AND VOLUMES: STREAMLINED AND CO-ORDINATED

By Robert C. Gilles, Ph. D.

The field of integers is dominated by three major numerical systems, the binary, the decimal and the duodecimal, which for present purposes will be called the triumvirate or the Big Three.

Eight, rather than two or four, may best be regarded as the basic number of the binary system. Certainly no one would recommend writing as 1 and 0 anything smaller. The decimal base of course is the number ten, and the duodecimal twelve, as denoted by their names.

The binary system, while the simplest of the three, is in many ways the most important because it epitomizes the balance of nature: positive and negative, male and female, left and right, etc. The English system of measures, so far as these relate to capacity, is also grounded mainly on binaries, four gills to the pint, two pints to the quart, eight quarts to the peck, etc. No matter what the unit, partition by halves – a half bottle, a quarter section – springs first to adult minds and is most easily grasped by children. A standard ruler thus divides the inch into 8th's, 16th's and possibly 32nd's or even 64th's, the plan favoured by architects and used regularly for thickness of lumber, steel, glass and other articles.

Two is also a cardinal element in the other systems. The decimal is built around two and five, which stand rather offish to each other and cause most of the awkwardness in the metric system. Its primary advantage is that only the decimal notation of numbers is universally recognized. The duodecimal tribe alone includes as a charter member the number three, along with two repeated, making it apparent at once why twelve is the most inclusive and at the same time the most flexible of the three bases.

Using the three prime numbers two, three and five, we find that of the first ten numbers only one, viz. seven, is foreign to the triumvirate, of the first twelve only two, and of the first score only six, all of these except 14 being themselves prime members. It is evident that by fashioning a scheme of money, or of time, or of weights and measures which combines successfully the merits of the Big Three we can achieve the utmost in factorability and adaptability. The oft-maligned English pound is a case in point. With 240 pence it admits binary factors as high as 16, binary-decimals up to 80 and decimal-duodecimals to 120, with a bewildering variety in between. Sixty, the number of minutes to the hour and the degree and the number of seconds to the minute, has no less than ten integral factors higher than unity. The decimal number 100, nearly twice as large, has only seven, all of them decimal except four. If we jump to 1,000, the next decimal landing, we introduce only one new non-decimal factor, eight. Rather a sorry showing for the metric system. Three is a complete metric stranger.

Seeking a three-sided solution for English measures, I came to the conclusion that the linear and surface measures in that system could hardly be improved to any extent worth the effort. We already have 12 inches to the foot and three feet to the yard. The rod, at 5½ yards, seems suspect at first glance until we consider that eleven is a factor of 5,280, the number of feet in a mile; and that 5½ is half of 11. It then works out that 320 rods = one mile and 160 square rods an acre. Since 640 acres = one square mile, the linear rod and the square rod give us excellent binary and decimal factors for both the mile and the square mile. Both English and American engineers work continually with the decimalized inch, producing decimal factors at the small end of the scale too. My problem was in this manner reduced to finding if possible some practical method of bringing order into the British-American world of weights and cubic measurements. As most of us know, the manifold and often conflicting standards of these types in the English-speaking world are an unholy mess. To tolerate them longer endangers our whole house.

In 1963 I published two articles on this general question, the earlier one called "Let's Not Go Metric!" and appearing in the January issue of the F B I Review (Federation of British Industries); and the second in the same journal for September. This article bore the title, "New Light on a Weighty Problem" and presented a table – Table 5 in the article – embodying a radical consolidation of the standards just discussed. The "Review" copyrighted this table in my name to protect us from commercial exploitation in the event that the table some day actually came into effect. We have no wish to discourage its reproduction, in whole or in part, merely for comment or for discussion, as that is the only way in which the table can become known. For the benefit of our readers I am therefore laying the table before them here. It should be understood that the weights shown are for so many cubic inches of pure water, the temperature of which would have to be agreed on by Britain and America:

PROPOSED NEW STANDARDS FOR BRITISH-AMERICAN WEIGHTS AND VOLUMES

One Ounce	= One-Sixteenth Pound	=	$\frac{27}{16}$ (1.6875)	Cubic Inches
" Gill	= Four Ounces	=	$\frac{27}{4}$ (6.75)	" "
" Pint	= One Pound	=	27 (= 3" Cubed)	" "
" Quart	= Two Pounds	=	54	" "
" Gallon	= Eight "	=	216 (= 6" Cubed)	" "
" Peck	= 16 "	=	432	" "
" Bushel	= 64 "	=	1728 (= 1 Cu. Ft.)	" "
" Barrel	= 256 "	=	6912 (= 4 " ")	" "
" Hogshead	= 512 "	=	13824 (= 8* " ")	" "
" Ton	= 2048 "	=	55296 (= 32 " " "	"

*= 24" Cubed

and (20 Grains = One Pennyweight
(20 Pennyweights = One Ounce
(64 Grains = One Drachm
(6400 Grains = 100 Drachms = One Pound

Note: The drachm does not appear in original table.

The preceding table can be fully successful only when the pound is altered so as to weigh 64 to a cubic foot of water, instead of 62.33 as in Britain or 62.4 as in America, where the standard temperature of the water for such purposes is 39.2° F. In Britain it is 62° F. The new pound would then be about 2.5% lighter than the avoirdupois pound, which is coeval with the metric system and has no great claim on our affection through hoary tradition. In considering the new ton, it must be emphasized that the controlling variable in the proposed system is the cubic content in pure water at a fixed temperature reached by official agreement. The weight of 32 cubic feet of water at 4°C. or 39.2°F. would be about 1997 pounds avoirdupois. If that temperature became the British-American standard, then the new ton would be the weight just stated. If 62° F. were adopted, the proposed ton would be a bit lighter. In any case it would have 2048 pounds, the eleventh power of two.

With this table we would slough off Troy weights, liquid ounces, scruples – who in this day and age wants to be bothered with scruples? – replace two gills and three drachms with one, and eliminate all differences between Britain, America and the Dominions. Only one ton, and no more distinction between wet measures and dry. The English system of weights and measures would be relieved for all time of its only valid reproach among men.

Let us now see to what extent the above table achieves the sought-for combining of binary, decimal and duodecimal systems under one roof. Binaries are certainly provided for in customary British fashion with successive units in the relation one to two, or one to four, etc. and in the same order as we know them today. Since there are 400 grains to the ounce, we can divide the latter repeatedly by two until we reach 25 grains without incurring fractions. Decimals are recognized by making 6400 the number of grains in the pound, and most of all by the introduction of the drachm of 64 grains. The pound is thereby decimalized. A weight of 325.72 pounds would be 325 pounds and 72 drachms. We also have 20 pwt. to the ounce. The new drachm would weight about 68 of the present grains. Although 15 of the new grains would be almost exactly the same weight as 16 of the old, the new grain would still be 14.5 times more precise than the gram, since it would require that many to counterbalance a gram. The new plan of weights therefore makes ample provision for fine weighing, as in pharmaceutical work. Whether it could recapture that field from the metric system, only time could tell.

It has already been stated that cubic capacities are the key to the table. All are in the duodecimal system, if we include the numerators in the ounce and the gill; and culminate with the bushel as one cubic foot. The stress given to water equivalents in determining weights may now be explained. Until now the metric system has had the unique advantage of easy convertibility from volumes to weights, e.g. a liter of water weighing a kilogram, etc. From the table it is seen that a pint of water would weigh an even pound, also that a cubic container three inches on a side would hold a pint. Six inches on a side it would hold a gallon, twelve inches a bushel, 24 inches a hogshead and with 32 cubic feet a ton. In the other direction, a cube 1½ inches on a side would weight two ounces in water, and one 3/4" on a side 100 grains. If we know the specific gravity of a substance, we can at once modify these figures to take care of the new substance; furthermore, a cubic foot of anything would be a bushel, as only one example, affording easy checks.

Standard containers in exact inches would surely come into general use. E.g., a quart carton for milk or ice cream would be 3" x 3" x 6". Two convenient sizes for bushel boxes would be available, 8" x 12" x 18" and 9" x 12" x 16". A gallon canister would be 4" x 6" x 9". The customer could protect himself from cheating with nothing handy but a foot rule. A tank 2' x 4' x 4' would hold a ton of water, and so ad infinitum.

FORM OF BEQUEST FOR WILLS

"I give or bequeath to the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island, New York, New York 10304, a voluntary nonprofit organization duly incorporated under the laws of the State of New York, the sum of $______________ ."

Contributions to the Society are deductible for income tax purposes.

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Parliament, but by a piece of bureaucratic legerdemain. This is a matter of the gravest importance for Britain, and for America as well. We have no information as to the measures contemplated to compel use of the metric units and to discourage the continued usage of the old familiar and convenient standards. But if there is going to be resort to compulsive legislation, we anticipate strong public reaction.