You can amuse yourself using other numbers as the base and you will find that 1/(b-1) will always come out as 0.1111.... Why this will always be so, no matter what the base is, provided it is greater than 2, is evident if we perform the division indicated in the fraction form of 1/b-1, as follows:

$$
\begin{array}{lllll}
b-1)\ 1 & & & & (1/b + 1/b^2 + 1/b^3 + 1/b^4 \\
\underline{1 - 1/b} & & & & \\
\quad + 1/b & & & & \\
& \underline{1/b - 1/b^2} & & & \\
& \quad + 1/b^2 & & & \\
& & \underline{1/b^2 - 1/b^3} & & \\
& & \quad 1/b^3 & & \\
& & & \underline{1/b^3 - 1/b^4} & \\
& & & \quad + 1/b^4 &
\end{array}
$$

Now let us substitute the proper number for b in the continued fraction we got by performing the division. And what is the proper number? In any system of positional notation that employs the zero, the base of the system will always be written 10, no matter what such base is. To be a proper positional system, it has to be written 10. This being so, the continued fraction $1/b + 1/b^2 + 1/b^3 + 1/b^4$ .... will have to be written $1/10 + 1/10^2 + 1/10^3 + 1/10^4$ .... and this is the same as 1/10 + 1/100 + 1/1000 + 1/10000 .... which, written in the fractional form using the numerator only without the denominator, comes out as 0.1 + 0.01 + 0.001 + 0.0001 .... and, doing the addition, the result is 0.1111....

The same thing will happen if the numerator is any other *single* number less than the base and less than the denominator (b-1). In such case, the numerator will be repeated: 0.222 or 0.333.... and so on.

I shall now leave you to draw any consequences you can out of this curious fact.

The Duodecimal Bulletin

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THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island 4, New York. F. Emerson Andrews, Chairman of the Board of Directors. Kingsland Camp, President. Ralph H. Beard, Editor. Separate subscriptions $2.00 a year, 50¢ a copy.

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The Duodecimal Bulletin

All figures in italics are duodecimal.

FIRST INTERNATIONAL DUODECIMAL CONFERENCE

M. Jean Essig extended the hospitality of his Normandie estate on Tuesday, 27 September 1960, to the representatives of the Duodecimal Society of America and of the Duodecimal Society of Great Britain for a conference that establishes another historic landmark for duodecimals. M. Essig is author of Douze, Notre Dix Futur, and since the publication in 1955 of this important book, has actively advocated the study of the duodecimal base in many public addresses.

Universality in the application of numeration is essential to the welfare of all humanity. And this is the great obstacle to be surmounted for the progress of duodecimals. It is well recognized, today, that duodecimals offer great advantages and simplifications, but the magnitude of the educational problem involved in establishing a new base for all number for world use, in itself defines the basic internationality of duodecimals.

The publication of M. Essig's book and the foundation of the Duodecimal Society of Great Britain were the initial steps in recognition of the dimensions of the problem. Subsequent correspondence and conversations paved the way for this important conference. The historic parallel will be apparent to everyone.

In the early centuries of the Christian Era, the Roman numerals were generally used. The Babylonians developed the concept of the zero and of a positional notation using the 60-base. The Moorish conquests carried the Arab's use of the Hindu-Arabic numerals for the ten-base into Spain. From there, Gerbert of Aurillac, a Frenchman who studied mathematics at Barcelona, Cordoba and Seville - and later became Pope Sylvester II - carried this notation into France and Italy. By 1500, the use of our present numerals had become general. Duodecimals face a similar long road of slow-but-sure progress, which we prefer to consider as starting with Simon Stevin in 1585.

Thus, this conference at La Herpiniere, Beaumontel, Eure, France, on the 27/28th of September, 1960, becomes an important event in our history. M. Essig consented to be Chairman of the Conference, and he delegated his secretary, M. Baillancourt, to act as Secretary to the Conference.

Confreres Camp, Bishop, Essig and Baillancourt

Kingsland Camp opened the session by extending, in English and French, formal greetings from the Duodecimal Society of America to his confreres. He announced that Doctor Eduardo Buda, Piazza Ennio 1, Rome, Italy, has written an important book, "The Duodecimal Arithmetic," in English, and seeks a publisher. He suggested that all develop as close a contact with Dr. Buda as possible, to develop a focus of duodecimal interest in Italy.

M. Essig remarked that the formation of a solid dozenal society in France now would be inopportune because of the present changes in the French way of life. Moreover, it is more difficult to develop a flourishing society of this type in France than in America. It is anticipated that the French press will adequately comment on the occurrence of this conference.

It is important that the duodecimal bibliography cover as comprehensively as possible, the duodecimal publications of all lands and languages. For example, our information on works in German and Italian is rather sparse. All agreed on immediate interchange of all duodecimal discoveries.

Mr. Camp inquired as to the latest developments in the selection of a unit of length. M. Essig commented that a

new unit is needed, as the inch and the foot are the only existing units with duodecimal coefficients. Mr. Bishop felt that, although the wide usage of the inch and the foot - of the penny and the shilling - should not be disregarded, there was need for a new international unit; but that independent proposals in our three countries had already arrived at one. This unit, he said, was the 10^{-7} part of the circumference of the earth. The separate determinations of this length were in fair agreement, with values between *1.149* and 1.14Ɛ meter, or *3.8* and *3.X* feet. All joined in recommending that the proponents should collaborate in bringing their separate proposals into agreement. It was also suggested that the feasibility of dividing this new unit into *40* sub-units should be explored.

Attention was given to the form and requirements of a duodecimal international currency. Such a currency, issued by the U. N. would naturally be an ideal. But M. Essig emphasized the great difficulties involved, because of the degree of financial agreement necessary among the member nations. He mentioned the franc-ancien, once proposed as a common currency for the Latin countries. The denominations desirable in such a dozenal currency were discussed.

In the discussion of the duodecimal division of the circle and the day, another step toward unanimity was notable. M. Essig now favors dividing the day into twelve bi-heures or duors. For this unit, Mr. Camp suggested a unique symbol combining 2 and π in this fashion: . He also mentioned that, since midnight and noon are indicated at two opposite places on the *10* duor clock-dial, in the D. S. A. there has been proposed a revised dial with *0* (midnight) at the bottom of the dial, and noon (*6*) at the top. Thus, facing south, the duor-hand would indicate the direction of the sun, or conversely, pointing the duor-hand at the sun, the zero would point south, as a sort of compass.

This led to consideration of an international duodecimal emblem, and the following suggestions were introduced:

Mr. Camp:	A circle with twelve points on the circumference joined up into 4 triangles, 3 squares, and 2 hexagons, the points numbered clockwise from zero at the bottom.
M. Essig:	Twelve blue stars on a golden background. This is the inverse of the flag of the Council of Europe.

M. Bishop: A twelve-pointed star with the four cardinal compass points elongated.

M. Baillancourt: A circle halved, the bottom half being shaded by a dozen (or multiple thereof) bars. With numerals numbered from zero at the bottom, it would resemble the clock-face cited above.

It was agreed that simplicity was essential, and proposals are solicited, embodying the above ideas.

The conferees paid particular attention to a review of numeral symbols for dozenal numbers. All current usages and proposals were carefully reviewed. The adequacy of the present usages of the Duodecimal Society of America has been much debated recently, especially in Great Britain where their Duodecimal Newscast has progressively published tables of the various proposals for names and symbols for dozenal numbers, and of the units of measure. The conferees agreed on three basic principles.

a) The numerals should be easy to write.
b) For the zero and the first nine numerals, current usage is acceptable for the present.
c) The new digits should not be confusable with other numerals or letters of the Greek or Latin alphabets.

Proposals are invited for ideas conforming to these principles.

Mr. Bishop advocated the utmost possible cooperation among our societies in the pooling of publicity, publications, facilities, funds and ideas. The present collaboration is excellent, and its extent and intensity should be enlarged.

The Conference then took the initial steps in the formation of the Association Duodecimal Internationale, to meet the need for international collaboration - to unite duodecimal research and development - and to foster international relations with other bodies.

Please turn to page 36.

ON TIME AND DISTANCE

by Peter Andrews
The Graduate College, Princeton, N.J.

In an earlier article in this Bulletin, the present writer argued that the duodecimal system of units of measurement eventually to be adopted should, insofar as possible, be a "natural" one, based on, and in harmony with, the underlying realities of the universe in which measurements are to be made. It now seems apparent that there is a natural relationship between units of time and units of distance which is so fundamental that any system of units not incorporating it will eventually be regarded as less than fully satisfactory. To gain proper perspective on this relationship, it is necessary, strangely enough, to consider the relationships between units of mass, energy, heat, and work,

One of the great achievements of science has been the gradual recognition that work, heat, kinetic energy, and mass are all different forms of the same thing---energy---and that our measure of the quantity of energy in a system need not depend on the particular forms that energy assumes at any given moment. Now for those concerned with designing systems of units of measurement, the Law of Conservation of Energy has a very specific meaning: we may use the same units of measurements to measure energy in all its forms. Then if a given quantity of electrical energy is converted into heat, we need not multiply by a conversion factor to determine the amount of heat that is produced, for the quantity of energy did not change when its form changed. The idea of measuring work, kinetic energy, and heat in the same units is not hard for the present generation to accept; the idea that mass and energy should be measured in the same units may take a little time to gain general acceptance, but as the use of atomic energy becomes more widespread, so will the idea of the equivalence of mass and energy, until eventually the idea of proposing a new system of scientific units with different units for mass and energy will be regarded as absurd. Of course, we may give different names to our units of mass, heat, kinetic energy, etc., if it pleases us to do so, since our knowledge that these are all different forms of the same thing does not prevent us from distinguishing between the forms; the important point is that the conversion factor between these units is unity.

Let us now consider Einstein's famous equation $E=mc^2$, which may be interpreted as follows: if we convert entirely to energy a quantity of matter whose mass in our system of units is m, the amount E of energy produced may be obtained in our system of units by multiplying m by c^2, where c is (by a remarkable circumstance) the velocity of light in a vacuum as measured in our system of units. Thus c^2 may be interpreted as a conversion factor between units of mass and units of energy. (This helps to explain the ubiquitousness of c in the formulas of physics.) Since we have just agreed that the conversion factor between units of mass and units of energy should be unity, we are led to require that $c^2=1$, and hence that $c=1$. That is, the velocity of light in a *natural* unit of velocity, in terms of which we may measure all velocities. Our feeling that this is a natural unit of velocity is enhanced when we recall that according to the Theory of Relativity, this is the upper limit to velocities that physical objects can ever attain. That it is therefore commonly regarded as an extremely large velocity need not bother us at all, for we can divide it by an appropriate power of twelve to obtain a satisfactory duodecimal unit for velocities near the magnitudes we commonly encounter.

It is generally accepted that unit distance should be the distance traveled in unit time by an object traveling at unit velocity. Thus, once our unit of time has been chosen, our unit of distance is determined. Since there now is a consensus that the duodecimal unit of time is the day, we conclude that the duodecimal unit of distance is the light day, the distance light travels in a day. Dividing this distance by 12^{12} to obtain a convenient order of magnitude, we obtain a natural duodecimal unit of length which is approximately 9.531 feet (2.905 meters) long. We can obtain other units from this, of course, by multiplying or dividing by various powers of twelve; in particular, there is a basic unit of length 9.531 inches long. It is hoped that the Duodecimal Society will settle upon names for these units in the near future.

One minor additional point concerning time may be mentioned. It is customary to write numbers from left to right, with the larger place values on the left. We gain a bonus of simplicity and order from our choice of the day as the duodecimal unit of time if we extend this convention to the way we write the date and time of day. For example, if we wish to record that it is now 2:15 P.M. on the eleventh of January, 1961 in duodecimal notation we may simply write

1175 January *Ɛ;716*

LOGICAL MONEY, WEIGHTS AND MEASURES

by Brian R. Bishop, Secretary,
Duodecimal Society of Great Britain

What Is A Rational System Of Units?

The many advantages - and more - of the decimal metric system can be enjoyed, yet its serious disadvantages avoided, if we adopt what we may call the dozenal or duodecimal metric system. This seemingly bold assertion is based on logical facts which have had many eminent advocates over the years, and indeed has precedents in British tradition.

Recent reports, principally that of the Committees of the British Association for the Advancement of Science and the Association of British Chambers of Commerce, have shown that there are high costs involved in the change-over to any logical system of units. Currency reform alone would involve costs "in excess of 100 millions" in Great Britain. With expenses of this order we want value for money - the best possible.

The decimal metric is neither the best nor the only rational solution available. The Hodgson Committee report says ($36), "... the metric units of length and mass are now defined by physical standards which are in fact as arbitrary as the Imperial Standards." Care should be taken not to confuse rationalization and decimalization. Rationalization is to arrange all units in multiples of any same number, whether the number be ten, twelve or any other. Decimalization is that particular form of rationalization which uses multiples of ten.

Ten Is A Poor Base For A Rational System

"Ten is only divisible by 2 and 5; twelve, by 6, 4, 3, and 2. The power of telling easily what is the price of a quarter of a thing, or a third of a thing, when you know the price of the thing itself, is very important in the daily transactions of the ordinary market; and no decimal system can be so easy in this respect as the system which we have ..." (The Economist, September, 1859). The British Association and the Chambers of Commerce recognize the importance today of the interests of the man-in-the-street

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Footnote: Prepared as text of a folder for the British Society.

and the housewife: "Any change in coinage or weights and measures would have far-reaching implications for the public and no change should be made without seeking the views of the public." The decimal metric system, founded on base ten, is restricted to only two whole fractions, one of which is not often required in everyday life.

It is natural and more convenient to divide by the three factors nearest unity - 2, 3, and 4. Factor 6, half of the twelve-unit, replaces factor 5, which is useful only because it is half of ten and useless because it does not divide exactly any further. Vulgar fractions continue in common use instead of decimal fractions largely because the latter need clumsier expressions for as frequent a fraction as 1/3; even 1/4 needs two decimal places. Percentages are based on the hundred, which is divided exactly by only seven numbers, and expresses common fractions awkwardly; pergrossages are based on the gross, which is divided exactly by fourteen numbers, so that 1/3 is 4 dozen pergross instead of 33.333%, and 1/16 is 9 pergross instead of 6 1/4%. Shops find it more practical to group articles 3 x 4, or 6 x 2, 3 x 2 x 2, ten being incapable of three dimensions or optional subdivisions. Time and again we use dozens and grosses (capable of universal applications) and their numerous whole-number fractions as quantities for selling, because it is inconvenient either to sell or pack items in quantities restricted by the poor divisibility of tens and hundreds.

Planning A System Of Units On Twelves

It is no wonder that the three most-used of our units, the foot, the shilling and the day are traditionally divisible into twelfth-part submultiples. The foot gives in whole inches the common fractions 1/2, 1/3, 1/4, 1/6 and 1/12 and can also represent 1/8 and 1/16 more easily than the decimal metre. The same applies to the shilling. If wished, a new basic metric length, from which other units of weights and measures can be derived, may be calculated (with greater accuracy this time!) from the circumference of the earth or any suitable constant, and decided in the light of modern scientific methods. The Hodgson Committee report has some suggestions (§§ 44, 45, 46, 55).

The Report of the New Zealand Decimal Coinage Committee (§ 3, 1959) gives first-page importance in its summary to the greater divisibility of our present coinage (mainly because of its dozenal aspect) and points out its advantageous connection with weights and measures: "The only significant disadvantage is that a decimal system has less divisibility than 10 and 100, which connects pounds, shillings and pence closer to present weights and measures." The report of the Australian Decimal Currency Committee stresses this matter of divisibility and connection with units of weight and measurement at more length (August, 1960, § 77). In fact, looking at a table of the Imperial Standards, the frequent occurrence of twelve or of factors of twelve (especially in Liquid Measure, Troy and Apothecaries' Weights as well as Length), shows that the British and American systems tend naturally towards dozenal, not decimal. Attempts to divide the circle and sphere into a hundred parts have failed because it is impracticable; but dozenal division is in fact less cumbersome in expression than the present 360°; e.g. one twelfth of a circle is the common angle now shown as 30°. No one can reasonably imagine a clock with hours other than by the dozen, or the year with other than twelve months,. corresponding with the division of the circle just mentioned.

The Australian Committee sees a conflict between divisibility and rationality when they say (§ 115): "It was noted from the outset that there was only one permanent and inherent disadvantage - divisibility - and this was a minor one which was easily outweighed by the inherent advantages of having the currency system on the same basis as the decimal notation." Yet these principles are really complimentary and can be so easily reconciled by having the currency on the basis of the dozenal notation. The twelve factor in units of distance, money and time facilitates all calculations, not least wages and salaries, which can be made even easier and more rational if shillings are grouped into denominations of twelve. Already the public counts in pennies and shillings, referring to pounds only when the number of shillings becomes too large to be easily written or spoken. To the penny and shilling a unit of twelve shillings can be added, well suited to the market-place, and a unit of one gross shillings, well suited to finance, banks, and those dealing with large sums of money. In this way rational currency can be achieved without the need to change any coin at all - only the short-lived banknotes - thus save the considerable expense of changing coin-operated machines and of reminting. A three-shilling piece could be minted, gradually replacing the half-crown, to obviate the present confusion between it and the florin.

Rationalization of currency by decimalization has provoked many conflicting proposals (as in The Financial Times correspondence in May, 1960), because of the decimal lack of factors. A comment by the Metric System and Decimal Coinage Committee of Ireland is very pertinent (Eire, 1959, §147): "So far as the public is concerned, the existing system of coinage not only works but works well. For everyday shopping, the shilling, based on the twelve units of the penny, is by reason of its great divisibility a very convenient coin for small transactions." This links with the statement of the British Association and Chambers of Commerce Report (p. 18): "The vast majority of these (i.e. retail or cash) transactions are concerned with sums under £1."

Easier Arithmetic

It will be seen that there are many advantages for everybody in arranging all units of money, weights and measures into denominations of twelve, and not ten. For the same reasons, just as the so-called Arabic decimal system of numbering which has ten symbols operating in a logical convention of "place value," is more efficient than the Roman system, so the dozenal system of numbering, which uses twelve symbols in an exactly similar convention, is more efficient than the decimal system.

The dozenal system of numbering has an extra symbol for ten and another for eleven. Then the two characters 1 and 0 when together (10) represent 1 dozen and 0 units, and 100 represents 1 gross, 0 dozens and 0 units. The natural limit of the decimal multiplication square is ten by ten; but we extend it to twelve by twelve, and learn it so. The mind, being quite capable of containing concepts up to a gross, can learn a rephrased square equally, if not more, competently. All mathematical functions possible in decimal numbering are just as possible in dozenal and tables of all such dozenal functions have already been published.

To sum up, dozenal arithmetic, both in practical measurings and abstract numbers, is both easier and more efficient. i) Mental calculations are aided; for example multiples of 3 or 9 end in 3, 6, 9, 0; of 4 or 8 in 4, 8, 0; and the digits of multiples of eleven add to eleven or a multiple of it. ii) We have shorter positional fractions; to compare some everyday vulgar and dozenal fractions:

1/2	decimal 0.5	dozenal 0;6		1/3	decimal 0.333	dozenal 0;4
1/4	0.25	0;3		1/6	0.1666	0;2
1/8	0.125	0;16		1/9	0.1111	0;14
1/16	0.0625	0;09		1/12	0.0833	0;1
1/32	0.03125	0;046		1/24	0.04166	0;06
1/64	0.015625	0;023		1/36	0.02777	0;04
1/128	0.0078125	0;0116		1/48	0.020833	0;03

iii) Greater quantities are contained in a given number of digits, so that using a four digit car registration index, the greatest number of cars is 9,999 in the decimal rendering, but in the duodecimal rendering 20,735, i.e. over double; and the same phenomenon has many applications such as in numbering licenses and banknotes, statistics, calculators, etc. iv) Most noteworthy is the finer metric system which puts into practical use all these advantages and makes the most of rationality.

Factors, Numerical And Logical

The purpose of this explanation is to suggest some of the principles which should be kept in mind when seeking that system of money, weights and measures which is best suited to the needs of modern man and his descendants. The choice of fundamental or basic units we consider to be within the capabilities and responsibilities of the appropriate government bodies, such as the Department of Scientific and Industrial Research, in consultation as necessary with representatives of science, commerce and people.

At the same time, and in considering a report of any democratic referendum such as that of the British Association and the Chambers of Commerce, or of any organization (such as a newspaper acting on its own initiative), it should be remembered always and insistently that respondents are acquainted with only that part of a present system which they happen to be using. Lacking any other experience, they cannot be expected to consider objectively or knowledgeably an alternative system, not to be capable, at least without special study and actual experimental trial, of asking themselves whether a better system than those existing could be devised to suit a far wider range of needs. The problem is of crucial importance in its long-term applications. Solutions must be approached with a completely unprejudiced mind if we are to avoid defective, even disastrous, solutions.

Let us not blindly accept the decimal metric system in ignorance of a fully-developed better alternative. Let us, on the other hand, not resign ourselves to accepting for all time the existing Imperial Standards system for fear of the temporary inconvenience and the cost of any transition. The dozenal metric system, reconciling the best in both, offers an assuredly better alternative which justifies the temporary inconvenience and cost. The longer we wait, the greater the the problems. If some countries have followed France in a second-best realization, how much more readily will countries follow Britain and America in the best. The future will soon amortize into insignificance our brief sacrifice as its benefits and esteem multiply.

FIRST INTERNATIONAL DUODECIMAL CONFERENCE

(continued from page 2X)

The following persons are to be invited to be the Honorary Committee:

Honorary President	M. Charles Volet, President of the International Bureau of Weights and
Honorary Vice-President	Sig. Eduardo Buda, author of duodecimal proposal in Rome.
Honorary Vice-President	Mr. Ralph Beard, Secretary of the Duodecimal Society of America.

The following Executive Committee was elected:

President	M. Jean Marie Essig
Vice-President	Mr. Kingsland Camp
Secretary and Treasurer	Mr. Brian R. Bishop

The Association will federate national Societies, and individuals in countries without a society. It was agreed that the former be requested to pay annually the equivalent of one-twelfth of their membership dues, and the individual members to pay annual dues at a rate averaging those of the national societies. Donations are to be invited. The President emphasized that the prompt establishment of a sound financial footing was essential.

The closing of the Conference was marked by Kingsland Camp's presentation of the Annual Award of the Duodecimal Society of America to Brian M. Bishop for his researches in the bibliography of duodecimals, for his establishment of the Duodecimal Society of Great Britain, and the foundation of its official organ, The Duodecimal Newscast.

THE DOZENALIST LOOKS AT COMPUTER ARITHMETIC

by George S. Cunningham
73½ Perley St., Concord, N.H.

The arrival of the modern digital computer with cells either "charged" or "not-charged" operationally uses the Hariot binary system because of its physical nature. In this article, the Hariot system will be explained in terms of the duodecimals advocated by the dozenalist in order to demonstrate that no real difficulty is presented by this modern machine. Numbers represented in binary notation will be roman type while italic representations will be duodecimals.

Binary arithmetic is a place-value system similar, in this respect, to our ordinary duodecimal system as well as to the obsolete but persistent decimal system. The radix of the binary system of notation is 2. The only digits used are 1 and 0. When used as a single-digit numbers 1 is exactly equivalent to *1* and 0 is exactly equivalent to *0*. 10 is used to represent *2*. The place value of a digit in the second position to the left is therefore *2*. Exactly as *10* represents *1* times a dozen plus *0* units in our system, 10 represents 1 times two plus 0 units. The analogy extends to further places also. 100 represents 1 times 2 squared plus 0 times two plus 0 units. A little calculation will convince the reader that 100 must be exactly the number represented by our *4*. The key to understanding is the realization that the binary numerals represent numbers, the same numbers represented, in a different way, by our dozenal numerals.

To further our understanding of Hariot numerals, we will determine the place values of several such numerals.

0 represents the number *0*
1 represents the number *1*
10 represents the number *2*
11 represents the number *3*
100 represents the number *4*

1000 represents 2^3 and equals *8*
10000 represents 2^4 and equals *14*
100000 represents 2^5 and equals *28*
1000000 represents 2^6 and equals *54*
10000000 represents 2^7 and equals *X8*

Like our dozenal numeration, the Hariot numerals may be extended indefinitely and are capable of representing any number, at least in theory, no matter how large. We note that the number represented simply by two digits, *ƐƐ* requires 8 digits for Hariot notation. *ƐƐ* equals *Ӿ8* plus *8* plus *4* plus 2 plus *1*. *14*, *28*, and *54*, not appearing will be represented by 0s. The corresponding Hariot numeration is 10001111. It is clear that our notation is much more compact. It is difficult to see how the computer using such an apparently awkward system can calculate so rapidly until it is pointed out that the computer will make from a Gro-mo calculations per grovic on the older models to 2.*6* Bi-mo calculations per grovic on the latest models. Humans operating at much slower speeds are well advised to stick to dozenals.

Operations with Hariot numerals are much simpler since the whole of addition is encompassed in four facts and the whole of multiplication in four more. These facts are as follows:

0 plus 0 is 0	0 times 0 is 0
0 plus 1 is 1	0 times 1 is 0
1 plus 0 is 1	1 times 0 is 0
1 plus 1 is 10	1 times 1 is 1

If it were not for the necessity of using cumbersome numeration for comparatively small numbers, our system would be endangered by this extreme simplicity of operation. Indeed this system may be better adapted for those who through lack of capacity are unable to learn the *200* facts involved in our computations. However, it is unlikely that such individuals can be trusted with any system of numeration however simple.

The computer programmer must translate numbers into Hariot numerals. The process is quite simple and involves continuous mediation (division by two). The remainders of the mediation process, if no remainder is considered as zero, will constitute the Hariot numerals from left to right. For example, we wish to know the Hariot numerals corresponding to this year, Anno Domini *1174*.

1174 ÷ 2 is *698* with remainder 0
698 ÷ 2 is *34Ӿ* with remainder 0
34Ӿ ÷ 2 is *185* with remainder 0
185 ÷ 2 is *Ӿ2* with remainder 1
Ӿ2 ÷ 2 is *51* with remainder 0
51 ÷ 2 is *26* with remainder 1
26 ÷ 2 is *13* with remainder 0
13 ÷ 2 is *7* with remainder 1
7 ÷ 2 is *3* with remainder 1
3 ÷ 2 is *1* with remainder 1
1 ÷ 2 is *0* with remainder 1

The Hariot numeral is then (reading the remainders from bottom to top) 11110101000 AD.

In order to render the machine answers intelligible to those accustomed to dozenals, the programmer must translate the Hariot numerals back into dozenal notation. The process is simply one of adding the various place values represented by 1's in the Hariot numeral. For example, if we wish to know the number represented by 111011111111. This is obviously the sum of $2^{Ɛ}$, $2^{Ӿ}$, 2^{9}, 2^{7}, etc. Note that 2^{8} represented by 0 is not added. Our process then is:

$2^{Ɛ}$ = *1 228*
$2^{Ӿ}$ = *714*
2^{9} = *368*
2^{8} = ----
2^{7} = *Ӿ8*
2^{6} = *54*
2^{5} = *28*
2^{4} = *14*
2^{3} = *8*
2^{2} = *4*
2^{1} = *2*
2^{0} = *1*
111011111111 = *2 27Ɛ*

The number we want is therefore *2 27Ɛ*.

This process of retranslating our Hariot results into dozenals is greatly aided by a table of powers of two. A short table is given. N = 2^{x} is computed below for values of x from *0* to *1Ɛ*.

x	N	x	N	x	N	x	N
0	*1*	*6*	*54*	*10*	*2454*	*16*	*107854*
1	*2*	*7*	*Ӿ8*	*11*	*48Ӿ8*	*17*	*2134Ӿ8*
2	*4*	*8*	*194*	*12*	*9594*	*18*	*426994*
3	*8*	*9*	*368*	*13*	*16Ɛ68*	*19*	*851768*
4	*14*	*Ӿ*	*714*	*14*	*31Ɛ14*	*1Ӿ*	*14Ӿ3314*
5	*28*	*Ɛ*	*1228*	*15*	*63Ӿ28*	*1Ɛ*	*2986628*

THE ANNUAL AWARD FOR 1959

At the meeting of the Board of Directors, 21 April 1960, George S. Terry as Chairman of the Committee recommended that the Annual Award for 1959 be conferred on Brian R. Bishop. This action was approved by the Board, though mention of this decision was eliminated from the published transactions, so that the presentation of the Award might come as a surprise to Mr. Bishop. It was planned that President Camp would personally make the presentation at the First International Dozenal Conference, scheduled for September 1960.

Our plans worked out most satisfactorily. At the close of that conference Mr. Camp presented to Mr. Bishop the engrossed scroll, which was worded as follows: "The Annual Award of the Duodecimal Society for the year 1959 is conferred upon Brian R. Bishop for his researches in the bibliography of duodecimals, and more especially as the founder of the Duodecimal Society of Great Britain, and editor of its official organ, The Duodecimal Newscast."

Brian Bishop is 26 years of age, and 6 feet 6 inches tall. Professionally, he is a member of Great Britain's Civil Service, and an accomplished linguist with honors in Spanish, French and Latin, certificates from the Universities of Valencia, and Barcelona, and the degree of Bachelor of Arts, with Honours, from London University. In addition to his preoccupation with duodecimals, he is intensely interested in music and dramatics.

His great accomplishment in founding the Duodecimal Society of Great Britain has earned our warm gratitude. To his manifold duties as its Secretary, he now adds those of Secretary of the Association Duodecimal Internationale. He will shoulder much of the task of endowing that important body with blood, breath and energy for its active functioning. We extend him our hearty support, and assure him that he will never stand alone.

DOREMIC SCIENTISTS SURFACES

by Henry Clarence Churchman
403 Wickham Bldg., Council Bluffs, Ia.

In the July 1956 issue of THE DUODECIMAL BULLETIN we suggested dozenal common measurements of length, surface, and capacity, based upon the United States standard foot (fixed officially since 1 July 1959 at 30.48 centimeters). In such system all English-speaking peoples would retain their present denominations but introduce the conveniences of a dozenal system of numeration without any act of any parliament, congress, or assembly.

If our foremost desire were only to initiate a crash-program of duodecimal counting, then the retention of our foot, pound, and gallon is perhaps the easiest and quickest way in which to indoctrinate English-speaking peoples in the simplicity of duodecimal arithmetic. If, before duodecimal arithmetic is generally accepted, exclusive use of the present metric system becomes law in the U.S. and the Commonwealth as it became compulsory 1 January 1840 in France, all of us must suffer from the decimetric debacle caused by its inherent inflexibility.

Disregarding haste, regrouping our present denominations by dozens is not the only way in which we may advance. In the August 1959 issue of this same bulletin, Doremic Scientists Dimensions were set forth in which 44 international inches are said to equal one "dometron." As a candidate to become the unifying international unit of measurement in place of both the international inch and centimeter, the dometron might surpass both the meter and the yard. Its units are quite natural, many of its dimensions being already applied in many places.

The MS 880 specifications of a (French) Morane-Saulnier sport plane first shown at the Paris Air Show in June 1959 describe a cabin width which is equal to 1 dometron (3-2/3 international feet). Power-lawnmowers in the U.S. are quite commonly designed with a blade rotating diameter equal to a half dometron (22 inches), and at least one U.S. 1960 compact stationwagon listed a rear window width which is equal to one dometron and a height above ground (and a front wheel track) which is equal to one and a quarter dometrons precisely.

Equals of the remetron or erenaire dimension (see metronic scales) were used in Early American architecture and in some home site measurements. No less a thinker than Thomas Jefferson, who at age 33 wrote the Declaration of Independence designed and erected a brick-walled, octagonal house at Poplar Forest, a farm he owned near Lynchburg, Virginia. Each of its eight sides was a half remtron (22 feet) long, the equal of a half erenaire arc or 6 dometrons. More than one gro-mo or cantozend or remi (take your choice, each shown *100,000;0*) early platted lots in city and town subdivisions in the U.S. are the equal of one by three remetrons (44 by 132 feet). These are twelve by thirty-six dometrons today. U.S. county and township roads are commonly 1½ remetrons (4 rods) wide. At St. Malo, in France, the extreme spread between high and low ties---perhaps a world record---is equal to 1 erenaire arc, 1 remetron or twelve dometrons. So much for these artifacts to indicate uninhibited conduct of our forebears, our contemporaries, and of nature in measuring distance.

The present paper describes the characteristics of doremic scientists surfaces in relation to square meters and yards, a binumeral system pinpointing locations of land, and a dozenal stadia rod.

Metronic Dimensions

Please bear in mind that doremic scientists dimensions are derived from the continuous dozenal subdividing of a great circle of the earth equal to one dominaire dimension or one "dominante" unit of length (131 383 296 international feet or 40 045 628 620 800 international microns) again and again divided by twelve until we get down to an Air Mile of 6336 international feet in length (1931.2128 meters); and again, again and again until we have only the dimension of one dometron equal to 44 international inches (1.1176 meters).

Metronic Scales

Dozenal Units			International Units		Geophysical Units	
1 domimetron	shown	m*10 000;0*,	6336 feet,	1931.2128m,	1 naire	arc.
1 mimetron	"	m*1 000;0*,	528 "	160.9344m,	1 edonaire	"
1 remetron	"	m*100;0*,	44 "	13.4112m,	1 erenaire	"
1 dometron	"	m*10;0*,	44 "	1.1176m,	1 eminaire	"
1 metron	"	m*1;0*,	3-2/3 "	0.931-1/3dm,	1 edominaire	"
1 edometron	"	m*0;1*,	1/12 metron,	1/144 eminaire,	1 ereminaire	"
1 eremetron	"	m*0;01*,	1/144	1/1728	1 ebiminaire	"
1 emimetron	"	m*0;001*,	1/1728	1/20736	1 edobiminaire	"
1 edomimetron	"	m*0;0001*,	1/20736	1/248832	1 erebiminaire	"

Decimal Relationship of Miles, Edons and Furlongs

Atlas Plate 2 of The World, which accompanied the November 1960 of the National Geographic Magazine, compiled and drawn in the Cartographic Division of the National Geographic Society, conforms to a scale of 1:63 360 000 or one thousand land miles to the inch at the equator.

On the ground, one-tenth of a land mile, or one-twelfth of an air mile, may be called one Edon and is the equal of one edonaire arc (528 international feet) in geophysical distances. One land mile may be said to equal ten edons, and one thousand edons are for all practical purposes equal to one hundred land miles. As those who follow horse racing know, a furlong is one-eighth of one land mile. With that scale in mind, please note the following map distances on Atlas Plate 2, shown decimally.

One foot	equals	12 000	land	miles	or	10 000	air	miles	or	96 000	furlongs
1/2 "	"	6 000	"	"	"	5 000	"	"	"	48 000	"
1/4 "	"	3 000	"	"	"	2 500	"	"	"	24 000	"
1/8 "	"	1 500	"	"	"	1 250	"	"	"	12 000	"
1/10 "	"	1 200	"	"	"	1 000	"	"	"	9 600	"
One inch	equals	1 000				10 000	edons			8 000	
1/2 "	"	500				5 000	"			4 000	
1/4 "	"	250				2 500	"			2 000	
1/8 "	"	125				1 250	"			1 000	
1/10 "	"	100				1 000	"			800	
1/100	"	10				100	"			80	
1/1000	"	1				10	"			8	

Land Descriptions

In that area of the U.S. lying south of the states of Tennessee or Georgia, or northwest of the Ohio River or west of the Mississippi River excepting only Hawaii, Texas and parts of the states of Ohio, rural fire fighters and disaster relief agencies already have available to them under the Department of Interior Bureau of Land Management public land surveys a method of dividing a congressional township (by law composed of 36 square land miles) into subdivisions no larger than one percent of a square mile, with each one-percent square bearing a distinct and directory four-digit number to differentiate it from any of its 3600 equal square surfaces of land in the whole congressional township.

In accomplishing those divisions each square land mile is assumed to be divided decimally into ten meridional stripes numbered from the left or west border from zero through 9, and each such stripe is in turn divided into ten equal squares numbered, beginning with the bottom or south border, zero through 9. See diagram of a Township; also a diagram of 1 Quadrain, the one hundred equal subdivisions of which are numbered in the same manner precisely as the Gardens in every Section of land.

A certain disaster point, for instance, may be described as Section 1890, pronounced eighteen nine zero, Township 75 North, Range 42 West of the 5th Principal Meridian (abbreviated to 1890-75N-42W5PM). A person in Chicago, in Washington, or in San Francisco might readily place this area within one Garden of land or that one-percent of a square land mile lying in the extreme southeast corner of Section 18, Hardin Township, in Pottawattamie County, Iowa. One Garden of land is equal to one percent of a square land mile or 6.4 acres.

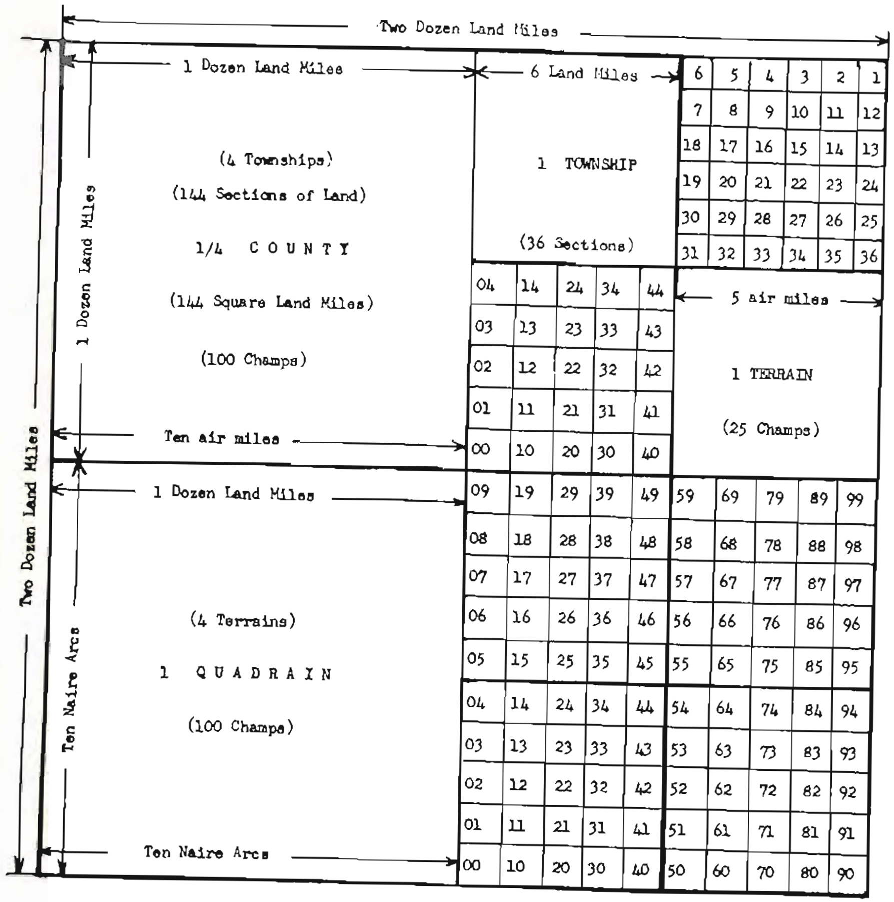
That same garden of land is the equal of one-twelfth of an AIR MILE (equal to one edon or one-tenth of a land mile) on every side. In order to aid your memory, you might recall the Garden of Eden---every garden of land equals 1 square edon (see metronic square scales).

Central Park in New York City is said to contain 840 acres of land. Since there are 640 acres in one square land mile, it follows the total area is equal to 1.3125 square miles. And since there are 100 gardens in one square land mile, we may determine the number of gardens of land comprising Central Park by merely moving the decimal point two places to our right. Thus, we find Central Park to equal 131.25 gardens of land. It would have to contain 144 gardens of land to equal the size of one Champ of land or one square air mile. This it lacks by only twelve and three-quarters Gardens of land, or 81.6 acres.

A sign on a public highway reading "Roadside Park .2 Mile" might with a slight amendment to our state statutes appear as "Roadside Park 2 edons," and "No Passing .3 Mile" is in practice the equal of "No Passing 3 edons." This indicates how interchangeable are our decimal and our doremic denominations of distance in the industrial English-speaking nations.

Metronic Square Scales

144 Gardens	equal	1 Champ or field 6336 (1.9312128km) by 6336	internat'l feet
144 Parks	"	1 Garden of land 528 (160.9344m) by 528	" "
144 Tables	"	1 Park of land 44 (13.4112m) by 44	" "
144 Tiles	"	1 Table surface 44 (1.1176m) by 44	" inches
		1 Tile surface 3-2/3 (0.931/1/3dm) by 3-2/3	" "



1 COUNTY or 4 QUADRAINS

Decimal and Duodecimal Land Area Interrelationship

144	Gardens	equal	1 Champ or one square air mile.
100	"	"	1 Section or one square land mile.
100	Champs	"	1 Quadrain (ten by ten air miles) or 4 Congressional Townships.
25	"	"	1 Terrain (5 by 5 air miles) or 1 Congressional Township.
4	Terrains	"	1 Quadrain (see above).
4	Quadrains	"	1 County, Parish or Shire (ideal 24 by 24 land miles or twenty by twenty air miles).

Those very "gardens of land" available to disaster area supervisors are the same size and shape as we would use in subdividing 1 Quadrain, 1 Terrain, or 1 Champ under the scale of metronic square surfaces. It has seemed fitting that the square air mile should be designated as one Champ (field) as a common appellation to describe that area of land larger than a whole Section of land, nearly equal to 4 square kilometers; and even more fitting if it in any way might be construed as some arithmetical part of the illustrious Champs Elysee of Paris.

The alliteration of Terrain and Township, the outside dimensions of which match each other, will enable English-speaking peoples quickly to associate one with the other for size. Of equal total area, the Terrain is divided into 25 squares, the Congressional Township into 36 squares. Four Quadrains (each containing 4 terrains) equal the most typical Iowa county in area.

Numbering of Champs of Land

There is a unique and universal way to number the Champs of land in all Quadrains. It is not dissimilar to the U.S. Army method of counting grids---COUNT RIGHT UP. In other words, you count right first, then up. In this fashion we can number each Champ in one Quadrain of land with just two digits in the manner shown in the subdivisions of one Section of land into 100 Gardens.

The FIRST of two digits of every numbered Garden of land in one Section gives us the number of one-tenths of a mile the southwest corner of that Garden lies to our right from the left of west boundary line of a Section of land. This may be zero to nine-tenths of a land mile.

And the SECOND digit indicates the number of one-tenths of a land mile this same corner of a Garden lies above

DOREMIQUE NUMBERING SYSTEM AND DIMENSIONS OF MORE COMMON SQUARE SURFACES (Compared with U.S.A. Land Measurements and with Meters)

0Ɛ	1Ɛ	2Ɛ	3Ɛ	4Ɛ	5Ɛ	6Ɛ	7Ɛ	8Ɛ	9Ɛ	XƐ	ƐƐ
0X	1X	2X	3X	4X	5X	6X	7X	8X	9X	XX	ƐX
09	19	29	39	49	59	69	79	89	99	X9	Ɛ9
08	18	28	38	48	58	68	78	88	98	X8	Ɛ8
07	17	27	37	47	57	67	77	87	97	X7	Ɛ7
06	16	26	36	46	56	66	76	86	96	X6	Ɛ6
05	15	25	35	45	55	65	75	85	95	X5	Ɛ5
04	14	24	34	44	54	64	74	84	94	X4	Ɛ4
03	13	23	33	43	53	63	73	83	93	X3	Ɛ3
02	12	22	32	42	52	62	72	82	92	X2	Ɛ2
01	11	21	31	41	51	61	71	81	91	X1	Ɛ1
00	10	20	30	40	50	60	70	80	90	X0	Ɛ0

1 side of TERRAIN
31680' or 9656.064m
1 side of TOWNSHIP
1 side of SECTION 5280' or 1609.344m
1 side of QUADRAIN 63360' or 19312.128m
1 side of CHAMP 6336' or 1931.2128m
1 side of GARDEN 528' or 160.9344m
1 side of PARK 44' or 13.4112m
1 side of TABLE 44" ou 1.1176m (contains 144 tiles)

' equals international feet
" equals international inches
m equals international meters
X equals ten (dix)

the bottom or south boundary line of such Section. Remember, we count right, then up. The Section subdivision diagram of 100 Gardens, like the Quadrain subdivision into 100 Champs, is not too difficult to memorize. (See Doremique Numbering System plate.)

Since all Champ two-digit numbers pinpoint the location of the lower lefthand corner coordinates of each particular Champ, it follows that Champ 00 (zero zero) lies in the southwest corner of one Quadrain of land. Similarly, Champ 99 (nine nine) lies in the northeast corner of one Quadrain, and Champ 06 (zero six) lies in the leftmost vertical stripe and in the 6th shelf above the base shelf of all Quadrains.

If we take any congressional township, composed of 36 Sections or 36 square land miles (see typical County plate), redesignate it as one Terrain of land, and subdivide it into 25 Champs, then all Gardens of land within it, without changing size or shape whatever, will be the equal of 144 subdivisions in each Champ of land, and will be numbered as are the subdivisions of Sections of land or of Quadrains of land excepting that their numbers will run higher---from *00* (zero zero) to *ƐƐ* (eleven eleven). See Doremique Numbering System plate.

While there are 25 Champs in one Terrain and 4 Terrains in one Quadrain, the unit which determines the descriptive or directory number to be borne by each Champ is the Quadrain, which contains one hundred Champs numbered from 00 to 99.

-U.S. Public Land Surveys Remain Valid

It perhaps should be noted here that by reason of the spheroidal shape of the earth it is not possible, especially in the north temperate zone, to lay out a series of abutting square counties parallel to any given latitude or longitude without shortening the northern boundary somewhat or adding to the length of the southern border. (See an atlas map of the state of Colorado or Wyoming.) Accordingly, instructions contained in the U.S. Public Lands survey manual not only call for this arrangement in every square area equal to 4 quadrains but this instruction is applied to each area the equal of one Terrain or Township.

This is accomplished, in general, by referring to all sections of land abutting the northern or the western boundary of a congressional township as "fractional sections." Legally a fractional section may be above or below the descriptive unit in actual dimensions. In each Terrain containing 25 Champs, the northernmost and the westernmost Champs would be of necessity designated as "fractional

Champs." So in all Quadrains of land (twelve by twelve land miles) all Champs whose number designation contains an initial zero or five or a terminal four or nine would be known as a "fractional Champ." See design of 4 Quadrains.

Fortunately, perhaps, the difference in length on the ground between one U.S. public land survey (Mendenhall) mile and the Canadian or international land mile is equal to only about 30 feet in 3000 land miles or about one foot in 100 land miles. This would equal about 3 inches of difference in the full length of 24 survey land miles. But observe that in all U.S. public land surveys the length of the northern and southern boundary lines of ALL townships (Terrains) were surveyed either over or under 6 actual land miles and their divergence from each other almost always exceeded 3 inches in length.

Therefore no surveys in the U.S. public land areas are disturbed by the adoption of Quadrains, Terrains, and Champs in the place of congressional townships and sections of land. The difference of 3 inches is absorbed by the method of accommodating squares of land to the surface of a spheroid.

Metes and Bounds

In the Original Thirteen Colonies of America, following an ancient English custom, all land areas were measured and described by "metes and bounds." That is, by linear distances in described directions from a point of beginning located a stated distance and direction from some well known landmark such as the fork of a river, a certain point of an island, an unusually large stone, a benchmark, and sometimes a large tree stump now long perished. Occasionally some small portion of a congressional section of land is significantly described by metes and bounds, especially if lying between a meandering stream or roadway and a not far distant railroad right of way.

A Word of Caution

Please note that the portion between the size of Sections of land, Champs, Terrains or Townships, and Quarter Counties or Quadrains, and their relationship to the 16-township most typical Iowa county, is normal on the plate showing the design of 1 County composed of 4 Quadrains. But in the plate showing the Doremique Numbering System and Dimensions of More Common Square Surfaces the method of numbering the subdivisions of the particular units there described, and not their proportional sizes, is the only reason for the design.

For instance, although each is 144 times larger than its next lower square unit, all Champs, Gardens, Parks, and Tables contain like-numbered subdivisions, from zero-zero to eleven-eleven. But 1 Quadrain, which is larger in area than any of these, is shown in a smaller space simply because its subdivisions are numbered from zero-zero to nine-nine. And 1 Terrain, which contains 25 Champs, while actually one-fourth the size of 1 Quadrain is 36 times the size of 1 Section of land.

If an indefinite Terrain be shown standing alone and not as a specific part of any Quadrain, its 25 Champs are numbered as if it lay in the lower lefthand or southwest quarter of a Quadrain of land. All land mile, air mile, and naire arc distances are related to international inches, feet, or meters, as indicated on the Doremique Numbering System plate. The Terrain and Quadrain square surfaces were constructed to be more indigenous and helpful to public lands survey areas in the U.S. The Champ (1 square naire arc) is the highest dozenal surface unit in doremic nomenclature, containing one gross gardens of land.

Systems of Dimensions Compared

One of the chief practical complaints against the present decimetric system of measurements, rests on the basic or congenital defect arising from the inability of the decimal base to subdivide unity or one circle into precise fractions of one-third, one-sixth, and several other very common geometric fractions. In other words, the kilometer unit is about 3/5 of the English land mile unit of distance and therefore quite unsuited for an itinerary measure.

While such claims are objectionable for the primary reason they are wholly irrelevant, it would seem nevertheless in an ideal system of measurements that meters (and bigger units) should be larger; and that decimeters and the finer graduations should be smaller and more atomic than we find in either of the currently used systems (yards and meters). This latter ideal characteristic is true of a naire arc (or domimetron, or navinaut, or double domotree, or kilometre duodecimale, according to the name given such unit by its author*), which is better than 1.9 kilometers in length. And respecting minute dimensions, observe that the metron (one side of a Tile) is about nine-tenths of a decimeter in length.

- - - - - - - - -

*Suggested, respectively from last to first, by Jean Essig of France, Horatio W. Hallwright of Canada, Charles S. Bagley, Henry C. Churchman, both the latter in the United States of America, this unit of dimension was reached by each of the independently of each other and, therefore, might be adjudged a not intricate or unnatural unit of measurement for this planet.

While the square yard is about 5/6 of the area of a square meter, note that the square meter in turn is only about 4/5 of the area of one Table surface.

One Table (1 dometron^2) is actually about 1-1/4 times the area of one square meter (1.249 029 76 square meter). One Champ (1 domimetron^2) or 1 naire arc^2 is about 3-3/4 times the area of one square kilometer (3.729 582 678 883 84 square kilometers).

Dropping below the square dometron, note that one Tile (1 metron^2) is about 6/7 of the area of one square decimeter (0.867 781-7/9 square decimeter).

And below the Tile, since these square surfaces are subdivided not into 100 but into 144 equal parts, the minuteness of every subdivision becomes, like the mills of the gods are said to grind, exceedingly fine with even fewer subdivisions in any particular scale.

Other Surfaces

It will avail English-speaking peoples nothing to run to the present decimetric system to escape from feet and inches, for the metric system as now used is on its way out as certain as daylight follows night. That system is striving to attain the flexibility of the English units of measurement, and could do so by swinging completely to a duodecimal or dometric system of numeration such as here described.

Unity is perhaps preferable to uniformity, and there is unity in the simple relationships of inches to centimeters (which required more than 100 years to achieve) and of microns to dometrons (precisely interchangeable from moment of conception). A metronic system of dimensions, like Esperanto among other languages, might prove less obnoxious than either one's own system is to the other. And since any dozenal numerical system of weights and measurements is more flexible than the present decimetric system of measurements and weights, all of us stand to advance with a metronic system and with a minimum of difficulties.

TERMINAL DIGITS OF $MN(M^2-N^2)$ IN THE DUODECIMAL SYSTEM

by Charles W. Trigg
Los Angeles City College

If M and N are integers, the unit's digit of

$$P = MN(M^2-N^2) = MN(M + N)(M - N)$$

is dependent upon the unit's digits of its four factors. Represent the unit's digits of M, N, P, by m, n, p, respectively. Now p will be zero if: $m = n$; or $m + n = 10$; or one of m, n, $m + n$, $m - n$, has the form $4k$, and another has the form $3r$.

These zeros form a symmetrical pattern in the square array of the values of p. Thus:

n \ m	1	2	3	4	5	6	7	8	9	X	Ɛ
1	0	6	0	0	0	6	0	0	0	6	0
2	6	0	6	0	6	0	6	0	6	0	6
3	0	6	0	0	0	6	0	0	0	6	0
4	0	0	0	0	0	0	0	0	0	0	0
5	0	6	0	0	0	6	0	0	0	6	0
6	6	0	6	0	6	0	6	0	6	0	6
7	0	6	0	0	0	6	0	0	0	6	0
8	0	0	0	0	0	0	0	0	0	0	0
9	0	6	0	0	0	6	0	0	0	6	0
X	6	0	6	0	6	0	6	0	6	0	6
Ɛ	0	6	0	0	0	6	0	0	0	6	0

The network of zeros consists of lines parallel to the sides for which m = 4 or 8 (i.e. 1/3 or 2/3 of 10) or n = 4 or 8, together with the diagonals and the alternate lines parallel to them. This network of 1X lines is symmetrical to each diagonal, to the perpendicular bisectors of the sides, and to the central element. 24 of the zeros lie on one line, 35 lie on two lines, and 14 lie on three lines.

Embedded in the network is a square (3 x 3) array of four-element squares with their diagonals parallel to the sides of the array. Each of these 30 elements is the digit 6 (i.e. 1/2 of 10).

Thus column-wise and row-wise, the array consists of three bands of three lines separated by zeros. Each band has a central line in which sixes and zeros alternate, bordered by lines in which three sixes are separate by groups of three zeros. Also, sixes appear in pairs in alternate lines parallel to the diagonals and are separated by zero pairs.

In addition to the X1 values of p exhibited in the array, there are 1Ɛ values of zero resulting from m = 0 or n = 0. Thus, if two integers M, N, are chosen at random, the probability that P will end in 6 is 30/100 or 1/4.

ONE OVER B MINUS ONE

by Alfonso Lomo, Ph.D.
P.O. Box 156, Goldens Bridge, N.Y.

The reader may be slightly puzzled by the title of this little sketch, but as the item unfolds he will understand the reason for it. Most of you will realize that there are certain features that are inherent in numbers -- that belong to them and stick to them regardless of the method used to represent them in writing. A prime number remains a prime, no matter whether it is written 17 or 15. On the other hand, there are some features that are inherent in the method of place notation with a zero that keep occurring regardless of the base used in the writing.

One feature that is inherent in the method of positional notation with zero is the fact that the decimal-form fraction corresponding to the common fraction that has 1 as the numerator, and the number 1 less than the base as the denominator, is a recurring series of 1's, provided the base is greater than 2. That is, 1/(b-1) = 0.111111..... In the case of the binary system where 1 = 1 and 10 = 2, 1/(b-1) is the same as 1/1 or 1. However, where the base is greater than 2, say 3, in which case the notation goes 1, 2, 10, 11, 12, 20, 21, 22, 100, etc. for 1, 2, 3, 4, 5, 6, 7, 8, 9 respectively, the fraction 1/(b-1) = 1/2, and if we perform this division we get the following:

```
2) 1.0  (0.1111.....
   2
   10
    2
    10
     2
     10
      2
      1
```

If the base had been 8, we would have had the following division:

```
7) 1.0  (0.111.....
   7
   10
    7
    10
     7
     1
```