COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	X	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5;9'
31	694	Three ft. two in.	3;2'
96	3Ɛ2	Two ft. eight in.	2;8'
19Ɛ	1000	Eleven ft. seven in.	Ɛ;7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
12 ) 30  + 5
12 ) 2   + 6
     0   + 2      Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *X*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *X*.

Numerical Progression

1	One		
10	Do	;1	Edo
100	Gro	;01	Egro
1,000	Mo	;001	Emo
10,000	Do-mo	;000,1	Edo-mo
100,000	Gro-mo	;000,01	Egro-mo
1,000,000	Bi-mo	;000,001	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	Ɛ
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
Ɛ	1X	29	38	47	56	65	74	83	92	X1

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THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

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The Duodecimal Bulletin

All figures in italics are duodecimal.

LETTER TO THE SECRETARY OF THE BEAVER COMMITTEE*

by Brian R. Bishop, Secretary
of the Duodecimal Society of Great Britain

Although I should have liked to express to you earlier our Society's views about the principle upon which to rationalize the systems of weights, measures and coinage of this country, you will now be better able to appreciate how our proposals meet the problems you have already discovered.

In this letter "fractional" means the expression of a part of a whole using the numerator alone, the denominator, decimal, duodecimal or otherwise being implicit. "Metric" is an alternative to "measuring" used adjectivally. On the typewriter I indicate the new symbols for ten and eleven as X and E, although formal consideration of number symbols may change their form. For clarity I distinguish duodecimal expressions with an asterisk.

The Duodecimal Society of Great Britain is convinced that, as twelve is the best-suited base for a system of numeration and mensuration, due recognition should be made and advantage taken of it. Among its members are academic and professional people of all kinds such as the Head of a University Mathematics Department, a surveyor, a dentist, Civil Servants, teachers, students, &c. A comparatively young Society, we cooperate with the older Duodecimal Society of America. Interest for our work has recently been shown in this country in, for example, 'The Times', 'The Sunday Times' and 'The Economist'. I have received many letters, all favourable. There is also known to be strong interest on the Continent and in the Commonwealth, not as yet organized.

Twelve is the best base and superior to ten for a numbering system -- and hence for measuring systems: this is a concrete, scientific fact in accordance with a natural and traditional British tendency. Mathematical and philosophical

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* *Editorial footnote: The Beaver Committee of the British Association for the Advancement of Science is directed to investigate the practicability, implications, consequences - domestic and foreign, and the cost of a changeover to the metric system or the decimalization of weights, measures and coinage. Please note that consideration of any other alternative is unauthorized.* RHB

writings recognizing this phenomenon go back hundreds of years in many lands. Here are a mere half-dozen random names ordered chronologically: Juan Caramuel, Blaise Pascal (both using Latin), Pierre Laplace (French), Sir Isaac Pitman, Herbert Spencer, H. G. Wells. It is unequivocally repeated today in popular works such as Arthur Mee's 'Childrens Encyclopedia' or Lancelot Hogben's 'Mathematics for the Million', and in specialist works such as Zdenek Kopal's 'Numerical Analysis' or Heinrich Teitze's 'Gelöste und ungelöste Mathematische Probleme'.

The reason for the duodecimal efficiency and decimal inadequacy is fundamentally, as the extract from a hundred years before on page 723 of 'The Economist' for 5th September, 1959, says: "10 is only divisible by 2 and 5; 12, by 6, 4, 3, and 2. The power of telling easily what is the price of a quarter of a thing, or a third of a thing, when you know the price of a thing itself, is very important in the daily transactions of the ordinary market; and no decimal system can be so easy in this respect as the system which we have..." It is natural to count with the factors nearest unity, i.e. 2, 3, and 4. Not only is there with twelve this 'remarkable divisibility by lower numbers', but the duodecimal factor 3 is intrinsically worth more than the decimal factor 5; quartering and thirding are each alone more frequent than division into fifths. Merchants find it more practical to group articles 3 x 4 or 3 x 2 x 2 instead of 2 x 5, ten being incapable of three dimensions or option.

As a result, arithmetic, mental and mechanical, is simpler when twelve is the number base. A brief mention of only three examples, affecting work of all kinds, will be sufficient illustration.

Firstly, multiplication and division operations are simplified. The new multiplication table is more symmetrical and memorable: e.g. more products end in nought because of the double number of exact divisors. "The multiplication table in our common arithmetic is generally carried as far as twelve times twelve, although its natural limit is nine times nine, which is clear proof that the mind is capable of working with the duodenary system without any inconvenience or embarrassment" (Peter Barlow, "An Elementary Investigation of the Theory of Numbers", 1811). Divisibility is far more easily recognizable, e.g. by the unit numbers 2, 3, 4, 6, 8, 9, and even E.

Secondly, duodecimal numbers whole and fractional, contain more quantity. As George Bernard Shaw observes, "... By adding two digits to our arithmetical tables we could make 16 figures do the work of 20 (a colossal saving of time for the world's book-keeping)". The duodecimal fractional *0;001 represents the decimal vulgar fraction 1/1728 and is more precise than the decimal fractional 0.001.

Thirdly, fractionals are generally shorter. We have a precise third, sixth and ninth (0;4, 0;2, 0;14). A comparison of decimal and duodecimal fractional reciprocals reveals only five non-recurring decimal fractions against seven duodecimal, three single-digit decimals against five duodecimal and one two-digit decimal against two duodecimal. Compare the decimal and duodecimal reciprocals of the so-frequent binary progression:

1/2	0.5	1/2	0;6
1/4	0.25	1/4	0;3
1/8	0.125	1/8	0;16
1/16	0.0625	1/14	0;09
1/32	0.03125	1/28	0;046
1/64	0.015625	1/54	0;023

Although I am not versed in this, I understand that computer specialists are considering shorter means of expressing numbers to reduced the size of machines and the number of components. Some favour a base-twelve solution (needing no conversion from duodecimal counting) or base twenty-four (easy to convert from duodecimal counting and also linked with alphabetical classification). I understand that two further advantages in digital computers are, firstly, that the second power of two being an exact divisor of twelve facilitates unmodified binary conversion and, secondly, that the smaller difference between the number base and the fourth power of two facilitates modified binary conversion.

Even the human hand is better employed counting in dozens. The fully opposable thumb distinguishes man from the beast. He can count up to a dozen -- not five -- on each hand by touching in turn the twelve phalanges of the four fingers as our Indo-European forefathers are thought to have done.

The operations of base-twelve arithmetic are, of course, no different from those of any other base, including ten. The enclosed leaflet gives a brief outline; but there are longer expositions in the bibliography. I also enclose a copy of an article 'An Excursion in Numbers' by an American, F. Emerson Andrews, which was extended into his later book, 'New Numbers'. There are logarithms and other tables, slide rules and other equipment.

You can see, therefore, that it is no coincidence that the two of our most used units, the shilling and the foot, are divisible into twelfth-part submultiples. The twelve-month (not thirteen-moon) year and the two-dozen-hour day play a very large part in everyday life. Attempts to re-divide the 360° circle into a hundred or even four hundred parts failed because of their fundamental inadequacy; for example neither can express the common 60° angle and its simple derivatives in natural numbers. In fact, considering also the multiples and sub-multiples using factors of twelve (e.g. 3-feet yard, 2-gill or 4-gill pint, 4-quarter hundredweight, 6-feet fathom, 60-second minute) it will be seen that the British system of coinage, weights and measures and tend naturally to be duodecimal, not decimal.

The whole of posterity, at school, work and leisure, will benefit from any steps we take. Easier arithmetic at school, both in abstract calculations and in concrete weights, measures and coins will leave time for more important and difficult things. (A number of our members were first introduced to duodecimals at school.) In this age of progress man at work is seeking greater efficiency, accuracy and speed. And when the day's study and toil is done, who wants unnecessarily complicated calculations to sew and saw?

Everybody, even the technical minority, deal most of their lives in dozens: eggs, tins of food, flowers, screws, bought wholesale (French "en gross") for the grocer (gross-er) and other retailers in grosses, as well as by the dozen. The octave has twelve semitones. The standard printing unit, the "pica em" is divided into twelve "points", and similarly on the Continent. Prices are frequently given in shillings where the pounds are involved. Approximations are usually expressed as "Oh, about half-a-dozen" rather than "About five".

In common with Teutonic tongues, our words for tens numbers do not use the suffix "-teen" until "thirteen". Although French gave us "dozen", we have nothing for "dizaine". In fine, as the Comte de Buffon said in 1970, "Less hommes ont si bien senti cette verite, qu'apres avoir adopte l'arithmetique denaire, ils ne laissent pas de sa servir de l'echelle duodenaire".

In any metric system the principle is to group the units according to the number-base of abstract counting. In this the choice of the best 'fundamental' units demands most careful and expert thought in every conceivable aspect. We need the best, the most convenient and efficient both in theory and practice. There is happily little risk of being carried away in the ferocious fervor which made the French Revolution Convention brush aside the current duodecimal recommendations.

We can rebuild our standards a posteriori with the advantages, already noted, of tradition and familiarity. Strong English traditional feeling for and extensive international use of English units (e.g. nautical miles, knots, thread units, aircraft design and instruments, &c), give us the opportunity to exploit the perfect duodecimal common link between the two extremes of stagnation in the quite chaotic imperial system or of uprooting to the quite inadequate decimal systems, yet retain their worth-while principles of base twelve and rationality.

Jean Essig, Inspecteur-General des Finances in France, proposes the metre for a 'fundamental' unit in 'Douze, notre dix future' and 'La duodecimalite: chimere ou verite futur'. The Roman Commonwealth and international use of feet and inches (reflected by 'The Times' in the article 'Inch and Penny under Fire' on April 20th: "Considerably more than half the world's engineering production is, in fact, in inch sizes".) suggests them to be more practicable.

All such historic units, however, are no longer self-defining or even accurate. For example, the metre is not a ten-millionth of the earth's polar quadrant and it now has to be defined in terms of the wavelength of red cadmium light waves. The recent redefining of the international inch in terms of the decimal metric metre merely uses the red cadmium line at second-hand.

Alternatively, an a priori duodecimal metric system could be constructed from one of the many defining quantities available, with the same logic as the French decimal metric system but without its mistakes, by exploiting twelve as the number base and the extra experience of the past century and a half. Absolute units such as volume and weight again derive from the length unit. The division of the clock and circle will probably not alter. Temperature can still depend on the freezing and boiling points of water, but be divided into a gross of parts.

The shilling is a convenient monetary unit, divided with equal convenience into twelve pence. Its continued use will avoid any noticeable change in our coins (perhaps a three-shilling piece would be less prone to confusion with the florin) - only in paper money. As coin-operated machines and the like work on low-denomination coins which need not change, there will be no inconvenience there. Recent conflicting proposals in correspondence to 'The Times' and 'The Guardian' show how decimal coinage cannot suit everyone. Duodecimal coinage will resolve the conflict. The thousand-mil pound has the cumbersomeness of the French franc in the unnecessary figures and unsuitable denominations.

Denominations for the hundred-cent pound will be equally difficult to find and subdivisions of the cent will be as ridiculous as the Spanish centimo. In common with other units, decimalized coinage makes division by three impossible.

The Duodecimal Society of Great Britain proposes that the change-over to a duodecimal system should be as soon, as short, and as complete as possible to avoid confusion and frustration. We are, however, reluctant to advocate any one metric system exclusively until there is a responsible formal agreement. Meanwhile, I personally use a 'foot-pound-second-shilling' system. This is merely an example of a temporary metric system and by no means implies that I or the Society necessarily favour either it or its premise of dependence on existing units for permanent adoption.

This system adopts an existing unit of each kind as its basic denomination from which multiples or sub-multiples are derived duodecimally. Length or distance is in terms of feet (f), a mile becoming *3 080f. (Even now air-heights are generally in feet and other distances in yards.) Weight is in pounds (p), a ton becoming *1 368p. (Cattle and bombs are now weighed in pounds.) Volume is in pints (pt). Large denominations are formed with prefixes on the principle of deca-/hecto-/kilo- -- do-/gro-/mo- -- foot-/pound-/pint. Decimal metric equivalents can be calculated back using the agreed definition of the new international foot and pound. Time is as at present, except counted duodecimally so that the year has *265 days and the day two-dozen hours.

I enclose a copy of the leaflet used by the Duodecimal Society of America. This gives an example of another a posteriori metric expedient which they find workable.

I have notes of many numbering and measuring proposals with a priori and a posteriori characteristics and shall gladly give you such details as you wish. You may also like to see a number of extracts from publications since 1644 and references to many more which we have.

Whatever duodecimal metric system we adopt, it will be easier for trade purposes to convert from one regular system to another and back again than from the present confusion. There is every chance that the rest of the world, still having some respect for the Anglo-Saxon countries and certainly having so much trade with them, when it realizes the great improvement not only on our own systems but also on their misguidedly decimal system, will follow us in due course as some governments have already followed France.

I know of evidence that some manufacturers agree that the difficulties of a change-over to a duodecimal form of measuring can be largely mitigated. Here are three ways: many instruments need not be scrapped but merely recalibrated, say with new dials; unadaptable machinery can be fitted with a conversion gadget; otherwise simple specialist tables can be reproduced. The increase in efficiency more than repays any expense.

We may nonetheless hesitate at the temporary inconvenience of a change, but not when we realize that our decision will last forever. We must act now; the longer we delay, the greater will grow the problems. As time advances, so the extent of our brief sacrifice will be amortized into insignificance, whilst the benefits and respect for its boldness will be increased. We have had Arabic notation in general use for only a few hundred years and imperial standards legal for only about a century. Here is a sound seed of British tradition which we should tend, discarding the weeds that have been choking it.

Your questionnaires will tell both needs and wishes. The wish each questionnee expresses, probably swayed by misconceptions or preconceptions, may not accord with the general or even the individual's need. The actual needs must therefore be laid clinically bare and remedies objectively outlined with disadvantages and advantages. Sense and experience demonstrate that a remedy with base ten as its number base is too inadequate and that our present system is too chaotic, whilst an efficient, practical remedy must have twelve as its most efficient and practical number base. The apparent novelty of approach and resemblance to what we are used to may sway individual wishes to accord with the general need. If our metric shoes pinch us, let us have a brand new set, to the familiar pattern which fits our feet, made to measure of the finest material existing, not a foreign s set a century and a half off the shelf, nor our present set patched and stretched. If we change, as change we must, let it be for the (duodecimal) best, not a for a (decimal) second-best.

POWERS OF TWELVE 12^n

n	12^n
1	12
2	144
3	1 728
4	20 736
5	248 832
6	2 985 984
7	35 831 808
8	429 981 696
9	5 159 780 352
10	61 917 364 224
11	743 008 370 688
12	8 916 100 448 256
13	106 993 205 379 072
14	1 283 918 464 548 864
15	15 407 021 574 586 368
16	184 884 258 895 036 416
17	2 218 611 106 740 436 992
18	26 623 333 280 885 243 904
19	319 479 999 370 622 926 848
20	3 833 759 992 447 475 122 176
21	46 005 119 909 369 701 466 112
22	552 061 438 912 436 417 593 344
23	6 624 737 266 949 237 011 120 128
24	79 496 847 203 390 844 133 441 536
25	953 962 166 440 690 129 601 298 432
26	11 447 545 997 288 281 555 215 581 184
27	137 370 551 967 459 378 662 586 974 208
28	1 648 446 623 609 512 543 951 043 690 496
29	19 781 359 483 314 150 527 412 524 285 952
30	237 376 313 799 769 806 328 950 291 431 424
31	2 848 515 765 597 237 675 947 403 497 177 088
32	34 182 189 187 166 852 111 368 841 966 125 056
33	410 186 270 246 002 225 336 426 103 593 500 672
34	4 922 235 242 952 026 704 037 113 243 122 008 064
35	59 066 822 915 424 320 448 445 358 917 464 096 768
36	708 801 874 985 091 845 381 344 307 009 569 161 216
37	8 505 622 499 821 102 144 576 131 684 114 829 934 592
38	102 067 469 997 853 225 734 913 580 209 377 959 215 104
39	1 224 809 639 974 238 708 818 962 962 512 535 510 581 248
40	14 697 715 679 690 864 505 827 555 550 150 426 126 974 976
41	176 372 588 156 290 374 069 930 666 601 805 113 523 699 712
42	2 116 471 057 875 484 488 839 167 999 221 661 362 284 396 544
43	25 397 652 694 505 813 866 070 015 990 659 936 347 412 758 528
44	304 771 832 334 069 766 392 840 191 887 919 236 168 953 102 336
45	3 657 261 988 008 837 196 714 082 302 655 030 834 027 437 228 032
46	43 887 143 856 106 046 360 568 987 631 860 370 008 329 246 736 384
47	526 645 726 273 272 556 326 827 851 582 324 440 099 950 960 836 608
48	6 319 748 715 279 270 675 921 934 218 987 893 281 199 411 530 039 296

POWERS OF TWELVE 12^n

THE SERIES *10^n ± 1* IS COMPOSITE

(with the exception of *Ɛ* and *11*)

by Ralph H. Beard

There has been noticeable emphasis on numbers resembling the form of *101*, *1001*, *10001*, etc., in recent papers by F. Emerson Andrews, and I. V. C. Valevsky. This led us to examine their factorability.

With the help of George S. Terry and D. H. Lehmer, we have found that *10^n ± 1* is composite, with the exception of *Ɛ* and *11*. And that their factors arrange such numbers into an interesting pattern of classes and subclasses. (Decimally, the number 101 is prime.)

The associated table of the Powers of Twelve shows how the end-figures of their decimal values repeat every fourth power.

. . . .

Numbers of the form *10^n - 1* resemble *ƐƐƐƐ*, and the have an *Ɛ* factor.

Numbers of the form *10^n + 1* fall into several categories as to their factors. When n is odd, the factors are *11*, and some variation of *Ɛ0Ɛ1*. *Ɛ0Ɛ0Ɛ1*, etc.

A glance at the table of the decimal Powers of Twelve will show that when n is singly even, (2, 6, 10, 14, etc.), these numbers end in 4, - and this ending with +1 becomes 5; revealing the presence of a 5 factor.

When n is doubly even, the factors arrange these numbers into an array of subclasses.

When n = *8*a + *4*, the factors include *75* and *175*. The series of such powers runs: *4*, *10*, *18*, *24*, *30*, and every further *8*th power.

When n = *14*a + *8*, the factors include *15* and *81*. The series of such powers is: *8*, *20*, *34*, *48*, *60*, and every further *14*th power.

When n = *28*a + *14*, the factors include *75115*. The series is: *14*, *40*, *68*, *94*, *100*, and every further *28*th power.

When n = *54*a + *28*, the factors include *541*. The series is: *28*, *80*, *114*, *168*, *200*, and every further *54*th power.

When n = X8a + 54; the factors include 18X81 and 3Ɛ055. This series is: 54, 140, 228, 314, 400, and every further X8th power.

When n = 194a + X8, the factors include 195. Some of the numbers of this series are: X8, 280, 454, 638, 800, and every further 194th power.

The principle underlying the subclasses has been well stated by George Terry: "All factors first occurring in $10^n + 1$, with n doubly even, recur in odd multiples of this n." As an instance, 35 appears first as a factor of $10^{18} + 1$, of the subclass 8a + 4. It next appears as a factor of $10^{50} + 1$, also of that class, and later in every further odd multiple of the 18th power.

The sequence of subclasses that has been used in the body of this paper, can be developed by assigning serial values to b in the general equation: $n = 2^{(b+2)}(2a+1)$. The way these groups interlace is surprisingly exhaustive. Their pattern is set forth in the following table.

Power	Subclass	Power	Subclass
4	8a + 4	64	8a + 4
8	14a + 8	68	28a + 14
10	8a + 4	70	8a + 4
14	28a + 14	74	14a + 8
18	8a + 4	78	8a + 4
20	14a + 8	80	54a + 28
24	8a + 4	84	8a + 4
28	54a + 28	88	14a + 8
30	8a + 4	90	8a + 4
34	14a + 8	94	28a + 14
38	8a + 4	98	8a + 4
40	28a + 14	X0	14a + 8
44	8a + 4	X4	8a + 4
48	14a + 8	X8	194a + X8
50	8a + 4	Ɛ0	8a + 4
54	X8a + 54	Ɛ4	14a + 8
58	8a + 4	Ɛ8	8a + 4
60	14a + 8	100	28a + 14

The factors of the subclasses 368a + 194 and beyond have not been developed. Yet short of n = 1000, there remains only three unfactored cases of this type; to clear them, we must find factors for $10^{194} + 1$, $10^{368} + 1$, and $10^{714} + 1$. 10^{1000} represents 12^{1728} on the ten-base, which is a number some 2000 figures in length.

It is worth noting that a block of nine numbers in series with 10^n are not prime: those ending 8 9 X Ɛ 0 1 2 3 4.

THE CHOICE OF UNITS OF MEASUREMENT

by Peter Andrews*

The question of how a system of units of measurement can best be devised is of vital importance to this Society, for no one can legitimately even hope for the universal adoption of duodecimals until a comprehensive and indisputably satisfactory system of duodecimal units is established and has gained unanimous support from the advocates of the duodecimal system of numbers. At the risk of repeating things that have already been said very well in these pages, but in the hope of challenging other opinions on this subject that have been expressed here, I should like to summarize what seem to be the most important points at issue.

Before doing this, however, it may be well to summarize the the points on which there seems to be general agreement, and to which we shall adhere at all costs. The first of these is that, whatever our fundamental unit of measurement in a dimension (such as length, time, mass, or even money) may be, all other basic units in that dimension must be multiples or subdivisions of that unit by some power of twelve, just as the centimeter, millimeter, and kilometer are all multiples or subdivisions of the meter.

Secondly, there seems to be general agreement that units which are logically dependent shall not be defined independently. For example, if we have defined basic units of time and distance, we shall not define another unit of velocity, but we shall define unit velocity as that velocity necessary to travel a unit distance in unit time. Of course, we could also start by defining units of time and velocity, and then define the unit distance as that distance traveled in unit time as unit velocity. Here again, we adopt the valuable features of the metric system while rejecting its undesirable emphasis on the base ten.

We now come to the crucial question: how may we best choose the fundamental independent units of measurement? There seem to be four basic approaches to this problem. First of all, arguments are often heard to the effect that the units of measurement must be "convenient" measures for practical situations. Secondly, it may be argued that the units must be such that calculations are facilitated. Third, it may be claimed that the duodecimal units of measurement should be based on units with which people are already

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*The Graduate College, Princeton, N.J.

familiar, since this will facilitate the transition to duodecimals. Thus it may be argued that the basic duodecimal units of measurement should be multiples or subdivisions of the foot. Fourth, it may be argued that a "natural" set of units (one suggested by phenomena or magnitudes found in nature) should be adopted, if there is any such. It is usually assumed that such a set of units will eventually be found to be the most convenient for both calculations and practical measurements. Unfortunately, there are some serious difficulties in all of these approaches, as can be seen by trying to implement them consistently and uniformly.

First of all, can any unit of measurement that is used in a variety of situations really be considered an especially convenient unit in itself? For example, a unit of measure forty four inches long may be quite convenient in building showers[1], but is it well suited to making chairs? We might as well recognize (thankfully) that for practical measurements any system of units involving multiples of twelve will do about as well as any other, and if we try to be kind to the architect we may not be doing justice to the furniture designer. In such a situation, we must look to broader considerations.

The attempt to make units of measurement such as to facilitate calculations may meet similar difficulties. For example, the surveyor or navigator may be very happy if we make the basic unit of length a subdivision of a great circle on the earth, but what shall we then say to the astronomer who would like it a subdivision of the light-year? It might also be noted in regard to this particular example that measurements of the earth are likely to be somewhat inaccurate, and that the size and shape of the earth may be changing slowly. So far as possible, the advantages of the duodecimal system of units finally chosen should be universal and permanent, and it would be remarkable indeed if this could be achieved simply by adopting a set of units suggested by some special problem. Nevertheless, in a few cases this method may be appropriate. For example, as long as men live on earth, they will probably wish their basic unit of time to be a convenient subdivision of the length of one day.

Next we must consider the argument that the basic duodecimal units should simply be the same as some units now in common use. Obviously this is a reasonable argument if it turns out that no set of units is, in itself, any better than any other. That is, if we find that in the long run every set of basic units is very convenient for some situations and problems but not quite so convenient for others, so that on balance the merits of each system seem to have about equal weight, and if we find that no system of units is unambiguously suggested by nature, then we might as well keep some basic units of measure with which people are already familiar, and build up the duodecimal system of units from these. Suppose, however, we find that some new and unfamiliar set of units actually does have inherent merits which outweigh those of any set now in common use; shall we adopt the new set because it is better or stick to the old one because people are used to it?

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1 See *The Duodecimal Bulletin* for August 1959, p. 25.

Obviously we would have to examine the new system of units before we could decide whether it was worth throwing out all familiar units for. However, suppose for the moment that a set of basic units has been proposed and that educated men agree that mankind will eventually want to adopt it. Would it not then be simplest in the long run to implement it at the same time that duodecimal numbers are adopted, so that there will never be more than one set of duodecimal units and the problems of changing from one system to another need never again arise? It may indeed take a little longer to persuade people to adopt duodecimals if they must also learn a completely unfamiliar system of units, but would this not be worth the assurance that, once duodecimals are adopted, men need never again argue over which system of units to use or convert measurements from one set of units to another (as must often be done now with both metric and British units in existence)? Moreover, if there is a really superior system of basic units, and if this were presented as an integral part of the duodecimal system, would not the advantages of the system of units provide strong arguments for the adoption of the whole system of duodecimals? Let us not hesitate to propose a great step forward for fear that the world will never take more than a small one; the greatest danger in this age lies not in looking ahead too far, but in not looking ahead far enough. As in the past, it will probably turn out that our most reckless striving for perfection falls short of what we eventually wish had been accomplished; compromising for reasons of expediency simply hastens the day when another reform must come.

It thus becomes necessary to consider the question: is there an inherently superior natural system of units, and if so how shall we find it? It will take a great deal of detailed, technical, and thoughtful work to answer this question adequately, but there is some hope that some such system does exist. For example, it is now fairly obvious that the basic unit of electrical charge should be the charge of an electron or proton. (The two charges are equal in magnitude.)

There seem to be other constants in the universe which, while always expressed in some particular system of units, have physical significance in themselves. As examples we may mention Planck's constant (which involves units of energy and time, or of mass, length, and time) and the velocity of light, to which the Theory of Relativity assigns a very fundamental place indeed.

Suppose, for example, that we choose to make our units of time, length, mass, and charge such that the length of one day, the velocity of light, Planck's constant, and the charge on the electron all have unit values (multiplied by an appropriate power of twelve, perhaps). Then the light-day is 12^{12} times a basic unit of length about 9.531 feet long, which means there is also a basic unit of length about 9.531 inches long. The requirement on Planck's constant necessitates a basic unit of mass approximately equal to 2.054 pounds. With basic units of distance, time, mass, and electrical charge defined, we have automatically defined units of velocity (distance/time), acceleration (distance/time^2), force (mass·distance/time^2), energy and heat (mass·distance^2/time^2), electric potential (mass·distance^2/time^2·charge), electrical current (charge/time), electrical resistance (mass·distance^2/time·charge^2), etc., by means of the elementary dimensional relationships.

There are, however, certain very great difficulties ahead if we decide that we should adopt some set of natural units. First of all, it may not be obvious just which basic scientific relationships should be taken as fundamental in defining the set of basic units, and there may be occasions when practical considerations incline us to compromise the ideal of a set of units in harmony with really fundamental scientific facts. For example, suppose we discover that there is a natural unit of time which is quite unrelated to the accident of how fast the earth is spinning (and hence the length of our day); how shall we then establish our basic unit of time? Once again the ugly prospect of a double system of units raises its head.

Even if we can agree in theory on a set of natural units, certain methodological problems must be overcome. We wish our units of measurement, once fixed, to remain the same for all time (or at least for a very long time), but it would also be nice to have them scientifically accurate (that is, defined according to our original scheme) at all times. Ideally, we should avoid situations such as that encountered when it was found that normal human body temperature is not quite 100°F as planned when the Fahrenheit scale was established. Therefore, to continue the example

where the velocity of light is set at unity, it does not seem desirable to define the unit of length as the length of a certain bar of metal so constructed that we expect the velocity of light to turn out to be unity, for as measurements improve we are almost certain to find that we have made a slight error. We might avoid this difficulty by simply defining unit velocity to be the speed of light, and then using our measurements at any time in future history to obtain the most accurate measure of unit length then available. In this case, however, we must recognize that scientific measurements of this sort often depend on scientific theories in a very complicated way, and cases are even known where measurements of what is presumably one constant by different theories yield slightly different values for the constant. Moreover, scientific theories do change from time to time (though we can expect well established theories not to change very much), and there is always the possibility that the very theory we use in defining some unit will be modified. Obviously, if we decide to define a set of natural units by direct measurement of physical quantities, we will do well to do so in such a way that these measurements need involve only quite well established theories.

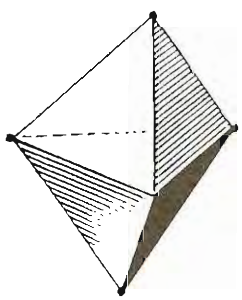
We might summarize the problem of using natural units in the following way. If natural units are available but we refuse to use them, we introduce arbitrariness where there could be reason and order, and we throw to chance and changing circumstances the hope that our system of units shall simplify calculations and measurements as much as possible throughout the entire history of their use. If, on the other hand, we attempt to base our system of units on those aspects of nature that seem most fundamental, we become painfully aware that at every moment our understanding of what is fundamental in nature is incomplete and susceptible to change.

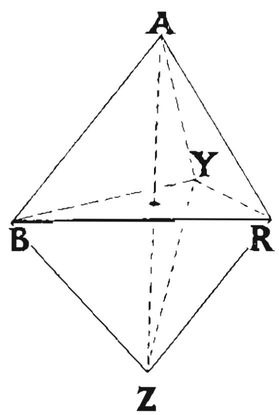
In the end, we shall have to recognize that no matter what system of units of measurement is devised, people will probably wish someday that it had been done differently, and the best we can do is make the system such that a new reform will not be necessary for a very long time. This would seem to demand considering every aspect of the problem in great detail and taking advantage of the most basic and sophisticated knowledge now available to mankind.

BOOK REVIEW

BYRAZ COLOUR NOTATION, by Louis Loynes, Byraz Colour Bureau, 6 Monmouth St., London, W.C.2. 8 x 10 paper, 132 p. 10 sh.

The Byraz System conceives all colors arranged into a double triangular pyramid, with blue, yellow and red at the corners of the median triangle, (or "magnomial"), with white, (called "alb" or A), at the apex, and black, (called "zero" or Z), at the nadir. The various values of gray are spaced along the central vertical axis in twelve steps from the magnomial to white, twelve descending steps from the magnomial to black. Thus, median gray is located at the center of the magnomial triangle.





There are also twelve steps in the "bivariant range" along each edge of the magnomial triangle between blue and yellow, - between yellow and red, - and between red and blue. Colors with increasing proportions of gray are ranged between the edges of the triangle and the center. The ascending levels, (or quilts), have the lighter colors and the blends with the lighter grays. The descending quilts have the darker values, tapering to black, or zero.

The numerical notation of these values is duodecimal. This avoids the distortion of values between the yellow and the red in the Munsell Color Notation, - to accommodate the decimal notation.

The work is quite comprehensive. The color systems of Goethe, Newton, Ostwald and Munsell are well described. And an excellent bibliography is provided. The varying factors of incident and transmitted light, - of filters and prism spectra, are thoroughly discussed. The material provided on the geometry and physics of light and optics is quite ample.

The plan of the book makes provision for work and experiment on the part of the student. Means to accumulate a personal color atlas is cleverly set forth. The Byraz Colour Bureau offers its cordial cooperation.

Mr. Loynes has developed an associated nomenclature for the system in amazing detail. His work is a major contribution to the literature of the difficult technique of color - where so many incommensurate variants are unavoidably involved.

RHB.

ARITHMOCRYPT

Mary Lloyd suggests that someone should get a medal.

```
               R T
          ---------
D E N S E ) G R O W T H
            O A E O A
            -----------
            D R D H T H
            D N G E G U
            -----------
              N W S D D
```

If you substitute the right figures, it makes sense.

AWAKENING MINDS

It is part of our undertaking to make men aware that we are using an inferior number system, and that the best is easily available. We are making some progress. But it is difficult to disturb the contentment of the average person with things as they are.

There are a few people with the gift of inserting a new line of thought into the mind as on a fresh page. It seems to be simply done, yet few can do it. Ben Franklin was one. He had a persuasive ability to transport his companions into the world of his own unfettered thinking. And to release their dormant energy into the work he planned. We have enjoyed recently a most active response to a suggestion of one of our members.

Design News is a lively, bi-weekly, technical journal in the field of product engineering. There has been current a discussion of "Millimeters vs Inches" in their editorial section, "The Sounding Board".

William C. Schumacher, 78-25 86th St., Glendale 27, N.Y., entered the discussion with a letter to the editors which they published under the title, "Duodecimal System," in their issue of August 21st. We have received over 50 responses to that letter and they are still coming in. We are as much impressed with the technical standing of the respondents as with the volume. The editors of Design News have warmly granted our request for permission to reprint Mr. Schumacher's letter.

DUODECIMAL SYSTEM

A Letter to the Editor of DESIGN NEWS*
by William C. Schumacher

Dear Sir:

The July 6 editorial, "Millimeters Versus Inches", raises once more the plaint that the United States is long overdue for adoption of the metric system. Acknowledgement is paid to the obvious fact that official adoption of this system would entail considerable inconvenience for an extended period of time. Let me be the first to acknowledge that manipulation of quantities is much easier in the metric system, at least where areas, volumes, densities and the like are concerned. These advantages stem from the fact that units are related to one another by multiples and submultiples of 10, the radix of our numbering system.

Our antiquated system (basically English) has some advantages, though. Generally speaking, when we need to express a third, a quarter, a sixth, or an eighth of some unit, either a whole number, or a whole number and a simple fraction of a smaller unit is just right. And I think there is no denying that it is more frequently necessary to deal with thirds, etc., than with fifths or tenths, where the advantage lies with the metric units. But if you are prepared to upset the apple cart by calling for the adoption of the space metric system, why not go a little farther and promote a system which, so far as I have been able to discover, is in all respects as good as the better of the two systems we have been discussing, and in some fairly important ways much better than either. I refer to the Duodecimal System, wherein the radix of the numbering system is the dozen instead of 10. I purposely did not write "12", because in that system the digits are *0, 1, 2, 3, 4, 5, 6, 7, 8, 9, X*, ("dek", having a value of ten) and *Ɛ* ("el", having a value of eleven). The quantity which we presently call 12 (or a dozen) becomes, in the proposed system, "*10*", wherein the "*1*" signifies a dozen and the "*0*" signifies no additional units. Lest anyone protest that "you can't have decimals," "log and trig functions won't work" or such other superficial objections, please be assured that not only can you have all of these things, but some pretty sharp individuals have already worked out these things; furthermore, multiplications, divisions, powers and roots become easier to handle than they now are. Other benefits accrue, such as the fact that many integral quantities

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use fewer digits than they now do, many useful fractions are exactly expressed by a few places of "duodecimals" (analogous to "decimals"). These benefits come from the greater factorability of 12 as compared with 10, and from the greater capacity of each place in the makeup of the numbers.

One area wherein duodecimals have a truly outstanding advantage is in the division of the circle. What is distinctive about 1/10 of a circle? Or about 10 deg? Or about 100 deg? However, if we regard a circle as a "unit" of circular measure, then 1/12 of a circle (written *0.1* in duodecimal notation) is a particularly significant angle which we now call 30 deg; 2/12 of a circle (written *0.2* in duodecimals) is our present 60 deg, 3/12 of a circle ("*0.3*") is what we are accustomed to call 90 deg, and so on. By subdividing further duodecimally, *0.01* circle is our present 2½ deg, *0.001* circle is equal to 12½ minutes, and *0.0001* circle is just barely over a minute of arc. Considering the refinements we are now beginning to achieve in optical work, gearing, navigation, intercontinental and interplanetary trajectories, and so on, doesn't it look inviting to have an angular measuring system wherein units of all sizes can be dealt with by straight uncomplicated arithmetic, instead of repeatedly converting by factors of 60 and 360, and wherein the sixth place (*0.000001*) denotes a resolution already well beyond one second of arc?

Similarly in the measure of time, if a day is the unit, and *0.1* is a smaller unit (two hours) and *0.01* is a still smaller unit (ten minutes), and the third place is already smaller than a minute, visualize the convenience of timetables with only three places of figures which supersede both the cumbersome A.M.-P.M. distinction and the awkward "twenty three hundred hours" folderol. Again, time differences involve only straight arithmetical subtraction.

If this concept interests you, more information can be had from the Duodecimal Society of America, located at 20 Carlton Place, Staten Island 4, N.Y. They welcome inquiries and are yearning for the day when enough educated and influential people are aware of this system to bring some weight to bear toward its widespread use.

ALL NUMBER SYSTEMS HAVE BASE 10

by Nelson B. Gray
Highbridge Road, RD #2, Oneida, N.Y.

Strange as it may seem, the base of all number systems is the symbol "10." If this is the case, how can different bases be designated? Just what does the symbol 10 really mean? Can it have different meanings in different numeration systems?

First, it is generally known that all number systems are based on a power series extending in both directions from unity, which we will designate by the general term n^o (n to the zero power). The series then takes on the general form of: n $\ldots n^5 + n^4 + n^3 + n^2 + n^1 + n^o + n^{-1} + n^{-2} + n^{-3} + n^{-4} + n^{-5} + \ldots n^{-m}$

The base or radix of any number system is determined by the number of the different symbols necessary to express all of the counting functions in that system. Another way of expressing this idea is that the radix is one more than the largest symbol in unit's place or the first order.

There are listed below the names and symbols used in all the present systems from binary to duodecimal, together with the unit representation of the radix, expressed in the terms of the decimal (our present) system.

Name	Radix	Symbols Necessary
Binary	2	0,1
Ternary	3	0,1,2
Quaternary	4	0,1,2,3
Quinary	5	0,1,2,3,4
Senary	6	0,1,2,3,4,5
Septenary	7	0,1,2,3,4,5,6
Octonary	8	0,1,2,3,4,5,6,7
Nonary	9	0,1,2,3,4,5,6,7,8
Decimal	10	0,1,2,3,4,5,6,7,8,9
Undecinary	11	0,1,2,3,4,5,6,7,8,9,X
Duodecimal	12	0,1,2,3,4,5,6,7,8,9,X,E

From the above table it will be noted that the total number of the symbols used in each counting system is the same in quantity as the radix of the system.

The radix is "10" in each group. Again, what is "10"? This really is the term "n^1" in our power series. What is "n^1"? It means one in the second order, (or some single-figure-multiple of it), and nothing in the first order.

Let us see if the following counting chart will be of any assistance in the further understanding of the method of operation in different bases. We will count from one to twelve (expressed decimally) in each base, placing the decimal system at the top for easy reference.

COUNTING CHART

Decimal	1	2	3	4	5	6	7	8	9	10	11	12
Binary	1	10	11	100	101	110	111	1000	1001	1010	1011	1100
Ternary	1	2	10	11	12	20	21	22	100	101	102	110
Quaternary	1	2	3	10	11	12	13	20	21	22	23	30
Quinary	1	2	3	4	10	11	12	13	14	20	21	22
Senary	1	2	3	4	5	10	11	12	13	14	15	20
Septenary	1	2	3	4	5	6	10	11	12	13	14	15
Octonary	1	2	3	4	5	6	7	10	11	12	13	14
Nonary	1	2	3	4	5	6	7	8	10	11	12	13
Decimal	1	2	3	4	5	6	7	8	9	10	11	12
Undecinary	1	2	3	4	5	6	7	8	9	X	10	11
Duodecimal	1	2	3	4	5	6	7	8	9	X	E	10

We have mentioned a power series and used a general expression of it. Actually, what is the numerical equivalence of each order of the power series in each of the numerated bases? The following chart gives the decimal notation for each order of the power series for each base through the fifth order, which is just far enough to give a rudimentary idea of its operation.

POWER SERIES CHART

	n^4	n^3	n^2	n^1	n^o
Binary	16	8	4	2	1
Ternary	81	27	9	3	1
Quaternary	256	64	16	4	1
Quinary	625	125	25	5	1
Senary	1296	216	36	6	1
Septenary	2401	343	49	7	1
Octonary	4096	512	64	8	1
Nonary	6561	729	81	9	1
Decimal	10000	1000	100	10	1
Undecinary	14641	1331	121	11	1
Duodecimal	20736	1728	144	12	1

What qualities make a good base for a system of numeration? First and foremost the base should have a high degree of factorability. Secondly, it should have as few symbols as possible and still be able to express large numbers without becoming too cumbersome. There appears, from the above specifications, to be only one of the listed bases which meets these requirements. This is the duodecimal system of numeration. With the exception of one, and itself, this base is twice as factorable as any other base. Large numbers can be expressed most easily; more easily, in fact, than in any of the other listed bases. There are only twelve symbols necessary to express all of the numbers in the counting process and this is not too cumbersome. Add to this the fact that 1/4, 1/3, 2/3, 3/4 can all be expressed as one-place decimals in this, and only this, system. This system also provides the best answer to the advocates of the change to a metric system of measurement. Grocers, who deal in dozens and buy by the gross, are using this system even today, to as large an extent as is possible within the limitations of our own "ten" base.

There is only one real factor which favors the base "ten" over the "dozen" base. That is the factor of familiarity; we now have it and apparently are stuck with it. There are many articles dealing in detail with all of the points on which duodecimals far exceed decimals in ease of computation and general usability.

We think and compute in the number system to the base "ten". How is it possible to convert to other bases? Conversion from one radix to another usually involves only those bases which are designated as binary, decimal and duodecimal. If other bases are being investigated conversions are made to the base-ten system and then to the desired base, by a series of divisions, using the remainders in reverse order, as the desired figure in the new base. The primary conversion to the base "ten" is made by the use of the power series as set up in the above-mentioned Power Series Chart.

A NEW METHOD OF CONVERSION

By following the method, which I am introducing here, conversions can be made easily from any base to any other base, directly without the interim conversion to the base "ten".

Let us take any random number to be converted. Designate the radix of the original number and the base to which the conversion is to be made.

This is the rule to follow: Starting at the left, or at the digit at the largest order, multiply this digit by a number which is the representation of the radix of the original number, as expressed in the notation of the base to which conversion is to be made; to this product add the digit next appearing in the original number; taking this sum as a multiplicand, multiply again by the same multiplier and to that product add the next digit in the original number; continue in this manner until all digits of the original number to be converted have been so used. The result is the conversion of the original number to its corresponding figure in the new base., e.g. Convert octonary 11232 to its corresponding quinary number. You first ask yourself the question, "What is the radix of the number to be converted?" The answer is "8". Then you express this figure in the number system of the new base. (5 in the example.) Consultation of the table for the counting series shows that the idea of "eight" units in the base "five" is the symbol "13" or one unit of the second order and three units of the first order. Multiply the digit at the extreme left by "13" thinking quinarily and add the second digit. The whole process follows:

```
     1 1 2 3 2            244
x   13              x      13
    13                   1342
+    1                    244
    14                   4332
x   13              +       3
   102                   4340
   14               x      13
   242                  24120
+    2                   4340
   244                 123020
                    +       2
```

Therefore $11232_8 = 123022_5$

To prove the calculation, reconvert the new number 123022_5 to the base "8", using the same method.

```
     1 2 3 0 2 2(5)
     5                    276
     5              x       5
+    2                   1666
     7              +       2
x    5                   1670
    43              x       5
+    3                  11230
    46              +       2
x    5
   276   123022(5)  =   11232(8)
+    0
   276
```


To further check the calculations, convert both of these figures to any base at all. The results should correspond exactly. As an example conversions will be made in three bases: -- binary, decimal and duodecimal.

DECIMAL CONVERSION

```
     1 1 2 3 2(8)          1 2 3 0 2 2(5)
x    8                x    5
     8                     5                  190
+    1                +    2             x      5
     9                     7                  950
x    8                x    5             +      2
    72                    35                  952
+    2                +    3             x      5
    74                    38                 4760
x    8                x    5             +      2
   592                   190                 4762
+    3                +    0
   595                   190
x    8
  4760
+    2
  4762
```

Both figures convert to 4762 in the base "ten".

DUODECIMAL CONVERSION

```
   1 1 2 3 2(8)      6          1 2 3 0 2 2(5)
x  8                       x    5
   8                 62         5                 13X
+  1           x      8    +    2            x      5
   9                414         7                 672
x  8           +      3    x    5            +      2
  60                417        2Ɛ                 674
+  2           x      8    +    3            x      5
  62               2908        32                2908
               +      2    x    5            +      2
                   290X       13X                290X
                           +    0
                              13X
```

Again 290X = 290X. This conversion started the same as the decimal conversion, but it must be remembered that you must think dozenally.

In the binary conversion, be sure to consult the counting chart to get the binary equivalent for the bases "8" and "5" and proced as before.

	$1\,1\,2\,3\,2_8$
x	1000
	1000
+	1
	1001
x	1000
	1001000
+	10
	1001010
x	1000
	1001010000
+	11
	1001010011
x	1000
	1001010011000
+	10
	1001010011010

	$1\,1\,2\,3\,2_5$
x	101
	101
+	10
	111
x	101
	111
	1110
	100011
+	11
	100110
x	101
	100110
	1001100
	10111110
+	0
	10111110 →

	10111110
x	101
	10111110
	101111100
	1110110110
+	10
	1110111000
x	101
	1110111000
	11101110000
	1001010011000
+	10
	1001010011010

Finally, 1 001 010 011 010 = 1 001 010 011 010

If nothing else is indicated by this last conversion, it certainly shows how cumbersome is the calculation done binarilly.

We now have five different expressions representing the same number in five different bases. By the method being investigated, we can convert this number in all of the used bases to the base "six". If the conversion method works, all of the answers should be the same.

SENARY CONVERSION

Remember that the thinking has to be done in the base "six" in all of these calculations.

	$1\,1\,2\,3\,2_8$
x	12
	12
+	1
	13
x	12
	30
	13
	200
+	2
	202
x	12
	404
	202
	2424
+	3
	2431
x	12
	5302
x	2431
	34012
+	2
	34014_6

	$1\,2\,3\,0\,2\,2_5$
x	5
	5
+	2
	11
x	5
	55
+	3
	102
x	5
	514
+	0
	514
x	5
	4222
+	2
	4224
x	5
	34012
+	2
	34014_6

	$4\,7\,6\,2_{10}$
x	14
	104
+	11
	115
x	14
	512
	115
	2102
+	10
	2112
x	14
	12452
	2112
	34012
+	2
	34014_6

	$2\,9\,0\,X_{12}$
x	20
	40
+	13
	53
x	20
	1500
+	0
	1500
x	20
	34000
+	14
	34014_6

	1 001 010 011 010
x	2
	2
+	0
	2
x	2
	4
+	0
	4
x	2
	12
+	1
	13
x	2
	30
+	0
	30 →

	30
x	2
	100
+	1
	101
x	2
	202
+	0
	202
x	2
	404
+	0
	404
x	2
	1212
+	1
	1213 →

	1213
x	2
	2430
+	1
	2431
x	2
	5302
+	0
	5302
x	2
	15004
+	1
	15005
x	2
	34014
+	0
	34014_6

As a final exercise in conversion, a small number, e.g., decimal 123, can be converted into each base, and then prove these figures by a conversion of the results into the duodecimal base. Each of these final conversions should result in the identical numbers.

	Base 12	11	10	9	8	7
	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
x	𝒳	𝒳	10	11	12	13
	𝒳	𝒳	10	11	12	13
+	2	2	2	2	2	2
	10	11	12	13	14	15
x	𝒳	𝒳	10	11	12	13
	𝒳0	𝒳𝒳	120	13	30	51
+	3	3	3	13	14	15
	𝒳3	102	123	143	170	231
+				3	3	3
				146	173	234

	Base 6	5	4	3	2
	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
x	14	20	22	101	1010
	14	20	22	101	1010
+	2	2	2	2	10
	20	22	30	110	1100
x	14	20	22	101	1010
	120	440	120	110	11000
	20		120	1100	11000
	320		1320	11110	1111000
+	3	3	3	10	11
	323	443	1323	11120	1111011

Reconversion of these results to the duodecimal base.

	Base 12	11	10	9	8	7
	𝒳 3	1 0 2	1 2 3	1 4 6	1 7 3	2 3 4
x	10	Ɛ	𝒳	9	8	7
	𝒳0	Ɛ	𝒳	9	8	12
+	3	0	2	4	7	3
	𝒳3	Ɛ	10	11	13	15
x		Ɛ	𝒳	9	8	7
		𝒳1	𝒳0	99	𝒳0	9Ɛ
+		2	3	6	3	3
		𝒳3	𝒳3	𝒳3	𝒳3	𝒳3

	Base 6	5	4		3			2				
	3 2 3	4 4 3	1 3 2 3		1 1 1 2 0			1 1 1 1 0 1 1				
x	6	4	4		3			2				
	16	18	4		3			2				
+	2	4	3		1			1				
	18	20	7	→26	4	→11	→35	3	→7	→13	→26	→51
x	6	5	4	4	3	3	3	2	2	2	2	2
	𝒳0	𝒳0	24	𝒳0	10	33	𝒳3	6	12	26	50	𝒳2
+	3	3	2	3	1	2	0	1	1	0	1	1
	𝒳3	𝒳3	26	𝒳3	11	35	𝒳3	7	13	26	51	𝒳3

Why does this method of conversion work? If any number $n^4+n^3+n^2+n^1+n^0$ to the base "7" is investigated and converted to the base "10" the following results are obtained.

$$\begin{array}{lrrrrr}
 & n^4 + & n^3 + & n^2 + & n^1 + & n^0 \\
\times & 7 & & & & \\
\hline
 & 7n^4 + & n^3 & & & \\
\times & & 7 & & & \\
\hline
 & 49n^4 + & 7n^3 + & n^2 & & \\
\times & & & 7 & & \\
\hline
 & 343n^4 + & 49n^3 + & 7n^2 + & n^1 & \\
\times & & & & 7 & \\
\hline
 & 2401n^4 + & 343n^3 + & 49n^2 + & 7n^1 + & n^0
\end{array}$$

Reference to the Power Series Chart will indicate that the constant coefficients correspond exactly with the Septenary conversion constants as set forth in this chart.

It now appears that it is demonstrated that by this method conversions can be made directly from any base to any other base without a preliminary conversion to the decimal base. All that is necessary is a counting chart or the ability to construct one, and the knowledge and perseverance to think in the base to which a conversion is desired.

Now refer to the counting chart. If the decimal expression of each base be examined, the following facts become clear, proving that the base of any number system is "10".

Binary	2 = 10	Octonary	8 = 10
Ternary	3 = 10	Nonary	9 = 10
Quaternary	4 = 10	Decimal	10 = 10
Quinary	5 = 10	Undecinary	11 = 10
Senary	6 = 10	Duodecimal	12 = 10
Septenary	7 = 10		