

COUNTING IN DOZENS

1 2 3 4 5 6 7 8 9
 one two three four five six seven eight nine d

Our common number system is decimal - based on ten. The dozen twelve as the base, which is written *10*, and is called *do*, for quantity *one gross* is written *100*, and is called *gro*. *1000* is representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens and the 3 applies to tens-of-tens, or hundreds. Place value is important in dozenal counting. For example, 265 represents 5 units, 2 dozen-dozen, or gross. This number would be called 2 *gro* 6 *d*. Coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying.

94	136	Five ft. nine in.	5.9
31	694	Three ft. two in.	3.2
96	3E2	Two ft. eight in.	2.8
19E	1000	Eleven ft. seven in.	2.7

You will not have to learn the dozenal multiplication tables as you already know the 12-times table. Mentally convert the quantities into decimal and set them down. For example, 7 times 9 is 63, which is 5 dozen and 3. Using this "which is" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication tables.

Conversion of small quantities is obvious. By simple inspection 35 years old, dozenally you are only 2E, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

12)	365	
		30	+ 5
12)	2	+ 6
		0	+ 2

Dozenal numbers may be converted to decimal numbers by setting down the first figure, adding to it 12 times the second figure, plus 12² (or 144) times the third figure, plus 12³ (or 1728) times the fourth figure, and so on. Or, to use a method corresponding to the illustration, keep dividing by 12, and the successive remainders are the desired decimal numbers.

Fractions may be similarly converted by using successive multiplication instead of divisions, by 12 or 1/12.

Numerical Progression		Multiplication	
1	One	1	2 3 4 5 6 7 8 9 10 11 12
10	Do	.1	Edo
100	Gro	.01	Egro
1,000	Mo	.001	Emo
10,000	Do-mo	.000.1	Edo-mo
100,000	Gro-mo	.000.01	Egro-mo
1,000,000	Bi-mo	.000.001	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

duodecimal Bulletin

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DUODECIMAL BIBLIOGRAPHY

DUODECIMAL SOCIETY OF AMERICA

Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of \$6, covering initiation fee (\$3) and one year's dues (\$3), must accompany applications.

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CONTENTS

The Annual Meeting	1
How We Began. F. Emerson Andrews	3
Citation.	8
Do-Metric Thermal Units	9
The Uncial System Paul Van Buskirk	2
Scales of Notation. D. W. Mynett	13
Hand Base Changing by Summation H. F. Stevens	15
The Reciprocal of 3937. Ralph H. Beard	19
I'm A Dozener Eugene J. Zirkel	12
A Note of Conversion Multiples. George S. Terry	23
A Mathematical Recreation	24
Science Fiction and Duodecimal. F. H. Ames, Jr.	25
Excerpt from "Our Money Words". Webb B. Garrison	26
The Curate & The Girls. Lewis Carl Seelbach	27

WHY CHANGE ?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numbers universally used. *"Why even try to learn to use these heathenish scrawly symbols, with their stubborn propensity for error, in place of the beautiful clear numbers which our fathers have used for untold generations? Think of the needless waste. We would have to change all of our counting boards and abaci. X is X, isn't it? And why do we need a symbol for nothing? You can't count it! No! Let us keep to our simple tried and true numerals, and let the barbarians scratch their heads, and rub themselves out. It will all come to 0 anyhow."*

Yet, although it took D years, the new notation became generally used, and man's thinking leapt forward like an arrow sped from a bow. The early years of the Renaissance marked a new stage in the use of symbols, with the advent of algebra, fractionals (decimals?), logarithms, analytical geometry, and the calculus. Can you imagine what it would be like to try to express the coordinates of points on a curve in Roman notation?

Mathematicians became conscious of a new dimension in symbolism, and the fundamental concepts of numbers were re-examined. Man awoke to the fact that different number bases could be used, and Simon Stevin stated in 1585 that the duodecimal base was to be preferred to the decimal.

The new Arabic notation accomodated mathematical statement better, and facilitated ideation. All thinking accelerated when released from the drag of the cumbrous Roman notation.

The parallel seems tenable. The notation of the dozen base accomodates mathematical statement better, and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base embodies a concurrent analysis and definition of numbers that stimulates classification and generalization. Yet this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatisfactory because it has "not-enough-factors."

Then shouldn't we change? No! No change should be made, and we urge no change. All the world uses decimals. But people of understanding should learn to use duodecimals to facilitate their thinking, and to ease the valuative processes of their minds. Duodecimals should be man's second mathematical language. They should be taught in all the schools. In any operation, that base should be used which is most advantageous, and best suited to the work involved. We expect

advocate a

that duodecimals will progressively earn their way into general popularity. But no change should be made. Perhaps by the year 2000, or maybe by 1200; which is 14 years later, duodecimals may be the more popular base. But then no change need be made, because people will already be using the better base.

When one is familiar with duodecimals, a number of accessory advantages become apparent. Percentage is a very useful tool, but many percentages come out in awkward figures because of the inflexibility of decimals. When based on the gross, twice as many ratios come out in even figures, and among them are some of those most used, as thirds, sixths, and twelfths, - eighths and sixteenths. There are advantages associated with time and the calendar. Monthly interest rates or charges are derived from annual rates, or the reverse, by simply moving the unit (decimal?) point. The price of a single item bears the same relation to the price of the dozen, and so does the inch to the foot.

The proper correlation of weights and measures has always been one of the world's serious problems. None of the present systems is completely satisfactory. The American and English standards are convenient to use since they are the final result of a long process of practical evolution in which many inconvenient measures have been adjusted or abandoned. The French decimal metric measures have the advantage of being set upon the same base as the number system, and are well systemized. But many of the units are awkward because of their arbitrary sizes, and because their decimal scale does not accommodate division into thirds and fourths readily.

The duodecimal system of weights and measures, based on the inch and yard, the pint and the pound, has the desirable elements of both systems, and few of their faults. This Do-Metric System retains the familiar units of the American and British standards in approximately their present size, and arranges them into an ordered metric system using the scale of twelve. This fits perfectly into the duodecimal notation, and the combination accomodates the inclusion of the units of time and of angular measure within the system, which hitherto has not been possible.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have thought staid and established, and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is cordially invited.

and we hope that a largly progression will bring duodecimals into popular preference

The Duodecimal Bulletin

All figures in italics are duodecimal.

THE ANNUAL MEETING

The Annual Meeting of the Duodecimal Society of America was held at the Gramercy Park Hotel, New York City, on Thursday, January 22nd. President Robert opened the meeting promptly at 2030 EST., and asked the Secretary to summarize the year's activities.

The notable feature of the year is the publicity achieved for duodecimals. Kingsland Camp's paper on "Looking Toward the Future" appeared in the Journal of Calendar Reform for July. A paper by Paul Van Buskirk, setting forth his "Nomenclature for Base Twelve Numbers," was published in Surveying and Mapping for July-September. A favorable review of the New Musical Notation of Velizar Godjevatz in the Musical Courier for November was much appreciated. And in the October issue of School Science and Mathematics, Donovan A. Johnson published an excellent paper on the use of duodecimals as an aid in teaching mathematics, under the title, "A Unit of Our Number System."

There were two issues in Volume 8 of the Duodecimal Bulletin. The October issue was devoted exclusively to The Duodecimal Bibliography, thus completing the initial stage of one of our major projects.

In June, F. Emerson Andrews, Chairman of our Board of Directors, was awarded the honorary degree of Doctor of Humane Letters, by his alma mater, Franklin and Marshall College. It is to be noted that Mr. Andrews' works on duodecimals played no small part in earning him this distinction.

There was a small gain in membership, and the Society can be well satisfied with these testimonials to its growth and influence.

Treasurer Humphrey reported that donations for the year totaled \$1058, contributed largely by the same loyal and generous members to whom the Society owes so much. However, expenditures of about \$1700 exceeded receipts for the year by about \$400.

As directed by the Board, the proposal for official endorsement of the World Calendar by the Society, has been submitted to the membership by mail. Secretary Beard reported that favorable replies have been received from about 25% of the members, and not one objection. Thereupon, the question was formally proposed to this meeting, and unanimously approved.

note out by Tom Lester's offering Do-Metric advertisement

ratios are

things are better by the dozen

President Robert then announced that the Board of Directors had this day elected Ralph H. Beard as President and Secretary for 1953, and asked Mr. Beard to take over the meeting.

The new president acknowledged his deep appreciation of responsibilities entrusted to him and pledged his best efforts in the Society's interests. He asked Chairman Andrews to review for the meeting the origins and development of the Society. Mr. Andrews' talk appears elsewhere in this issue, entitled, "How It Began."

President Beard then outlined the recent accomplishments of the Society and his plans for the current year. He will endeavor to develop more frequent and more intimate contact with our members, and better use of their talents and abilities. He announced the appointment of Paul and Camilla Adams as Assistant Secretaries to get this correspondence with members under way.

Kingsland Camp reported that he will produce his new planisphere on a commercial basis this spring, in a form adaptable to any latitude. The time scales used will be the duodecimal day and the year divided according to the months of the World Calendar.

Edward Pharo, of the Eckert Mauchly Computer Co., announced that the three-stable-stage relay (the flip-flap-flop,) has now been achieved, and has been described in one of the recent issues of the Proceedings of the I.R.E. In a period of open discussion, he described many of the details of operation of the amazing electronic computers.

With the closing of the formal program, the meeting dissolved into the familiar kaffee-klatch that we all enjoy so much.

OFFICERS AND COMMITTEES, 1953: *Chairman of the Board*, F. Emerson Andrews, 34 Oak St., Tenafly, N.J.; *Directors Class of 1954*, F. Emerson Andrews, William Shaw Crosby, Nathan Lazar, Louis Paul D'Autremont; *Class of 1955*, H. C. Robert, Jr., Kingsland Camp, Paul E. Friedemann; *Class of 1956*, George S. Terry, H. K. Humphrey, Ralph H. Beard; *President & Secretary*, Ralph H. Beard, 20 Carlton Place, Staten Island, N.Y.; *Vice President*, Paul E. Friedemann, 904 Mifflin Ave., Pittsburgh 21, Pa.; *Treasurer*, H. K. Humphrey, 520 Ash St., Winnetka, Ill.; *Committee on Awards*, George S. Terry, Chairman, 507 Main St., Hingham, Mass.; F. Emerson Andrews, Ralph H. Beard; *Committee on Bibliography*, Lewis Carl Seelbach, 163 Davidson Ave., Buffalo 15, N.Y.; *Committee on Correspondence*, Camilla Adams, Assistant Secretary, Paul Adams, Assistant Secretary, 90-10-149th St., Jamaica 2, N.Y.; *Committee on Finance*, George S. Terry, Chairman, F. Emerson Andrews, H. K. Humphrey, Louis Paul d'Autremont; *Committee on Membership*, Doris Burke Lloyd, 2505 Pinebrush Road, Baltimore 9, Md.; *Nominating Committee*, H. F. Stevens, Chairman, 744 Lexington Ave., R.F.D.1, Union, N.J., Dudley A. George, F. H. Ames, Jr.; *Committee on Weights & Measures*, Dallas H. Lien, Chairman, 2641 Sutherland Ave., Indianapolis 5, Ind., Paul Adams, Albert De Valve, Eugene M. Scifres; *Editor, The Duodecimal Bulletin*, George S. Terry.

HOW WE BEGAN

F. Emerson Andrews

This is the story of how the Duodecimal Society of America started. It is in part a personal record, but I can tell it without embarrassment, for I have been only the catalyst, the one who has interested the other and abler persons who have made possible the growth of this Society and its substantial accomplishments.

The idea of counting by dozens antedates this Society by at least several centuries. Indeed, fragments of duodecimals (in the sexagesimal system and elsewhere) are found among the Babylonians. The Society's new Bibliography indicates that the subject may have been treated in a book by Simon Stevin as early as 1585. In England, prominent proponents included Sir Isaac Pitman, who was trying to induce his shorthand students to use duodecimal counting as early as 1855; Thomas Leech whose *Dozens versus Tens* published in 1866 was the earliest substantial book on the subject in the English language, and Herbert Spencer, who is said to have left a provision in his will which would promote the duodecimal system and oppose the metric if ever the British Parliament attempts to introduce the latter.

In America, a few minor efforts had been made looking toward duodecimal counting, including Nystrom's duodenal arithmetic proposal (the name was not happy), and Perry's pamphlet, *The American System of Mathematics*. Some school arithmetics suggested the idea, usually along with other possible bases. But in the 1930's, few people had ever heard of the idea, much less tried it out.

i

One blizzardy afternoon in January, 1934, I sat at my home writing desk with nothing pressing to do. An important article had just been finished; two hours were not enough to start another. I remembered hearing long ago, possibly from a teacher in high school, that one could count by twelves easier than by tens. This questioned one of the few remaining things in this world of change that nearly everyone takes for granted. I tried out this half-remembered idea. Soon I found that all I needed was to invent two new numerals, one for the old quantity *ten* and another for *eleven*. All that afternoon papers covered with scribbled figures multiplied on my desk as unaccountably as tropical fish.

Mrs. Andrews, who was at that time an editor in a publishing house, came home at dusk to find a writer's desk covered with no writing, just a mad array of figures, some of them not even honest Arabic numerals.

"Well," she said cheerfully, "what a way to waste your time!"

I had rather intended to waste my time, but this was too much of a challenge. In pretended dudgeon I announced that these figures represented an important discovery--not, perhaps, a completely new discovery, but one that had never been adequately interpreted to a possibly waiting world. I, a writer, would make this discovery known.

As a result of this rash boast I delved much deeper into the duodecimal system, uncovering some of the earlier writings, and in a few weeks finished an article which I hopefully sent off to *Scientific American*, which seemed the logical market. I got a most distressing valentine from the associate editor, a letter dated February 14, 1934, from which I quote some of the more horrible portions:

Dear Mr. Andrews:

Unquestionably the duodecimal system would be far superior to our decimal system. I recall discovering this from some old book when in college and spending a good solid hour trying to drill it into the head of my roommate, who had won the highest honors in his class as an engineer, the fact that there is no special magic, nothing at all special, in ten as a base. . . Plainly you have immersed yourself in this study for a long time (it had been two weeks) and lost your ability to see it as a reader would see it *de novo*.

Few would read your article through, because it would be labor, giving most readers a severe headache. . . It would take a real dabster of a writer to put it over.

Cordially yours,
Albert G. Ingalls

Clearly, I had failed as a writer and managed only to pass myself off as a mathematician! So I scrapped the article, rewrote it three more times, and finally, at least reasonably content with it, sent it off to another likely magazine, christened "An Excursion in Numbers."

Four additional magazines would have none of it, making five rejections in all. The likely magazines, with scientific readership, were exhausted; where now? At this point I sent it to one of the most unlikely magazines, but one in which I had been previously published, the *Atlantic Monthly*. Back it came, but this time with a letter from assistant editor Aswell, reporting he had thought it one of the most interesting articles of the year, but the *Atlantic* never had published mathematics and probably never would; certainly he could not accept such a manuscript in the absence of the editor, Ellery Sedgwick, in Japan. But if I wished to send it back after Mr. Sedgwick returned--

I became a reader of ship news reports. Mr. Sedgwick and my manuscript returned to Boston practically simultaneously. Soon came another letter. The *Atlantic* would publish, provided I would permit insertion of an italic paragraph warning off non-mathematical readers at the point where the article began to be mostly arithmetic. The italics went in, and the October, 1934, *Atlantic* carried the "Excursion."

It was an excursion in a sense I had not anticipated. Before my own author's copies came, piles of letters began arriving. They were from engineers, sea captains, missionaries, sugar planters, Wall Street bankers, teachers of mathematics, lunatics. The *Atlantic* was getting a similar flood, and, from a magazine which never published mathematics, became suddenly one which urged me to do two more articles, which I eventually did. We shall return to this correspondence, out of which grew the Duodecimal Society, but first a few words on the literary aftermath.

In the careful preparation of the "Excursion" I had accumulated far more material than could there be used--enough, indeed, for a small book. So I wrote the book, titled it *New Numbers*, and took it to a publisher for whom I had worked five years, where all the editors were personal friends. It came back promptly. Indeed, six publishers were unanimously uninterested. The lucky seventh attempt was to Harcourt, Brace and Company where the book was accepted by an editor I had never seen before. The day the book was published, September 19, 1935, Faber and Faber of London cabled an offer for a special British edition, which was issued next year. Soon the British stock was burned upon in the Nazi fire raids on London, but not before Mr. J. Halcro Johnston had received a copy and was stimulated to write his own version of how counting by dozens should be done, *The Reverse Notation*, published by Blackie and Son, Limited, in 1937.

It would be unfair to leave the impression that all of this correspondence was important or historic. Some of it was just amusing, some of it a nuisance. One correspondent denounced the whole proposal as blasphemous, alleging that counting by tens was God-sent from Sinai, as could be proved by the Ten Commandments. Realizing that with this person mathematical argument was useless, I replied stoutly that in the New Testament there were *twelve* Apostles.

ii

One of the earliest letters growing out of the "Excursion" deserves permanent place in the Society's files. I quote it in part:

29 October 1934

Dear Sir:

With reference to your interesting article on Numbers in the October *Atlantic*, as I understand it, the advantages of the duodecimal system for everyday purposes in order of their importance are:--

1. Measurement of length in feet and inches as stated in the article. . . (and so on for two pages)

Will you please tell me of a book on this subject, not too abstruse, but containing log. tables?

Yours sincerely,
George S. Terry

I told Mr. Terry of the Leech logarithms for certain primes, their inaccuracies, and my belief that "no really serviceable table of 12-base logarithms is in existence. If your interest is keen enough to support the considerable labor involved, may I suggest that you might be making a very real contribution to the future by developing such a table?"

Mr. Terry rose to that challenge in the magnificent way this Society knows. His monumental *Duodecimal Arithmetic* was published by Longmans, Green and Company in 1938, making available for the first time in history adequate tables of logarithms to the twelve base, trigonometric functions in terms of the duodecimal circle, and other needed apparatus.

Meanwhile Mr. Terry with his logarithms, Mr. F. Howard Seely of California with an arithmetic he was preparing, and Mr. Charles Q. DeFrance of Nebraska were becoming steady correspondents. Other correspondence usually ended after one or two interchanges, but this group wrote regularly for years. The top row of keys on my typewriter were nearly worn out by letters that began "Dear Sir" but continued from there on almost entirely with figures. Mr. DeFrance thought nothing of dabbling in reciprocals of primes running to 180 places. This small group usually made carbons of letters, sending to the whole circle, and gradually began calling itself, humorously, The Duodecimal Society of America. Then finally came the letter which may be called the real starting point of the Society. I quote the significant paragraph:

21 September 1939

Dear Mr. Andrews:

. . . One other point. In recent circular letters between the four of us, "The Duodecimal Society of America" has been spoken of, with you as President and me as Treasurer. This seems to me very sound and capable of being put to a lot of use--especially as I am willing to finance it if necessary. But what is the procedure? Who are you going to elect as

original members? It should publish a bulletin. You know all about that business. Should it be "The Duodecimal Society (of America)?"

Yours sincerely,
George S. Terry

A substantial endowment and other money gifts came from Mr. Terry for the founding and operation of the Society, to be added to the great intellectual contribution he had made and was making. We now had an idea, a name, and financial backing; we needed an organizer.

New Numbers went out of print in 1940 (it was later reprinted by another publisher) but library copies were still having their influence. A most significant letter reached me in 1941 via the former publisher, Harcourt, Brace:

September 14th, 1941

Dear Mr. Andrews:

In consideration of our mutual interest in duodecimals, I would like to have your response to certain questions that have occurred to me.

As a result of the current war, there will probably be established an international government of some type or other. This may reasonably involve the establishment of an international coinage, new standards of weights and measures, and an integrated telephone service. All of these stress the importance of making rapid progress in the education of the public in the advantages of duodecimals.

Hence, my questions. Is there an organization for this purpose? Can you give me the address of its correspondent? Is it national or international?

If no organization exists, I would like to help, unprofessionally, in starting one. I am forwarding letters similar to this to the names following. (Messrs. Terry, Seely, Perry, Rear Admiral Elbrow.)

Truly yours,
Ralph H. Beard

The infant Society had found its organizer, its human dynamo. Mr. Beard's magnificent contributions to this Society as an organizer, as a mathematician and an expert in weights and measures, as an indefatigable secretary, Bulletin editor, and now president, are known to every member.

On the 5th of April, 1944, the executive committee of the Society held its first official meeting. It was a notable day in several respects. Eight inches of snow had fallen, most unusual in New York so late in the spring; and the Society decided to incorporate formally under the laws of New York State. This

incorporation was finally effected as of 18 July 1944, with these incorporators: George S. Terry, F. Howard Seely, Ralph H. Beard, F. Morton Smith, and F. Emerson Andrews. The rest of the history of the Society is chronicled in its *Bulletin*, which began publication in 1945.

On 8 January of that year I sent to the Society members a New Year's greeting which may appropriately close this informal narrative of our beginnings:

Whether this year or any near year our newborn Society will become large in numbers, I do not know. Whether it was courage or mere rashness which led us to launch our venture in the middle of a desperate war, only the long future will decide. But when I see the long continuing enthusiasm of our first small group, and read the highly interesting notes of some of our newer members, I am much encouraged. No one can measure the power of an idea, or be sure when its hour has come

F. Emerson Andrews

COPY OF CITATION OF F. EMERSON ANDREWS ON THE OCCASION OF HIS RECEIVING THE DEGREE OF DOCTOR OF HUMANE LETTERS AT THE 165th ANNUAL COMMENCEMENT OF FRANKLIN AND MARSHALL COLLEGE, LANCASTER, PENNSYLVANIA, JUNE 6, 1952.

Mr. President:

I am delighted to present F. Emerson Andrews, author and consultant on publications

In 1934 Mr. Andrews published an article, "An Excursion in Numbers"; this was developed into a book, *NEW NUMBERS*, published in 1935; out of this book, in 1943, grew an organized movement for research in counting by dozens, the Duodecimal Society of America, of which society Mr. Andrews has been a member since its founding, president from 1944 to 1950, chairman of the Board since 1950, and recipient of the society's first annual award in 1944. Let no one henceforth underestimate the power of a few well chosen words.

For his distinguished work as a creative and expository writer and for his devotion to the production of worthwhile books, the Board of Trustees is pleased to present Mr. Andrews for the honorary degree of

DOCTOR OF HUMANE LETTERS

DO-METRIC THERMAL UNITS

Heat is a form of energy. Energy can be converted into heat, and heat can be converted into energy. The mechanical equivalent of heat is the quantity of energy which, transformed into heat, will raise the temperature of a unit mass of water one degree within a specified temperature range. This equivalent is called the calorie or the thermal unit. Its values are different for different levels of temperature. The value generally given is the "mean" value, which is the average of all values between the freezing point and the boiling point of water.

Thus, the Calorie is the energy required to raise the temperature of one gram of water 1° Centigrade; - the British Thermal Unit is that required for 1° Fahrenheit in one pound avoirdupois; - and the Do-Metric Thermal Unit is that required for 1° Do-Metric in one do-metric pound of water. The mean value of the do-metric thermal unit corresponds with that for the change from 19° to 1X°.

TEMPERATURE							
	Thermometer scales						
	Fahr.	Cent.	Kelvin	Do-Metric			
				Popular		Absolute	
Dec.Nos.	Duodec.	Dec.Nos.	Duodec.	Dec.Nos.	Duodec.	Dec.Nos.	Duodec.
Absolute Zero	-459.72	-273.18	0	-393.38	-289.47	0	0
Oxygen Boils	-297.35	-182.97	90.21	-263.48	-192.59	129.90	289.47
Water Freezes	32.00	0	273.18	0	0	393.38	289.47
Water, Max. Dens.	39.16	3.98	277.16	5.73	5.89	399.11	293.14
Approx. Room Temp.	68	20	293	29	25	422	282
Blood Heat	98.60	37.00	310.18	53.28	45.34	446.66	312.72
Water Boils	212.00	100.00	373.18	144.00	100.00	537.38	389.47
Sulphur Boils	832.28	444.60	717.78	640.22	454.28	1033.60	721.73
Silver Melts	1761.44	960.80	1233.98	1383.55	973.68	1776.93	1040.82
Gold Melts	1945.40	1063.00	1336.18	1530.72	776.88	1924.10	1144.12
Degree Equivalents							
	Decimal			Duodecimal			
	Fahr.	C. or K.	Do-M.	Fahr.	C. or K.	Do-M.	Do-M.
1° Fahrenheit*	1	.5	.8	1	.68	.9724	
1° Centigrade or K.	1.8	1	1.44	.9724	1	1.5344	
1° Do-Metric	1.25	.694	1	1.3	.84	1	
*Allow 32° for freezing point.							

CONVERSIONS

- From Centigrade to Fahrenheit: Multiply by 1.8 and add 32°.
- From Centigrade to Do-Metric : Multiplying by 1.44 will give degrees in decimal figures. Convert to duodecimal figures.
- From Fahrenheit to Centigrade: Subtract 32° and multiply by .5, or 5 9.
- From Fahrenheit to Do-Metric : Subtract 32°. Multiplying by .8 will give degrees in decimal figures. Convert to duodecimal.
- From Do-Metric to Centigrade : Convert to decimal figures and multiply by .694 .
- From Do-Metric to Fahrenheit : Convert to decimal figures and multiply by 1.25 . Add 32°.

The Duodecimal Bulletin

THERMAL UNITS (mean values) $(\frac{ml^2}{t^2})$						
	Decimal					
	B. T. U.	Gram.Cal.	D. T. U.	Joules	Ft. Pds.	Yd. Podyed.
British Thermal Unit	1	251.996	.820 163	1 054.85	25 031.9	343.777
Gram Calorie	.003 968	1	.003 255	4.185 99	99.334 6	1.364.22
Do-Metric Thermal Unit	1.219 27	307.251	1	3 286.15	30 520.6	419.157
Joule (abs.)	.000 948	.238 892	.000 778	1	23.730 3	.325 901
Foot Poundal	.000 040	.010 067	.000 033	.042 140	1	.013 734
Yard Podyed	.002 909	.773 021	.002 386	3.068 42	72.814 4	1
Duodecimal						
British Thermal Unit	1	182.225	.961 272	732.228	12 592.2	247.932
Gram Calorie	.006 735	1	.005 752	4.229 48	83.402 3	1.445 45
Do-Metric Thermal Unit	1.276 79	217.301	1	822.194	15 724.8	272.177
Joule (abs.)	.001 772	.274 980	.001 416	1	12.891 2	.372 177
Foot Poundal	.000 092	.015 490	.000 082	.060 992	1	.012 894
Yard Podyed	.005 032	.896 720	.004 148	3.092 29	60.993 3	1

THE UNICIAL SYSTEM

by Paul Van Buskirk,*

The uncial terminology as set forth is the end product of years of research and "rumination" on my part. It is meant to provide Joe Doaks with an almost effortless transition into the dozenal system and at the same time give the expert a complete place value nomenclature in uncial terms. If we can get Joe to say "ten new system" the first time he runs into our methods, we have broken the "semantic blockage". He can then shorten to ten-new, or ten-uncial system or ten-u, and will soon accept the letter u as a "symbol" for the system. On the typewriter, the letters T and U can be used during the transition period.

The loose use of the Latin enables him to have familiar prefixes for place values. When he learns that the do-zens are all in the second place, it is not hard for him to accept tri-zens as a name for the third place magnitudes.

The root relationship of inch and ounce helps him to comprehend the uncial fractions as a general case of the inch-foot and hour-day relationship. The contraction to "unx" is inevitable, and again his meagre Latin suffices to cover the place values. If he is a shop man, he will appreciate the three place final values inserted in the exact decimal equivalents of an inch table. Given uncial scales that can be edge-compared with his present rules, he will venture to use them. He will want uncially graduated micrometers to free him from bothering with the excess figures. He will ask for uncially-dimensioned plans and be able to demonstrate their superiority.

The general problem that we are attempting to solve has only two phases-(1) perfection of a dozenal system (2) obtaining public adoption of that system.

The whole history of numbers and measures is one of evolution with the exception of the French - Metric. It would seem best to go back to where the orderly evolution stopped and attempt to carry on from that period. The small table of weights and measures is an attempt to show what the pattern of that time was, how it has been modified and how it can be restored to the original path of progressive changes.

Since linear measure is most used and has such an important place in our land title descriptions, it must be the keystone of our system. Our original chain and link surveys are gradually being converted into feet and decimals when new deeds are written. A simple multiplication by 66 does the job. Further

TABLE OF DECIMAL EQUIVALENTS

$\frac{1}{64}$.015625 .023	$\frac{33}{64}$.515625 .623
$\frac{1}{32}$.03125 .046	$\frac{17}{32}$.53125 .646
$\frac{3}{64}$.046875 .069	$\frac{35}{64}$.546875 .669
$\frac{1}{16}$.09 .0625	$\frac{9}{16}$.69 .5625
$\frac{5}{64}$.078125 .083	$\frac{37}{64}$.578125 .683
$\frac{3}{32}$.09375 .116	$\frac{19}{32}$.59375 .716
$\frac{7}{64}$.109375 .139	$\frac{39}{64}$.609375 .739
$\frac{1}{8}$.16 .125	$\frac{5}{8}$.76 .625
$\frac{9}{64}$.140625 .183	$\frac{41}{64}$.640625 .783
$\frac{5}{32}$.15625 .186	$\frac{21}{32}$.65625 .786
$\frac{11}{64}$.171875 .209	$\frac{43}{64}$.671875 .809
$\frac{3}{16}$.23 .1875	$\frac{11}{16}$.83 .6875
$\frac{13}{64}$.203125 .253	$\frac{45}{64}$.703125 .853
$\frac{7}{32}$.21875 .276	$\frac{23}{32}$.71875 .876
$\frac{15}{64}$.234375 .299	$\frac{47}{64}$.734375 .899
$\frac{1}{4}$.3 .25	$\frac{3}{4}$.9 .75
$\frac{17}{64}$.265625 .323	$\frac{49}{64}$.765625 .923
$\frac{9}{32}$.28125 .346	$\frac{25}{32}$.78125 .946
$\frac{19}{64}$.296875 .369	$\frac{51}{64}$.796875 .969
$\frac{5}{16}$.39 .3125	$\frac{13}{16}$.99 .8125
$\frac{21}{64}$.328125 .383	$\frac{53}{64}$.828125 .983
$\frac{11}{32}$.34375 .416	$\frac{27}{32}$.84375 .916
$\frac{23}{64}$.359375 .439	$\frac{55}{64}$.859375 .939
$\frac{3}{8}$.46 .375	$\frac{7}{8}$.96 .875
$\frac{25}{64}$.390625 .483	$\frac{57}{64}$.890625 .983
$\frac{13}{32}$.40625 .486	$\frac{29}{32}$.90625 .986
$\frac{27}{64}$.421875 .509	$\frac{59}{64}$.921875 .909
$\frac{7}{16}$.53 .4375	$\frac{15}{16}$.83 .9375
$\frac{29}{64}$.453125 .553	$\frac{61}{64}$.953125 .953
$\frac{15}{32}$.46875 .576	$\frac{31}{32}$.96875 .976
$\frac{31}{64}$.484375 .599	$\frac{63}{64}$.984375 .999
$\frac{1}{2}$.6 .5	1 ./. .1.

SCALES OF NOTATION

D. W. Mynett

Standard practice, when converting integral numbers from one base to another, is to divide the given number by the required base as often as is necessary to arrive at a remainder less than the required base. The remainders of each successive division are set down in order and this gives the required transformation.

Example 1.

12)	19375	
12)	1614	(7
12)	134	(6
		11	(2
			<u>Ans. £267</u>

The amount of computation when working manually is considerable and increases as the required base decreases. To find the equivalent to 19375 in base 2 requires 14 successive divisions. An effort was made, therefore, to discover a shorter method of conversion and this resulted in the discovery of the following technique.

Example 2. Convert 19375 from decimals to duodecimals.

	19	In brief: 173
Less 2 x 1	<u>2</u>	<u>32</u>
	17	1417
	173	<u>282</u>
Less 2 x 17	<u>32</u>	11555
	141	<u>2222</u>
	1417	<u>£267</u>
Less 2 x 141	<u>282</u>	
	1155	Append 5
	11555	
Less 2 x 1155	<u>2222</u>	
	<u>£267</u>	<u>Ans. £267</u>

This rapid method of conversion of numbers of any magnitude removes one of the principle obstacles to the general acceptability of the duodecimal system in ordinary life.

The method is applicable to any base and conversion from duodecimals to decimals is thus equally easy.

Example 3. Convert £267 from duodecimals to decimals.

	£ = 11	In brief: 112
	112	<u>22</u>
Add 2 x 11	<u>22</u>	1346
	134	Append 6
	1346	<u>268</u>
Add 2 x 134	<u>268</u>	16147
	1614	Append 7
	16147	<u>3228</u>
Add 2 x 1614	<u>3228</u>	
	<u>19375</u>	<u>Ans. 19375</u>

In every case the addition or subtraction is performed in the required base and this also applies to the multiplication. Applied to the duodenary scale, the method affords considerable economy of effort, but the savings are quite spectacular when dealing with the binary scale.

Example 4. Convert 19375 from base ten to the binary scale.

$19 = 2^4 + 2^1 + 1$	$=$	$\frac{10011}{100113}$	Append 3 (3 = 11)		$2 \) \ 19375$
Add 8×10011		$\frac{10011000}{110000017}$	(7 = 111)		$2 \) \ \underline{9687} \ (1)$
Add 11000001000		$\frac{11000001000}{111100100015}$	(5 = 101)		$2 \) \ \underline{4843} \ (1)$
Add $8 \times$ 11110010001		$\frac{11110010001000}{100101110101111}$			$2 \) \ \underline{2421} \ (1)$
					$2 \) \ \underline{1210} \ (1)$
					$2 \) \ \underline{605} \ (0)$
					$2 \) \ \underline{302} \ (1)$
					$2 \) \ \underline{151} \ (0)$
					$2 \) \ \underline{75} \ (1)$
					$2 \) \ \underline{37} \ (1)$
					$2 \) \ \underline{18} \ (1)$
					$2 \) \ \underline{9} \ (0)$
					$2 \) \ \underline{4} \ (1)$
					$2 \) \ \underline{2} \ (0)$
					$1 \ (0)$

Ans. 100,101,110,101,111

100,101,110,101,111

The general rule is:

Convert the first two digits of the given number to the new base and append the third digit. Multiply the converted digits by the difference between the original base and the required base. If the difference is positive add the product; if negative deduct the product from the above converted number. Append to the answer the next digit from the given number and repeat the process. Continue until all the digits of the given number have been dealt with.

Where the first digit of the given number is greater than the required base, it is convenient to commence by converting that digit only.

I am indebted to Dr. D. C. Gilles, B.Sc., Ph.D., D.I.C. of Messrs. Scientific Computation Service Ltd., London, for the following mathematical analysis of the method.

$$\text{Let } a_0A^r + a_1A^{r-1} + \dots + a_r \equiv \alpha_0B^p + \alpha_1B^{p-1} + \alpha_2B^{p-2} \dots + \alpha_p$$

Then the next digit is a_{r+1} we wish to find

$$a_0A^{r+1} + a_1A^r + \dots + a_rA + a_{r+1}$$

Take $(\alpha_0B^p + \alpha_1B^{p-1} + \alpha_2B^{p-2} + \dots + \alpha_p)B + a_{r+1}$

Add $(\alpha_0B^p + \alpha_1B^{p-1} + \alpha_2B^{p-2} + \dots + \alpha_p)A - B$

Sum is $(\alpha_0B^p + \alpha_1B^{p-1} + \alpha_2B^{p-2} + \dots + \alpha_p)A + a_{r+1}$
 or $a_0A^{r+1} + a_1A^r + \dots + a_rA + a_{r+1}$

HAND BASE CHANGING BY SUMMATION

by H. F. Stevens

When dozenal is adopted, considerable scattered and/or occasional hand base changing will become necessary. At times conditions will not warrant logarithmic or other indirect approaches especially by persons busy or not mathematically inclined.

Of course a cumbersome 1-2-----999999 or a 1-2-----999999999 etc. reference chart containing 1999998 or 199999998 numerical expressions could be provided for all six-place or nine-place numbers.

However,

	dec.	doz.	dec.	doz.	dec.	doz.					
(A)	900000	equals	374X00	(B)	990000	equals	328E00	(C)	999000	equals	402180
	90000	"	44100		9900	"	5890		999	"	623
	9000	"	5260		99	"	83		999999	"	402853
	900	"	630		999999	"	402853				
	90	"	76								
	9	"	9								
	999999	"	402853								

A three-term arithmetical summation obtained from the following decimal to dozenal reference chart will change any six-place decimal integer to its equal dozenal integer.

The reference chart was derived from (B) above and contains 594 numerical expressions. If it had been derived from (C) above it would have employed a two-term arithmetical summation for six-place integers.

Any reference chart of this type can easily be expanded in either direction indefinitely.

Note: The following conversion tables compiled by the late F. Howard Sealy are reproduced for comparison, since either method has certain advantages.

SIX PLACE TABLE FOR DEC. TO DOZ. CONVERSION BY 10² SUMMATION

Dec.	Doz.	Dec.	Doz.	Dec.	Doz.	Dec.	Doz.	Dec.	Doz.
990000-328200	490000-127694	9900-5890	4900-2X04	99-83	49-41				
980000-323168	480000-121940	9800-5808	4800-2940	98-82	48-40				
970000-3X9414	470000-1X72X8	9700-5744	4700-2878	97-81	47-32				
960000-3X3680	460000-1X2254	9600-5680	4600-2724	96-80	46-3X				
950000-399928	450000-198500	9500-5528	4500-2730	95-72	45-39				
940000-393294	440000-192768	9400-5534	4400-2668	94-7X	44-38				
930000-38X240	430000-188X14	9300-5470	4300-25X4	93-79	43-37				
920000-3844X8	420000-183080	9200-53X8	4200-2520	92-78	42-36				
910000-37X754	410000-179328	9100-5324	4100-2458	91-77	41-35				
900000-374X00	400000-173594	9000-5260	4000-2394	90-76	40-34				
890000-362068	390000-169840	8900-5198	3900-2310	89-75	39-33				
880000-365314	380000-163X28	8800-5114	3800-2248	88-74	38-32				
870000-352580	370000-15X154	8700-5050	3700-2184	87-73	37-31				
860000-355828	360000-154400	8600-4288	3600-2100	86-72	36-30				
850000-342X94	350000-14X668	8500-4204	3500-2038	85-71	35-22				
840000-346140	340000-144914	8400-4X40	3400-1274	84-70	34-2X				
830000-3403X8	330000-13X280	8300-4978	3300-1X20	83-62	33-29				
820000-336654	320000-135228	8200-4824	3200-1X28	82-6X	32-28				
810000-330900	310000-122494	8100-4830	3100-1964	81-69	31-27				
800000-326268	300000-122740	8000-4768	3000-18X0	80-68	30-26				
790000-321214	290000-1129X8	7900-46X4	2900-1818	79-67	29-25				
780000-317480	280000-116054	7800-4620	2800-1754	78-66	28-24				
770000-311728	270000-110300	7700-4558	2700-1690	77-65	27-23				
760000-307994	260000-106568	7600-4494	2600-1608	76-64	26-22				
750000-302040	250000-100814	7500-4410	2500-1544	75-63	25-21				
740000-2982X8	240000-96X80	7400-4348	2400-1480	74-62	24-20				
730000-292554	230000-91128	7300-4284	2300-1328	73-61	23-12				
720000-2X8800	220000-87394	7200-4200	2200-1334	72-60	22-1X				
710000-2X2X68	210000-81640	7100-4138	2100-1270	71-52	21-19				
700000-299114	200000-978X8	7000-4074	2000-11X8	70-5X	20-18				
690000-293380	190000-91254	6900-3220	1900-1124	69-59	19-17				
680000-289628	180000-88200	6800-3228	1800-1060	68-58	18-16				
670000-283894	170000-82468	6700-3X24	1700-998	67-57	17-15				
660000-279240	160000-78714	6600-39X0	1600-924	66-56	16-14				
650000-2741X8	150000-72980	6500-3918	1500-850	65-55	15-13				
640000-26X454	140000-69028	6400-3854	1400-988	64-54	14-12				
630000-264700	130000-63294	6300-3790	1300-904	63-53	13-11				
620000-25X968	120000-59540	6200-3708	1200-840	62-52	12-10				
610000-255014	110000-537X8	6100-3644	1100-778	61-51	11-9				
600000-242280	100000-49X54	6000-3580	1000-624	60-50	10-X				
590000-245528	90000-44100	5900-3428	900-630	59-42	9-9				
580000-232794	80000-3X368	5800-3434	800-568	58-4X	8-8				
570000-235X40	70000-34614	5700-3370	700-4X4	57-49	7-7				
560000-2300X8	60000-2X880	5600-32X8	600-420	56-48	6-6				
550000-226354	50000-24228	5500-3224	500-358	55-47	5-5				
540000-220600	40000-12194	5400-3160	400-294	54-46	4-4				
530000-216868	30000-12440	5300-3098	300-210	53-45	3-3				
520000-210214	20000-26X8	5200-3014	200-148	52-44	2-2				
510000-207180	10000-5954	5100-2250	100-84	51-43	1-1				
500000-201428		5000-2X88		50-42					

CONVERSION TABLES

billions		hunds.		tens millions		millions		hun. thous.		tens thous.		thous.		hunds.		tens		units			
000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000	000		
23X	293	854	29	5X6	454	3	423	054	402	854	49	254	5	954	624	84	X	1	1		
479	967	428	56	290	828	6	846	0X8	805	428	97	828	2	628	1	1X8	148	18	2		
628	832	140	84	577	140	X	069	140	1	008	140	125	740	15	440	1	820	210	3		
937	712	994	21	261	594	11	490	194	1	40X	994	173	394	12	194	2	394	294	4		
276	5X6	628	142	561	228	14	823	228	1	811	628	201	428	24	228	2	X88	358	5		
1	125	47X	280	148	232	280	18	116	280	2	014	280	242	280	2X	880	3	580	420	6	
1	634	351	214	175	218	714	12	339	314	2	616	214	299	114	34	614	4	074	4X4	7	
1	673	225	768	1X3	202	268	22	960	368	2	819	768	326	368	3X	368	4	768	568	8	
1	822	029	400	211	429	400	26	183	400	3	020	400	374	X00	44	100	5	260	630	9	
1	230	291	054	=	ten	billions	(10,000,000,000)														

A - INTEGERS - DECIMAL TO DOZENAL

B - INTEGERS - DOZENAL TO DECIMAL

gro meg megros		zen meg megros		gro megros		zen megros		megros		tens		units									
000	000	000	000	000	000	000	000	000	000	000	000	000	000								
429	981	696	35	831	808	2	985	984	248	832	20	736	1	728	144	12	1	1	1		
1	289	945	088	107	495	424	8	957	952	746	496	62	208	3	184	432	36	3	3		
1	719	926	784	143	327	232	11	943	936	995	328	82	944	6	912	576	48	4	4		
2	149	908	480	179	159	040	14	929	920	1	244	160	103	680	8	640	720	60	5		
2	579	890	176	214	990	848	17	915	904	1	492	992	124	416	10	368	864	72	6		
3	009	871	872	250	822	656	20	901	888	1	741	824	145	152	12	096	1	008	84	7	
3	439	853	568	286	654	464	23	887	872	1	990	656	165	888	13	824	1	152	96	8	
3	869	835	264	322	486	272	26	873	856	2	239	488	186	624	15	552	1	296	108	9	
4	299	816	960	358	318	080	29	859	840	2	488	320	207	360	17	280	1	440	120	10	
4	739	798	656	394	149	888	32	845	824	2	737	152	228	096	19	008	1	584	132	11	
5	159	780	352	=	fbi-megro	(1,000,000,000)															

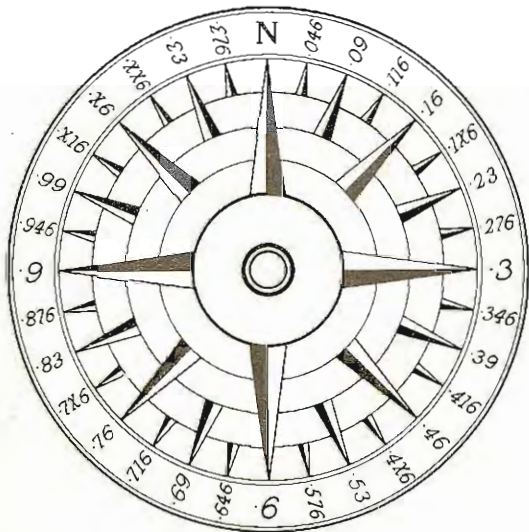
Ed.

TABLE C FRACTIONS - DECIMAL TO DOZENAL

	.	.0	.00	.000	.000 0	.000 00
1	.124 972	.015 344	.001 88X	.000 20E	.000 026	.000 003
2	.249 725	.02X 688	.003 558	.000 419	.000 050	.000 006
3	.372 497	.043 X10	.005 226	.000 628	.000 076	.000 009
4	.497 24X	.059 153	.006 XE4	.000 836	.000 09E	.000 010
5	.600 000	.072 497	.008 782	.000 X45	.000 105	.000 013
6	.724 972	.087 81E	.00X 450	.001 054	.000 12E	.000 016
7	.849 725	.0X0 E63	.010 11X	.001 262	.000 155	.000 019
8	.972 497	.0E6 2X7	.011 9X8	.001 471	.000 17E	.000 020
9	.X97 24X	.10E 62E	.013 676	.001 67E	.000 1X5	.000 023

TABLE D FRACTIONS - DOZENAL TO DECIMAL

	.	.0	.00	.000	.000 0	.000 00
1	.083 333	.006 944	.000 579	.000 048	.000 004	.000 000
2	.166 667	.013 889	.001 157	.000 096	.000 008	.000 001
3	.250 000	.020 833	.001 736	.000 145	.000 012	.000 001
4	.333 333	.027 778	.002 315	.000 193	.000 016	.000 001
5	.416 667	.034 722	.002 894	.000 241	.000 020	.000 002
6	.500 000	.041 667	.003 472	.000 289	.000 024	.000 002
7	.583 333	.048 611	.004 051	.000 338	.000 028	.000 002
8	.666 667	.055 555	.004 630	.000 386	.000 032	.000 003
9	.750 000	.062 500	.005 208	.000 434	.000 036	.000 003
X	.833 333	.069 444	.005 787	.000 482	.000 040	.000 003
E	.916 667	.076 389	.006 366	.000 530	.000 044	.000 004



THE RECIPROCAL OF 3937

by Ralph H. Beard

The ratio of the inch to the meter has been determined by Act of Congress to be 1 : 39.37 . In working with the relations between the measures of the do-metric system and those now in general use, there is frequent need to apply the reciprocal of 3937. This number possesses many interesting characteristics.

The number 3937 is composite, having the factors 31 and 127. The reciprocal of 3937 is a circulant of 210 terms. Its outstanding characteristic is that its terms double in sets of six. Terms 7 to 12 are the double of terms 1 to 6, - and this doubling goes through the entire 210 terms. The multiples of the circulant that appear are 5, 2, 4, and 8.

254000					
508001	256032				
016002	512065	386080			
032004	024130	772161	709169		
064008	048260	544323	418338	386842	
128016	096520	088646	836677	773685	757937
	193040	177292	673355	547371	515875
		354584	346710	094742	031750
			693421	189484	063500
				378968	127000

As to the factors of 3937, the reciprocal of 31 cycles in 15 terms, doubling each 6 terms, - and the reciprocal of 127 cycles in 42 terms, also doubling each 6 terms. As for many primes, all multiples of the circulant appear.

787401
574803
322580
645161
290
598425
196850
393700

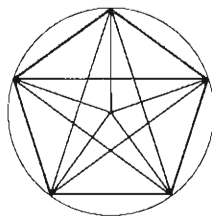
When expressed in duodecimals, the pattern of each of these numbers is far more complex. Often the duodecimal pattern of numbers is simpler and more condensed.

The reciprocal of 2341 is a circulant of 446 (630) terms, which doubles in 160 (216) terms. The multiples of the circulant that appear are 9, 3, 6, 2, 4, and 8. Since these are the products of the successive doubling, a more extensive group develops than decimally.

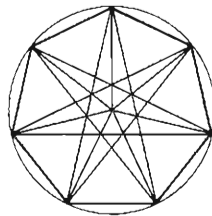
532536	9E72E3	674003	617646	789E64	4X8024	110305	1X782E	314168
882035	3115E6	0X9059	954236	08E280	7203X6	369640	5E2944	942702
464283	EEX2E2	298817	029967	E29E55	X59508	1X6453	E2777X	E2X3455
302E67	591133	X2E276	01E84E	X08X27	29E900	13973X	85X989	X7X000
X64X71	7E25X7	128007	033091	357E08	994048	22060X	39345X	628315
54406X	622E20	1960E7	6X8470	1E2E41	240790	717080	E2E689	685204
908547	E2E854	575432	057713	E97XXE	8E6X14	3908X7	X53339	E868XX
605E12	E62267	985E30	03E49E	815852	57E600	277279	4E9757	938001
909923	3X4E92	254012	066162	6E3X15	768094	441018	766829	05462X
X88119	045E20	3701E3	194920	2E2X82	481361	232141	E2E157	14X409
614X93	E24E88	E2X864	0E3227	E7399E	5E1828	761593	8X6677	E51599
00E25	E04513	74E260	07X97E	4E24X4	E3E000			

Examining the factors of 2341, we find that the reciprocal of 27 yields a circulant of the full period of 26 terms (n-1, which is probable for primes ending in 5 or 7,) doubling each 6 terms, - and the reciprocal of X7 has X6 terms, doubling in 76 terms.

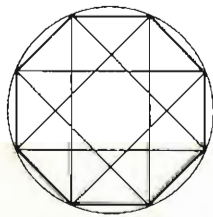
	117338	278E87	220E40
	90X255	91E989	547686
478XX0	06977X	13X5X3	710580
935981	465128	X6E2X4	883943
66E743	03499E	07E2E1	9662X0
11E286			
23X550	232674	535E52	441X81
	6184XE	63E756	X93150



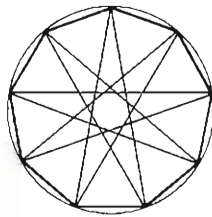
Angle .37249



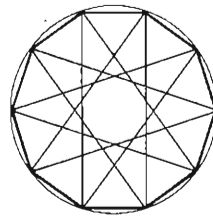
Angle .435186X



Angle .46



Angle .48



Angle .49724

I'M A DOZENER

by Eugene J. Zirkel

$$5 \times 4 = 18 \quad 14 \div 2 = 8 \quad 8 + 7 = 13 \quad 169 - 92 = 97$$

$$7^2 = 41 \quad (69)^{1/2} = \pm 9 \quad 3^3 = 23 \quad (54)^{1/3} = 4$$

What grade would you give to a student who turned in a paper with the above problems in arithmetic? Zero? I'd give him a perfect mark. All his calculations are correct, it's just that he's working with twelve symbols instead of our ordinary ten. He's counting in the duodecimal system, a number system that counts by dozens rather than by tens. His numbers proceed as follows:

1	2	3	4	5	6	7	8	9	X	E	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do
10	12	13	etc.							
do-one	do-two	do-three									

If you now count off four groups units with five units in each group you will see that five multiplied by four equals do-eight (see the first problem above). All the other problems above can also be verified in this way. In fact this is the way your multiplication tables were originally constructed. However, there is a simpler way to check these problems (or do some others).

Any series of digits merely means a sum of a power series where the digits are the coefficients of a power of the base of the number system. e.g.

- A 123 in a system of 5 symbols $1 \cdot 5^2 + 2 \cdot 5^1 + 3 \cdot 5^0$ or 38
- B 123 " " " " 10 " $1 \cdot 10^2 + 2 \cdot 10^1 + 3 \cdot 10^0$ or 123
- C 123 " " " " 12 " $1 \cdot 12^2 + 2 \cdot 12^1 + 3 \cdot 12^0$ or 171

Thus the problem above $169 - 92$ becomes

$$1 \cdot 12^2 + 6 \cdot 12 + 9$$

$$- 9 \cdot 12 - 2$$

$$1 \cdot 12^2 - 3 \cdot 12 + 7 = 12(12 - 3) + 7 = 9 \cdot 12 + 7 = 97 \text{ in the scale of twelve.}$$

We have now seen how to change a number from the scale of twelve to the scale ten (C above). In the reverse process we can change any number in the scale ten to the scale twelve by dividing that number by twelve, the remainders being the new

digits. Thus 437 in the ten scale is changed to the scale as follows:

$$\begin{array}{r} 12 \) \ 437 \\ \underline{ \ 36} \ + \ 5 \\ \ 3 \ + \ 0 \end{array}$$

hence 437 in the scale ten is 305 in the scale twelve.

What does all this amount to? What is the practical value of a new number system? Why should we change when our system of ten symbols is apparently just as good?

The answer to these questions lies in the word apparently. Have you ever studied any other system or for that matter even your own? You may have noticed that you have ten fingers and ten symbols in your counting system. This is no coincidence. The first counting was done on fingers and when man ran out of fingers he started over again, saying one ten fingers and one etc. until he got to two ten fingers. Someone started a symbolism of vertical lines so that we had 1, 11, 111, 1111, 11111, but this became too unwieldy and so a symbol for five was invented, namely V. Twice five became two V's one inverted under the other as X or X. Thus the system of Roman Numerals came into existence. Following this we had the invention of individual symbols 1, 2, 3, 4, 5, 6, 7, 8, 9 and so our counting became

no ten fingers & none	1 ten fingers & none	2 ten fingers & none
" & 1	1 " & 1	2 " & 1
" & 2	1 " & 2	2 " & 2
" & 3	1 " & 3	2 " & 3
.	.	.
.	.	.
.	.	.

which soon became

none	1 & none	2 & none
1	11	21
2	12	22
3	13	23
.	.	.
.	.	.

This was a convenient symbolism for all values save one in each ten which led to the development of the zero.

Thus our number system today is a combination of nine digits and a zero for place. The symbols and zero were derived by necessity but the base ten was purely accidental and most

inconvenient. Most of our measures which were derived for practicality, use twelve as a base. Thus we have twelve inches in a foot, twelve months in a year, twelve *objects in a pound*, ounces in a pound (Troy), just to name a very few. But the base of our system of counting which was not derived by practical use is ten. Why did grocers (the word comes from the same root as gross) sell things in dozens and why did carpenters put twelve divisions in a foot? Simply to facilitate the use of the common fractions $1/2$, $1/3$, and $1/4$. So by *experience it was learned it was easier to count by twelve's*.

Let's look at the advantages of the duodecimal system.

1. In the duodecimal system we count 143 units in only two digits, 44 more than in the decimal system; and in general all numbers have less digits in the duodecimal system.
2. The multiplication table is easier to learn in the new system with more repetition than in the decimal system. The table has only one three-digit number in the duodecimal system but eleven three-digit numbers in the decimal system.
3. The base of the duodecimal system has twice as many factors as the base of the decimal system. That is $1/2$, $1/3$, $1/4$, and $1/6$ of 12 are all whole numbers while only $1/2$ and $1/5$ of 10 are whole numbers.
4. Corresponding to the decimal point we have a *more* convenient duodecimal point which gives an exact value for $1/3$ and $1/9$ which were repeated decimals in the former system. It also simplifies $1/4$ from $.25_{10}$ to $.3_{12}$ and $1/8$ from $.125_{10}$ to $.16_{12}$.
5. Many practical problems are simplified, e. g.

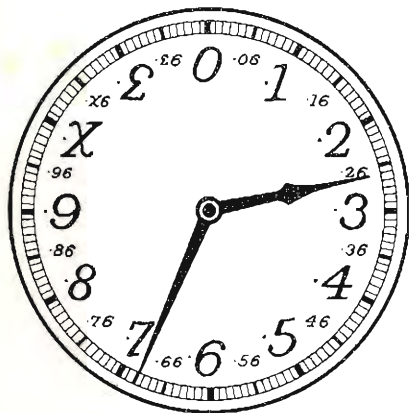
Find the area of a rectangle 4'3" long and 6'7" wide.

<u>decimal</u>	<u>duodecimal</u>
$4 \cdot 12 + 3 = 51''$	$(4.3') (6.7') = 23.29 \text{ ft}^2$ or
$6 \cdot 12 + 7 = 79''$	$23 \text{ ft}^2 \ 29 \text{ in}^2$
$(51'')(79'') = 4029 \text{ in}^2$	
$\frac{4029}{144} = 27 \text{ ft}^2 \ 141 \text{ in}^2$	
<u>4 Steps</u>	<u>1 Step</u>

<u>decimal</u>	<u>duodecimal</u>
<u>4 Steps</u>	<u>1 Step</u>
Add 3 yrs. + 10 mos.	3.X yrs.
2 " + 5 "	2.5 "
6 " + 9 "	6.9 "
5 " + 8 "	5.8 "
16 " + 32 "	16.8 " or 16 yrs. + 8 mos.
16 yrs. + 2 yrs. + 8 mos.	
18 yrs. + 8 mos.	
<u>3 Steps</u>	<u>1 Step</u>

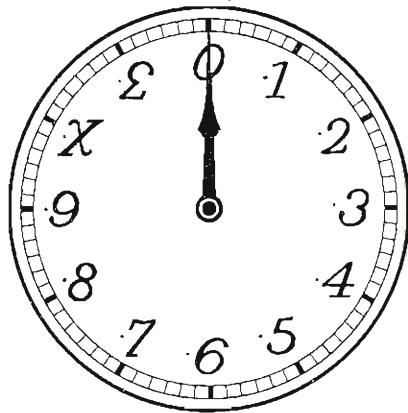
In conclusion then the duodecimal system is less complex in both learning and application. It has many advantages and only one so-called disadvantage, namely it is a change and many people don't want to change. But then the current cumbersome denary system was opposed by narrow minded people who used the Roman numerals and who were too lazy to improve themselves. The better system eventually won out and SO WILL DUODECIMALS.

MIDNIGHT



NOON

Diurnal Clock



Local Washington Clock

With Time expressed as a duodecimal of the Day

A NOTE OF CONVERSION MULTIPLES

by George S. Terry

As suggested (p. 17, Oct. 1951 Bulletin) it has been shown impossible to find any conversion Trebles.

When we come to Quadruples, the first power of twelve which is greater than 4 times the corresponding power of Ten is 12⁸, namely 4299 81696.

Following the formula given on p. 16 of the above article, we have

$$\begin{aligned}
 &a_0 + 12a_1 + 12^2a_2 + 12^3a_3 + 12^4a_4 + 12^5a_5 + 12^6a_6 + 12^7a_7 + 12^8a_8 \\
 &= 4(a_0 + 10a_1 + 10^2a_2 + 10^3a_3 + 10^4a_4 + 10^5a_5 + 10^6a_6 + 10^7a_7 + 10^8a_8) \\
 \text{or } &299\ 81696.a_8 = 41\ 68192.a_7 + 10\ 14016.a_6 + 1\ 51168.a_5 + 19264.a_4 \\
 &\quad + 2272.a_3 + 256.a_2 + 28.a_1 + 3.a_0
 \end{aligned}$$

so putting $a_8 = 1$, if $a_7 = 5$ $5(4168192) = 208\ 40960$

& $a_6 = 9$ $9(10\ 14016) = 91\ 26144$

we have near a balance.

Continuing with	$a_5 = 0$	=	0
	$a_4 = 0$	=	0
	$a_3 = 6$	$6(2272) =$	13632
	$a_2 = 3$	$3(256) =$	768
	$a_1 = 6$	$6(28) =$	168
	$a_0 = 8$	$8(3) =$	24
			299 81696

Since this gives an exact balance, the number a_8 to a_0 , i.e., 159 006 368 has four times the volume read duodecimally than it has read decimally. To check the result, multiply by 4 and convert. There are doubtless others.

A MATHEMATICAL RECREATION

What fractions equal $\frac{\text{product of numerator}}{\text{product of denominator}}$? e.g. $\frac{26}{65} = \frac{12}{30}$

The seven decimal results in two figures are given in Scripta Mathematica for March 1949. $\frac{14}{63} \cdot \frac{15}{24} \cdot \frac{16}{64} \cdot \frac{26}{65} \cdot \frac{18}{45} \cdot \frac{19}{95} \cdot \frac{49}{98}$ and a method by listing reductions of all numbers from 11 to 99.

The reduction is $\frac{\text{product}}{\text{number}}$ in simplest form. Two equal reductions give a result. e.g. The reduction of 26 is $\frac{6}{13}$ So is that of 65, hence $\frac{26}{65}$ is a result.

This listing is laborious and a formula is desirable. The following though incomplete, gives ten of the eleven duodecimal results. It is hoped that a more complete expression may be forthcoming.

If the factors of the base are $(p_1q_1), p_2q_2)$ etc.

$$\frac{(p_1-1)(10-p_2)}{(q_2-1)(10-q_1)} \text{ gives } \begin{array}{l} \frac{18}{26} \quad \frac{19}{36} \quad \frac{1\cancel{X}}{56} \quad \frac{1\cancel{E}}{26} \\ - \quad \frac{29}{38} \quad \frac{2\cancel{X}}{58} \quad \frac{22\cancel{E}}{28} \\ - \quad - \quad \frac{3\cancel{X}}{59} \quad \frac{3\cancel{E}}{29} \\ - \quad - \quad - \quad \frac{5\cancel{E}}{2\cancel{X}} \end{array} \text{ omitting } \frac{39}{68} \text{ i.e. } \left(\frac{3}{4}\right)^2$$

the blanks being unity or repetitions as reciprocals. Similarly for base eight it gives $\frac{16}{34} \cdot \frac{17}{74} \cdot \frac{37}{76}$ omitting $\frac{14}{33}$ i.e. $\left(\frac{2}{3}\right)^2$. But for base ten it gives only the last three of those listed above.

SCIENCE FICTION AND DUODECIMAL

F. H. Ames, Jr.

Perhaps it would be best to first define science fiction for the benefit of those who are not too familiar with this increasingly popular type of literature. It may be roughly defined as being fiction based upon known or predicted scientific facts and phenomena. Do not confuse this type of writing with the multitude of lurid paper backed pocket books whose covers usually depict a half nude girl in the clutches of some weird monster. As one editor put it "If the stories' main theme is love or a woman then it is not science fiction but just a cowboy and Indian story in a strange setting."

Please realize that this type of writing usually follows a definite pattern along expected lines of evolution and progress in the fields of science. Quite a few of the authors are men with a good technical background who write along these lines for their own entertainment and as a hobby.

The usual plot has Man, as we know him now or as we expect him to be in the next century or so, encountering inhabitants of another world, planet or what-have-you. Almost without exception the "stranger" comes from a more advanced civilization having more complex and advanced sciences than those of earth. Quite often the alien also possesses abnormal physical and mental powers that mankind would like to have himself.

The point of this dissertation is the method often used by the author to first exchange intelligence between Man and the being from another world. He uses the idea of comparing number systems. Since it is probable that all rational thinking animals have a system of counting it is a logical starting point for a comparison of civilizations. Mathematical notations may be expressed by writing, drawing in the sand or by groups of pebbles and the like so there is a media of expression available under practically all conditions.

Invariably when this "gimmick" is used it develops that the stranger, who is much more advanced than we, uses a duodecimal system; he apparently has no difficulty in understanding our system of counting by tens as soon as he sees it. The same can hardly be true of our hero as there is no expansion of the duodecimal theory on the part of the author. In the majority of these interchanges of information the stranger is possessed of six digits on his hands, feet, tenacles or other appendages so as to justify his usage of a base of twelve. However this is not always true so that it would appear that some science fiction writers believe that a more advanced civilization would be using duodecimals regardless of the number of digits.

This concept is quite interesting and I believe that we should encourage the usage of duodecimals in this type of literature.

These stories have a very large audience and should be exploited so as to make more people familiar with our mathematical hobby. It might be a good idea for one of our members to write up an article on duodecimals and point out the possibilities of its usage in science fiction writing. Conceivably a story could be written indicating the mass confusion that would result from an abrupt changeover from a system of ten to one based upon twelve. A basic article would probably be of interest to one of the better publications as they are seriously interested in improving the quality of their work.

EXCERPT FROM "OUR MONEY WORDS"

By Webb B. Garrison

in July 1952 issue of Bankers Monthly:-

Cent

No device in everyday use gained its name by a more tortuous route than the ordinary cent.

In 1871, the U. S. Congress set about establishing a system of coinage. Business was greatly hampered by lack of uniformity among the states. Some of the major colonies used a cumbersome system based on the famous Spanish "piece of eight," which was worth 90 pence of English money. It circulated widely, especially in Maryland and Pennsylvania, so that as late as 1790 many merchants kept books in terms of "dollars and ninetieths."

Robert Morris learned that Congress was working on the creation of a new system, so wrote to lawmakers proposing that the dollar be divided into 1440 parts. A coin equal to 100 of these parts should be struck, said he. Smallest piece of money in his proposed system, he suggested that it be called cent--from Latin centum (one hundred).

Congress never gave serious consideration to dividing the dollar into 1440 parts. Instead, lawmakers divided it into 100. But they salved the feelings of Morris by adopting his name for the smallest coin. That is how a term for 100 gave its name to 1/100th of a dollar.

THE CURATE & THE GIRLS

Lewis Carl Seelbach

"Here is a problem which looks rather baffling, because there doesn't seem to be the slightest connection between what you are told and what you have to discover. A certain number of girls - you are not told how many - spent a certain number of days - you are not told how many days either - making a charity patchwork quilt.

When it was finished, they decided to celebrate the occasion by each girl exchanging a kiss with every other.

It was intended, at first, to include in the celebration the local curate also, but, as he was a bashful young man, it was finally decided that he should exchange kisses with only those girls who were his sisters but not with the others.

The total number of kisses exchanged was six dozen.

Supposing that the curate's sisters had not helped with the quilt, how much longer would it have taken to complete the same?"

Quoted from the Stargazer Talks by Robert Gould July 1944, reprinted 1946. Geoffrey Bles, 52 Douherly St., London W.C.1.

Solution:	Possible Number of Girls	Inter-Girls Kisses	Other Kisses to Total 6 Dozen
	1	0.	60:
	2	1	52:
	3	3	59:
	4	6	56:
	5	X	52:
	6	13:	49:
	7	19:	43:
	8	24:	38:
	9	30:	30:
	X	39:	23:
	2	47:	15:
	10	56:	6:
	11	66:	minus 6.

Assuming \mathcal{E} number of workers, the androgynous kisses are 15: or more than \mathcal{E} . Therefore \mathcal{E} is not a possible answer, as sisters are less than \mathcal{E} . Assuming 11: workers, the interfemale kisses are 66: or half a dozen more than the total of actual kisses, 6 dozen by count and hypothesis. Therefore the number of workers must be between \mathcal{E} and 11: or exactly a dozen, in which the interfemale kisses plus androgynous kisses meet the 6 dozen as stated in the problem.

6 girls would do the work in twice the time required by a dozen girls, there are 6 sisters and 6 non-sisters.