COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	X	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5.9'
31	694	Three ft. two in.	3.2'
96	3Ɛ2	Two ft. eight in.	2.8'
19Ɛ	1000	Eleven ft. seven in.	Ɛ.7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
 12 ) 30 + 5
  12 ) 2 + 6
       0 + 2    Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by X, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or X.

Numerical Progression

1	One		
10	Do	.1	Edo
100	Gro	.01	Egro
1,000	Mo	.001	Emo
10,000	Do-mo	.000,1	Edo-mo
100,000	Gro-mo	.000,01	Egro-mo
1,000,000	Bi-mo	.000,001	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	Ɛ
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
Ɛ	1X	29	38	47	56	65	74	83	92	X1

The Duodecimal Bulletin

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THE DUODECIMAL SOCIETY OF AMERICA
20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island 4, New York. F. Emerson Andrews, Chairman of the Board of Directors. H. C. Robert Jr., President. George S. Terry, Editor. Separate subscriptions $2.00 a year, 50¢ a copy.

CONTENTS

The Annual Meeting *1*

The Ecclesiastical Symbolism of the Dozen
Katharine L. Tait *3*

Constants ....George S. Terry *7*

Elements of Primitive Right Triangles
Col. Robert S. Beard & Ralph H. Beard *Ɛ*

Of Maps, Time, The Stars, and The Calendar
Kingsland Camp *15*

The Duodecimal System Helps Teach the Teachers
Dr. Charles T. Dieffenbach *26*

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numbers universally used. *"Why even try to learn to use these heathenish scrawly symbols, with their stubborn propensity for error, in place of the beautiful clear numbers which our fathers have used for untold generations? Think of the needless waste. We would have to change all of our counting boards and abaci. X is X, isn't it? And why do we need a symbol for nothing? You can't count it! No! Let us keep to our simple tried and true numerals, and let the barbarians scratch their heads, and rub themselves out. It will all come to 0 anyhow."*

Yet, although it took D years, the new notation became generally used, and man's thinking leapt forward like an arrow sped from a bow. The early years of the Renaissance marked a new stage in the use of symbols, with the advent of algebra, fractionals (decimals?), logarithms, analytical geometry, and the calculus. Can you imagine what it would be like to try to express the coordinates of points on a curve in Roman notation?

Mathematicians became conscious of a new dimension in symbolism, and the fundamental concepts of numbers were reexamined. Man awoke to the fact that different number bases could be used, and Simon Stevin stated in 1585 that the duodecimal base was to be preferred to the decimal.

The new Arabic notation accomodated mathematical statement better, and facilitated ideation. All thinking accelerated when released from the drag of the cumbrous Roman notation.

The parallel seems tenable. The notation of the dozen base accomodates mathematical statement better, and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base embodies a concurrent analysis and definition of numbers that stimulates classification and generalization. Yet this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis-factory because it has "not-enough-factors."

Then shouldn't we change? No! No change should be made, and we urge no change. All the world uses decimals. But people of understanding should learn to use duodecimals to facilitate their thinking, and to ease the valuative processes of their minds. Duodecimals should be man's second mathematical language. They should be taught in all the schools. In any operation, that base should be used which is most advantageous, and best suited to the work involved. We expect

that duodecimals will progressively earn their way into
general popularity. But no change should be made. Perhaps
by the year 2000, or maybe by *1200*, which is *14* years later,
duodecimals may be the more popular base. But then no change
need be made, because people will already be using the
better base.
When one is familiar with duodecimals, a number of accessory
advantages become apparent. Percentage is a very useful tool,
but many percentages come out in awkward figures because of
the inflexibility of decimals. When based on the gross, twice
as many ratios come out in even figures, and among them are
some of those most used, as thirds, sixths, and twelfths, -
eighths and sixteenths. There are advantages associated with
time and the calendar. Monthly interest rates or charges are
derived from annual rates, or the reverse, by simply moving
the unit (decimal?) point. The price of a single item bears
the same relation to the price of the dozen, and so does
the inch to the foot.
The proper correlation of weights and measures has always
been one of the world's serious problems. None of the present
systems is completely satisfactory. The American and English
standards are convenient to use since they are the final
result of a long process of practical evolution in which many
inconvenient measures have been adjusted or abandoned. The
French decimal metric measures have the advantage of being
set upon the same base as the number system, and are well
systemized. But many of the units are awkward because of their
arbitrary sizes, and because their decimal scale does not
accomodate division into thirds and fourths readily.
The duodecimal system of weights and measures, based on the
inch and yard, the pint and the pound, has the desirable ele-
ments of both systems, and few of their faults. This Do-Metric
System retains the familiar units of the American and British
standards in approximately their present size, and arranges
them into an ordered metric system using the scale of twelve.
This fits perfectly into the duodecimal notation, and the
combination accomodates the inclusion of the units of time
and of angular measure within the system, which hitherto
has not been possible.
If "playing with numbers" has sometimes fascinated you, if
the idea of experimenting with a new number base seems intrigu-
ing, if you think you might like to be one of the adventurers
along new trails in a science which some have thought staid
and established, and without new trails, then whether you
are a professor of mathematics of international reputation, or
merely an interested pedestrian who can add and subtract,
multiply and divide, your membership in the Society may
prove mutually profitable, and is cordially invited.

The Duodecimal Bulletin

All figures in italics are duodecimal.

THE ANNUAL MEETING

As has become customary, the Board of Directors met on Thursday afternoon, January 24th, just prior to the Annual Meeting, in the Board Room of the Russell Sage Foundation, 505 Park Ave., N.Y. The Board decided that greater attention should be given to the organization and personnel of our committees, that closer liaison should be effected between the chairmen and the numbers of their committees, and that the committees should establish more definite programs for the years activities. It was suggested that any new developments and other matters of new value be relayed to Editor Terry for publication in the Bulletin.

The Board unanimously endorsed the proposal for the change to the perpetual World Calendar, and arranged that the proposal, with adequate supporting information, be submitted to our members, recommending ratification of the Board's action, in order that the Society's official endorsement of The World Calendar may be announced at the 1953 Annual Meeting.

After re-electing the officers to their present posts for the ensuing year, the Board repaired to the Gramercy Park Hotel for the Directors' Dinner, and for the Annual Meeting in the roof terrace of the hotel, which followed.

The eighth Annual Meeting of the Society was called to order by President Robert at 8:30 P.M. Secretary Beard reported that, while there was little that was notable in the year's developments, the Society had continued its progress nevertheless. Since the Society's formal organization in 1944, we have spent about \$9200 and still have \$4000 in the Endowment Fund and \$600 in the bank. We have distributed 37,000 pieces of duodecimal literature, and have 1150 names on our mailing list, reaching all over the world, and including 38 teachers colleges and 100 libraries. The current printings of the Duodecimal Bulletin are in 1800 copies, and this year we have gained four members for a total of seventy-two.

Treasurer Humphrey presented his formal report, and commented that our budget for the year approximated \$1265 and our receipts nearly equaled that amount. This excellent performance was largely due to the generosity of our members, whose donations of about $1000 were received with the Society's thanks and gratitude. The number of this year's donors set a new record.

The Nominating Committee recommended the re-election of the Directors of the Class of 1952 as the Class of 1955, and proposed that the 1953 Nominating Committee consist of Paul Adams, Chairman, Albert De Valve, and Raymond Kassler. Since Secretary had received no other nominations, these nominees were duly elected.

Editor Terry made a plea for more papers from our members for publication in the Bulletin. It is not necessary that papers be technically perfect. What is wanted is more papers of popular interest. Where some modification seems desirable, the material will be put into proper shape by the staff.

The President reviewed the actions of the Board as covered above and announced its endorsement of the World Calendar, with the course of action to be taken.

After the completion of the official business, talks were delivered by Dr. Charles T. Dieffenbach, visiting lecturer at the Paterson (N.J.) State Teachers College, by Col. Robert S., and Ralph H. Beard, and by Kingsland Camp. The texts of their talks will appear in the Bulletin.

An informal talk of great interest was volunteered by Edward W. Pharo, design engineer with the Eckert-Mauchly Co., engaged in the planning and production of the new electronic computing machines. In elaborating on the operational arithmetic of these machines, Mr. Pharo commented on President Robert's paper on Split Base Arithmetic, and explained how the four-place binary unit, used in such machines, is as readily arranged for operation with the duodecimal base as the decimal.

Thereafter, President Robert declared the formal part of the meeting closed, and, with the serving of refreshments, the meeting dissolved into the usual delightful hodge-podge of group and corner discussions, with constant flow and interchange between them, which continued well toward midnight.

THE ECCLESIASTICAL SYMBOLISM OF THE DOZEN

by Katharine Lamb Tait

The interest in duodecimals seems to be as varied as the interests and occupations of the members of the Society and the contributors to this Bulletin. Witness its use in musical notation, as a basis for better weights and measures, or as a device for teaching teachers. Not all its advocates are even mathematicians. But all are in love with the beauty of the wholeness of the number of twelve. As I am.

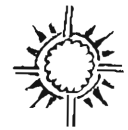
My interest springs from the fact that I design stained glass windows and that ecclesiastical symbolism is full of twelve and its factors. These are woven through all religious design, from the church itself and its sculpture, to the last bit of inside decoration or color.

These symbols have been used since the early days of the church and are not confined to any one denomination. They are like another language, a simpler one, understood by those who cannot read as well as by the most literate, and often express deep thoughts more quickly than a book filled with words. They set the imagination free to go behind and beyond the emblem. In fact, since the ultimate truth cannot be defined, but only expressed by symbols, the use of symbolism to express the unexpressable is one of the functions of religious art. And since beauty is so akin to truth, there always is a relationship between true artistic expression and mathematical laws and geometric formulae, as in the upsurge of a great cathedral that so perfectly fulfills them both.

Most of these symbols based on numbers are very old, very interesting, and capable of infinite variation and intermingling. But since many of their symbolic meanings are not widely known, a few words about them may be of interest.

Twelve has always been considered the perfect number in the symbolism of Christianity, as it embraces almost all the basic ideas. It stands for the whole Christian church, because twelve was the number of the first disciples that followed the Christ, and it was twelve that found the first Christian Community after His death. (Another disciple, Mathias, being elected to fill the place of Judas.) So twelve also stood for the Twelve

Tribes of Israel, the twelve sons of Jacob, the ancient Hebraic nation. These were represented as twelve stars in a circle around the Sun and Moon. Twelve were the old Biblical patriarchs as well: Adam, Noah, Abraham, Isaacs, Jacob, etc.



Twelve now represents the complete circle of Christian Brotherhood, and is often shown as a Sailboat holding a group of twelve passengers, the boat with its cross-armed mast being an old symbol of the church, sailing the Waters of Life.

The twelve disciples when treated symbolically are either shown in a circle around the figure of Christ, the Son, the Sun, the Source of Light, or in four groups of three each, as in Leonardo's painting of The Last Supper, where he has two groups on each side of the central Christ. Since tradition assigns a month or Sign of the Zodiac to each disciple, this recalls the moving of the sun through the twelve months and four seasons, from Spring to Winter to Spring again, and echoes the Eastern idea of the turning of the wheel of life from birth to death to rebirth, from dark to light to dark again. Thus, the circle becomes the symbol of Immortality or Eternal Life.

The Sun-circle, the twelve-rayed Sun, or great "Day Star" stands for Christ Himself. He is always shown with a halo divided into four parts by a crossing from (three arms of which radiate from His head), each division being rayed three times. He is "The Sun of Righteousness" prophesied by Malachi, "the Light of the World".

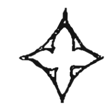
The six-pointed Star or double triangle is the symbol of God the Father and is called "The Creator's Star." This is a doubling of the ancient Hebrew equilateral triangle, one of their signs for Jehovah, which is often shown with an eye inside it, the all-seeing "Eye of God." This Eye of God by the way appears in a triangle at the top of a pyramid on our dollar bills.

Ancient Jehovah was thought to have the three attributes of a king: Power, Majesty, Wisdom, which were the three sides of the triangle. To these were added three more by the early church to embody the idea of a loving Father as well as King of all; Power becoming Justice; Majesty, Mercy; and Wisdom, Love. These are called the "Six Attributes of the Deity", marking the hexagon within His star.

Three, or the triangle, today of course stands for the Holy Trinity; Father, Son, and Holy Spirit. This is used in many beautiful ways; as interwoven

circle and triangle, as three interlaced circles, as a trefoil, or trequetra, or even as three circling fish, or a three petaled clover leaf.

Four, or the four-pointed star is the Star of Earth, representing the four directions, the four winds, the four elements: earth, air, fire water. It also stands for the four Evangelists, and the four Gospels through which the Word was spread. It symbolizes the four rivers of Paradise spoken of in Revelation which flowed over the earth from Mount Zion, on which stood the Lamb of God. It is the Cross itself, the greatest of all Christian symbols, the cross of Faith, with its four arms piercing the sky, plunging into the earth, and spreading over mankind. It is the ground plan of all Christian churches from the basilica with its circular center, to the great cruciform cathedrals.

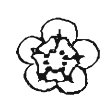


There are over four hundred recognized forms of the cross, about fifty of which are used frequently. Each form has a distinctive meaning and name, such as, the Latin cross of Calvary, the floriated cross of Hope, the radiant cross of Resurrection, the Greek cross, the Tau cross, the ancient Egyptian cross of Life-after-Death, the well-loved Celtic cross with its circle of Eternity woven around the arms, etc.

Eight, or the cross with four rays becomes the eight-rayed Star of Regeneration or rebirth, the promise of the coming of the Kingdom of Heaven on earth. Baptismal fonts are octagonal to recall this idea of renewal.

To complete the symbolism of twelve or "do" we shall have to go into the odd-numbered forms, or stars, which do not "factor" but are necessary to round out the picture.

Five, or the five-pointed star is the Star of Jesus, the new star, the star of Epiphany that the wise men saw in the East, the promise of a new kind of King, the new kind of kingdom, and a new kind of man. This star is the "measure of man" with his head and four limbs making five points, a new God-like man, head erect. Five is also the number of the petals of the rose of Love, the rose of Jesus and of Mary, the rose of Messianic promise.




Seven, or the seven-pointed star is the symbol of the Holy Ghost, as there were traditionally seven "Gifts" of the Holy Spirit given to anyone who received the Word. This is also called the Pentecostal Star, used at Pentecost to denote the descent of the flames of the Holy Spirit at that time. Seven were the churches of the early Christiandom spoken of by St. John in Revelation as representing the whole Christian world. This is the meaning of the seven-branched candlesticks still used on altars.

Nine is the number of the fruits of the Holy Spirit enumerated by St. Paul, the result of a holy life, and is represented by a nine-pointed star. There are also traditionally, nine choirs of angels, three circles of three each, surrounding the throne of God, starting with seraphim and cherubim, and ending with the Archangels and angels headed by the great Michael.

Ten seems to have no significance in Christian symbolism, other than that of the ten commandments of Moses, and then they are always divided, six and four, or three and seven, or five and five, on two tablets. The ten Sephiroth of the cabala are also divided three and seven. Eleven stands only for the group of disciples after Judas' betrayal, which goes to prove that the ancients felt as we do, that twelve was the all-embracing quantity, the complete number that held all truth within itself.

All these symbols, and many, many more will be found throughout any church or cathedral, both old and modern, starting with the basic cross-shape of its nave, chancel, and transepts; woven into its architecture and carvings, as well as intermingled throughout its decoration and windows. They are a universal language that is as eloquent in a French Gothic cathedral as in an Italian Romanesque church or English chapel to those who understand their meaning.

This is a very short exposition of a large body of accumulated traditions, beliefs and ideas connected with the first twelve numbers, all of which can be depicted by simple geometric forms of great beauty. I feel that it gives life to the study of duodecimals.

CONSTANTS

The following table of constants to two dozen places has been used for some years. Many of them were originally obtained by conversion of decimal figures, rather than from first principles by duodecimal calculation.

This weakness has now been overcome by the publication in the latest Bulletin of extended values of Natural Logarithms.

[Table of Constants]

Note that the table of natural logarithsm may be checked at any point from the formula

$$\text{nat. log. } (1 + A) = A - \frac{A^2}{2} + \frac{A^3}{3} - \frac{A^4}{4} \text{ etc.}$$

$$\text{nat. log. } (1 - A) = -A - \frac{A^2}{2} - \frac{A^3}{3} - \frac{A^4}{4} \text{ etc.}$$

which expansions are easy if A is small, for example:

$$27 = \frac{6^3(1+.008)}{7} \quad \text{or} \quad 27 = \frac{10^7(1+.0000007)}{5 \cdot 17 \cdot 1\mathcal{E}^3}$$

To "factorise" primes in this way, find factors of the approximate reciprocal, the nat. logs. of which factors are known.

Thus $\frac{1}{27}$ = approx: $.048$ and $27(.048) = 1.008$

or $\frac{1}{27}$ = approx: $.0478XX1$ and $27(.0478XX1) = 1.0000007$

This method is not confined to whole numbers, e.g. nat.log. π.

To find nat. log. π.

The reciprocal of π is approxinately $.39X$

$\pi(.39X) = .\mathcal{EEE}X61 \;\; 7X8\mathcal{E}85 \;\; 105740 \;\; 9177\mathcal{E}0 \;\; 16 = 1\text{-}A$

$$\text{Thus } \pi = \frac{10^3}{2 \cdot 5^2 \cdot \mathcal{E}}\,(1 - .00015X \;\; 413036 \;\; X\mathcal{E}647\mathcal{E} \;\; 2X440\mathcal{E} \;\; X6)$$

$$\begin{array}{rcrrrrr}
-\,A & = - & .00015X & 413036 & X\mathcal{E}647\mathcal{E} & 2X440\mathcal{E} & X6 \\
-\,\frac{A^2}{2} & = - & & 11363 & 30447X & 7955X9 & 50 \\
-\,\frac{A^3}{3} & = - & & 1 & 1236\mathcal{E}1 & XX7X27 & 60
\end{array}$$

$-\dfrac{A^4}{4}$	=	−			128	875577	29
$-\dfrac{A^5}{5}$	=	−				15652	67
$-\dfrac{A^6}{6}$	=	−				1	99
	adding the six terms						
ln 1-A	=	− .00015X	42439Ɛ	322558	634834	47	
− ln 2	=	− .839912	483369	XƐ2137	42X346	79	
− ln 5^2	=	− 3.276273	864823	924519	7758ƐƐ	00	
− ln Ɛ	=	− 2.493690	Ɛ028X4	5Ɛ3468	207X5X	36	
		− 6.3877Ɛ5	453055	530459	824718	3X	
ln 10^3	=	+ 7.55590X	5Ɛ0483	39318Ɛ	203245	16	
ln π	=	+ 1.18X115	159429	X62931	59X729		

This result may be checked from $\frac{1}{\pi}$ whose reciprocal is approximately 3.185 and $\frac{3.185}{\pi}$ = 1.000130431 etc. also 3.185 = $\frac{51(75)}{10^3}$ whence ln $\frac{1}{\pi}$ = −1.18X115 etc. and checks the above result.

The extended natural logarithms may be used to get extended values of common logs. From the table, nat. log. 10, i.e. 2 ln 2 + ln 3 is 2.599Ɛ etc. whose reciprocal .49Ɛ4 etc. is recorded as M the multiplier to convert natural to common logs. Whence by multiplication.

$\log_{10} 2$	=	.34201Ɛ	20X371	X7X232	04520X	694
$\log_{10} 3$	=	.537Ɛ81	7X3498	283757	Ɛ317X2	X54

The following calculation of Euler's Constant is recorded here as a tribute to J. C. Adams who by this method obtained two hundred and sixty places, a herculean task. (Proc: Royal Soc. Vol. 27, 1878). Euler's constant C = 1 + 1/2 + 1/3 + ... + 1/m − ln m when m is infinite and its value may be found from the formula:

$C = S_n - R_n - 1/2 \ln n(n+1)$ where S_n is the sum to $\frac{1}{n}$

and $R_n = \dfrac{B_1}{n(n+1)} - \dfrac{B_2}{3}\dfrac{(n+1)^3-n^3}{n^3(n+1)^3} + \dfrac{B_3}{5}\dfrac{(n+1)^5-n^5}{n^5(n+1)^5}$ − etc.

The Bernoulli numbers B, etc. being

$\frac{1}{6} \cdot \frac{1}{26} \cdot \frac{1}{36} \cdot \frac{1}{26} \cdot \frac{5}{56} \cdot \frac{497}{16Ɛ6} \cdot \frac{7}{6}$ etc.

so taking n = 53 since 53(54) is convenient.

EULER'S CONSTANT (C)

$\dfrac{B_1}{53(54)}$	=	.0000X3	5186X3	5186X3	5186X3	5	+
			X6Ɛ7	067722	950252	8	−
			3	2X5939	3Ɛ8Ɛ2Ɛ	4	+
				1Ɛ3Ɛ	5Ɛ9909	Ɛ	−
				1	X893X2	7	+
					2869	0	−
					5	4	+
− R	−	.0000X3	509ƐXƐ	754980	4521Ɛ1	1	
− 1/2 ln 2400	−	4.198Ɛ3X	210430	008949	ƐX0914	X	
Sum to $\frac{1}{53}$	+	4.88X53X	404Ɛ80	699905	97624Ɛ	2	
C	=	.6Ɛ1518	8X6760	Ɛ381Ɛ7	543345		

A Correction for the Value of $Ɛ^{Ɛ^{Ɛ}}$

The last paragraph of p. 10 of the October 1951 Bulletin contains the following errors.

1. The number of integers recorded is evidently far too small. The correct figure is 45 438 624 114.
2. To give the nine figures beginning this large number we need two dozen places of $\log_{10}Ɛ$ which as now obtained from the natural log are: .Ɛ6Ɛ5XX Ɛ08546 192Ɛ29 608803 the corrected first nine figures are 941267046.
3. The last figure of the number is Ɛ as recorded at the end of the paragraph.

For these errors your Editor is entirely to blame and is most contrite.

CONSTANTS

π	3.184809	493Ɛ91	866457	3χ6212
$1:\pi$	0.39χ058	2886Ɛ3	742178	522878
π^2	9.χ52814	9Ɛ2906	279651	763699
$1:\pi^2$	0.1270ƐƐ	524723	33953Ɛ	794353
$\sqrt{\pi}$	1.932972	χ0χ88Ɛ	8Ɛ3107	939185
$1:\sqrt{\pi}$	0.692Ɛ05	09ƐƐχƐ	623359	205893
$\sqrt{\frac{2}{\pi}}$	0.96χ8Ɛ2	647ƐƐ2	Ɛ77334	129416
$\log \pi$	0.564062	958849	980059	1Ɛ8χχ1
$\ln \pi$	1.18χ115	159429	χ62931	59χ729
e	2.875236	069821	9Ɛχ719	71009Ɛ
$1:e$	0.44Ɛ842	160564	χ444Ɛ6	χ08Ɛ59
e^{π}	1Ɛ.183149	Ɛ4χ351	682985	97Ɛ590
$e^{-\pi}$	0.062810	09χ482	626Ɛ25	579420
$e^{\pi/2}$	4.988608	5χχ066	62Ɛχχ0	χ7χ200
$e^{-\pi/2}$	0.25Ɛ271	109778	366χ20	χχ4006
M	0.49Ɛ494	439904	549679	Ɛ59Ɛ60
1:M	2.599Ɛ03	5Ɛ8169	13106Ɛ	881096
C(Euler)	0.6Ɛ1518	8χ6760	Ɛ381Ɛ7	543345
$\sqrt{2}$	1.4Ɛ7917	0χ07Ɛ8	573770	4χƐƐƐ7
$\sqrt{3}$	1.894Ɛ97	ƐƐ9687	04χ153	9χ82Ɛ5
$\sqrt{10}$	3.569Ɛ73	ƐƐ7152	0982χ7	7945χχ
$\sqrt[3]{2}$	1.315188	11χ39χ	χ22801	χ49415
$\sqrt[3]{3}$	1.53825χ	183676	χχ7907	9388χ2
$\sqrt[3]{10}$	2.358170	9χχ919	6Ɛ5574	34641Ɛ
$\sqrt[3]{\pi}$	1.56χ993	Ɛ3917Ɛ	494Ɛ94	205χ0χ

ELEMENTS OF PRIMITIVE RIGHT TRIANGLES

by Col. Robt. S. Beard and Ralph H. Beard

The study of primitive right triangles has been going on for some thousands of years. There is a clay tablet in the Plimpton Collection at Columbia University which dates from about 1750 B.C., and records in Babylonian script the dimensions of a number of primitive right triangles. At this late date, it would be a little optimistic to expect to discover much that is new about them.

But you may not remember all that went on in the mean time. And the papers of Harry Robert in recent Bulletins about the perimeters and the inscribed circles of such triangles, let us to explore some of the details of their construction. We found some interesting things, and we believe you will share our enjoyment of them.

A primitive right triangle is one in which the three sides have no common factor. The general equation is:

$$a^2 + b^2 = c^2 \qquad \text{(I)}$$

and this can be transformed into the equation of the generators:

$$(m^2 - n^2)^2 + (2mn)^2 = (m^2 + n^2)^2 \qquad \text{(II)}$$

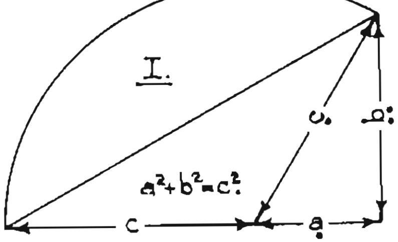
in which a is represented by $(m^2 - n^2)$, b by $2mn$, and c by $(m^2 + n^2)$. Since b is $2mn$, and either m or n is always even, b is always divisible by 4. The other sides, a and c, are always odd. Either a or b is divisible by 3 and one of either a, b or c is divisible by 5.

Another form of the equation of the generators uses the difference between m and n; i.e., $m - n = p$, and $m = n + p$. If we substitute this value for m, this equation becomes:

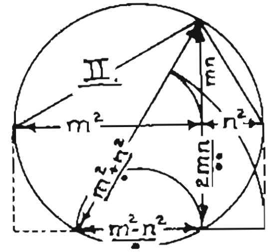
$$(p^2 + 2pn)^2 + (2pn + 2n^2)^2 = (p^2 + 2pn + 2n^2)^2 \qquad \text{(III)}$$

in which n may be odd or even, but p is always odd, and the quantities in the parentheses represent a, b, and c, respectively. The diagram on the following page shows the geometrical definition of these terms, the relation a,b,c, to m,n being shown in Figure I. Reference to Plate III will show that the diameter of the inscribed circle is 2pn, an essential element of the dimension of each of the three sides.

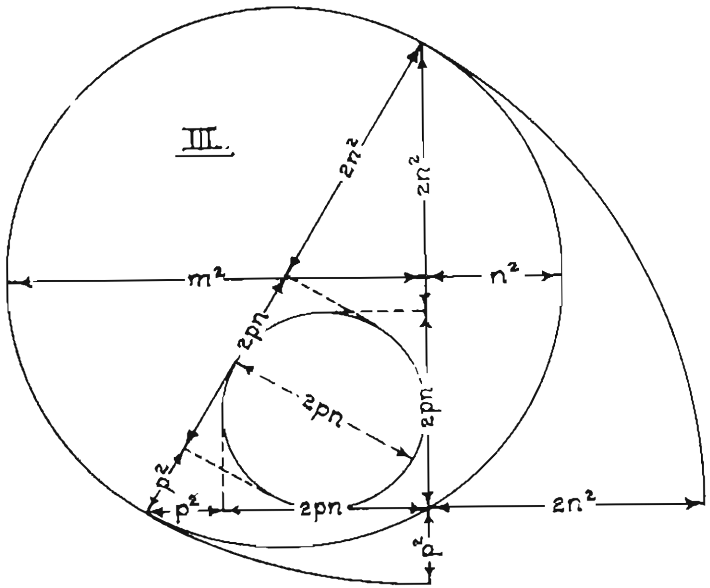
PRIMITIVE RIGHT TRIANGLES.



$$\frac{c+a}{b} = \frac{m}{n}.$$



$$(m^2-n^2)^2+(2mn)^2=(m^2+n^2)^2.$$
$$m^2:mn::mn:n^2.$$



$a = m^2-n^2 = p^2+2pn.$ $\qquad m-n=p.$

$b = 2mn = 2pn+n^2.$ $\qquad m=p+n.$

$c = m^2+n^2 = p^2+2pn+2n^2.$ $\qquad 2pn=a+b-c.$

$$\frac{c+a}{b} = \frac{(m^2+n^2)+(m^2-n^2)}{2mn} = \frac{2m^2}{2mn} = \frac{m}{n}.$$

Using the equation (III), we develop values for a, b, and c, by assigning successive values for p and n, as shown in the following tables.

INDEXED BY VALUES OF p

$a = (2n + p)p \qquad b = 2n(n + p) \qquad c = b + p^2$

	p = 1			p = 3			p = 5			p = 7			p = 9			p = 11		
n	a	b	c	a	b	c	a	b	c	a	b	c	a	b	c	a	b	c
1	3	4	5	15	8	17	35	12	37	63	16	65	99	20	101	143	24	145
2	5	12	13	21	20	29	45	28	53	77	36	85	117	44	125	165	52	173
3	7	24	25	21	20	29	55	48	73	91	60	109				187	84	205
4	9	40	41	33	56	65	65	72	97	105	88	137	153	104	185	209	120	241
5	11	60	61	39	80	89				119	120	169	171	140	221	231	160	281
6	13	84	85				85	132	157	133	156	205				253	204	325
7	15	112	113	51	140	149	95	168	193				207	224	305	275	252	373
8	17	144	145	57	176	185	105	208	233	161	240	289	225	272	353	297	304	425

INDEXED BY VALUES OF n

	n = 1			n = 2			n = 3			n = 4			n = 5			n = 6		
p	a	b	c	a	b	c	a	b	c	a	b	c	a	b	c	a	b	c
1	3	4	5	5	12	13	7	24	25	9	40	41	11	60	61	13	84	85
3	15	8	17	21	20	29				33	56	65	39	80	89			
5	35	12	37	45	28	53	55	48	73	65	72	97				85	132	157
7	63	16	65	77	36	85	91	60	109	105	88	137	119	120	169	133	156	205
9	99	20	101	117	44	125				153	104	185	171	140	221			
11	143	24	145	165	52	173	187	84	205	209	120	241	231	160	281	253	204	325
13	195	28	197	221	60	229	247	96	265	273	136	305	299	180	349	325	228	397
15	255	32	257	285	68	293				345	152	377						

To facilitate analysis of the resulting values, the triangles were listed in two tables, one indexed on the odd, or a, side, and the other indexed on the even, or b, side. These tables are shown below in abbreviated form, giving the decimal and duodecimal values.

Indexed by a Side						Indexed by b Side					
Decimal			Duodecimal			Decimal			Duodecimal		
a	b	c	a	b	c	a	b	c	a	b	c
3	4	5	3	4	5	3	4	5	3	4	5
5	12	13	5	10	11	15	8	17	13	8	15
7	24	25	7	20	21	5	12	13	5	10	11
9	40	41	9	34	35	35	12	37	2Ɛ	10	31
11	60	61	Ɛ	50	51	63	16	65	53	14	55
13	84	85	11	70	71	21	20	29	19	18	25
15	8	17	13	8	15	99	20	101	83	18	85
17	144	145	15	100	101	7	24	25	7	20	21
19	180	181	17	130	131	143	24	145	ƐƐ	20	101
21	20	29	19	18	25	45	28	53	39	24	45
23	264	265	1Ɛ	1X0	1X1	195	28	197	143	24	145
25	312	313	21	220	221	255	32	257	193	28	195
27	364	365	23	264	265	77	36	85	65	30	71
29	420	421	25	2Ɛ0	2Ɛ1	323	36	325	22Ɛ	30	231
31	480	481	27	340	341	9	40	41	9	34	35
33	56	65	29	48	55	399	40	401	293	34	295

Where gaps occurred because the tables were not sufficiently extended to embrace all the cases within range, they were filled in by factorizations, as follows:

FACTORIZATION OF $(m^2 - n^2)$, OR a
(when equal to 31)

Since 31 is a prime, there is only one possible case.

$$m^2 - n^2 = (m + n)(m - n) = 31 \times 1$$
$$m + n = 31$$
$$m - n = 1$$
$$2m = 32 \quad \text{and} \quad m = 16$$
$$2n = 30 \quad \text{and} \quad n = 15$$
$$2mn \text{ or } b = 2(15 \times 16) = 480$$
$$(m^2 + n^2) \text{ or } c = 225 + 256 = 481$$

FACTORIZATION OF 2mn, OR b
(when equal to 60)

n	m	$(m + n)(m - n)$ =		$(m^2 - n^2)$ or a	$(m^2 + n^2)$ or c		
1	30	31 x	29	899	900	1	901
2	15	17	13	321	225	4	229
3	10	13	7	91	100	9	109
5	6	11	1	11	36	25	61

Thus, there are 4, and only 4, primitive right triangles, where b = 60.

Since such factorizations will permit the determination of a set of primitive values for every case, it is possible to state that there is at least one primitive triangle for every odd value for a, and at least one primitive triangle for every multiple of 4, as a value of the b side.

It is at once noticed that the values in duodecimals for the hypotenuse always end in *1* or *5*. To develop the governing principle, separate tables were prepared for the *1* and for the *5* endings, as follows:

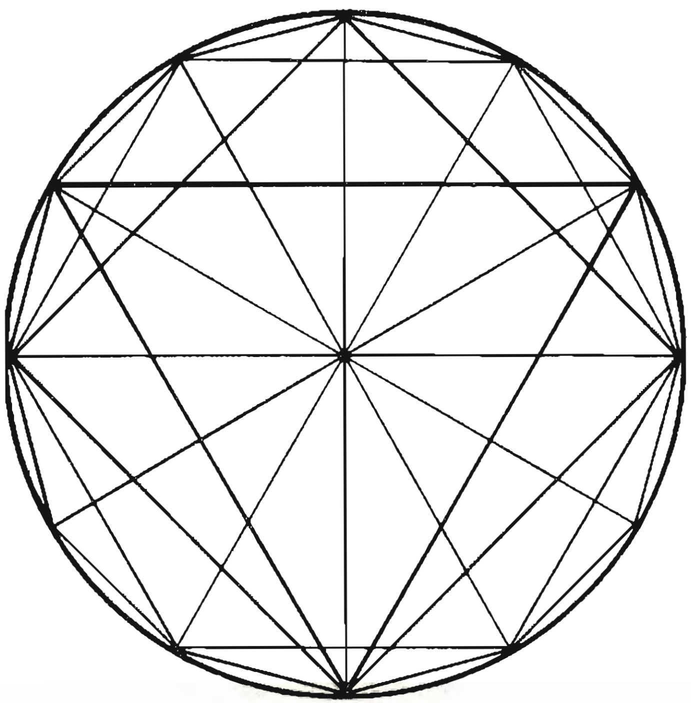
Duodecimal Hypotenuse Ends *5*, a Side Has *3*-Factor						Duodecimal Hypotenuse Ends *1*, b Side Has *3*-Factor					
Decimal			Duodecimal			Decimal			Duodecimal		
a	b	c	a	b	c	a	b	c	a	b	c
3	4	5	*3*	*4*	*5*	5	12	13	*5*	*10*	*11*
15	8	17	*13*	*8*	*15*	7	24	25	*7*	*20*	*21*
21	20	29	*19*	*18*	*25*	35	12	37	*2Ɛ*	*10*	*31*
9	40	41	*9*	*34*	*35*						
45	28	53	*39*	*24*	*45*	11	60	61	*Ɛ*	*50*	*51*
63	16	65	*53*	*14*	*55*	55	48	73	*47*	*40*	*61*
33	56	65	*29*	*48*	*55*	13	84	85	*11*	*70*	*71*
						77	36	85	*65*	*30*	*71*
39	80	89	*33*	*68*	*75*	65	72	97	*55*	*60*	*81*
99	20	101	*83*	*18*	*85*	91	60	109	*77*	*50*	*91*
15	112	113	*13*	*94*	*95*						
117	44	125	*99*	*38*	*X5*						
105	88	137	*89*	*74*	*Ɛ5*	17	144	145	*15*	*100*	*101*
51	140	149	*43*	*Ɛ8*	*105*	143	24	145	*ƐƐ*	*20*	*101*
						85	132	157	*71*	*Ɛ0*	*111*
165	52	173	*119*	*44*	*125*	119	120	169	*9Ɛ*	*X0*	*121*

The separation clearly shows that, that where the even (b) sides includes the 3-factor, the value for c ends in *1* duodecimally, and the even side will end in *0*; and where the 3-factor is included in the odd (a) side, the c side ends in 5. The 3-factor is always present in either the a or the b leg.

The effect of the location of the 5-factor is most conveniently analyzed on the decimal base. Since it can occur in any one of the three sides, a, b, or c, we find that its presence in either of the odd sides is apparent through the 5 ending; - in the case of the even side, which is always divisible by 4, the presence of the 5-factor will produce multiples of twenty.

That group of cases in which all of the three primary factors, 3, 4, and 5, are included in the b side, was also examined. Since the product of 3, 4, and 5, is five-do, or sixty, the b side will then display some multiple of it. Duodecimally, the hypotenuse will always end in *1*, and the a value will end in *1*, *5*, *7*, or *Ɛ*. The decimal parallel is that the hypotenuse will end in 1 or 9, and the odd leg will end in 1, 3, 7, or 9.

Perhaps it will emphasize this peculiar characteristic if we think of the duodecimal dimensions of these triangles as being so many feet and inches. The lengths of the hypotenuses of all primitive right triangles will always be - so many feet and one inch, or so many feet and five inches. In the five-inch cases, the odd side will always measure - a certain number of feet and either three, or nine, inches; and the even side will measure so many feet and either four, or eight, inches. In the cases where the hypotenuses are a certain number of feet and one inch, the even sides will measure exactly so many feet.



OF MAPS, TIME, THE STARS, AND THE CALENDAR

by K. Camp

(Extended from an address at the annual meeting of the Duodecimal Society, January 24, 1952.)

I. The Day, Longitude, and Maps.

We who live on this planet are obliged to regulate our lives by two principal units of time, the first of which is the interval from each midnight to the next called the day. Since the day is caused by the earth's rotation, it is inextricably involved with longitude, or the fraction of the earth's circumference measured from some assigned initial meridian: most logically the meridian of the international dateline.

It is unfortunate that the obvious idea of measuring longitude in one direction only, and preferably by fractions of a circumference that correspond to the divisions of a day, was not recognized when world maps were first made and published. Even now it is recognized only dimly and somewhat unofficially, as when we say "New York is five hours west of London." For serious map-making purposes we still reckon longitude in degrees and sixtyfold divisions thereof, to conform with the clumsy accepted usage for subdividing angles. And the practice of reckoning it both east and west from the prime meridian of Greenwich is worse still: this makes it more difficult for school children to grasp how there can be two different dates at the dateline. I recall my own difficulty with this as a schoolboy although I must have been as bright as the average.

No advocate of duodecimal arithmetic can contemplate this situation without reiterating the advantages of measuring fractions of a day, of longitude, of circumference, or of an arc by a single rational system of measurement. If by a "duor" we signify two hours of time or 1/12 (duodecimally, *0.1*) of a day, then a "duor of arc," of a circumference, or of longitude signifies 30° by the system now in general use. Moreover when finer divisions are required, they are expressed simply by longer duodecimal fractions and not clumsily by sixty-fold subdivisions used in no other fields. And the longitude of a place, measured always west from the international dateline, will equal at noontime of the place, the time in duors since the earth presented the dateline meridian to the sun.

So much for divisions of a day and maps of our earth's surface.

II. *The Year and the Calendar.*

Our other principal unit of time is the year. This unit defies exact expression in terms of any other, despite many futile harmonizing attempts throughout recorded history. Workable compromises have been devised; we live under one of them albeit not at all a neat one, with deliberately perpetrated irregularities to satisfy the vanity of Roman emperors who are now long dead and quite unlamented. By this present calendar of ours, there is nothing rhythmical about the lengths of the quarters or the months, nor are the months, the quarters, or the year itself rationally related to the days of the week. To borrow words from a current humorous book-title, this calendar helps us somewhat "to get from January to December," and that is about all.

The United Nations and all the world's parliaments now have before them a practical first step out of this clumsy arrangement, and a very easy step to take: the "World Calendar," ably presented by the World Calendar Association. By this plan, one day in the usual year, but two days in every leap year, are holidays that are not parts of any week. The remaining 364 (duodecimally *264*) days of the year fall into four equal quarters of 13 (*11*) weeks each. Every such quarter, and therefore the regularized 364 (*264*)-day bulk of every year, is to begin on a Sunday and end on a Saturday. For illustration it is therefore sufficient to show how a single quarter - say the first one of the year - would appear if scheduled vertically. This first quarter would follow the annual detached holiday after the last quarter of the previous year; the Association suggests calling that "World Day." Leap-year Day, as often as it comes, would be sandwiched in very similar fashion between the second and third quarters, and also not belong to any week.

The left side of the following illustration - or for that matter, any specimen World Calendar from the Association's literature - convincingly demonstrates how easy the change will be, especially on a year such as 1956 which will begin on a Sunday by both calendars. Dates within the year will be shifted only slightly from the old (many being identical), but months will thenceforth begin only on Sundays, Wednesdays or Fridays, and for six-day work weeks every month will contain 26 (duodecimally *22*) working days.

Several governments have already given definite approval to the new calendar, conditional of course on general concurrence. Many Chambers of Commerce, business organizations, and learned societies throughout our own country as well as the world at large, have also recorded their approval. The directors of our

Illustration: The First Quarter of the Year

By Proposed World Calendar — World Day = W

By Duodecimal Notation, Extended — *World Day = 0.00*

	Sun	Mon	Tue	Wed	Thu	Fri	Sat	Sun	Mon	Tue	Wed	Thu	Fri	Sat
January	1	2	3	4	5	6	7	*0.01*	*0.02*	*0.03*	*0.04*	*0.05*	*0.06*	*0.07*
	8	9	10	11	12	13	14	*0.11*	*0.12*	*0.13*	*0.14*	*0.15*	*0.16*	*0.17*
	15	16	17	18	19	20	21	*0.21*	*0.22*	*0.23*	*0.24*	*0.25*	*0.26*	*0.27*
	22	23	24	25	26	27	28	*0.31*	*0.32*	*0.33*	*0.34*	*0.35*	*0.36*	*0.37*
	29	30	31											
February				1	2	3	4	*0.41*	*0.42*	*0.43*	*0.44*	*0.45*	*0.46*	*0.47*
	5	6	7	8	9	10	11	*0.51*	*0.52*	*0.53*	*0.54*	*0.55*	*0.56*	*0.57*
	12	13	14	15	16	17	18	*0.61*	*0.62*	*0.63*	*0.64*	*0.65*	*0.66*	*0.67*
	19	20	21	22	23	24	25	*0.71*	*0.72*	*0.73*	*0.74*	*0.75*	*0.76*	*0.77*
	26	27	28	29	30			*0.81*	*0.82*	*0.83*	*0.84*	*0.85*	*0.86*	*0.87*
March						1	2							
	3	4	5	6	7	8	9	*0.91*	*0.92*	*0.93*	*0.94*	*0.95*	*0.96*	*0.97*
	10	11	12	13	14	15	16	*0.X1*	*0.X2*	*0.X3*	*0.X4*	*0.X5*	*0.X6*	*0.X7*
	17	18	19	20	21	22	23	*0.Ɛ1*	*0.Ɛ2*	*0.Ɛ3*	*0.Ɛ4*	*0.Ɛ5*	*0.Ɛ6*	*0.Ɛ7*
	24	25	26	27	28	29	30	*0.D1*	*0.D2*	*0.D3*	*0.D4*	*0.D5*	*0.D6*	*0.D7*

Duodecimal Society have just endorsed, and are submitting to the members for general approval, a resolution calling for adoption of the new calendar. Altogether, the project has strong and steadily increasing support.

I am led to venture three predictions about all this:

(1) As might be expected from the foregoing remarks, I predict general adoption of this calendar within the lifetime of most of us who are present this evening.

(2) My second prediction will take much longer to realize; probably none of us will live to see months fall into general disuse. But the new calendar makes each quarter contain an exact number of weeks and will inevitably strongly promote the practice, already perceptibly beginning in some fields, of making appointments and designating both past and future dates by the day of the week and the week of the quarter in which they fall. It is as easy to say "the thirteenth Tuesday of such-and-such a quarter" as "the twenty-sixth day of such-and-such a month," and the practice will have the great advantage of telling at once the day's position within the week: practically, more important information than its exact position within a month, for which its position within the quarter will prove an amply satisfactory substitute.

Note, that it will be futile to try to abolish months within the near future. It will be equally futile to attempt to prevent their gradual obsolescence within the remoter future.

(3) My third prediction will take very long indeed to be realized: probably centuries. It is this: Some day at long last, we shall adopt a logical feature from an ancient Chinese calendar and begin the year (or celebrate World Day, *0.00* of the year) at the winter solstice and not several days later on January 1. Then the quarters may very simply be named by the seasons with which they will very closely coincide: Winter, Spring, Summer and Fall.

Viewed independently, it is certainly a curious tradition of Western civilization to peg an odd day in the latter part of March to the spring equinox, but to set the calendar first of the year (January 1) at a meaningless point of the yearly cycle. True, that day is just now the average date (it oscillates somewhat with leap years) when the earth in its orbit is nearest the sun - its "perihelion date" in astronomical language. But this is temporary: the year's schedule is really set to the equinox, as just stated, so that the perihelion date changes by about one day in three centuries and in the course of a thousand centuries will return to its present day of the year, which assuredly will not, in that distant time, be called the first of January.

Duodecimal arithmetic and notation, if they come into general use, can easily hasten these trends. For consider how often we nowadays briefly designate a day by the number of its month, followed by its day in the month (or, confusingly, we sometimes reverse that order). Either way, we express it by two, three, or four digits with a dash or sloping bar to separate the units of time. Now turn back to the illustration of a quarter-year and look at the right hand side, that exhibits the days of the quarter in slightly extended duodecimal notation. (Disregard the separation of the 13-line column into three unequal blocks that keep weeks intact but abreast of their counterparts to the left, broken up to fit the months.) This enables designating any day in such a year of equal quarters by three digits that suggest in order (1) the quarter-years so far completed, i.e., *.0, .3, .6,* or *.9*; (2) the weeks so far completed within the quarter, i.e., *0, 1. 2. 3. 4. 5. 6, 7, 8, 9, Ӿ, Ɛ, D,* thus introducing a single additional symbol or monogram *D* for "do"; and (3) the day of the week (*1, 2, 3, 4, 5, 6, 7*). "World Day" preceding the first quarter, would be written *0.00* and Leap-year Day whenever it comes would precede the third quarter and be written *6.00*: the terminal figure of each of these is a cipher since neither belongs within a week.

Otherwise every subsequent quarter-year will have the same plan as the first one illustrated above, except of course that its leading digit throughout would differ in each case to suggest the duodecimal fraction (*.3, .6,* or *.9*) for one, two, or three quarters completed. A period separating this digit from the pair that follow, would indicate change to a mixed base of numbers and be more grammatical, if that word can be used of number-systems. Usage will very likely discard such a detail, unimportant wherever context plainly shows that a date is meant, as it nearly always does.

To point out and encourage use of such practical notation will be entirely in accord with the basic purpose of our Duodecimal Society, which is to make numerals and number-systems our servants and in no sense our masters. It is as logical to designate weeks and days in this fashion as it is to designate space in a hotel or office building with thirteen floors of seven rooms each, by giving the number of the the floor followed by that of the room. This familiar practice makes use of numbers so as to simplify finding any room in the place.

A final comment on our present calendar: as we all know, every fourth year is a leapyear unless its number in our era is a multiple of 100; in that case it is not a leapyear unless it is a multiple of 400. This plan is said to be in error by about one day in 5000 years.

A good while ago I made a calculation, now lost or misfiled, to the effect that if we reckoned the years of our era duodecimally, every fourth year could be a leapyear except for multiples of 144 (duodecimally of course *100*); and this simple plan would have an error of only one day in about 20,000 years. Note that 30736 equals *10,000* duodecimally; if the calculation was correct, further correction would be required probably only once in a geologic era - if human civilization endures so long!

I record this in the hope of catching the eye of someone who will give it an independent check and let me know if it is in error.

III. A New Planisphere.

I believe this preliminary model of planisphere is on a really new plan. It is unfortunate that the quality of the draftsmanship is not even distantly comparable with the beautiful diagrams and lettering of our friend Col. Robert Beard, posted on the wall just now; unfortunately also, there are several mistakes in the charting and most southern and several northern constellations were left undesignated for want of time or else not outlined because of doubt as to best form. However, the model does sufficiently exhibit its own principle.

Each side of the enclosed disc is a map of one celestial hemisphere, centered at its pole, and extended sufficiently into the opposite hemisphere to include the entire ecliptic: that is, the whole ecliptic appears on each side of this disc. The enclosing masks, in this model, are figured for 40° north latitude (*1.8* duors of circumference from the earth's north pole). One, overlying the north celestial chart of the disc, discloses the portion of the sky visible to an observer standing and facing north, with considerable overlap beyond the prime vertical (which passes through the observer's zenith and east and west points of horizon). The other mask in similar fashion discloses most (but at this latitude not all) of the sky visible below the prime vertical, as seen facing south; it also has overlap to east and west.

Commercial planispheres, on the other hand - at least the great majority of them - are essentially designed for an observer lying flat on the ground: they instruct him, for instance, to "hold map above head with compass letters (N,W,S,E) to the corresponding directions." A serious objection to this usual arrangement for teaching purposes, is the distortion of the constellations near the southern horizon, inevitable in a planisphere designed for this latitude if constellations near the pole are on a sufficient scale to be recognizable. In this instrument, however, the principle of stereographic projection is consistently followed from the pole to the very margin of each chart, so that each star group is as nearly correct in shape and in relation to its immediate neighbors as possible. This makes circumpolar constellations small but, I hope, still usefully recognizable. Of course it would be nice if so large a figure as the Great Dipper in the sky did not look so small on the northern chart as compared, say, with distant Capricornus or Corvus, on or beyond the ecliptic. But these latter we should ordinarily look for on the southern chart, and one of the educational advantages of this mapping principle is that this disc could equally well be fitted between a pair of appropriate masks for any other latitude desired, north or south.

A feature that should be mentioned, justifying the generous overlap of the two sides of the disc: every constellation will be included complete (in a finished model, of course) on at least one side; several will even appear complete on both sides.

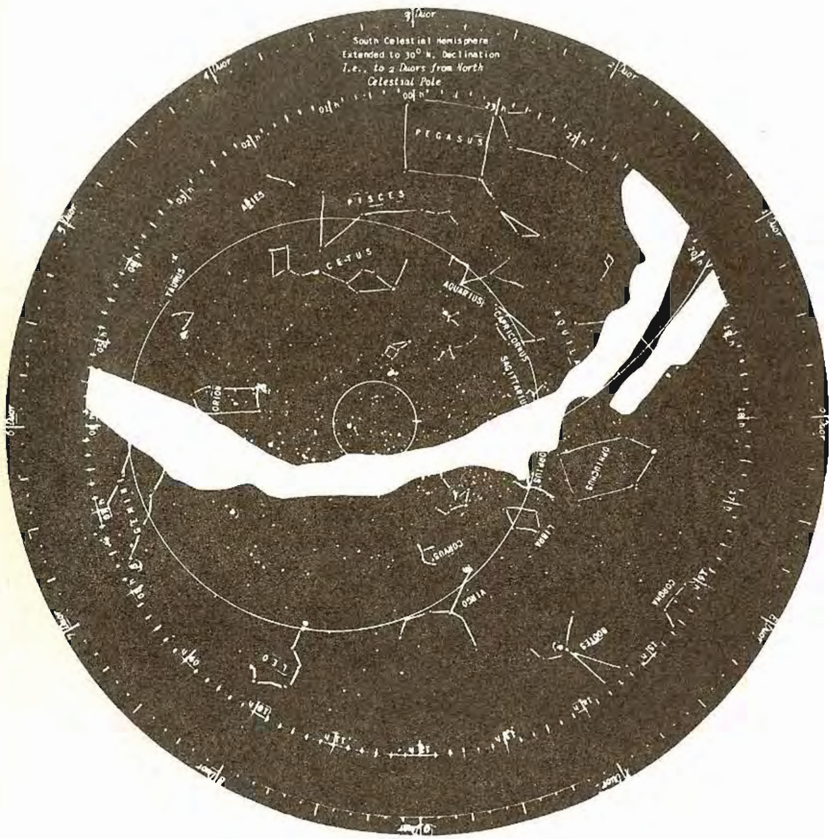
This planisphere may be used in traditional fashion, accepting the traditional calendar and the twenty-four hour day: turn

the disc to bring together the hour of day (or night) at the edge of either chart, with the day of the year inscribed on the inner rim of the enclosing ring of the mask. Either side of the device works just the same, and produces for both sides the aspect of the sky at that hour of that day or night. The meridian line of either mask then indicates corresponding sidereal time, since the hours inscribed just within the edge of the chart show right ascension and are interpreted as in astronomy, the army, or in Continental Europe for time of day, i.e., from ooh for midnight through 12h for noon to 23h for 11 p.m.

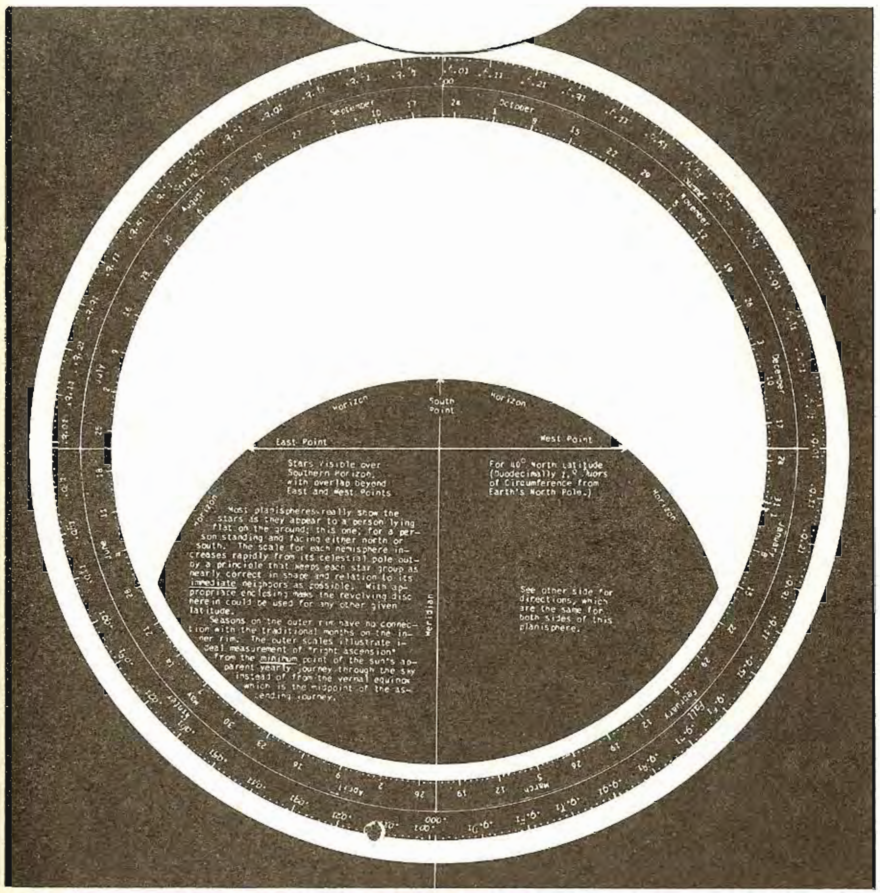
But this particular model illustrates also the ideal duodecimal way (in duors) of reckoning time of the day and right ascension, and the symmetrical and logical reference of both the zero of right ascension and World Day (*0.00* of the year) to the winter solstice. Such time of the day is calibrated on the outermost rim of the disc, and such a version of the World Calendar on the outer rim of the enclosing ring of the mask. The planisphere operates just the same way with these measures of time and right ascension as with the traditional ones, and shows more symmetry. Note how this puts *0.00* day at the very foot, *6.00* day at the very top, of the calendar ring. Each quarter is a quarter-circumference and is named by a season. These advanced features, by the way, would simplify the construction of a type of sundial designed to show time correct within a minute throughout the year whenever the sun is shining. Some day I hope to discuss such sundials in detail.

Unfortunately, however, it is not practicable to try to introduce too many changes to the public at once. This model, with a very few less expensive copies, is to be examined and experimented with by certain experts in popularizing astronomy, to get the benefit of their suggestions. After thus serving its immediate purpose it is to repose indefinitely in the archives of our Duodecimal Society in the hope of proving to a research worker some long time hence, that rational divisions of the day and the year and of right ascension were essentially anticipated at the present date of history.

For the public, if the plan of this instrument appeals to the educational authorities mentioned, and manufacture and sale in reasonable quantity seems feasible, a finished model will be constructed and produced. On the calendar ring of each mask, it will not show the traditional calendar at all, but the World Calendar as proposed by the Association, with direct equivalents in quarters, weeks and days immediately next to them on the plan in the second section of this paper. (It dates shift so slightly from our present clumsy calendar as to make negligible difference in the aspect of the sky so found.) Unfortunately, with the



year beginning at the present initial date (January 1), and with hours or duors of right ascension measured from the vernal equinox, the quarters will not merit the names of the seasons and the axis of the year (as we may call it) from World Day to Leapyear Day on the calendar ring will be oblique. Nevertheless such a device should prove an effective educational instrument and introduce the new calendar and duodecimal concepts to a fairly wide and open-minded section of the public.



Discussion.

An auditor: Haven't days been dropped to rationalize the calendar before? It shouldn't be too hard to shift January 1 back to the winter solstice and then name the quarters by the seasons.

Reply: Yes, Julius Caesar inserted about fifty days some years before Christ, and centuries later a pontiff in Rome took out some to give the world the present Gregorian calendar named after him. But it takes a very strong personality to accomplish such a thing, and public inertia is something appalling when you can't conclusively prove very substantial advantage to everybody. I don't see how we could convince the world of the advantage of that just now.

Mr. R. H. Beard: Wouldn't it be a better idea to make March the first month and keep the vernal equinox as the technical beginning of the year, so that the months from September to December inclusive would signify seventh to tenth, as they originally did in Latin?

Reply: As I've remarked, it isn't likely that months will stay with the world so very long, and it isn't worth while to try to save them. Also, for my part at any rate, I feel an ungenerous sort of personal animus against the old Romans for considering themselves practical geniuses because they could manage a clumsy calendar and arithmetic long after they should have outgrown them, and even bequeath a good deal of such trouble to us.

Mr. H. K. Humphrey: I know of a bank whose annual directors' meetings are held on a certain Tuesday of the first quarter of every year. This partly bears out what you said about reckoning time by the weeks of the quarters.

Reply: Yes, and I believe there are several New York banks with similar by-law provisions. The practicality of making arrangements primarily with weeks and days of the weeks in mind, has long been recognized in the weekly payment of cash wages, of course. A number of large business organizations such as my own (the Equitable Life) formerly paid all salaries twice a month on the fifteenth and last days thereof, or else on the fifteenth and last days thereof, or else on the working days just preceding. This is now superseded and even large salaries are paid every other Thursday afternoon, to the great convenience of everyone concerned.

Written Discussion.

Miss Elisabeth Achelis, President of the World Calendar Association (in a letter to the author appreciating the Duodecimal Society's meeting and anticipated endorsement of the World Calendar):

"There was one point in your talk, however, on which I disagree. I do not believe the people will ever abandon the monthly division in the calendar making it a purely numerical system, such as a railroad schedule. The division of the year into months and weeks is too convenient and practical to be lightly given up. To blot out months, as you and others have suggested, would be a real loss to civilization; and I, personally, hope it will never come to pass. We continue to have the 12 zodiacal signs of antiquity; and so I hope it will be with the 12 months."

Reply: Probably a lot of the old Romans regarded the disappearance of the nones, the ides, and the calends as "a loss to civilization" too, and even the best of poets sometimes stray so far from common sense as to sing

> Do not all charms fly
> At the mere touch of cold philosophy?

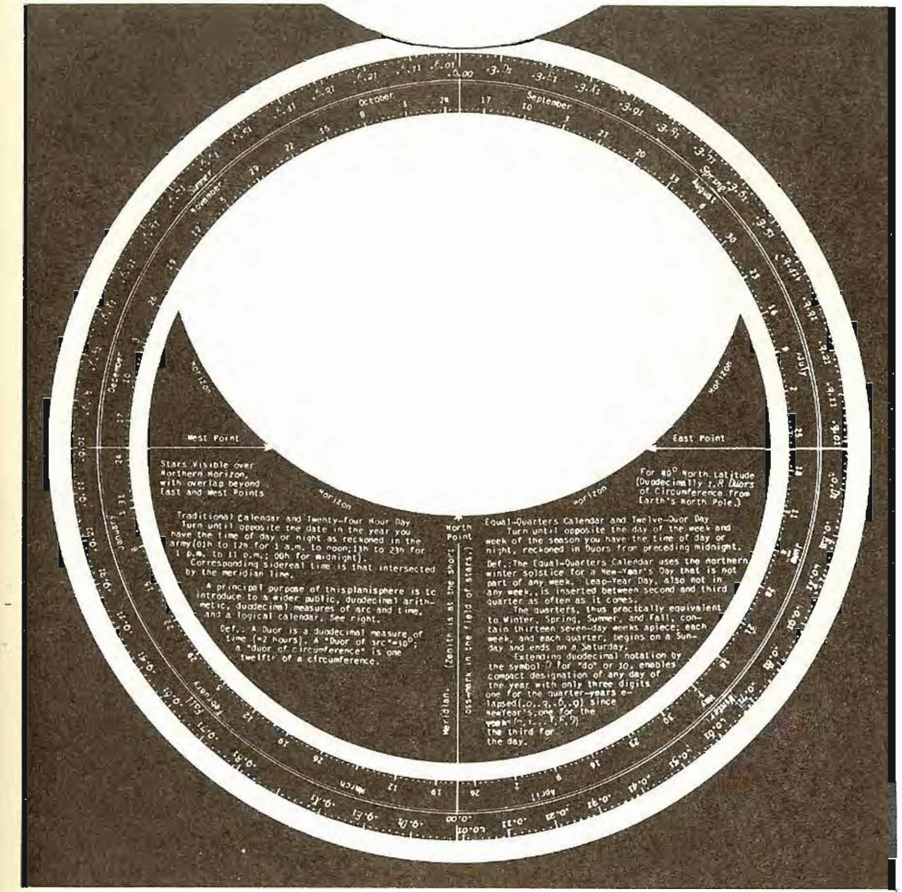
But the actual universe of science, immutable laws and rigorous reasoning has in itself far more of beauty, grandeur and real poetry than all the little attempts we humans make to "explain" things we do not understand, and the little usages we insist on clinging to long after they should be outgrown. Your reference to the zodiacal signs of antiquity seems somewhat unfortunate; according to any textbook beyond the most elementary, precession has made all the "signs" move over into adjacent constellations by now; in particular, that of Aries into Pisces, where the vernal equinox is now to be found. As "the undevout astronomer is made," watchers of the skies do not swear at the unprepossessing hodge-podge of signs and constellations that don't resemble what they're named for, bequeathed to us by the ancients. But they certainly never feel like swearing by them.

Probably most of us on the earth today have slight notion of how far precise division of time has utterly replaced ancient vaguenesses. For the latter, recall Isaac Watts' hymn based on the 90th Psalm: "an evening gone" or "the watch that ends the night" were as short intervals as they could express in the days of the Psalmist. Other awkward usages: "the time of greatest fullness of the market" had to do for "about 10 a.m." when the sun wasn't shining; "when passersby look keenly at each other" for the point in twilight when recognition grew difficult. Is it a loss, that we now replace names and expressions by numbers read from clock and watch dials?

We were all glad you were at the meeting and we wish your Association every success, for your proposed World Calendar is an important and firm step in the right direction. But that very calendar will inevitably hasten the day when recognized convenience compels the months to fall into "innocuous desuetude" (Grover Cleveland): they are not equal fractions of a quarter nor simple multiples of a week and cannot be made so. Even by your calendar, which we of the Duodecimal

Society will almost certainly approve very shortly as the best next step, a given weekday can be any one of thirteen month-days, and a given month-day (except for the 31st) can be any one of three weekdays. Most of us will still have to stop to look at a calendar when we want to set a date.

But I do not understand why you should regret or object to the eventual obsolescence of months. It will be a magnificent life's achievement for you, to have persuaded the world to



adopt so decided an advance as your proposed World Calendar. We pathetically finite humans cannot ask for absolute permanence in anything we do. As one of the great Victorians reverently sang,

> Our little systems have their day,
> They have their day and cease to be.
> They are but broken lights of Thee,
> And Thou, O Lord, art more than they!

THE DUODECIMAL SYSTEM HELPS TEACH TEACHERS

by Dr. Charles T. Dieffenbach.

My first introduction to your fascinating number system came from a college text-book, Morton's Teaching Arithmetic in the Elementary School - Part III. A grudging five pages in the four hundred and fifty page volume gave me the idea that brought me ultimately to the Duodecimal Society of America.

My work is with college graduates who are preparing to be teachers in the elementary grades. Some of my students are brilliant; all of them intelligent; but very few of them would be better than mediocre teachers. They were not seeing their tasks as teachers through the eyes of the children. To help them understand the difficulties of the student, I enlisted the aid of the duodecimals.

Knowing no more of the "new numbers" than their names and Morton's five pages, I faced my group to study further the elementary arithmetic curriculum. I placed the digits (do they have another name when there are twelve of them?) on the board, and said that I had tucked in two new numbers in our regular list and that I wanted them to memorize the series with a view toward computing with it. I gave no reason for my action.

After a few minutes the members of the class said they were ready. We began to add. Up to "1 + 8" they were perfect. But after that----these mature, high-I.Q.'d, intellectuals floundered to a greater and greater degree.

"We'd get this if there were any sense to it," came the excuse.

"There is." And I told the story of the duodecimals as I knew it.

"Why didn't you tell us this beforehand?"

"Because I was imitating the way too many teachers teach their children arithmetic. No reason, no meaning, just blind following of senseless assignments."

I assigned as homework that the group work out the multiplication tables for 2, 3, 4, and 5, duodecimally, which would bring them to a third grader's level - the level of an eight year old. The products entertained us immensely next day.

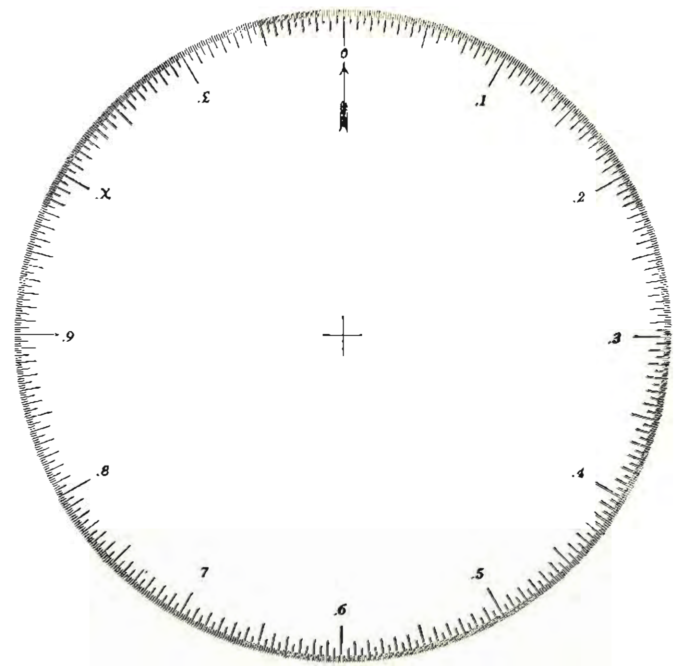
And that was all.

Our discussion indicated that I had gained one rarely achieved accomplishment with the help of your "numbers." I had put a bunch of teachers and would-be teachers in the shoes of the pupils. Just how much meaning will be introduced in thirty-five classrooms this fall because of this experience, I cannot say. But the matter deserves further study and next time I try the experiment I shall plan to evaluate the end products.

To find out what the masters in the field of teaching teachers to teach arithmetic have done with duodecimals, I communicated with six of them with interesting results.

Brueckner of the University of Minnesota, Clark of Teachers College, Grossnickel of Jersey City State Teachers College, Schuster of Trenton State Teachers College, and others use bases different than ten in order to each future teachers of math. the principles underlying number bases in general. But none of them has used your system for the purpose described above, and all of them agree that the idea has merit.

Incidentally, the Andrews volume and the Terry tome and your society were mentioned by practically all of my correspondents as sources for my future elucidation. And rightly!



Duodecimal Protractor reading .002 Circle