COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	X	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	*136*	Five ft. nine in.	*5.9'*
31	*694*	Three ft. two in.	*3.2'*
96	*3Ɛ2*	Two ft. eight in.	*2.8'*
19Ɛ	*1000*	Eleven ft. seven in.	*Ɛ.7'*

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

```
12 ) 365
 12 ) 30 + 5
  12 ) 2 + 6
        0 + 2     Answer: 265
```

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by X, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or X.

Numerical Progression

1	One		
10	Do	*.1*	Edo
100	Gro	*.01*	Egro
1,000	Mo	*.001*	Emo
10,000	Do-mo	*.000,1*	Edo-mo
100,000	Gro-mo	*.000,01*	Egro-mo
1,000,000	Bi-mo	*.000,001*	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	Ɛ
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
Ɛ	1X	29	38	47	56	65	74	83	92	X1

The Duodecimal Bulletin

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THE DUODECIMAL SOCIETY OF AMERICA

20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island 4, New York. F. Emerson Andrews, Chairman of the Board of Directors. H. C. Robert Jr., President. George S. Terry, Editor. Separate subscriptions $2.00 a year, 50¢ a copy.

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WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numbers universally used. *"Why even try to learn to use these heathenish scrawly symbols, with their stubborn propensity for error, in place of the beautiful clear numbers which our fathers have used for untold generations? Think of the needless waste. We would have to change all of our counting boards and abaci. X is X, isn't it? And why do we need a symbol for nothing? You can't count it! No! Let us keep to our simple tried and true numerals, and let the barbarians scratch their heads, and rub themselves out. It will all come to 0 anyhow."*

Yet, although it took D years, the new notation became generally used, and man's thinking leapt forward like an arrow sped from a bow. The early years of the Renaissance marked a new stage in the use of symbols, with the advent of algebra, fractionals (decimals?), logarithms, analytical geometry, and the calculus. Can you imagine what it would be like to try to express the coordinates of points on a curve in Roman notation?

Mathematicians became conscious of a new dimension in symbolism, and the fundamental concepts of numbers were re-examined. Man awoke to the fact that different number bases could be used, and Simon Stevin stated in 1585 that the duodecimal base was to be preferred to the decimal.

The new Arabic notation accomodated mathematical statement better, and facilitated ideation. All thinking accelerated when released from the drag of the cumbrous Roman notation.

The parallel seems tenable. The notation of the dozen base accomodates mathematical statement better, and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base embodies a concurrent analysis and definition of numbers that stimulates classification and generalization. Yet this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis-factory because it has "not-enough-factors."

Then shouldn't we change? No! No change should be made, and we urge no change. All the world uses decimals. But people of understanding should learn to use duodecimals to facilitate their thinking, and to ease the valuative processes of their minds. Duodecimals should be man's second mathematical language. They should be taught in all the schools. In any operation, that base should be used which is most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity. But no change should be made. Perhaps by the year 2000, or maybe by *1200*, which is *14* years later, duodecimals may be the more popular base. But then no change need be made, because people will already be using the better base.

When one is familiar with duodecimals, a number of accessory advantages become apparent. Percentage is a very useful tool, but many percentages come out in awkward figures because of the inflexibility of decimals. When based on the gross, twice as many ratios come out in even figures, and among them are some of those most used, as thirds, sixths, and twelfths, - eighths and sixteenths. There are advantages associated with time and the calendar. Monthly interest rates or charges are derived from annual rates, or the reverse, by simply moving the unit (decimal?) point. The price of a single item bears the same relation to the price of the dozen, and so does the inch to the foot.

The proper correlation of weights and measures has always been one of the world's serious problems. None of the present systems is completely satisfactory. The American and English standards are convenient to use since they are the final result of a long process of practical evolution in which many inconvenient measures have been adjusted or abandoned. The French decimal metric measures have the advantage of being set upon the same base as the number system, and are well systemized. But many of the units are awkward because of their arbitrary sizes, and because their decimal scale does not accomodate division into thirds and fourths readily.

The duodecimal system of weights and measures, based on the inch and yard, the pint and the pound, has the desirable elements of both systems, and few of their faults. This Do-Metric System retains the familiar units of the American and British standards in approximately their present size, and arranges them into an ordered metric system using the scale of twelve. This fits perfectly into the duodecimal notation, and the combination accomodates the inclusion of the units of time and of angular measure within the system, which hitherto has not been possible.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have thought staid and established, and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is cordially invited.

The Duodecimal Bulletin

All figures in italics are duodecimal.

THE ANNUAL MEETING

The summit of the Society's year is the Annual Meeting, with its stimulating assembly of purposeful duodecimal minds. Our 7th Annual Meeting was held this year on January 25th, at the Gramercy Park Hotel in New York, as usual, with President Robert in the chair. Secretary Beard was asked to summarize the year's developments.

The Society suffered a grave loss in the death of George Bernard Shaw in November. G.B.S. has paid many a tribute to the advantages of duodecimals, and his comments have been valuable to us because of the worldwide recognition of his independent thinking and keen intelligence.

Our most notable performance this year has been in the distribution of literature. Over 2500 kits of introductory duodecimal material have been distributed through the teachers of mathematics in high schools and colleges. It is the gage of our progress that fourteen state teachers colleges and three teachers' institutes were included in this distribution.

While only two issues of the Duodecimal Bulletin were published, over 2000 copies of each issue have been required. In addition there were two issues of the news-letter - Dozenal Doings - sent to our members.

The material of the Bulletin continues its recognized high standard. Mr. Robert's recent article on Modern Computing Machines and Split Base Arithmetic is important to us as an entering wedge in this active field, where present practice is confined almost exclusively to combined decimal-binary base operations.

In Weights and Measures, the program of continued development of the Do-Metric System, and publication of tables of conversion factors, is well under way.

Mrs. Doris Burke Lloyd has done an excellent job in assembling the current literature on Color Notation and in establishing contact with those active in current research. However, there seems to be no immediate prospect that duodecimals can offer any material advantages in this work in its present stage.

Lewis Carl Seelbach, our Bibliographer, is to be credited with uncovering the duodecimal works of J. F. Montucla (1799) and John Leslie (1817) in addition to the identification of John Playfair as the author of the unsigned duodecimal article in the Edinburgh Review of 1807.

Our correspondence continues to come from the four quarters of the earth, though in slightly lessened volume recently, with concentrations of interest in South America, Central Europe, and the British Isles.

Treasurer Humphrey reported that the year's budget approximated $1825 as compared with $1750 for last year. Receipts were $1710, of which over $1400 was donated by devoted members. The deficit of $115 has reduced our cash balance somewhat, but required no encroachment on the Endowment Fund.

Mrs. Doris Burke Lloyd, Chairman of the Membership Committee, announced a gain of six members, for a present total membership of sixty-six. Three Aspirants were advanced to Members during the year.

For the Committee on Awards, Mr. Terry reviewed the history of the Annual Award from its inception in 1944. He announced that the Annual Award for 1951 had been conferred upon J. Halcro Johnston of Orkney, Scotland, in recognition of his work, The Reverse Notation, which describes this unique development in numerical notation as employed with the twelve base. The Certificate of Award was displayed for inspection.

President Robert announced that, at a meeting of the Board of Directors, earlier in the day, the incumbent officers had been re-elected to serve for 1951, and that the committee arrangements and personnel would continue without change, as listed in the Duodecimal Bulletin, Vol. 6, No. 2.

Mr. Frederick Condit, reporting for the Nominating Committee, submitted the recommendation that the Directors of the Class of 1951 be re-elected as the Class of 1954. Also, that, following the foregoing precedent, the Nominating Committee recommended that it be continued as the Nominating Committee of 1952.

With some amusement, this recommendation was accepted. As there were no other nominations, these nominees were elected.

Following the completion of our official business, Mr. F. Emerson Andrews addressed the meeting in a review of Mr. Johnston's Reverse Notation. He gave a clear and graphic exposition of the more evident applications of its particular advantages.

His cogent illustration of the standard mathematical operations in this unusual notation system interested and enlightened his hearers.

Mr. Velizar Godjevatz followed with a narration of some of the more recent developments and reactions to his new duodecimal notation for music.

In the discussion following these talks, Mr. Terry gave the meeting an interesting item in number theory as to the largest number representable by three figures. In the decimal base, this is the 9th power of the 9th power of 9, and is written with an exponent to the exponent. While this number, expressed in duodecimals has *97* bimo figures, the parallel construction for Ɛ requires over *45* trimo figures, or enough (as written on this typewriter,) to stretch more than *2X* times around the earth. To lend emphasis to his comments, Mr. Terry gave the first and last six figures of the decimal number, and the first and last nine figures of the duodecimal number using Ɛ.

This concluded the formal part of the meeting, and president Robert pronounced it formally adjourned at 2230 EST. However, all were invited to remain and partake of the refreshment, both alimentary and cerebral, that would follow. The invitation was unanimously accepted and the usual absorbing discussions continued well into the evening.

~ ~ ~ ~ ~ ~ ~ ~ ~ ~ ~

TWELVE TIME

by R. T. Lynhart

What would be, I wonder,
If time went by twelves?
Twelve whispers each breather
That once was ten minutes,
Twelve breathers an hour;
Each noonday, twelve hours.
A twelfth of a whisper,
A midge, is twelve murmurs,
And each little murmur
Is twelve tiny slivers.

AN ITEM ON TRIANGULAR NUMBERS

by Lewis Carl Seelbach

Triangular numbers are formed by taking the successive sums of the terms of an arithmetical series whose differences is *1*.

Unit Series

1 2 3 4 5 6 7 8 9 X Ɛ 10

Triangular Numbers

1 3 6 X 13 19 24 30 39 47 56 66

But, if we take the cubes of the terms of the unit series,

1 8 23 54 X5 160 247 368 509 6Ɛ4 92Ɛ 1000

And summarize the successive terms,

1 9 30 84 169 309 554 900 1209 1901 2630 3630

Then take the square roots of these sums,

1 3 6 X 13 19 24 30 39 47 56 66

We have developed the triangular numbers from a different approach.

NOTE. This connection between polygonal and polyhedral numbers (cubes being hexahedral) suggests others. To stimulate enquiry, the following are listed with general term to provide for extension. The vertical difference for polygonal numbers is the previous triangular number. What triangular numbers are also square?

Polygonal Numbers							
Triangular	1	3	6	X	13	...	$\frac{n(n+1)}{2}$
Square	1	4	9	14	21	...	n^2
Pentagonal	1	5	10	1X	2Ɛ	...	$n+\frac{3n(n-1)}{2}$
Hexagonal	1	6	13	24	39	...	$n+2n(n-1)$
Polyhedral Numbers							
4-hedral	1	4	X	8	2X	...	$\frac{n(n+1)(n+2)}{6}$
6-hedral	1	8	23	54	160	...	n^3
8-hedral	1	6	17	38	71	...	$\frac{n(2n^2+1)}{3}$

NATURAL LOGARITHMS

COMPUTING THE NATURAL LOGARITHMS OF THE FIRST EIGHT PRIMES TO SIX DOZEN AND THREE PLACES

by Harry C. Robert, Jr.

At the annual meeting of the Duodecimal Society in New York last January, Mr. Terry pointed out that the only way of converting the first figures of very large numbers from one base to another was the use of logarithms. Mr. Terry has published some extended values to *13* placed and has available, unpublished but available in microfilm, an eighteen place common log table. While these values would serve many purposes, the writer felt that the evaluation of such numbers as $Ɛ^{Ɛ^{Ɛ}}$ made it desirable to consider a condensed table of extended values, with a greater number of places than has previously been published.

Being a novice at such computations, the writer undertook as a preliminary exercise, the calculation of the natural logs of the first few primes carrying the several expansions to *43* places. The selection of the number of places was arbitrary, being simply the maximum number that could be entered on the quadrille ruled notebook paper generally used by the writer in computations. On submitting the results of this work to Mr.Terry, it was learned that the four dozen places, which had appeared to be a good length for a condensed table, would be little improvement over published forty-eight place decimal tables by Peters. Mr. Terry suggested extending the tables to *42* or *46* duodecimal places.

On pasting two sheets of notebook paper together it appeared that the expansions could conveniently be carried to *68* places. The work involved seemed considerable when it was realized that the expansion for log *2* involved *48* terms for just *43* places and the expansion for log *9* had required *1X* terms. It was therefore decided to drop these two expansions and use instead two of the check expansions from the original computations instead. A total of eight expansions of $\frac{\log(z/z-1)}{2}$ were used, where all factors of both z and (z-*1*) are found among the first eight primes, *2, 3, 5, 7, Ɛ, 11, 15* and *17*. Simultaneous equations for the several expansions were then written in terms of the logs of the factors of (z/z-*1*) and the equations solved for the logs of the several primes.

The several expansions used were, as follows,-

z		z-1		2z-1	Log (z/z-1)
$(3^3)(17)$ =	369	(2^9) =	368	715	$2a_1$
$(2^6)(17)$ =	854	$(3^5)(5)$ =	853	14X7	$2a_2$
(7^4) =	1481	$(2^5)(3)(5^2)$ =	1480	2941	$2a_3$
$(3^4)(Ɛ^2)$ =	5809	$(2^3)(5^2)(7^2)$ =	5808	Ɛ415	$2a_4$
$(3^4)(15^2)$ =	11669	$(2^4)(7)(Ɛ)(17)$ =	11668	23115	$2a_5$
(11^4) =	14641	$(2^4)(3)(5)(7)(15)$ =	14640	29081	$2a_6$
$(7)(11)(17)$ =	1001	$(2^6)(3^3)$ =	1000	2001	$2a_7$
$(5^2)(7^2)$ =	861	$(2^3)(3^2)(15)$ =	860	1501	$2a_8$

These lead to the following equations,-

1.	- 9L2	+ 3L3						+ L17	=	$2a_1$
2.	- 6L2	- 5L3	- L5					+ L17	=	$2a_2$
3.	- 5L2	- L3	+ 2L5	+ 4L7					=	$2a_3$
4.	- 3L2	+ 4L3	- 2L5	- 2L7	+ 2LƐ				=	$2a_4$
5.	- 4L2	+ 4L3		- L7	- LƐ		+ 2L15	- L17	=	$2a_5$
6.	- 4L2	- L3	- L5	- L7		+ 4L11	- L15		=	$2a_6$
7.	- 6L2	- 3L3		+ L7		+ L11		+ L17	=	$2a_7$
8.	- 3L2	- 2L3	+ 2L5	+ 2L7			- L15		=	$2a_8$

The solutions are, as follows,-

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8
3L2 =	+ 5X0	+ 1238	+ Ɛ8	+ 422	+ 844	+ 2Ɛ4	- Ɛ94	+ 1194
L3 =	+ 310	+ 768	+ 62	+ 226	+ 450	+ 168	- 628	+ 734
L5 =	+ 462	+ Ɛ0X	+ 90	+ 32X	+ 658	+ 234	- 914	+ X80
3L7 =	+ 1446	+ 341X	+ 28X	+ Ɛ8X	+ 1Ɛ58	+ 832	- 2908	+ 3282
3LƐ =	+ 1820	+ 415X	+ 344	+ 125X	+ 24Ɛ2	+ X22	- 3488	+ 3Ɛ82
3L11 =	+ 1970	+ 44Ɛ2	+ 372	+ 1358	+ 26Ɛ4	+ XXX	- 376X	+ 42ƐX
3L15 =	+ 1ƐX0	+ 4X58	+ 3Ɛ8	+ 1512	+ 2X24	+ 1004	- 4014	+ 483X
L17 =	+ 832	+ 1830	+ 146	+ 5Ɛ0	+ ƐX0	+ 420	- 1480	+ 1760

The eight expansions were then computed. The longest has only *12* terms for *68* places. The total number of terms for all eight is only *78*. The computations were made in two steps. First, the terms of the geometric progression, $1/(2z-1)^{2y+1}$, were computed, the results being written on alternate lines of quadrille ruled paper. The several terms were then totaled and the sum compared with the sum of the series, $(2z-1)/4z(z-1)$. This comparison served to check this portion of the work. The several terms were then divided by $(2y+1)$, the exponent for the respective terms. The value of y is *0, 1, 2, etc.* The values of the several quotients were entered in the lines left blank in the first part of the operation, using red ink. The quotients were then totaled, the sum being the desired expansion.

The desired logarithms were then computed by substitution in the solutions of the eight equations. The resulting values of L2, L3 and L5 were then compared with values converted from the base ten values given by Uhler in the Proceedings of the National Academy of Sciences, Vol. 26, 1940, pages 205-212. Each of the three values check with Uhler's results to six dozen and four places. This is the accuracy to be expected from the magnitude of the coefficients in the several solutions by which the expansions are multiplied. Instead of checking results with Uhler, a ninth expansion could have been computed for a completely independent check.

The methods used here were similar to those used by Uhler in computing the logs of 2, 3, 5, 7 and 17 decimally to 330 places. The equations can be used for computing any desired number of places depending entirely on the number of places in the eight expansions.

While the writer carried on this investigation, Mr Terry developed the natural logarithms of the primes in the first gross to four dozen and six places. The two operations have laid the foundation for an interesting and useful table of extended values of natural logarithms.

The results follow.–

log 2.	.839912	483369	XƐ2137	42X346	792537	88658X	
	1402X5	405318	4931XƐ	084X36	251989	743533	5Ɛ3
log 3.	1.12249X	871651	3489Ɛ9	024604	4Ɛ5626	1Ɛ7Ɛ00	
	047X0Ɛ	422361	727X44	Ɛ30438	356805	61Ɛ327	55Ɛ
log 5.	1.739137	X32411	X7226X	998X5Ɛ	5ƐƐX13	4Ɛ88ƐƐ	
	Ɛ19504	61X286	46Ɛ162	485Ɛ6Ɛ	67Ɛ3Ɛ3	X85439	126
log 7.	1.Ɛ42648	6X1X29	Ɛ8Ɛ305	9X29X8	Ɛ3Ɛ846	8903ƐƐ	
	335072	5X09X1	Ɛ63717	X12607	353436	Ɛ2505X	Ɛ24
log Ɛ.	2.493690	Ɛ028X4	5Ɛ3468	207X5X	35X987	5462Ɛ4	
	617006	742314	602447	182Ɛ34	X496Ɛ8	X18034	4X6
log 11.	2.694295	8XX608	8Ɛ6713	81589Ɛ	31Ɛ085	465376	
	896465	172616	Ɛ34880	333050	75X385	996383	4XX
log 15.	2.9ƐƐ961	869828	613561	89Ɛ57Ɛ	962681	066605	
	7ƐX513	1065Ɛ4	750973	Ɛ590Ɛ8	Ɛ69320	61X900	109
log 17.	2.Ɛ3ƐƐX7	820448	4095ƐƐ	12Ɛ6Ɛ6	068X99	122272	
	757X07	2Ɛ1095	88992Ɛ	8X7177	X3Ɛ763	18X616	7X3
log 1Ɛ.	3.176173	6891Ɛ1	ƐX126X	051784	666674	2ƐX292	
	486XƐ8	8Ɛ6016	5878				
log 25.	3.44X82Ɛ	57XƐ28	Ɛ63ƐX0	283466	4XX348	X27X45	
	Ɛ69528	07X195	2777				
log 27.	3.525Ɛ1X	X2287Ɛ	469Ɛ96	1204Ɛ8	8X6486	097964	
	X1Ɛ343	2337Ɛ6	156X				
log 31.	3.73Ɛ7ƐƐ	14241X	958147	669963	40Ɛ566	441XƐƐ	
	1077Ɛ3	Ɛ88037	8583				
log 35.	3.869076	935625	2X52X4	1Ɛ807Ɛ	663260	48Ɛ408	
	7X22ƐX	5X45Ɛ4	2X10				

log 37.	3.91742Ɛ	448X8Ɛ	X83754	8Ɛ1359	30ƐƐ1X	711455
	1XƐ034	55X157	5Ɛ53			
log 3Ɛ.	3.X2507Ɛ	17772X	997357	210894	X2704X	Ɛ74005
	850181	500Ɛ6Ɛ	7097			
log 45.	3.Ɛ787Ɛ8	167301	2Ɛ424Ɛ	1ƐX198	237650	Ɛ22X31
	X42Ɛ63	430806	Ɛ889			
log 4Ɛ.	4.0Ɛ1Ɛ99	697738	4XƐX59	58Ɛ633	4X2549	261X02
	648ƐƐ3	6Ɛ2Ɛ23	0Ɛ95			
log 51.	4.13Ɛ70Ɛ	701XƐ3	650Ɛ79	078684	5X38X3	1XƐ548
	414568	42367Ɛ	26X2			
log 57.	4.255860	X77896	5X2035	336082	446787	7X34X7
	16XX17	217688	ƐXX7			
log 5Ɛ.	4.319XƐ1	XƐ050Ɛ	XX1087	618858	2Ɛ8124	128378
	X44523	578XX0	3X58			
log 61.	4.359XƐ7	2Ɛ15X7	4356ƐƐ	300ƐƐ9	85Ɛ184	63X084
	8186Ɛ9	803891	1154			
log 67.	4.4524X5	4622Ɛ8	515823	647838	031301	064631
	929Ɛ7X	367438	8024			
log 6Ɛ.	4.50390Ɛ	42X88Ɛ	08Ɛ659	787195	747959	1X34Ɛ4
	Ɛ63320	177624	9070			
log 75.	4.5X4444	46Ɛ996	XƐƐ9X2	818716	936Ɛ55	612453
	Ɛ663X5	064Ɛ87	2218			
log 81.	4.6X9125	952Ɛ63	7X55ƐX	Ɛ09Ɛ5Ɛ	812834	977445
	574252	4X7885	9X63			
log 85.	4.746Ɛ18	030XXƐ	99581Ɛ	7229X4	80Ɛ2X9	59X71X
	7466X4	89X790	9007			
log 87.	4.77498X	72X38Ɛ	3ƐƐ1X1	855577	00X357	37Ɛ063
	814326	824X07	5192			
log 8Ɛ.	4.80X794	174395	444484	147244	90Ɛ819	Ɛ08Ɛ53
	X44330	Ɛ91340	030Ɛ			
log 91.	4.836795	86Ɛ158	13058X	53ƐƐ52	254398	148648
	344631	Ɛ36895	2818			
log 95.	4.888Ɛ14	47Ɛ339	078Ɛ92	7041XƐ	6555X3	5ƐƐ327
	698047	ƐƐ499Ɛ	5Ɛ25			

log X7.	4.X16909	172155	457365	921049	244786	644XX3
	504404	6990XX	1289			
log XƐ.	4.X60411	254462	2X4352	X07633	2503X3	17165Ɛ
	X46136	071768	4261			
log Ɛ5.	4.Ɛ05888	3X9377	145918	363637	815033	6Ɛ624Ɛ
	6909Ɛ1	36X77X	3906			
log Ɛ7.	4.Ɛ26930	26775X	136X61	4059Ɛ0	7233Ɛ0	476339
	838171	26Ɛ930	6110			

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A MEASURING WHEEL

Warren H. Chapin of Eden, Mich., author of the article in the last Bulletin on the use of 96 in place of the basic, two-place 100, as a "Convenient Compromise Between Binary Scales and Decimals," submits the following interesting suggestion.

"If anyone desires to experiment with a 96° circle, etc., economically, cut a circle out of 3/4" plywood on a 7 5/8" radius, and paste a 10¢ tape measure around it. This will have a 48" circumference, and each half-inch mark will indicate 1°. With it, one can make cardboard protractors, sundials, and similar graduated circles. It can also be used as a measuring wheel to demonstrate some of the fundamental facts of measurement, and many of the simple advantages of 96 as a base."

Were Mr. Chapin's plywood disc cut out on a 7 23/32" radius,* its circumference would be more exactly one yard. Each quarter-inch mark would indicate *.01*^{c} as well as the quan, and each three inches would be a palm.

A similar arrangement might be much appreciated were it used to form the bottom of your wife's sewing kit.

* Archimedes might have said 7 7/11", Adrian of Metz $7\frac{227''}{335}$ and and we dozeners might say $7\frac{797''}{103Ɛ}$ and still none of us be right.

CONVERSION AND LARGE NUMBERS

by George S. Terry

Hand conversion of very large and very small numbers is laborious. The following table has been found useful in economy of time, and in showing the relative position of the units' place.

Decimal	Duodecimal		Decimal	Duodecimal	
10^{3}	.6Ɛ4	10^{3}	10^{-3}	1.889 Ɛ98	10^{-3}
10^{6}	.402 854	10^{6}	10^{-6}	2.Ɛ9Ɛ 944	10^{-6}
10^{9}	.23X X94	10^{9}	10^{-9}	5.1Ɛ0 126	10^{-9}
10^{12}	.141 982	10^{10}	10^{-12}	8.XƐƐ 031	10^{-10}
10^{15}	.094 1X7	10^{13}	10^{-15}	13.4X7 400	10^{-13}
10^{18}	.054 XX5	10^{16}	10^{-18}	22.759 153	10^{-16}

For example,

If the diameter of a Hydrogen molecule is decimally 8.07×10^{-9} inches the duodecimal value is *8.0X(5.1Ɛ)* 10^{-9} = *35.8* 10^{-9} inches.

If the volume of the Earth is 1415.10^{18} cubic yards the duodecimal value is *99Ɛ(.054XX)* 10^{16} = *45.19* 10^{16} cubic yards.

The largest known prime is $2^{127}-1$ and contains thirty-nine digits. The first of which are 170 141. It may be written 170.151 10^{36}. The duodecimal value is *122.184 (.054XX5)*^{2}.10^{30} = *.2X696* 10^{30}, thus containing three dozen integers. The complete number is

2X6 959 258 068 187 353 99X 37X 20X 31Ɛ 353 4X7.

If we need the last integers of a large number the table is less useful but since such numbers are usually derived from large powers of small numbers, let us look at the endings of powers of 2. These match the last figure every 2nd power, match two figures 6th power, match three figures every $2(3^2)$ power, match n figures every $2(3^{n-1})$th power.

Thus 2^{X7} matches four figures every 46th power, ending as 2^{61} and 2^{17}, – and 2^{17} being 2134X8, our number ends 34X7.

The largest number which can be expressed decimally with three digits is 9^{9^9} that is 9 to the 387 420 489th power. This number contains more than 369 million digits and, as pointed out in Math. Teacher for Dec. 1950 p. 418, would require almost 1167 miles of paper to write with five digits to the inch. Duodecimally the number contains 96 885 541 integers beginning 261Ɛ9X ... ending X55 809. Note the matching endings of powers of 9. Two figures match every 2nd power, three every 2^3 power, four every 2^5 power and n figures every 2^{2n-3} power. Thus our power which is X9 8Ɛ5 809 matches four figures with the power's remainder when divided by 28 i.e. with the 9th power, or six figures with its remainder when divided by 368 i.e. with the 235th power.

The number $Ɛ^{Ɛ^Ɛ}$ is very large even compared with the last. It contains 45 438 624 integers and if written as above would stretch over 2X times round the earth. It begins 941 27X 60X ... and ends ...9X5 42Ɛ 3X3, the first from the antilog of $Ɛ^Ɛ(\log Ɛ)$, the last from matching powers and (more easily) from end terms of the expansion of $(10-1)^{Ɛ^Ɛ}$. For example:-

The last terms of $(10-1)^n$ with n odd are $-1 + 10n - \frac{10^2 n(n-1)}{2} + \text{etc.}$

Our large $n = Ɛ^Ɛ$ which ends XƐ. Also $\frac{XƐ(XX)}{2}$ ends 7.

So the last three figures of $Ɛ^{Ɛ^Ɛ}$ are $-1 + XƐ0 - 700$ i.e. 3XƐ.

DO-METRIC DYNAMIC MEASURES

by Ralph H. Beard

Basic concepts of work and energy are derived from our own muscular efforts. We learn that differences in effort are required to move bodies of different weights, – in vertical or horizontal directions, – for different distances, – at different speeds or within different times. We come to know that to produce an effect of a certain extent requires a cause of a cercertain size.

These forces and motions may be measured in various combinations of the simple elemental measures of length, weight or mass, and time. Each type of these derived measures may be analyzed or described by its dimensional formula, which states its elements of m, l, and t. These formulas are given for each of the tables which follow, because of their great aid to clear thinking in this area where confused thought is widely present.

The measures of acceleration, force, work and energy, and power for the do-metric system are presented in the selected forms with considerable diffidence. These forms have been chosen as the most practical, after consideration of the many other possibilities. Designations have been used which are abbreviations of the component factors. They will serve as neces-symbols, – rather than names, – until proper terms are found.

A sharp and clear distinction must be drawn between absolute units and gravitational units. The old familiar British-American foot-pound and horse-power include the gravitational factor of 32.174 feet per second per second in their basic definitions. They belong therefore to the gravitational system. The foot-poundal, on the other hand, was designed expressly to omit the gravity factor. For this reason 32.174 foot-poundals are, in general terms, equivalent to one foot-pound. The foot-poundal is defined as the action of a force, capable of accelerating a mass of one pound one foot per second per second, moving a mass of one pound through a horizontal distance of one foot. Since its elements are all unit quantities, it is a unit of the absolute system. The characteristic of the absolute measures is the combination of only unit quantities in their established values.

Variables are excluded from the definitions of the absolute measures. The acceleration due to gravity is definitely variable, ranging from 32.091 feet at sea level at the equator, to 32.255 feet at the poles. An accompanying table gives value of the gravity factor for several American cities of different latitudes. Where accuracy is essential, correction must be made for this variation.

The do-metric dynamic measures conform to the absolute system, as do the measures of the French metric system. Where the gravity factor is involved, it can readily be introduced as a simple quantity. Its value in do-metric units is an acceleration of 1.293 *(1.3624)* yard per dovic per dovic. (In the duodecimal nomenclature this becomes, "yard e dovic e dovic." The dovic is selected as the time unit of these dynamic measures, as it is the unit of the unified time-circle measure which falls between the second of time and the second of arc. The second of time equals 2.88 dovics.

It may develop that the dovic will ultimately be called "the second," since, in relation to the minette, (which is 50 seconds of time, 12½ minutes of arc, or, duodecimally *.001*^{c},) the grovic may be considered as the prime, the dovic the second, and the vic the third, or trice. This is old and familiar practice, associated with the use of the symbols ′, ″, and ‴.

Enter tables from the left.

VELOCITY $(\frac{l}{t})$

	DECIMAL			DUODECIMAL		
	Cm.p.Sec.	Ft.p.Sec.	Yd. e Dovic	Cm.p.Sec.	Ft.p.Sec.	Yd. e Dovic
Centimeters per Sec.	1	.032 808	.003 797	*1*	*.048 839*	*.006 68Ɛ*
Feet per Second	30.480 061	1	.115 7̇40̇	*26.591 666*	*1*	*.1480̇*
Yards e Dovic	263.347 727	8.640	1	*19Ɛ.420 X56*	*8.781 206*	*1*

ACCELERATION $(\frac{l}{t^2})$

	DECIMAL			DUODECIMAL		
	Cm. p. Sec.^{2}	Ft. p. Sec.^{2}	Yd. e Dovic^2	Cm. p. Sec.^{2}	Ft. p. Sec.^{2}	Yd. e Dovic^2
Cm.p.Sec.p.Sec.	1	.032 808	.001 318	*1*	*.048 839*	*.002341*
Ft.p.Sec.p.Sec.	30.480 061	1	.040 188	*26.591 666*	*1*	*.059 540̇*
Yd. e Dovic e Dovic	758.441 453	24.883 20̇	1	*532.536 X11*	*20.X72 205*	*1*
Accel. Force of Gravity	980.6550̇	32.1740̇	1.293 000	*698.7Ɛ9 153*	*28.210 809*	*1.362 37Ɛ*

Enter tables from the left.

FORCE $(\frac{ml}{t^2})$

	DECIMAL			DUODECIMAL		
	Dyne	Poundal	Pd. Yd.	Dyne	Poundal	Pd. Yd.
DYNE Gram cm.p.Sec.^{2}	1	.000 072 33	.000 002 98	*1*	*.000 15Ɛ Ɛ9*	*.000 008 X9*
POUNDAL *Pd.*Ft. p Sec.^{2}	13 825.525	1	.041 200 66	*8 001.637*	*1*	*.05Ɛ 240 63*
PD. YD. Pd.Yd. e Dovic^2	335 565.487	24.271 454	1	*142 239.711*	*20.331 025*	*1*
G. UNITS	Kgm.Wt. 980 665.0̇	Pd. Wt. 32.1740̇	G. Pd. Yd. 1.293 000	Kgm.Wt. *3Ɛ3 621.0̇*	Pd. Wt. *28.210 809*	G. Pd. Yd. *1.362 37Ɛ*

WORK $(\frac{ml^2}{t^2})$

	DECIMAL			DUODECIMAL		
	Joules	Ft.Poundals	Yd.Pdyds.	Joules	Ft.Poundals	Yd.Pdyds.
JOULE 10 000 000 Dyne Cms. or Ergs	1	23.730 262	.325 901	*1*	*1Ɛ 891 X87*	*.3XƐ 1X7*
FT. POUNDAL	.042 140	1	.013 734	*.060 99X*	*1*	*.01Ɛ 894*
YD. PDYD.	3.068 417	72.814 340	1	*3.09X 287*	*60.993 274*	*1*
G. UNITS	Kgm.Meter 9.806 650̇	Foot Pound 32.174 0̇	G. Yd. Pdyd. 1.293 000	Kgm.Meter *9.981 X84*	Foot Pound *28.210 809*	G. Yd. Pdyd. *1.362 37Ɛ*

POWER $(\frac{ml^2}{t^3})$ Rate of work.

	DECIMAL			DUODECIMAL		
	Watts	Ft.Pdls p. sec.	P Units	Watts	Ft.Pdls p. sec.	P Units
WATT 1 Joule per Sec.	1	23.730 262	.938 594	*1*	*1Ɛ.891 X87*	*.Ɛ31 X84*
Ft. Poundals p. Sec.	.042 140	1	.039 553	*.060 99X*	*1*	*.058 375*
POWER UNITS Yd. Pdyd. e Dovic	1.065 423	25.282 757	1	*1.095 074*	*21.348 74Ɛ*	*1*
G. UNITS	Kgm. M. p. sec. 9.806 650̇	Ft.Pounds p. sec. 32.174 0̇	G.P.Units 1.293 000	Kgm. M. p. sec. *9.981 X84*	Ft.Pounds p. sec. *28.210 809*	G.P.Units *1.362 37Ɛ*

LARGE POWERS $(\frac{ml^2}{t^3})$

	DECIMAL			DUODECIMAL		
	Kw.	H. P.	M. P.	Kw.	H. P.	M. P.
KILOWATT 1000 watts	1	1.341 019	.543 168	*1*	*1.411 345*	*.662 717*
HORSE POWER 550 Ft.Pd. per Sec.	.745 702	1	.405 041	*.8Ɛ4 6X6*	*1*	*.4X3 XƐ3*
MO. POWER UNITS 1728 Power Units	1.841 051	2.468 883	1	*1.X11 407*	*2.576 292*	*1*

VARIOUS VALUES FOR G. FACTOR

	Ft/sec.^{2}	Cm/sec.^{2}	Yd/dovic^2
New York	32.161	980.269	1.292 48
Boston	32.166	980.422	1.292 68
Miami	32.123	979.111	1.290 95
Seward, Alaska	32.214	981.885	1.294 61

CORRECTION FOR UNITS OF MASS

In the last issue of the Bulletin, Vol. *6*, No. *3*, p. *66*, we published a table of the equivalents of the units of mass which has been found to be seriously inaccurate. The basic error was reliance on a text book value given for the mass of the U. S. pound avoirdupois as equivalent to that of 27.692 cubic inches of water. The correct equivalent is 27,6805 cubic inches of water, derived as follows:

U. S. pound avoirdupois	= .4535924 kilogram
Kilogram	= 1.000027 cubic decimeter water
Decimeter	= 3.937 inches
Pound avoirdupois	= (1.000 027)(.4535924)(3.937)^3
	= 27.6805 cubic inches water
or duodecimal	= *(1.000 07)(.5539838)(3.££177)*^{3}
	= *23.81££* cubic inches water

In order to correct the error, the following corrected table of equivalents is published for use in place of the inaccurate table. Please mark the incorrect table to refer to this one.

The following table is to be entered from the left

	D E C I M A L		
	Pound (A)	Pound (D)	Kilogram
Pound Avoirdupois	1	1.025 204	.453 592
Pound Duodecimal	.975 415	1	.442 440
Kilogram	2.204 622	2.260 188	1
	D U O D E C I M A L		
Pound (A)	*1*	*1.037 678*	*.553 984*
Pound (D)	*.£85 626*	*1*	*.538 656*
Kilogram	*2.255 707*	*2.315 732*	*1*

CONVERSION DOUBLES

or

DUODECIMAL NUMBERS WHICH REPRESENT TWICE AS MUCH AS THE SAME DIGITS INTERPRETED DECIMALLY

by H. K. Humphrey

One of the advantages of the duodecimal number system is the fact that larger quantities are represented by smaller numbers. Often this results in a saving of digits. I am especially sensitive to this fact just now, for the bank for which I work has finally reached totals of 410 million, and so for the first time we have used the last column in the adding machine which we bought with this possibility in mind. But, if we had been able to keep our books duodecimally, we could have continued to use the old 9-column machine (7 for dollars, plus 2 for cents) until we reached about three and a half times our present size, since 12^7 is about 3.5 times 10^7.

But even when this feature of duodecimals does not result in the saving of a column of digits, it can lead to interesting results. In particular, Ralph H. Beard pointed out in the last bulletin, Vol. 6, No. 3, p. *6𝒳*, that there are numbers for which duodecimal interpretation of the digits (or of their place value) gives a quantity just twice as great as the result of decimal interpretation of the same digits. He gave several examples of such doublets and then raised the question whether there might not be more.

The answer to this question is a definite yes - there seem to be *19* (21) such numbers, and they have some fascinating characteristics. They occur in pairs, the difference between the members of each pair being always 4, there are just a dozen such pairs, but three are imperfect in that one member of each of these pairs involves the digits *𝒳* or *£*. These *10*, (12) pairs are shown in Table I, with an identifying letter assigned to each pair for later reference. When it is necessary to refer to just one of the members of a pair, the subscript 1 will be used for the smaller number, 2 for the larger.

TABLE I

A	*11788* = 23576 *11790* = 23580	B	*11818* = 23636 *11820* = 23640	C	*12298* = 24596 ..... = ...	D	*12328* = 24656 *12330* = 24660
E	*24658* = 49316 *24660* = 49320	F	*25168* = 50336 *25170* = 50340	G	..... = ... *25200* = 50400	H	*36988* = 73976 *36990* = 73980
I	*37498* = 74996 ..... = ...	J	*37528* = 75056 *37530* = 75060	K	*38038* = 76076 *38040* = 76080	L	*49858* = 99716 *49860* = 99720

In addition to the constant difference of 4 between members of the same pair, the differences between pairs show an interesting pattern of repetition. The difference *50;* (60) occurs 4 times; *710;* (1020) occurs 8 times, *760;* (1080) 6 times; *11820;* (23640), 4 times; *12330;* (24660), 8 times; and *12𝒳90* (25740) all of 𝒳 (10) times. Table II may help to make finding these differences easier.

TABLE II

Difference		Occurrence	Times
50;	(60)	$D_1 - C_1$, B – A, J_1 – I	4
710;	(1020)	$C_1 - A_1$, D – B, F – E, $I_1 - H_1$, K – J	8
760;	(1080)	D – A, $G_2 - E_2$, J – H, $K_1 - I_1$	6
11820;	(23640)	H – F, L – K	4
12330;	(24660)	E – D, H – E, $J_2 - G_2$, $I_1 - F_1$, L – J	8
12𝒳90;	(25740)	E – A, $F_1 - C_1$, $G_2 - D_2$, J – E, K – F, L – H	10

Note that B – A means BOTH $B_1 - A_1$ and $B_2 - A_2$.

I thought I had a sure-fire method of finding such numbers, but by following out these differences, found a few which had been missed. Although I did commit some errors of omission, the method looks good, and may be interesting. The conditions are satisfied if

$$a + 12\ a + 12^2 a_2 + 12^3 a_3 + 12^4 = 2(a_0 + 10\ a_1 + 10^2 a_2 + 10^3 a_3 + 10^4 a_4)$$

or

$$a_0 + 12\ a_1 + 144 a_2 + 1728 a_3 + 20636 a_4 = 2a_0 + 20\ a_1 + 200 a_2 + 2000 a_3 + 20000 a_4$$

or

$$-a_0 - 8a_1 - 56a_2 - 272a_3 + 736a_4 = 0$$

This makes it clear at once that all such numbers must consist of at least five digits, since the 4th is the first power of 12 which is as much as double the corresponding power of 10. The problem then reduces to taking the excess for the 4th power, 736, a_4 times, and balancing this excess by assigning values to a_3, a_2, a_1, and a_0 such that the total excess is equal to the total deficiency, limiting all choices to 9 or less. For example, if a_4 be 1, try a_3 + 1, the remaining excess is 736 - 272 + 464, and this is a possible choice since we can still use up as much as 9(56) = 504. If 6 be then chosen for a_2, the remaining excess will be reduced to 464 - 336 = 128, and it will be impossible to reach a balance, since we have left only 9(8+1) = 81. So a_2 must be at least 7, in which case the excess will be reduced to 72. If 8 be then chosen for a_1, 72 - 8(8) = 8 will remain, and that can be balanced by letting a_0 = 8. Finally, 8 + 64 + 392 + 272 = 736, and the smallest number, *11788,* has been found. If 4 be added to this number, the last digit will become 0 and the next to that will be increased by 1, since *8 + 4 = 10,* so *11790* will also satisfy the conditions. This explains the difference of 4 between the members of each pair. But it will not work if the penultimate digit is 9, for in that case this digit becomes 𝒳, it is for this reason that pairs C and I are defective. It is interesting to note that the larger member of each pair ends in 0. Apparently for any combination of three digits, a_4, a_3, and a_2, which will work, complete balance can be secured by properly choosing a_1, leaving a_0 = 0, then, if a_1 be reduced by 1, the resulting discrepancy can be cured by letting a_0 = 8, which is the same as subtracting 4, and with the result that all the smaller numbers end in 8. But if the last digit and the penultimate of the larger number are 0, then subtracting 4 produces an Ɛ in the penultimate column, and it is for this reason that pair G is defective.

It has been shown above that these numbers must have at least 5 digits, that is must be greater than 10000. There is an upper limit, too. There cannot be more than 5 digits, and the first one (a_4) cannot be greater than 4. For 5 would create an excess of 5 (736) = 3680. This is greater than the total deficiency which can be provided even when all the other digits are 9, since 9 (272 + 56 + 8 + 1) = 9 (337) = 3033. So the largest must be less than 50000.

I have found all this a fascinating method of teasing myself into doing some duodecimal arithmetic than I otherwise might have done. and it is by no means finished. Can the same thing be done for a factor of 3? and 4? and 5? etc? I venture the guess that it can, and am in hope that some will be encouraged to try. But lest discouragement be met right at the start, I suggest that 3 be skipped for a while. The first power of 12 which is more than 3 times the corresponding power of 10 is the 7th, which is 35,831,808, leaving an excess of nearly 6 million. But the total deficiency for the powers from 0 to 6 is only 75,896 and if all the first 7 digits were 9's, only 683,064 of the excess could be balanced. Consequently, this seems to be hopeless. But 4, 5, and 6 do seem to be possible, and I hope that some will enjoy trying them.

MEMBERS CAN HELP AUGMENT THE DUODECIMAL BIBLIOGRAPHY

by Lewis Carl Seelbach
Chairman, Committee on Bibliography

The initial stage of collection of the records of duodecimal works is about completed. The indexes and files of the larger libraries have been sieved. The complete bibliography in its present condition as an approximately complete record will be published in an early edition of the Bulletin.

Further discoveries will depend on a much wider search. Each member, who may have the inclination and the time, is urged to explore the local libraries within his reach. Here and there, about the country, there are special libraries devoted to some particular purpose. Do not neglect these special collections.

Perhaps some suggestions as to how to go about the search may prove useful. First determine what system of classification the library uses, and review the listings of the class that will include the duodecimal works. Make up a set of cards, showing the title and author of those works that appear interesting. The shelves of many libraries are open to the public. Where otherwise, consult the librarian, and request permission for access to the shelves containing these works for personal inspection.

As a rule, the material desired for the Duodecimal Bibliography consists of the title, the author, place of publication, name of publisher, the year of publication or copyright, brief comment on the nature of the work, and the pages where the duodecimal material appears.

We are interested in all duodecimal references, favorable and unfavorable. It is quite possible that references will be found in works that are not directly concerned with this subject. We want to list such references. Do not let the apparent triviality of the reference deter you. While duplication is to be avoided, we will do the necessary screening for this purpose. Send us what you find.

The Library of Congress has co-operatively furnished the following notes on the various classification systems you may encounter.

Library of Congress class number QA141 stands for systems of numeration.

Dewey Decimal Classification number 511.2 includes notation, numeration, and other related topics.

James Duff Brown's Subject Classification uses A402 in the same way the Library of Congress uses QA141.

Henry E. Bliss' Classification uses for duodecimal notation, the special symbol ANE.

Cutter Classification uses LCA comprehensively for number concept, cumbers, and numbering.

Dictionary catalogs using Library of Congress subject headings, will have a heading of "Duodecimal System," for books dealing specifically with this subject.

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SHAKE HANDS WITH OCTAL

There are now available Octal calculating machines giving results of addition, subtraction, multiplication and division on base VIII on which twice five is 12, three times seven is 25. This is a useful and constructive development. Machines may be rented and it is hoped that the opportunity will be made use of to do more computation on this base in order to explore its strengths and weaknesses. Any base is the ideal base for certain operations and it is only by use that this can be evaluated.

The purpose of the machine, however, as set forth in the circular, is stated to be for easy conversion of whole numbers from Base VIII to Base II (Binary) for feeding Binary results into the large electronic computers.

A single integer (Base VIII) is represented exactly by three integers on Base II. The only conversion table required for immediate conversion of numbers of any size is the following:–

TABLE I

Octal	Binary
0	000
1	001
2	010
3	011
4	100
5	101
6	110
7	111

Thus 4365 Octal = 110 011 110 101 Binary. Similarly for Octal π 3.1104 Octal = 11.001 001 000 100 Binary.

It may be questioned if use of the machine for conversion from either Base X or Base XII to Octal is not more liable to error than use of a simple conversion table to Octal. The machine's importance, however, is in the increased interest in conversion base to base and, as indicated above, in the opportunity for actual Octal computation.

Conversion Base XII to Binary may be accomplished (1) by division by 2 and listing remainders (2) by use of a conversion table direct from Base XII to Binary which would be unwieldy. (3) By use of Table II given below converting duodecimal to Octal and thence to Binary using Table I. This last is the easiest method.

DUODECIMAL	Units	10 etc	100 etc	1 000 etc	10 000 etc
1	1	14	220	3300	50400
2	2	30	440	6600	121000
3	3	44	660	12100	171400
4	4	60	1100	15400	242000
5	5	74	1320	20700	312400
6	6	110	1540	24200	363000
7	7	124	1760	27500	433400
8	10	140	2200	33000	504000
9	11	154	2420	36300	554400
X	12	170	2640	41600	625000
Ɛ	13	204	3060	45100	675400

OCTAL

THE MAN WITH TWELVE FINGERS

by F. H. Ames, Jr.

The title of this paper was not selected because of its humorous possibilities – it presents a question that has captured the fancy of numerous people: Would a numerical system using a base of twelve be superior to our present decimal system? Would we be better off now, and more advanced in our sciences, if ancient man had been possessed of twelve fingers – not ten?

There is no valid record of man's first attempt to count but it is logical to assume that such a system evolved in an attempt to count personal possessions. Since normal man possessed 10 fingers (and toes) a system finally evolved using 10 as a base; this of course has been true only a comparatively short period of time. Some ancient tribes counted by pairs (as do our new modern mechanical brains), a Brazilian tribe counted on their knuckles thereby creating a system based on 3, an Indian tribe used the sacred quarters of the sky. The Babylonians used a system with 60 as a base; the Malayans developed a numerical system based upon 20 and used it to make extremely complex astronomical calculations.

These systems were developed because of various physiological reasons, not because the system was particularly adapted to mathematical calculations. It was not until the invention and acceptance of a symbol for zero and the principle of position that mankind possessed a numerical system which could be readily used for complex mathematical calculations. The acceptance of our present Arabic number system took place over a period of nearly five centuries. About the time Jamestown was settled the last important addition was made; this was the invention of the decimal point.

It may come as a surprise to find that our present system, as we know it, has been in use less than half as long as the period in which the usage of Roman Numerals dominated civilization. Imagine having to compute percentage problems – such as taxes – using the laborious Roman system which represented simple quantities only.

Our present number system is far from perfect, its achievements are based primarily upon its unanimous acceptance by the population of the world. Religion, government, art, economics, language and morals may vary from one portion of the globe to the next, but each person uses the same system of counting. The usefulness of our present system lies largely in the usage of

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the symbol zero. This same simplicity will be preserved in any number system that utilizes zero.

Let us consider the present system, let us try to ascertain if our worship of tradition is holding us back. The most common usage of numbers is for the purpose of measurement. How many items can you think of that are sold by tens; how many that are sold by twelves? Time is divided into twelve months or four seasons of three months each. Admittedly, the present division of months into 28 to 31 days is quite awkward. However, the day consists of two divisions of twelve hours; the minutes result from combining a system of 12 and 10, the lowest common denominator being 60. On our compass we encounter another compromise – there are four cardinal directions and we bow to the magic number of 10 from that point on. Incidentally, the term "grocer" comes from the same root as "gross" – meaning a man who deals by the gross. Perhaps the tradesman is a better mathematician than the scholar.

Any number system must fulfill a few basic requirements. It must be able to represent exactly any conceivable quantity. It should have a small number of different symbols; it should be capable of expressing large quantities with relatively few figures; it must be able to express fractional quantities and relations between quantities in an efficient manner. It must permit simplified mathematical processes; it should be adapted to as many laws of natural phenomena as possible, so that computations involving them may be facilitated.

An examination of the table "Factors of Numbers" reveals that six is the first number with two factors; twelve is the first number having four factors. Further study of the table reveals that the lowest number having a maximum of factors is always a multiple of twelve. At first glance, a number system based on 60 may appear favorable as it has ten factors. After visualizing the necessity for memorizing a multiplication table of 60 I believe all will agree that twelve is a more practical base. On the other hand a smaller base would require longer figures for the expression of comparatively small quantities. A multiplication table, base of 12 is attached. Examine it and notice the recurrence of various numbers, 12, 24, 36, 6, 8, 10, 18, 20, 30, 40, etc.

Let us now "invent" a "new" system (a duodecimal system) using 12 as a base. First, let us learn to count by twelves. The Roman Numeral *X*, pronounced "dek", will be used for our old ten; the symbol Ɛ called "el" for eleven, and *10* will be used for old twelve – this is "do" (pronounced "dough") to remind us that it represents a dozen.

COUNTING BY TWELVES

1	one	*11*	do-one	*99*	ninedo-nine
2	two	*12*	do-two	*9X*	ninedo-dek
3	three	*13*	do-three	*9Ɛ*	ninedo-el
4	four	*14*	do-four	*X0*	dekdo
5	five	*15*	do-five	*X1*	dekdo-one
6	six	*16*	do-six	*X2*	dekdo-two
7	seven	*17*	do-seven		
8	eight	*18*	do-eight	*XƐ*	dekdo-el
9	nine	*19*	do-nine	*Ɛ0*	eldo
X	dek	*1X*	do-dek		
Ɛ	el	*1Ɛ*	do-el	*ƐƐ*	eldo-el
10	do	*20*	twodo	*100*	gro,(for gross)

In our new system, *10* represents one dozen plus zero units. A very simple system can be used to convert any number to a base 10 number. Use 2346 as an example in a system of base five. To convert this figure to a system based on ten, we translate the figure as follows:

$$\begin{aligned} 2346 = \quad & 6 \times (5)^0 = 6 \\ \text{plus } & 4 \times (5)^1 = 20 \\ \text{plus } & 3 \times (5)^2 = 75 \\ \text{plus } & 2 \times (5)^3 = \underline{250} \\ & 351 \text{ in 10 system} \end{aligned}$$

The new system of counting seems more difficult simply because it is new. However, those with small children probably realize that teaching this system would be no more difficult than encountered at present if the basic principles are studied. In our new system, *12* means one dozen plus two units or the quantity we now express as 14. Any new number can be easily understood if t this basic principle is remembered. Remember that the new number *123* means one gross – plus two dozen – plus three units or 171 in our present system.

An examination of the multiplication table shows that three digit numbers are not necessary until we reach (eldo-el) *ƐƐ* – number 143 in our present system. A multiplication table obtained by utilizing the first 12 quantities will have 144 totals in each case. In the system of base 10, eleven of these totals are in three figures; in a system based on 12, only one of these totals is in three figures.

Let us carry this examination further – in our present number systems there are a few helpful factoring "facts":

1) All even numbers are divisible by 2.
2) All numbers ending in 0 are divisible by 10.

3) All numbers ending in 0 are divisible by 5.
4) All numbers ending in 5 are divisible by 5.

In a duodecimal system our "facts" are more numerous:
1) All even numbers are divisible by 2.
2) All numbers ending in 0 are divisible by *10*.
3) All numbers ending in *0* are divisible by *6*.
4) All numbers ending in *0* are divisible by *4*.
5) All numbers ending in *0* are divisible by *3*.
6) All numbers ending in *9* are divisible by *3*.
7) All numbers ending in *8* are divisible by *4*.
8) All numbers ending in *6* are divisible by *6*.
9) All numbers ending in *6* are divisible by *3*.
X) All numbers ending in *4* are divisible by *4*.
Ɛ) All numbers ending in *3* are divisible by *3*.

Sample problems in changing to the duodecimal system are given below:

Change 1492 into the duodecimal system.

```
12 | 1492
 12 | 124 + 4
  12 | 10 + 4
        0 + X
```

Answer *X44*

Change 38,686,575 into the duodecimal system.

```
12 | 38,686,575
 12 | 3,223,881 + 3
  12 |   268,656 + 9
   12 |   22,388 + 0
    12 |   1,865 + 8
     12 |    155 + 5
      12 |    12 + Ɛ
               1 + 0
```

Answer *10,Ɛ58,093*

Examples of the basic processes are given below – refer to the addition and subtraction table if necessary.

I. Addition – remember that the column must be added to "do" before carrying the 1.

```
Add
  315
  716
  410
 ----
 123Ɛ
```

II. Subtraction – remember that when 1 is borrowed that it represents 12 units instead of 10.

```
Subtract
 1,Ɛ62,50X
   Ɛ70,9Ɛ6
 ---------
   ƐƐ1,714
```


III. Multiplication

```
      X89
      3Ɛ7
   ------
    6,313
   9,X03
  2,823
 --------
  366,643
```

IV. Division

```
          475
  36 / 141Ɛ6
       120
        21Ɛ
        206
         156
         156
```

V. Decimals and duodecimals for common fractions –

one	1.	*1.*
one-half	.5	*.6*
one-third	.3333 · · ·	*.4*
one-fourth	.25	*.3*
one-fifth	.2	*.249724* · · ·
one-sixth	.16666 · · ·	*.2*
one-seventh	.142857 · · ·	.186X35 · · ·
one-eighth	.125	*.16*
one-ninth	.1111 ·	*.14*
one-tenth	.1	*.124972* · · ·
one-eleventh	.0909 · · · ·	*.1111* · · · ·
one-twelfth	.08333 · · ·	*.1*

If this short discussion has aroused your interest and curiosity in a new number system then the time has been well spent. The subject is fascinating and offers many possibilities to those interested in playing with numbers. More comprehensive information on the duodecimal system will be found in "New Numbers," by F. Emerson Andrews, and "The Dozen System," by George S. Terry.