COUNTING IN DOZENS

1	2	3	4	5	6	7	8	9	X	Ɛ	10
one	two	three	four	five	six	seven	eight	nine	dek	el	do

Our common number system is decimal - based on ten. The dozen system uses twelve as the base, which is written *10*, and is called *do*, for dozen. The quantity *one gross* is written *100*, and is called *gro*. *1000* is called *mo*, representing the meg-gross, or great-gross.

In our customary counting, the places in our numbers represent successive powers of ten; that is, in 365, the 5 applies to units, the 6 applies to tens, and the 3 applies to tens-of-tens, or hundreds. Place value is even more important in dozenal counting. For example, *265* represents *5* units, *6* dozen, and *2* dozen-dozen, or gross. This number would be called *2 gro 6 do 5*, and by a coincidence, represents the same quantity normally expressed as 365.

Place value is the whole key to dozenal arithmetic. Observe the following additions, remembering that we add up to a dozen before carrying one.

94	136	Five ft. nine in.	5.9'
31	694	Three ft. two in.	3.2'
96	3Ɛ2	Two ft. eight in.	2.8'
19Ɛ	1000	Eleven ft. seven in.	Ɛ.7'

You will not have to learn the dozenal multiplication tables since you already know the 12-times table. Mentally convert the quantities into dozens, and set them down. For example, 7 times 9 is 63, *which is* 5 dozen and 3; so set down *53*. Using this "*which is*" step, you will be able to multiply and divide dozenal numbers without referring to the dozenal multiplication table.

Conversion of small quantities is obvious. By simple inspection, if you are 35 years old, dozenally you are only *2Ɛ*, which is two dozen and eleven. For larger numbers, keep dividing by 12, and the successive remainders are the desired dozenal numbers.

12) 365
12) 30 + *5*
12) 2 + *6*
0 + *2* Answer: *265*

Dozenal numbers may be converted to decimal numbers by setting down the units figure, adding to it 12 times the second figure, plus 12^2 (or 144) times the third figure, plus 12^3 (or 1728) times the fourth figure, and so on as far as needed. Or, to use a method corresponding to the illustration, keep dividing by *X*, and the successive remainders are the desired decimal number.

Fractions may be similarly converted by using successive multiplications, instead of divisions, by 12 or *X*.

Numerical Progression

1	One		
10	Do	*.1*	Edo
100	Gro	*.01*	Egro
1,000	Mo	*.001*	Emo
10,000	Do-mo	*.000,1*	Edo-mo
100,000	Gro-mo	*.000,01*	Egro-mo
1,000,000	Bi-mo	*.000,001*	Ebi-mo
1,000,000,000	Tri-mo	and so on.	

Multiplication Table

1	2	3	4	5	6	7	8	9	X	Ɛ
2	4	6	8	X	10	12	14	16	18	1X
3	6	9	10	13	16	19	20	23	26	29
4	8	10	14	18	20	24	28	30	34	38
5	X	13	18	21	26	2Ɛ	34	39	42	47
6	10	16	20	26	30	36	40	46	50	56
7	12	19	24	2Ɛ	36	41	48	53	5X	65
8	14	20	28	34	40	48	54	60	68	74
9	16	23	30	39	46	53	60	69	76	83
X	18	26	34	42	50	5X	68	76	84	92
Ɛ	1X	29	38	47	56	65	74	83	92	X1

The Duodecimal Bulletin

Volume 6 - No. 2
August - 1950

THE DUODECIMAL SOCIETY OF AMERICA
20 Carlton Place ~ ~ ~ ~ ~ Staten Island 4, N. Y.

THE DUODECIMAL SOCIETY OF AMERICA

is a voluntary nonprofit organization for the conduct of research and education of the public in the use of Base Twelve in numeration, mathematics, weights and measures, and other branches of pure and applied science.

Full membership with voting privileges requires the passing of elementary tests in the performance of twelve-base arithmetic. The lessons and examinations are free to those whose entrance applications are accepted. Remittance of $6, covering initiation fee ($3) and one year's dues ($3), must accompany applications.

The Duodecimal Bulletin is the official publication of the Duodecimal Society of America, Inc., 20 Carlton Place, Staten Island 4, New York. F. Emerson Andrews, Chairman of the Board of Directors. H. C. Robert Jr., President. George S. Terry, Editor. Copyrighted 1950 by the Duodecimal Society of America, Inc. Permission for reproduction is granted upon application. Separate subscriptions $2.00 a year, 50¢ a copy.

CONTENTS

WHY CHANGE?

This same question was probably rife in Europe between the years 1000 and 1500, when the new Arabic numerals were slowly making their inching progress in displacing the comfortable and familiar Roman numbers universally used. *"Why even try to learn to use these heathenish scrawly symbols, with their stubborn propensity for error, in place of the beautiful clear numbers which our fathers have used for untold generations? Think of the needless waste. We would have to change all of our counting boards and abaci. X is X, isn't it? And why do we need a symbol for nothing? You can't count it! No! Let us keep to our simple tried and true numerals, and let the barbarians scratch their heads, and rub themselves out. It will all come to 0 anyhow."*

Yet, although it took D years, the new notation became generally used, and man's thinking leapt forward like an arrow sped from a bow. The early years of the Renaissance marked a new stage in the use of symbols, with the advent of algebra, fractionals (decimals?), logarithms, analytical geometry, and the calculus. Can you imagine what it would be like to try to express the coordinates of points on a curve in Roman notation?

Mathematicians became conscious of a new dimension in symbolism, and the fundamental concepts of numbers were re-examined. Man awoke to the fact that different number bases could be used, and Simon Stevin stated in 1585 that the duodecimal base was to be preferred to the decimal.

The new Arabic notation accomodated mathematical statement better, and facilitated ideation. All thinking accelerated when released from the drag of the cumbrous Roman notation.

The parallel seems tenable. The notation of the dozen base accomodates mathematical statement better, and facilitates ideation. It, too, is a step forward in numerical symbolism. The factorable base embodies a concurrent analysis and definition of numbers that stimulates classification and generalization. Yet this is accomplished by such simple means that students in the primary grades can tell why they are better. Literally, the decimal base is unsatis-factory because it has "not-enough-factors."

Then shouldn't we change? No! No change should be made, and we urge no change. All the world uses decimals. But people of understanding should learn to use duodecimals to facilitate their thinking, and to ease the valuative processes of their minds. Duodecimals should be man's second mathematical language. They should be taught in all the schools. In any operation, that base should be used which is most advantageous, and best suited to the work involved. We expect that duodecimals will progressively earn their way into general popularity. But no change should be made. Perhaps by the year 2000, or maybe by *1200*, which is *14* years later, duodecimals may be the more popular base. But then no change need be made, because people will already be using the better base.

When one is familiar with duodecimals, a number of accessory advantages become apparent. Percentage is a very useful tool, but many percentages come out in awkward figures because of the inflexibility of decimals. When based on the gross, twice as many ratios come out in even figures, and among them are some of those most used, as thirds, sixths, and twelfths, - eighths and sixteenths. There are advantages associated with time and the calendar. Monthly interest rates or charges are derived from annual rates, or the reverse, by simply moving the unit (decimal?) point. The price of a single item bears the same relation to the price of the dozen, and so does the inch to the foot.

The proper correlation of weights and measures has always been one of the world's serious problems. None of the present systems is completely satisfactory. The American and English standards are convenient to use since they are the final result of a long process of practical evolution in which many inconvenient measures have been adjusted or abandoned. The French decimal metric measures have the advantage of being set upon the same base as the number system, and are well systemized. But many of the units are awkward because of their arbitrary sizes, and because their decimal scale does not accomodate division into thirds and fourths readily.

The duodecimal system of weights and measures, based on the inch and yard, the pint and the pound, has the desirable elements of both systems, and few of their faults. This Do-Metric System retains the familiar units of the American and British standards in approximately their present size, and arranges them into an ordered metric system using the scale of twelve. This fits perfectly into the duodecimal notation, and the combination accomodates the inclusion of the units of time and of angular measure within the system, which hitherto has not been possible.

If "playing with numbers" has sometimes fascinated you, if the idea of experimenting with a new number base seems intriguing, if you think you might like to be one of the adventurers along new trails in a science which some have thought staid and established, and without new trails, then whether you are a professor of mathematics of international reputation, or merely an interested pedestrian who can add and subtract, multiply and divide, your membership in the Society may prove mutually profitable, and is cordially invited.

The Duodecimal Bulletin

All figures in italics are duodecimal.

THE REVERSE NOTATION

by J. Halcro Johnston

That the Arabic Notation is not perfect is well known to readers of The Duodecimal Bulletin: the absence of the common factors *3*, *4*, and *6* from the base on which it is founded is the most obvious of its shortcomings. Had twelve been adopted in place of ten for that base the necessity for either The Bulletin or the Society that it represents could hardly have arisen.

That the notation fails in another respect is, however, not so well known and it is to this other shortcoming that I wish to draw attention. The Arabic Notation is a one-way system and can best be pictured as a one-way street. This is due to the fact that all the digits used - *1* to *9* - operate in the same direction: if *4*, for instance, represents *4* paces to the east, *3* will represent *3* paces in the same direction and there is no digit to represent *3* paces to the west. We are, of course, quite free to label the street for traffic in either direction, east-going or west-going, but having once decided on the direction it is a troublesome operation to change it, equivalent to having another street for traffic in the opposite direction; in the language of arithmetic we introduce negative numbers and negative numbers are well known to be troublesome.

The reader may remark here that negative numbers are not often required in every-day life. This, however, is not quite correct: negative numbers are used to a large extent but are not usually labelled as such. Subtraction indicates that they are present and subtraction is a common operation. In book-keeping two sides of the folio or two columns are used, one for the debits and the other for the credits; if the latter are represented by positive numbers, the former should be represented by negative ones, and vice versa. To find the balance of an account the positive and negative entries have to be kept separate, totalled separately, and the totals then subtracted - a lengthly and cumbersome operation due entirely to the defective notation used. The surveyor engaged in levelling to find the heights above sea level of different points on the Earth's surface meets with the same difficulty: separate columns in his level-book have to be kept for positive and negative readings and differences.

There is another disadvantage in the use of digits all of the same sign: half the number used, 6 to 9 in the decimal notation and *6* to *ℰ* in the duodecimal, are unnecessarily large and cumbersome and the resulting notation is heavy and unbalanced and does not lend itself to approximate methods of arithmetic. This is illustrated in the use of 3.55 for 5 minutes to 4 o'clock and 999 for 1 less than 1000.

The mistakes of the Arabic Notation are all eliminated in the Reverse Notation which is based on twelve and has equal numbers of positive and negative digits. No digit larger than *6* is used, and subtraction is no longer a separate operation: it is replaced by addition. Only one column or one side of the folio is needed in book-keeping, the sum of a number of debits and credits is their algebraic sum and gives the balance directly. As there is no digit greater than *6* approximate results can be got simply by replacing the unwanted digits on the right of the number by *0*'s. Numbers of both signs are treated on exactly the same footing and, once the pupil has learned the rules of addition and the new multiplication table, the old difficulties associated with negative numbers may be forgotten.

The digits used are:

$\bar{6}$ called	rix
$\bar{5}$	riv
$\bar{4}$	ror
$\bar{3}$	re
$\bar{2}$	ru
$\bar{1}$	ron
0 to *6* as at present	

Arranged symmetrically about *0* the first five dozen numbers with their names are:

$\bar{2}\bar{5}$	ru-riv	$\bar{1}\bar{5}$	ron-riv	$\bar{5}$	riv	$1\bar{5}$	seven	$2\bar{5}$	two-riv
$\bar{2}\bar{4}$	ru-ror	$\bar{1}\bar{4}$	ron-ror	$\bar{4}$	ror	$1\bar{4}$	eight	$2\bar{4}$	two-ror
$\bar{2}\bar{3}$	ru-re	$\bar{1}\bar{3}$	ron-re	$\bar{3}$	re	$1\bar{3}$	nine	$2\bar{3}$	two-re
$\bar{2}\bar{2}$	ru-ru	$\bar{1}\bar{2}$	ron-ru	$\bar{2}$	ru	$1\bar{2}$	ten	$2\bar{2}$	two-ru
$\bar{2}\bar{1}$	ru-ron	$\bar{1}\bar{1}$	ron-ron	$\bar{1}$	ron	$1\bar{1}$	eleven	$2\bar{1}$	two-ron
$\bar{2}0$	ru dozen	$\bar{1}0$	ron dozen	0	nought	10	dozen	20	two-dozen
$\bar{2}1$	ru-one	$\bar{1}1$	ron-one	1	one	11	one-one	21	two-one
$\bar{2}2$	ru-two	$\bar{1}2$	ron-two	2	two	12	one-two	22	two-two
$\bar{2}3$	ru-three	$\bar{1}3$	ron-three	3	three	13	one-three	23	two-three
$\bar{2}4$	ru-four	$\bar{1}4$	ron-four	4	four	14	one-four	24	two-four
$\bar{2}5$	ru-five	$\bar{1}5$	ron-five	5	five	15	one-five	25	two-five
$\bar{2}6$	ru-six	$\bar{1}6$	ron-six	6	six	16	one-six	26	two-six

The numbers from *1* to *100* are given in Table I. Note that *6* can be replaced by $\bar{6}$ and vice versa provided *1* or $\bar{1}$ is added to the digit on the left; *6* and $1\bar{6}$ represent the same number, the

former being used if followed by any of the digits $\bar{1}$ to $\bar{6}$ and the latter if followed by any of the digits *1* to *6*.

The symbols adopted to represent the new digits will be recognized as those already in use for the characteristics of the logarithms of fractions. Each consists of two parts, a digit and a bar. A single new symbol would have been preferable but it will be appreciated that considerations of typing and printing limit our choice in the matter. For manuscript use I suggest the following:

ADDITION Just as we had to learn at school that *2* and $\bar{2}$ make *4*, so also is it necessary to memorize the sums of every pair of digits, thus:

$\bar{5}$	$\bar{2}$	$\bar{4}$	3	$\bar{5}$
4	6	4	6	$\bar{3}$
$\bar{1}$	4	0	$1\bar{3}$	$\bar{1}4$

Having learned to sum the digits in pairs sums of any size should present no difficulty:

EXAMPLE

$$
\begin{array}{r}
43\ 02\bar{1} \\
4\ \bar{4}0\bar{4} \\
1\bar{5}\bar{4}\ 403 \\
\bar{1}\ \bar{4}\bar{3}3 \\
\bar{3}4\bar{2} \\
1\bar{3}\ 532 \\
\bar{1}\bar{2}\ 060 \\
1\ \bar{1}\bar{5}3 \\
1\ \bar{1}15 \\
10\ \bar{5}\bar{1}5 \\
\bar{5}\bar{4}\bar{5} \\
\hline
10\bar{2}\ \bar{2}4\bar{3}
\end{array}
$$

The total of the units is $1\bar{3}$ and *1* is carried forward to the dozens: the total of the third column from the left is $\bar{1}\bar{2}$ and $\bar{1}$ is carried forward: the totals of all the other columns are merely the digits shown below them. Note that the carry-forward is usually either *0* or a small number and that groups of digits often cancel out. If the sum to the nearest *1,000* had been all that was required it could have been got though the two columns on the right were omitted.

SUBTRACTION Subtraction is replaced by addition: the number to be subtracted is replaced by one of the same magnitude but opposite sign which can then be added in the usual way. To find the opposite number the digits are reversed, thus: to subtract $14\bar{3}.6$ add $\bar{1}\bar{4}3.\bar{6}$, to subtract $\bar{2}2\bar{5}$ add $2\bar{2}5$.

MULTIPLICATION Unfortunately multiplication calls for the memorizing of a new multiplication table - Table II. But it is not a difficult table to memorize: a cursory examination will reveal a remarkable degree of symmetry and order: in the $1\bar{4}4$ products the number of *0*'s is *24* compared with *15* in the duodecimal table and *8* in the corresponding *54* products of the decimal table. Having learned the multiplication table we proceed as in ordinary arithmetic.

EXAMPLE.- Multiply $1,\bar{5}32$ by $2,1\bar{1}3$.

$$
\begin{array}{r}
1\bar{5}32 \\
\underline{21\bar{1}3} \\
2\bar{2}\bar{3}6 \\
\bar{1}5\bar{3}\bar{2} \\
1\bar{5}32 \\
\underline{1264} \\
1312\bar{3}\bar{5}6
\end{array}
$$

If the result to three significant figures was all that was required we should proceed as follows:

$$
\begin{array}{r}
1\bar{5}32 \\
\underline{21\bar{1}3} \\
1264 \\
1\bar{5}3 \\
\bar{1}5 \\
\underline{3} \\
1313
\end{array}
$$

which gives the approximate product *1,310,000*.

DIVISION As in ordinary arithmetic division is a matter of trial and error. We try different divisors and select that that gives the least remainder. But whereas in ordinary arithmetic the remainder must be the least positive one and may be any number less than the divisor, in the Reverse Notation it may be positive or negative but must be less than half the divisor.

Example.- Divide $2\bar{2},3\bar{3}1$ by *3*.

This will be worked out in long division to show all the steps:

$$
\begin{array}{l}
3)\,2\bar{2}3\bar{3}1\,(1\bar{5}3\bar{1}0 \\
\quad\ \underline{\bar{3}} \\
\quad\ \bar{1}\bar{2} \\
\quad\ \underline{13} \\
\qquad 1\bar{3} \\
\qquad \underline{\bar{1}3} \\
\qquad\ \ \bar{3} \\
\qquad\ \ \underline{3} \\
\qquad\quad 1
\end{array}
$$

Example(ii) Divide $1\bar{3}14\bar{1}$ by $512.\bar{2}$ and obtain the result correct to one duodecimal place:

$$
\begin{array}{r}
512.\bar{2}\ )\,1\bar{3}14\bar{1}\,(\ 2\bar{3}.56 \\
\underline{\bar{1}2\bar{2}\bar{4}4} \\
\bar{1}\bar{1}03 \\
\underline{1336} \\
24\bar{3} \\
\underline{\bar{2}\bar{1}\bar{5}} \\
24 \\
\underline{\bar{2}\bar{6}} \\
\bar{2}
\end{array}
$$

The required answer is $2\bar{3}.5$

FRACTIONS If p is a prime number, $2n + 1$, by Fermat's Theorem it will be a factor of either $10^n + 1$ or $10^n - 1$ and when $\frac{1}{p}$ expressed as a recurring duodecimal it will belong to one or other of two corresponding types. If p is a factor of $10^n + 1$ it can readily be shown that its period will alternate in sign, thus:

$$\frac{1}{7} = 0.2\bar{3}\bar{5},\bar{2}35,2\bar{3}\bar{5},\ldots..$$

Factors of $10^n - 1$, on the other hand, may or may not alternate, thus:

$$\frac{1}{1\bar{1}} = 0.111\ldots.$$

which does not alternate (where n is prime I have not so far found any factor of $10n - 1$ that alternates)

Expressed as duodecimals the simple fractions are as follows. Alternating duodecimals are shown thus: (a)

$\frac{1}{2} = 0.6$	$\frac{1}{3} = 0.4$,	$\frac{1}{4} = 0.3$
$\frac{1}{5} = 0.\dot{2}\dot{5}..$ (a)	$\frac{1}{6} = 0.2$	$\frac{1}{7} = 0.\dot{2}\bar{3}\dot{\bar{5}}..$ (a)
$\frac{1}{8} = 0.16$	$\frac{1}{9} = 0.14$	$\frac{1}{1\bar{2}} = 0.1\dot{2}\dot{5}..$ (a)
$\frac{1}{1\bar{1}} = 0.\dot{1}\ldots$	$\frac{1}{10} = 0.1$	$\frac{2}{3} = 1.\bar{4}$
$\frac{3}{4} = 1.\bar{3}$	$\frac{5}{6} = 1.\bar{2}$	$\frac{3}{8} = 0.5\bar{6}$
$\frac{5}{8} = 1.\bar{5}6$		

LOGARITHMS Some of the advantages of the Reverse Notation are found in the use of reverse logs. The troublesome negative characteristic of the ordinary notation which always looks so lonely in a one-way system falls naturally into place in the homely surroundings of the two-way system, while complicated calculations involving multiplication and division are replaced by the single operation, addition.

EXAMPLE.- Find d where $d^2 = \dfrac{4 \times 600}{\pi \times 3.\bar{4} \times 6.3\bar{3}5}$

$$\begin{aligned} \log 600 &= 3.\bar{3}\bar{4}\bar{2}0 \\ \text{"} \quad 4 &= 1.\bar{5}\bar{4}41 \\ - \text{"} \quad \pi &= 0.\bar{5}6\bar{4}0 \\ - \text{"} \quad 3.\bar{4} &= 0.\bar{5}32\bar{1} \\ - \text{"} \quad 6.3\bar{3}5 &= \bar{1}.320\bar{5} \end{aligned}$$

$$2 \,\underline{|\, 2.\bar{4}30\bar{5}}$$

$$1.\bar{2}16\bar{2}$$

From the tables we find that this is the log. of $1\bar{4}.14$ which is the value of d required.

The logs used in the above example are from a brief table given in my book The Reverse Notation which was compiled with the help of the table from Mr. Andrews' New Numbers.

Like a new language the Reverse Notation may appear strange and forbidding on first acquaintance and, as in the learning of a new language, this aspect will only disappear after a period of constant use. Before even simple calculations can be made one must be able to add and multiply any pair of digits without stopping to think - $3+\bar{5}=\bar{2}$ or $4\times\bar{6}=\bar{2}0$ - Let us assume that we have acquired equal facility in the use of both notations, the old and the new, and let us see how they will compare. Addition should be quicker and easier in the two-way system due to the cancelling of groups of two or more digits which total 0, and the smaller numbers, if any, to carry forward. In bookkeeping and logarithms time would be saved due to the elimination of subtraction as a separate operation. Multiplication and division would benefit from the simpler multiplication table and the greater number of factors in base twelve than in base ten. On the other hand, apart from the loss of 5 as a base factor, I have not discovered any way in which we would lose by giving up the present system in favour of the Reverse Notation.

TABLE I

1	2	3	4	5	6	$1\bar{5}$	$1\bar{4}$	$1\bar{3}$	$1\bar{2}$	$1\bar{1}$	10
11	12	13	14	15	16	$2\bar{5}$	$2\bar{4}$	$2\bar{3}$	$2\bar{2}$	$2\bar{1}$	20
21	22	23	24	25	26	$3\bar{5}$	$3\bar{4}$	$3\bar{3}$	$3\bar{2}$	$3\bar{1}$	30
31	32	33	34	35	36	$4\bar{5}$	$4\bar{4}$	$4\bar{3}$	$4\bar{2}$	$4\bar{1}$	40
41	42	43	44	45	46	$5\bar{5}$	$5\bar{4}$	$5\bar{3}$	$5\bar{2}$	$5\bar{1}$	50
51	52	53	54	55	56	$6\bar{5}$	$6\bar{4}$	$6\bar{3}$	$6\bar{2}$	$6\bar{1}$	60
$1\bar{6}1$	$1\bar{6}2$	$1\bar{6}3$	$1\bar{6}4$	$1\bar{6}5$	$1\bar{6}6$	$1\bar{5}\bar{5}$	$1\bar{5}\bar{4}$	$1\bar{5}\bar{3}$	$1\bar{5}\bar{2}$	$1\bar{5}\bar{1}$	$1\bar{5}0$
$1\bar{5}1$	$1\bar{5}2$	$1\bar{5}3$	$1\bar{5}4$	$1\bar{5}5$	$1\bar{5}6$	$1\bar{4}\bar{5}$	$1\bar{4}\bar{4}$	$1\bar{4}\bar{3}$	$1\bar{4}\bar{2}$	$1\bar{4}\bar{1}$	$1\bar{4}0$
$1\bar{4}1$	$1\bar{4}2$	$1\bar{4}3$	$1\bar{4}4$	$1\bar{4}5$	$1\bar{4}6$	$1\bar{3}\bar{5}$	$1\bar{3}\bar{4}$	$1\bar{3}\bar{3}$	$1\bar{3}\bar{2}$	$1\bar{3}\bar{1}$	$1\bar{3}0$
$1\bar{3}1$	$1\bar{3}2$	$1\bar{3}3$	$1\bar{3}4$	$1\bar{3}5$	$1\bar{3}6$	$1\bar{2}\bar{5}$	$1\bar{2}\bar{4}$	$1\bar{2}\bar{3}$	$1\bar{2}\bar{2}$	$1\bar{2}\bar{1}$	$1\bar{2}0$
$1\bar{2}1$	$1\bar{2}2$	$1\bar{2}3$	$1\bar{2}4$	$1\bar{2}5$	$1\bar{2}6$	$1\bar{1}\bar{5}$	$1\bar{1}\bar{4}$	$1\bar{1}\bar{3}$	$1\bar{1}\bar{2}$	$1\bar{1}\bar{1}$	$1\bar{1}0$
$1\bar{1}1$	$1\bar{1}2$	$1\bar{1}3$	$1\bar{1}4$	$1\bar{1}5$	$1\bar{1}6$	$10\bar{5}$	$10\bar{4}$	$10\bar{3}$	$10\bar{2}$	$10\bar{1}$	100

TABLE II

	$\bar{6}$	$\bar{5}$	$\bar{4}$	$\bar{3}$	$\bar{2}$	2	3	4	5	6
$\bar{6}$	30	26	20	16	10	$\bar{1}0$	$\bar{1}\bar{6}$	$\bar{2}0$	$\bar{2}\bar{6}$	$\bar{3}0$
$\bar{5}$	26	21	$2\bar{4}$	13	$1\bar{2}$	$\bar{1}2$	$\bar{1}\bar{3}$	$\bar{2}4$	$\bar{2}\bar{1}$	$\bar{2}\bar{6}$
$\bar{4}$	20	$2\bar{4}$	14	10	$1\bar{4}$	$\bar{1}4$	$\bar{1}0$	$\bar{1}\bar{4}$	$\bar{2}4$	$\bar{2}0$
$\bar{3}$	16	13	10	$1\bar{3}$	6	$\bar{1}6$	$\bar{1}3$	$\bar{1}0$	$\bar{1}\bar{3}$	$\bar{1}\bar{6}$
$\bar{2}$	10	$1\bar{2}$	$1\bar{4}$	6	4	$\bar{4}$	$\bar{6}$	$\bar{1}4$	$\bar{1}2$	$\bar{1}0$
2	$\bar{1}0$	$\bar{1}2$	$\bar{1}4$	$\bar{6}$	$\bar{4}$	4	6	$1\bar{4}$	$1\bar{2}$	10
3	$\bar{1}\bar{6}$	$\bar{1}\bar{3}$	$\bar{1}0$	$\bar{1}3$	$\bar{1}6$	6	$1\bar{3}$	10	13	16
4	$\bar{2}0$	$\bar{2}4$	$\bar{1}\bar{4}$	$\bar{1}0$	$\bar{1}4$	$1\bar{4}$	10	14	$2\bar{4}$	20
5	$\bar{2}\bar{6}$	$\bar{2}\bar{1}$	$\bar{2}4$	$\bar{1}\bar{3}$	$\bar{1}2$	$1\bar{2}$	13	$2\bar{4}$	21	26
6	$\bar{3}0$	$\bar{2}\bar{6}$	$\bar{2}0$	$\bar{1}\bar{6}$	$\bar{1}0$	10	16	20	26	30

ONE PLUS ONE

by George S. Murphy

It was most disappointing to the writer to have missed Dr. Nathan Lazar's discourse on counting techniques through the ages, as presented at the January meeting of the Society, but judging from its title and from the speakers reputation it must have been most thought-provoking. Several related notions from various sources are suggested as an interesting complement to his remarks.

The chances are that many readers of this article have been introduced to the famous crow that would not stay around to be shot, but flew from his favorite perch whenever the farmer went to the barn-loft with his rifle, and returned only after the man appeared again at the door and walked away. Even when two men went up and one only departed, he waited until he counted two going away. The same with three and with four. But when five mounted to the loft and four went away - that was too much. The farmer ate crow for supper.

Modern experimental psychology has come through with accurate verification of this phenomenon on the human level, indicating that there are three general processes of counting, with two distinctive psychoneural "mechanisms" for the visual determination of "numerousness". It has not as yet been determined what the nature of these two mechanisms are, but the evidence for the duality is reasonably conclusive to the student of the science of psychology.

In a test at Mt. Holyoke College on the "Discrimination of Visual Number", reported by Kauffman, Lord, Reese, and Volkmann, the American Journal of Psychology, Vol. LXII, No.4, Oct. 1949, pp. 498-, two groups of subjects, one instructed to attain speed, the other accuracy, were set to the task of determining quickly, but without counting, the number of round spots contained within areas of equal size and shape. With each determination the subject rated his own confidence in his accuracy. A number of environmental conditions such as light-intensity, color of spots and of background, etc., were also varied. Curves were fitted to plots of estimated stimulus number against time of response and against confidence of accuracy for each of the instructed groups. Despite environmental changes, invariably the curve, or rather its derivative or slope, was discontinuous at a point corresponding to a stimulus number about equal to 6, being practically a horizontal straight line of full confidence or of constant reaction time up to that point, but changing suddenly, with increasing value of stimulus number, to a sharp slope of decreasing confidence with a curve asymtotic to the line of zero confidence, or to a slope of increasing reaction time curved to level off and approach a maximum. This sharp discontinuity was presented as clear evidence of a radical change of mechanism in the process of determination of number, as between those below and those above the number six.

To identify this distinctively different mental process the experimenters coined a new word "subitize" (from the Latin, subitare: to arrive suddenly) for determinations of stimulus number less than the point of discontinuity, and retained the word "estimate" for those of greater number. It was found that under instructions to attain accuracy, six was "subitized", whereas under instructions to attain speed, six was "estimated". All stimulus numbers below or above six could of course be determined by the third method, that of counting, but at a cost, in time, that would be a function of the number.

The ancient epigram quoted in support of decimal numeration at the time duodecimals were seriously proposed for the metric system, to the effect that, "Man is the measure of all things", or the later idealistic fragment, "Beauty (et omnes alii) is in the eye of the beholder", may be transferred even to a crow and his eye. For the crow looks at his claws and sees four, the man looks at his hand or his foot and sees five. Is it simply the lifelong association with the appearance of the extremities that determines the limit of "subitizing", with the further easy recognition of "one-greater-than-ness" giving to man that extra notch? Or is it something more fundamental in the structure-function complex of the thinking mechanism?

The physiology of the nervous system, from the transmission of nervous stimuli to thought processes and memory, does not hint at the answer. For nerve impulses are transmitted across the synapses to the next length of nerve by a trigger arrangement. A limiting minimum number of stimulated nerve endings entering a given synapse are required to set off the next lap of the relay. Habit is the reduction of this limit to the point of developing a hair-trigger effect. But each segment of nerve between synapses, right through all the ramifications of the thinking, remembering (habituated) brain, has only a choice either of going off or of not going off, with no "phizzers". What could be simpler? This presents the absolute minimum of complication to the business of choosing between possibles numbering more than one. Anything less would mean no choice at all, simply pure determinism. So Nature, operating on its usual policy of least work, goes into mass production, in an offhand sort of way, of billions of think-machines, every one guaranteed to last a life-time.

The brain is like a computing machine operating with a number system based on two. The modern electronic computer is directly copied after this physiological method of sorting, filing and answering messages. Its number system has only the digits zero and one, instead of zero to nine as in the decimal system. The science that deals with the workings of these natural and artificial brains is termed by Norbert Wiener "Cybernetics", a word of Greek derivation whose relation to the Latin "Guvernor", as used on steam engines and automobiles, is fairly evident.

So we are left with habit alone, as defined and illustrated above, as the only presently revealed factor in the differentiation between "subitizing" and "estimating."

We are struck, however, with the fact that the Duodecimalist is reinforced in his grossness by the realization of his coming in under the wire, as it were, of subition, since the number six is right at the point of discontinuity of the curve of facility.

Even in manual sign language an open hand may represent five and a closed fist six. The Arabic numeral six may well be a pictogram of such a fist.

The next chapter in the Story of Number,- that concerned with the abacus,- shows a ready adaptability to the substitution of twelve (or do) for ten as the base of the system, within the subitizing capabilities of man. This may well have a bearing on the growth of the Duodecimal Society in Asiatic lands, though perhaps paper and mental arithmetic will carry the field even there. As Dr. Lazar developed the theme at the Annual Meeting, the original 10-bead wire of the abacus was modified to employ the use of two classes of beads, - one class being a unit group of that number of beads within the capability of being immediately apprehended without counting; that is to say, of being "subitized"; - and the other class being a group representing the number of such unit groups required to define all the digits in that column. This first modification of the abacus provided one class with five beads, and a second class with two beads, thus representing the ten digits in one column.

A second modification came with the discovery that removing one bead from each class, leaving four beads and one bead in each column, permitted all the digits from zero to nine to be just as readily represented in each column. This removal may be termed a decomposition of each class by one bead.

It was then apparent that the 2-5 abacus of the first modification of the decimal abacus could just as readily be used to count by dozens. By analogy with the parallel decomposition in the second modification of the decimal abacus, a reduction to a

1-5 arrangement is seen to be the proper construction for duodecimal counting.

To apply these modifications, by classes and by decomposition, to an abacus for the *60*-base numeration, ready "subitizing" would require three classes of beads, probably with a unit group of five, the next class representing four unit-groups, with three beads in the last class. This would triply decompose into four-three-two, or, expressed analogously to the previous examples, result in a *2-3-4* arrangement. Such an instrument would permit several variations of the arrangement of the counting process. One form would facilitate the ten and score counting, another the use of the dozen.

One shudders at the thought of attempting to "do sums" by mental arithmetic with this triple-class method of operation. Single-class arithmetic provides enough of a whirl between Time's stork and the Ides of March. But, for an abacus, thanks to "subitizing", it would be just a matter of practice, and, for a machine, just a few more wheels. This is quite a departure from the ridiculously simple *2*-base, *1*-class device we call brains, but the latter, such as they are, will continue to do business at the old stand, at least till superman confounds the robots. For:

BRAINS WE HAVE, YES!

Need of the hour, lest mankind be doomed,
Is the brain that can synthesize myriad factors,
Composing the whole of the social cosmos,
Sufficiently well to provide us some guidance.

No problem yet posed has been more of a stumbling-block
Set in the path of orderly progress
Than this search for the secret of social symbiosis-
Problem to faze the electronic computer.

But never shall one brain assuage the World's ills -
Biological unit or integral combine.
Co-operative effort put forth by the many
In small ways, shall stake out the true path of progress.

CYCLIC SEQUENCES

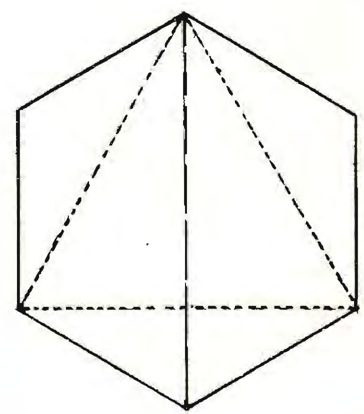
by George S. Terry

In any place system, a number requires for its interpretation the use of simple arithmetic processes applied both to the number base and to the integers involved. Thus on any base B, the number pqr = $pB^2 + qB + r$. We use continued multiplication (powers) and addition.

Continued use of a process usually produces a cyclic sequence; e.g., division of *1* by n to give reciprocals. Here the maximum length of the sequence, (n - *1*), is determined by n, and is independent of B. The length of the sequence for any specific base will be of maximum length, or some fraction thereof, - but just which it will be, it is less easy to foretell. The length of the sequence may be represented diagramatically in a polygon of n-*1* sides.

For example, - the maximum length of the reciprocal of *7* is *6*, and for any base the length of the reciprocal will be *6*, or some fraction of *6*. Thus, for Base XII and Base X the length is *6*, ($.\dot{1}86X3\dot{5}$ and $.\dot{1}4285\dot{7}$), for Base II it is *3*, ($.\dot{0}0\dot{1}$), for Base VI it is *2*, ($.\dot{0}\dot{5}$), and for Base VIII it is *1*, ($.\dot{1}$).

The length of these reciprocals may be represented as a hexagon for Bases XII and X, as an inscribed triangle for Base VI, as the diameter for Base II, and as a vertex or an integer for Base VIII.



In the Duodecimal Bulletin, Vol. *6*, No. *1*, p. *11*, sequences from continued use of powers and addition were examined. The rules governing the lengths of sequences are not clear. Note that, in the article quoted, the Robert sequences are the "reduces" of the Terry sequences. The "reduce" of any number is defined (Scripta Mathematica XIII, p. *175*) as the continued addition of the integers of a number until a single integer is obtained; e.g., (Base XII) *32Ɛ*→*14*→*5*. The reduce of *32Ɛ* is *5*. The above article dealt with repeated multiplication (powers) followed by addition following the process of interpretation of pqr.

To take a simpler case, consider the repeated process of single multiplication plus addition; that is: M(a), where M is a constant multiplier, and a is any number. Since we "reduce" the product, we need consider only single integers.

Base XII   *X(3)* = *26*→*8*   *X(8)* = *68*→*12*→*3*   *X(3)* = *26*→*8*
The sequence *(8,3)* being of length 2.

From this simpler case, certain rules appear. On any base, if M = B - *1*, we have a sequence of unit length. The reduce is B - *1*, and this integer cannot appear in any longer sequence. Therefore the length of any sequence on any base cannot exceed B - 2. That there must be a cyclic sequence (including zero repeated, as in the division case cited above,) is clear. On any base, if M = B - 2, the length of the sequence cannot exceed 2.

Full length sequences (B - 2,) can occur only where B - *1* is prime, and for any such base the length of the maximum sequence is B - 2 or a fraction thereof.

To examine the rules for maximum length of sequence for enough bases to permit positive definition has not been attempted because of the magnitude of the task, but certain rules seem valid as far as we have gone. These are to be regarded as tentative.

Where B - *1* = n^2, with n prime, the maximum length appears to be n(n - *1*), and with n composite, the length is n.

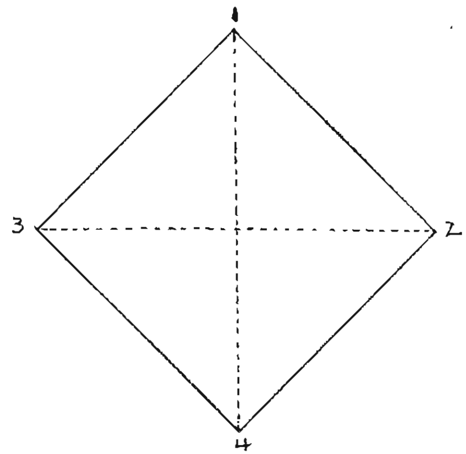
Where B - *1* is composite and odd, the maximum length seems to be one less than the greatest factor. When composite and even, most frequently the maximum length is 2, though there are cases where it again is one less than the greatest factor, as in the case of Base XXXV, where B - *1* = *2(15)*, and the maximum length is *14*, conforming to the formula B - x - y.

If the sequence length is even, the integers of the first half are complementary (to B - *1*,) to the corresponding integers of the second half. For odd bases, since B - *1* is even, it follows that any sequence is less than B - 2 in length.

Sequences with B - *1* Prime

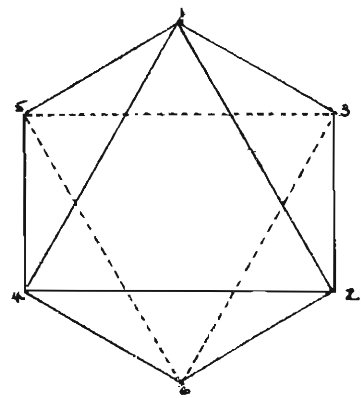
Base VI is a simple example, for with M equal to any integer less than B - 2, the sequence when:-

M = 2   is *1*, *2*, *4*, *3*
M = *3*   is *1*, *3*, *4*, *2*
M = B - 2 is *1*, *4*, or *3*, *5*.

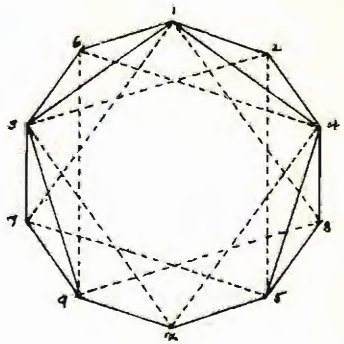


This may be represented geometrically by a square with the angles numbered from *1* to B - 2. For M = 2, go round clockwise, for M = *3* counter-clockwise, for M = B - 2, or *4*, go across a diagonal. Note that in any case no lesser polygon can be drawn.

Base VIII. The full sequence will be on a hexagon. To find the sequences, take M = 2. The sequence *1*, *2*, *4*, is on the triangle through *1*. Opposite to 2, and *4*, place their complements to B - *1*, namely *5*, and *3*, - and we have the order of the full sequences, and of the half sequences. Which values of M give the full length sequence? Those not on the lesser polygon through *1*, namely *3*, and *5*, (omitting *6*, which as B - 2 uses diameters). The order of the sequences is clockwise for M = *3*, and counter, for M = *5*.

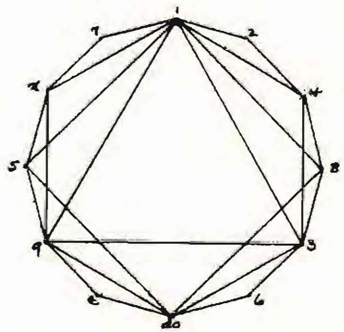


Base XII. The B - 2 sequence is on a decagon, the order given by M = 2 being *1, 2, 4, 8, 5, X, 9, 7, 3, 6*. The pentagon through *1* shows that if M = *3, 4, 5,* or *9,* we have the half length sequences. Since *2, 6, 7,* and *8,* are not on the pentagon, they yield full length sequences, - that for M = *6* being the reverse of that for *2,* and that for M = *7* being on the interior stellated decagon in the *1, 7,* order, while the M = *8* order is the reverse.



Any a of the *1, 3, 9, 5, 4,* sequence will reproduce that series with M = *3* and the same is true for the alternate sequence *2, 6, 7, X, 8.* For M = *4* these series are in reversed order.

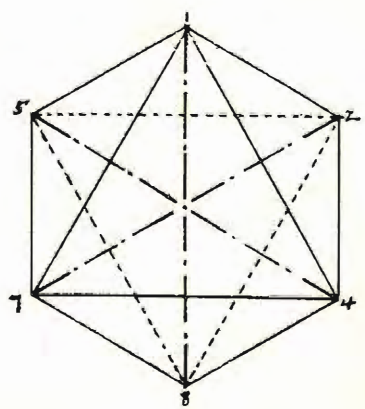
Base XIV. M = *2* gives a full length sequence. What others? Draw the hexagon, square and triangle through *1.* The omitted points of the dodecagon, *2, 7, Ɛ,* and *6,* give full length sequences. *4,* and *X,* give length *6; 3,* and *9,* being on the least polygon, give length *3.*



Sequence with B - 1 Square

Base X. B - 1 = n^2, with n prime. In this case we get unit sequences not only when B - 1 = M, but also for all multiples of n. So the maximum length of the sequence is 6, or (B - 1) - (n - 1). This resolves to B - n, and can also be represented as n(n - 1).

We get a full sequence when M = 2, namely 1, 2, 4, 8, 7, 5. As before, draw the hexagon and triangle through 1, and another dotted triangle starting with 2.



These show that 5 will also produce the full sequence, but in reverse order. When M = 4 or 7 we get a 3-sequence around the full triangle, with a = 1, 4, or 7 and similarly with a = 2, 5, or 8 for the other triangle. When M = 8, we find four sequences of length 2, including a 6, 3, sequence not illustrated in the figure. All these sequence pairs are complementary to B - 1.

The composite sets may be instanced by Base XVI, where B - *1* equals xy, both prime. x = *3,* and y = *5,* the maximum length of sequence being y - *1,* or *4.*

Note that some of the same sequences are obtained from the reduces of continual addition of a number and its reversal.

e.g.	Base VI	Base VIII		Base XII
	24→1	12→3	35→1	*39→1*
	42	21	53	*93*
	110→2	33→6	110→2	*110→2*
	011	33	011	*011*
	121→4	66→5	121→4	*121→4*
	121	66	121	*121*
	242→3	154→3	242→1	*242→8*
	242			*242*
	524→1			*484→5*
				etc.

Sequences by Subtraction (not reduced)

It is pointed out by Kaprekar, in Scripta Mathematica, No. XV, p. *245*, that on Base X, for any four integers, not all the same, arranged to form the greatest and the least number and then subtracted, - the operation being repeated with the four new integers, and so on, - we finally arrive at the number 6174, which continues to repeat. For instance:

4321 - 1234 = 3087, 8730 - 0378 = 8352, 8532 - 2358 = 6174

The same process on Base XII results in either a cycling triplet or sextet: *8284, 6376, 3ƐƐ8,* or *7ƐƐ4, 7375, 4198, 8374, 5287, 6196.*

ENIAC π AND e

Nothing could more strongly accent the pace of today's scientific progress than the recent report of the recalculation of π and e on the ENIAC at Aberdeen. The quarterly, "Mathematical Tables and Other Aids to Computation", published by the National Research Council, carried the report in the January 1950 issue, and a supplement in the April 1950 issue.

This was a project of Drs. N. C. Metropolis, of the Los Alamos Scientific Laboratory, G. Reitwiesner, of the Ballistic Research Laboratories, and J. von Neumann, of the Institute for Advanced Study. They secured permission to use ENIAC over several holiday week ends when, otherwise, the machine would have been idle.

After careful planning of procedures, e was calculated to 2010 decimal places over the July 4th, 1949, week-end, and later extended to 2500 decimal places. π was calculated over the Labor Day week-end to 2035 decimal places. These results may be found in the January and April issues of MTAC cited above.

Two items in the report are particularly noteworthy. The method used for the calculation of π is valid for about 7000 decimal places, - and a statistical survey of the resultant figures discovered significant indications of deviation from randomness in the figures for e, but none for π.

This amazing performance startles us into a degree of realization of the capacities of the new calculating machines, and into a sort of pained sympathy for the many mathematicians who have, through thousands of laborious hours, struggled for a more accurate determination of π since the days of Babylon.

The discovery of evidence of a departure from randomness in the figures for e is equally startling. We are advised that more detailed investigations are in progress. We cannot avoid the suggestion that these decimal figures should be transposed into duodecimals for this analysis, because of the known ability of the dozen base to expose the underlying patterns in numbers.

Drs. Metropolis, Reitwiesner, and von Neumann are to be congratulated for this outstanding achievement in precision of planning and performance.

DO-METRIC STANDARDS

To facilitate the work of the members of the society, and of its Committee on Weights and Measures, the definitions of the basic standards of the Do-Metric System will be stated from time to time, and revised as may become necessary.

The elaboration of the measures of the system is a continuing and sizable operation. Since it is essentially a collaborative undertaking, the co-operation of all interested members of the society is desired.

Linear Measure

The basic units of measure are the units of length, mass, and time, (l, m, t.) From these, in various combinations of their powers, are derived all measurement units.

The Do-Metric unit of length is the U.S. Standard Yard. This is officially defined in terms of the standard meter, as follows:

The primary standard of length in the United States is the United States Prototype Meter 27, a platinum-iridium line standard having an x-shaped cross section. The length of this bar which is deposited at the National Bureau of Standards in Washington, is known in terms of the International Prototype Meter, which is deposited at the International Bureau of Weights and Measures at Sevres, near Paris, France.

A supplementary definition of the meter in terms of the wave length of light was adopted provisionally by the Seventh General (International) Conference on Weights and Measures in 1927. According to this definition the relation for red cadmium light-waves (6438.4696A) under specified conditions of temperature, pressure, and humidity, is:

1 meter = 1 553 164.13 wave lengths
= *62X 9X4.169* wave lengths

1 yard = exactly $\frac{3600}{3937}$ meter
= .914 401 8 meter
= *.XƐ8 105 3* meter
= 1 420 216.12 wave lengths
= *585 X74.15* wave lengths

For industrial purposes a simpler relation between the yard and the meter (1 inch = 25.4 millimeters exactly,) has been adopted by the American Standards Association, (A.S.A. B48.1-1933,) and by similar organizations in 15 other countries.

The adoption of this industrial standard of conversion does not change the official definition of the yard, and the Do-Metric Yard is the U.S. Standard Yard, as previously defined.

Recently, a much finer measurement standard than the cadmium red line has been developed in the green line of the purified 198 isotope of mercury. Dr. William F. Meggers has measured this radiation with an accuracy of one part in 100 million, and announces its wave-length as 5460.752 A. The following constants are derived from this figure.

1 yard = 1 674 497.97 wave lengths
689 055.Ɛ8 wave lengths

5460.752 A = *1.949 4X* ultrau or cad.
6438.4696 A = *2.129 14* ultrau or cad.

LINEAR TABLE

Basic Units

Arranged in steps of *1000*

1000 Yards = *1* Mile = *1* M^1 Yd.
1000 Karl = *1* Yard = *1* M^0 Yd.
1000 Ultrau (Cads) = *1* Karl = *1* M^{-1} Yd.
1000 Rentrau = *1* Ultrau = *1* M^{-2} Yd.
1000 Gamrau = *1* Rentrau = *1* M^{-3} Yd.
1000 Cosrau = *1* Gamrau = *1* M^{-4} Yd.

Ordinate Units

Arranged in steps of *10*

10 Palm = *1 Yard*
10 Quan = *1* Palm = *.1* Yd.
10 Karl = *1* Quan = *.01* Yd.

The foot, inch, line, and point form an interior dozenal series, convenient for industrial use, called The Mechanic's Scale as follows:

Mechanics Scale

10 Inches = *1* Foot
10 Lines = *1* Inch = *.1* Ft.
10 Points = *1* Line = *.01* Ft.

Combined Arrangement

3 Points = *1* Karl
3 Lines = *1* Quan
3 Inches = *1* Palm
3 Feet = *1* Yard

4 Karl = *1* Line
4 Quan = *1* Inch
4 Palm = *1* Foot

The following table of equivalents is to be entered from the left

	Decimal			Duodecimal		
	$Mile_A$	$Mile_D$	Km	$Mile_A$	$Mile_D$	Km
American Mile	1	1.01851	1.60951	*1*	*1.028*	*1.73929*
Do-Metric Mile	.98560	1	1.58009	*.Ɛ9Ɛ16*	*1*	*1.63648*
Kilometer	.62137	.63288	1	*.75588*	*.76X89*	*1*
	Foot	Yard	Meter	Foot	Yard	Meter
Foot	1	.33333	.30480	*1*	*.4*	*.3794X*
Yard	3	1	.91440	*3*	*1*	*.XƐ810*
Meter	3.28083	1.09361	1	*3.34534*	*1.11591*	*1*
	Mil	mm	Karl	Mil	mm	Karl
Mil	1	.02540	.048	*1*	*.03773*	*.06XƐ5*
Millimeter	39.37	1	1.89289	*33.45344*	*1*	*1.X868X*
Karl	20.833	.52832	1	*18.X*	*.640Ɛ3*	*1*
	Micron	Mc-Inch	Cad	Micron	Mc-Inch	Cad
Micron	1	39.37	3.26551	*1*	*33.45344*	*3.32296*
Micro-Inch	.02540	1	.08294	*.037X8*	*1*	*.0ƐƐ3Ɛ*
Cad or Ultrau	.30623	12.05633	1	*.38120*	*10.0814*	*1*
	Angstrom	mMicron	Rentrau	Angstrom	mMicron	Rentrau
Angstrom	1	.1	.56428	*1*	*.12497*	*.69310*
MilliMicron	10	1	5.64283	*X*	*1*	*5.78699*
Rentrau	1.77216	.17722	1	*1.93238*	*.21629*	*1*
	mAngs.	X-Unit	Gamrau	mAngs.	X-Unit	Gamrau
MilliAngstrom	1	.99798	.97508	*1*	*.ƐƐ85Ɛ*	*.Ɛ84Ɛ3*
X-Unit	1.00203	1	5.64283	*1.00361*	*1*	*.Ɛ8843*
Gamrau	1.02556	1.02348	1	*1.0364X*	*1.0346Ɛ*	*1*

1 Millimeter = .03937 Inch
1 Karl = .02083 Inch
1 Mil = .001 Inch
1 Cad or Ultrau = .000012056 Inch
1 Edocad = .000001005 Inch
1 Micro-Inch = .000001 Inch

THE X-UNIT

by Dallas H. Lien

The X-Unit was originally intended to be 10^{-13} meters, which is a convenient size for measuring crystal lattice spacings and X rays. Its length was defined from early measurements made on lattice spacings of rock salt. Since that time the ruled diffraction grating has been perfected enough to make direct measurements of X ray wavelengths and has revealed a serious error in rock salt measurements, which arose from using an inaccurate value for Avogadro's Number in computing the number of crystal lattices per centimeter in thickness. Furthermore calcite crystals have been shown to be superior to rock salt as a measure standard. The X unit has been retained as originally defined, but is generally not referred to when working with meters or subdivisions thereof. When it is desired to convert a length from X-units to meters, a conversion factor of 3.03560/3.02945 x 10^{-13} is used. This is the ratio of new and old values for calcite grating space; that is: 3.03560 x 10^{-13} meters = 3.02945 X.U.

It has been found convenient to assume that one crystal lattice spacing is known and compare others to it without regard to its length in relation to the meter. There is a reluctance to changing the length of the measure standard to keep up with continually increasing accuracy in comparing it to the meter. Measurements of various crystals are generally made with X ray photographs and compared with similar results from a crystal chosen as standard. These measurements are generally quite accurate in terms of the standard, but it is the standard itself that can't be so easily compared to the meter. Some comparisons of crystals in terms of both X.U.'s and meter subdivisions (angstroms) are shown below.

Rock salt	NaCl	2.81971 A	2,814.00 X.U.
Calcite	$CaCO_3$	3.03560 A	3,029.45 X.U.
Quartz	SiO_2	4.25465 A	4,246.02 X.U.
Gypsum	$CaSO \cdot 2H_2O$	7.6001 A	7,584.70 X.U.
Mica	......	9.9629 A	9,942.72 X.U.*

Conversion factors for X unit.

Decimal	Duodecimal	Unit
1.00203 x 10^{-13}	*X.879E4 x 10^{-11}*	meters (U.S.)
1.09583 x 10^{-13}	*E.88425 x 10^{-11}*	yards (U.S.)
3.28749 x 10^{-13}	*2.E2107 x 10^{-10}*	feet (U.S.)
3.94499 x 10^{-12}	*2.E2107 x 10^{-E}*	inches (U.S.)

*Richtmyer and Kennard, Introduction to Modern Physics, fourth edition, 1947, McGraw-Hill, New York. p. 477

MATHEMATICAL RECREATIONS

D. M. Brown, Editor

If our readers will send their solutions to posed problems, we will publish them. In hopes that some of you will solve them, we will not publish the answers to the problems in Vol. 5, No. 2, September 1949, until the next issue.

The following teaser was suggested by Mr. Battell Loomis.

Cut an *8* x *8* square along lines as in Fig. 1, and reassemble as in Fig. 2. Note that the area in Fig. 1 is *54*, but in Fig. 2 it is *55*. Where did the extra unit go to? You will have to cut it out before you will believe it.

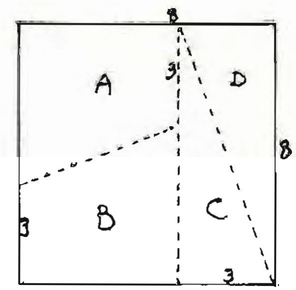


FIG. 1

Mr. Loomis also suggests cutting an *8* x *8* x *8* cube by planes perpendicular to a face as in Fig. 1. Upon reassembling as in Fig. 2, you will note that the volume is now *5* x *11* x *8* = *374*, where the original volume was *8* x *8* x *8* = *368*, a difference of *8* cubic units. Why?

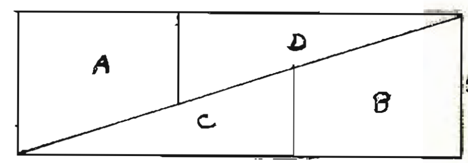


FIG. 2

The following problems were suggested by these examples. Cut a rectangle with dimensions a x b along lines indicated in Fig. 3, and reassemble as in Fig. 4. (If a = b = *8* and y = *3*, we have Mr. Loomis' problem.)

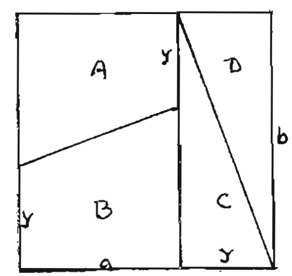


FIG. 3

I. Find integers a, b, and y, such that the excess area (E) is zero. For example, let a = *9*, b = *4* and y = *1*.

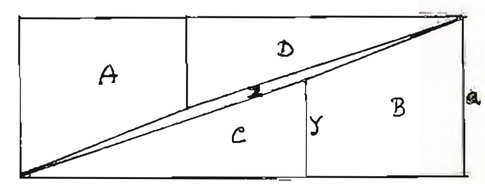


FIG. 4

II. Let the area (E) be a positive integer, z. For a given z, find integers a, b, and y. For example, in Mr. Loomis' problem, z = *1*, a = b = *3*, and y = *3*. Again, if z = *2*, then let a = *7*, b = *4* and y = *1*.

Send in your answers! Can you find a general form which the numbers a, b, and y, must have, to satisfy the conditions imposed for a specified z?

Mary Lloyd sent the following divisocrypt. Each letter represents a different digit, and when the letters are arranged in ascending order, they spell two words. Can you do it?

```
                 A E L U
SHINE ) M A R V E L B E
        S H I N E
        I M U M M L
        I M S R M S
          V M I M B E
          V U A L R E
            A V U E L
```

COMMITTEE APPOINTMENTS FOR THE YEAR 1950

Committee on Awards = George S. Terry, Chairman, 507 Main St., Hingham, Mass. ÷ Ralph H. Beard ÷ F. Emerson Andrews.
To decide whether a 1951 Award is to be made, and if so, to recommend for that award a person of outstanding achievement in mathematical research, with relation to duodecimals.

Committee on Bibliography = Lewis Carl Seelbach, Chairman, 163 Davidson Ave., Buffalo 15, N.Y.
To continue the excellent work in developing a bibliography.

Committee on Finance = George S. Terry, Chairman ÷ Ralph H. Beard ÷ H. K. Humphrey.
To review the Society's budget, act upon major expenditure proposals, and audit the report of the treasurer.

Committee on Mathematical Recreations = Donald Meeker Brown, Chairman, 12345 Jackson Road, Dexter, Mich. ÷ Mary B. Lloyd ÷ William A. Richards ÷ Dudley A. George ÷ Thomas W. Delaney.
To prepare material in this field for the Bulletin, be a center of member correspondence, and collect illustration + visual aid material.

Committee on Mathematical Research = Nathan Lazar, Chairman, 1601 Perry Street, Columbus, Ohio ÷ William Shaw Crosby ÷ Thomas H. Nash ÷ Leon L'Heureux ÷ Edward W. Pharo, Jr. ÷ Arthur A. Le Pori.
To conduct original research, to check Bulletin submissions for accuracy and validity, and to be a central advisory committee on application of duodecimals to various fields of mathematical research.

Committee on Membership = Doris Burke Lloyd, Chairman, 2505 Pinebrush Road, Baltimore 9, Md. ÷ Paul E. Friedemann ÷ Yvette L'Heureux ÷ Jamison Handy, Jr. ÷ H. Harold Hartzler.
To promote new memberships, to handle membership correspondence and encourage qualification for advanced standing, and to endeavor to help members into useful work and acquaintanceships.

Committee on Public Information = Paul Van Buskirk, Chairman, 1524 Clairmount Ave., Detroit 6, Mich. ÷ Jorge Carreras Codrington ÷ Frederick Condit ÷ John E. Whiteside.
To spread information on the duodecimal system through correspondence, meetings, discussion groups, preparation of news materials, publication, encouragement of special articles, work through the schools and colleges, and other means, including relations with dictionaries and encyclopedias.

Committee on Weights and Measures = Ralph H. Beard, Chairman, 20 Carlton Place, Staten Island 4, N.Y. ÷ Paul Adams ÷ Dallas H. Lien ÷ Eugene M. Scifres ÷ Albert De Valve.
To promote discussion and use of the Do-Metric System, to investigate further proposals in weights and measures, including problems of navigation and aviation, and to be an advisory board in this field.

Committee on Slide Rule = Kingsland Camp, Chairman, Rm. 914, 393 7th Ave., N.Y. 1. ÷ R. C. Williamson ÷ Eugene M. Scifres ÷ H. K. Humphries.
To continue the planning of a duodecimal slide rule.

Committee on Special Applications = Louis P. d'Autremont, Chairman, 1401 East 1st St., Duluth 5, Minn. ÷ Velizar Godjevatz ÷ Marcus I. Chriswell.
The function of this new committee is to investigate those matters in the Arts and Sciences where twelve and its principal factors either occur naturally or could be introduced to simplify enumeration, classification or measurements. In some cases the investigation may indicate the desirability of transfering some subject to the Mathematical Research or Weights and Measures Committees for further study but there would appear to be many subjects where this new committee could accomplish much more than an initial appraisal. Examples of the type of subject which lie in the sphere of this committee are,- d'Autremont's work on the Calendar, Godjevatz's notation for Music and Chriswell's work on the practical features of design of tools for using Do-Metric units.

MAIL BAG

In continuation of William Shaw Crosby's recent paper on "The Rounding Off of Uncials," a further convention is sometimes used to curtail additive errors, and covers all cases of omitted figures. A high dot means that omitted figures lie between *1/6* and *1/2*; a low dot represents figures between *1/2* and *5/6*; and no dot is used for figures between *-1/6* and *+ 1/6*. Duodecimally this would provide results accurate within $\pm$ .2 instead of $\pm$.*6*. Thus: *3.184809* could be written *3.2.* or *3.18·* of *3.1848* (see Thompson and Comrie, "Standard Four-figure Tables."). . . . . .

We are quite familiar with the fact that duodecimal figures "hold more" than decimal figures. For example, *ƐƐƐƐ* = 20735, which is more than double 9999. From this, it can be stated that four duodecimal places can represent more than double the maximum quantity expressible in four decimal places.

Being curious to know just where duodecimal figures would gain an entire place over decimal figures, we carried our computations a little further, and found that this occurs where the thirteenth duodecimal place represents a little more than the fourteenth decimal place. For purposes of comparison:-

$12^{13} = 1.0699 \times 10^{14}$, or, duodecimally, *Ɛ.2708* $M^4 = \chi^{12}$.

106 993 205 379 072 = *10 000 000 000 000*
100 000 000 000 000 = *Ɛ 270 805 802 454*

Dr. Alois Cibulka, of the University of Houston, has asked us to review his recent book, "All this Could Happen Only to an Engineer," which is a refreshing account of his life and works, presented in unrestrained and unconventional form. This is Dr. Cibulka's third book. His other publications are The Practical Design Handbook for Engineers, and a work on Calculus for Practical Engineers.

Dr. Cibulka was born in 1890 in the small Bohemian town of Civice, of Czech parents, in the reign of the Emperor Franz Joseph. He has lived through the wierd pattern of the years before and after the Serb incident, and now, as a naturalized American citizen, professional engineer and inventor, he writes the record of his life and thoughts. It is a strange and absorbing document.

But, "All This Could Happen Only to an Engineer," is in no sense a duodecimal work, nor are duodecimals mentioned in it; - though Dr. Cibulka informs us that he has been interested in duodecimals throughout his life, and is quite aware of their

many advantages. For this reason, we have decided against presenting a formal review of the book, and have so informed Dr. Cibulka. His address is 3801 St. Bernard St., Houston 4, Texas, and the price of the new book is \$ 4.50. . . . .

A prized item in our duodecimal bibliography has been the editorial which appeared in the Edinburgh Review, Vol. 9, 1807, p 376, on the "Base du Systeme Metrique Decimal." This paper was reprinted in the Duodecimal Bulletin, Vol. I, No. 3, October 1945.

Our intrepid bibliographer, Lewis Carl Seelbach, has now discovered that this unsigned editorial was written by John Playfair, (1748-1819,) who was Professor of Natural Philosophy at the University of Edinburgh, President of the Astronomical Institution of Edinburgh, and Fellow of the Royal Society of London. This editorial has been included in the Works of John Playfair, published by Archibald Constable and Co., of Edinburgh, in 1822. . . . .

Recently, there have been submitted to the Banking and Currency Sub-Committee of the U. S. Senate, a number of proposals for the minting of 2½¢ and 7½¢ coins. The lack of a coin of the value of the quarter dime, has repeatedly been noted by dodekaphiles. A duodecimal coinage would provide a six-cent piece to replace the nickel, and would include a new three-cent piece, to correspond (in its range,) with the quarter dollar.

The use of two- and three-cent pieces has been experimented with by the U. S. Mint in the past, but these coins have both fallen into unpopularity and disuse. The present proposals for the quarter dime have an additional adverse factor, in that the ultimate need for a new half-cent piece would be involved. Nellie Tayloe Ross, Director of the Mint, has strongly opposed the current proposals on the bases of cost and inconvenience. She has stated that new coins should be considered only when there is an insistent popular demand for their use.

Ye Ed.